

Adiabatic hydrodynamization in the rapidly-expanding quark-gluon plasma

Jasmine Brewer

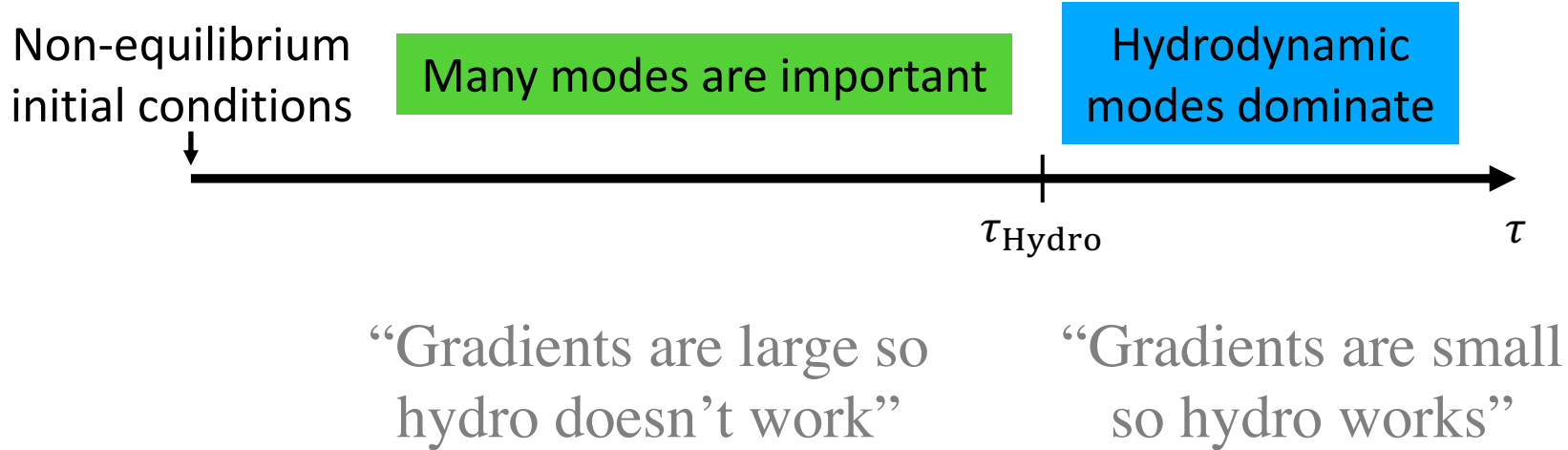


Based on:

JB, Li Yan, and Yi Yin [arXiv:1910.00021]

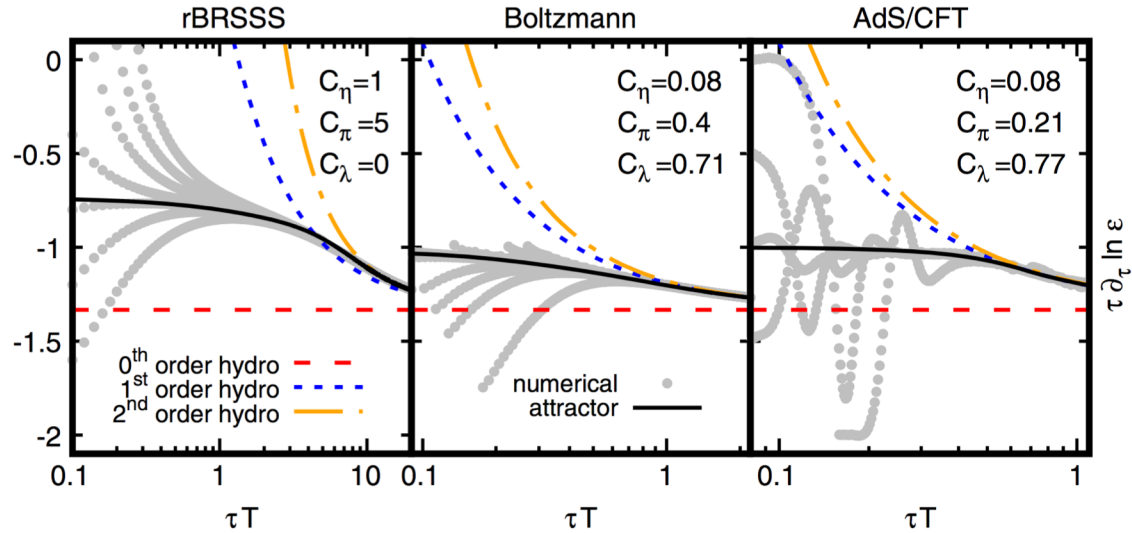
and ongoing work

Expected applicability of hydrodynamics

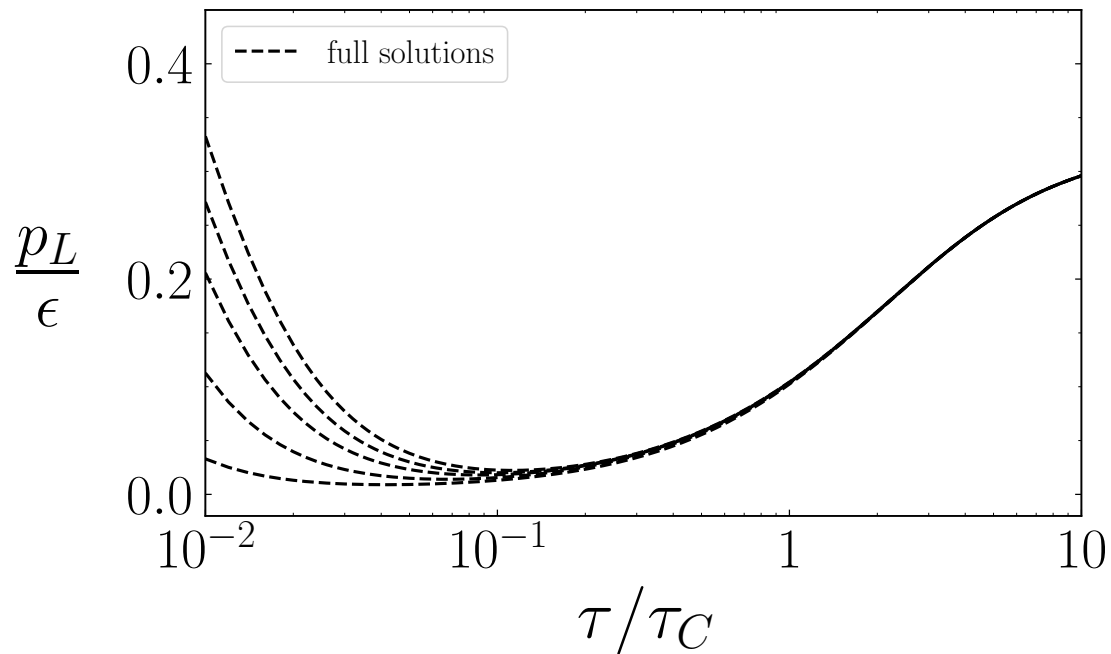


Observation of attractor behavior

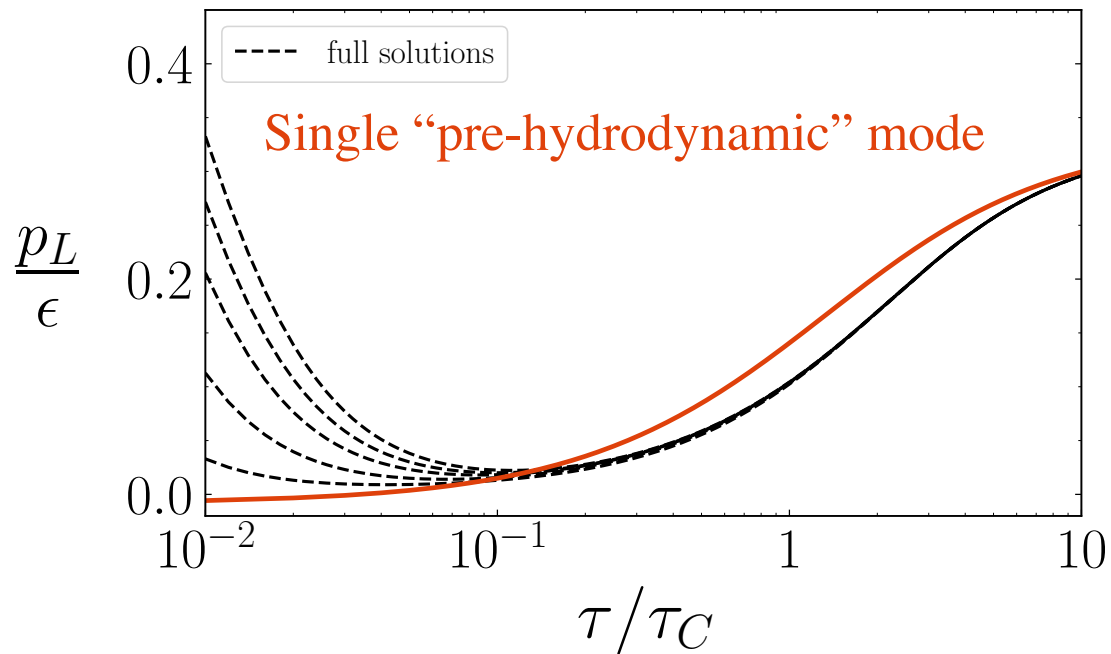
Heller and Spalinski [1503.07514], many follow-ups



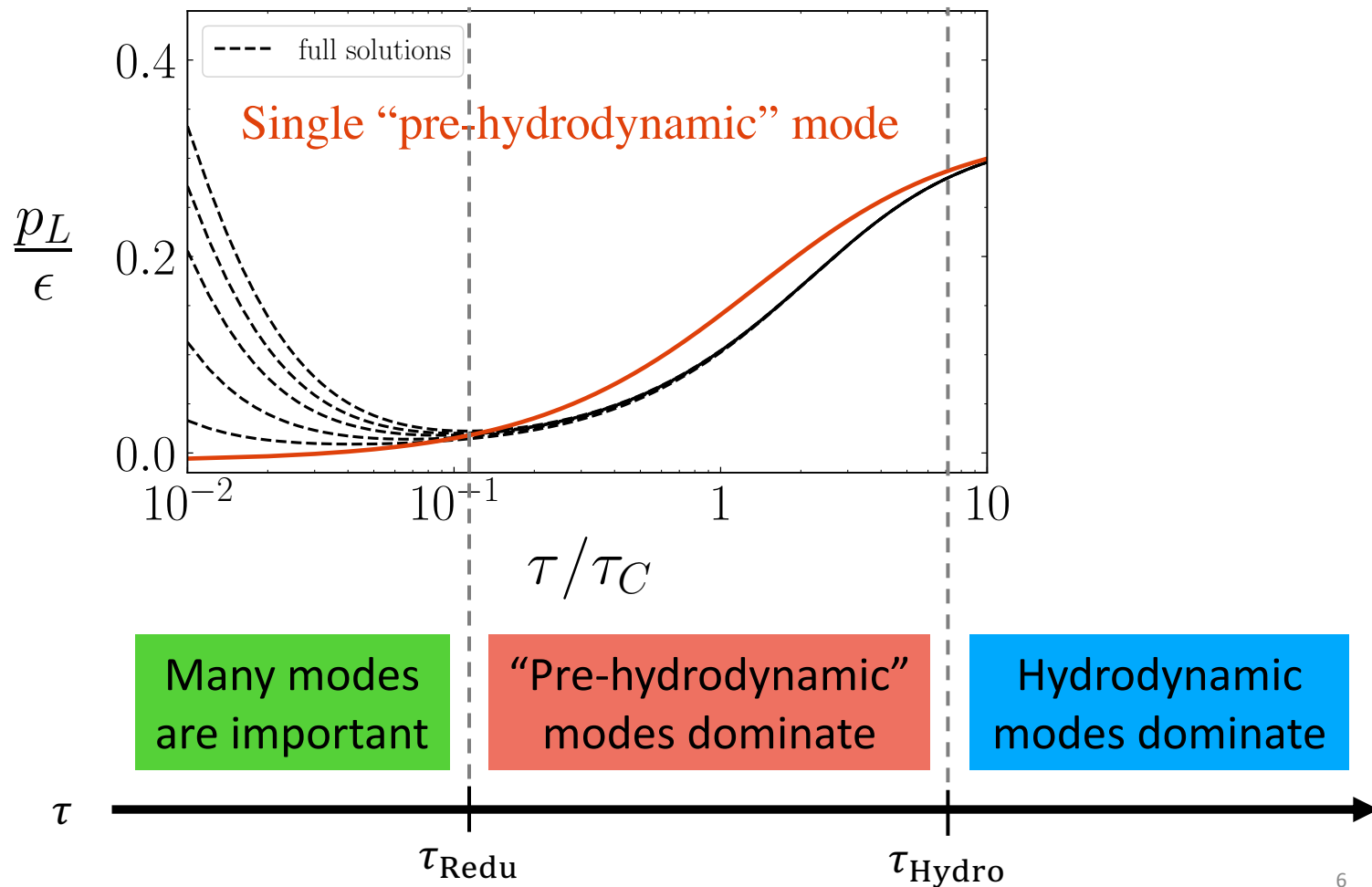
Suggestive of simplified bulk description before τ_{Hydro}



Bjorken-expanding kinetic theory in the relaxation time approximation



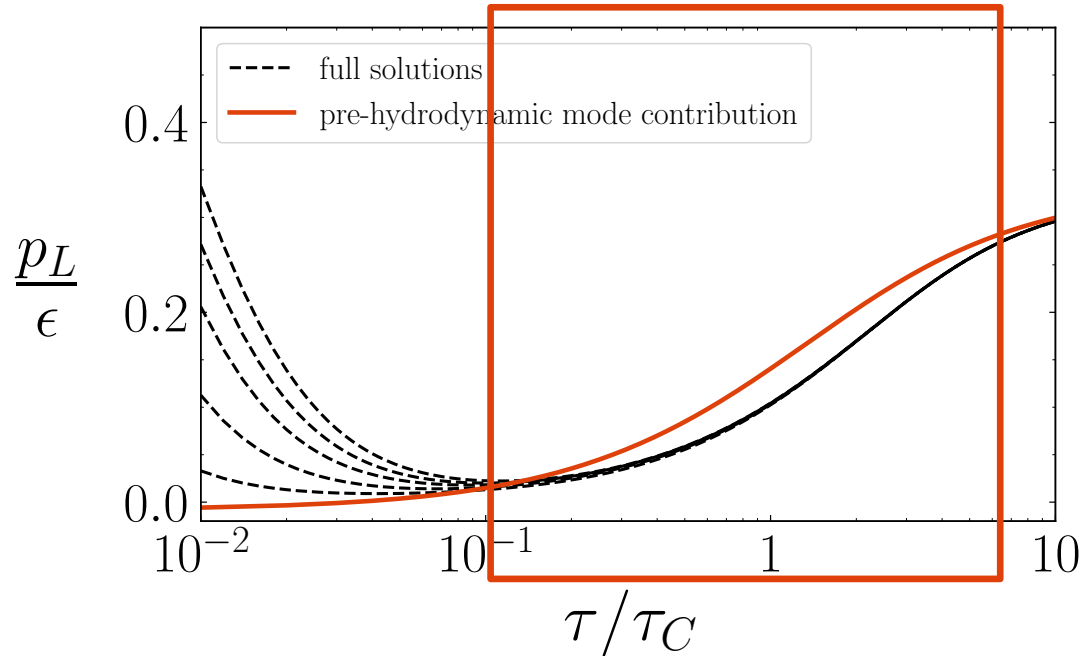
Bjorken-expanding kinetic theory in the relaxation time approximation



Adiabatic theorem:

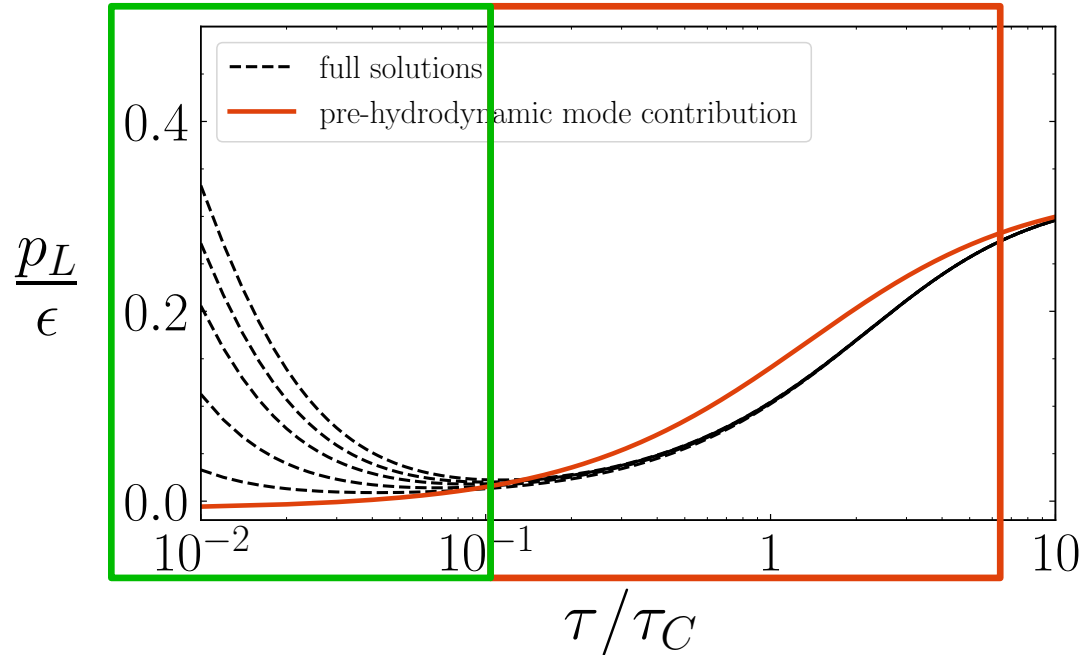
“A system prepared in its (instantaneous) ground state will remain in its (instantaneous) ground state under adiabatic evolution of the Hamiltonian”

Adiabatic interpretation of far-from-equilibrium behavior



Pre-hydrodynamic mode is instantaneous ground state of an effective Hamiltonian

Adiabatic interpretation of far-from-equilibrium behavior



Dominance of ground state at early times driven by rapid longitudinal expansion

Hamiltonian formulation from kinetic theory

Bjorken-expanding kinetic theory

$$\frac{\partial}{\partial \tau} f(p_z, p_\perp; \tau) = -\frac{p_z}{\tau} \frac{\partial}{\partial p_z} f(p_z, p_\perp; \tau) - \hat{C}[f]$$

longitudinal expansion \longleftrightarrow collisions

Hamiltonian formulation from kinetic theory

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Moments of kinetic equation give evolution of more macroscopic quantities

$$\int_{\mathbf{p}} |\mathbf{p}| f(p_z, p_\perp; \tau) = \epsilon(\tau)$$

Energy density

$$\int_{|\mathbf{p}|} |\mathbf{p}| f(p_z, p_\perp; \tau) = F_\epsilon(\cos \theta; \tau)$$

Angular distribution contributing to energy density

Hamiltonian formulation from kinetic theory

$$\int_{|\mathbf{p}|} |\mathbf{p}| \left(\underbrace{\frac{\partial}{\partial \tau} f(p_z, p_\perp; \tau)}_{\frac{\partial}{\partial \tau} F_\epsilon} = - \frac{p_z}{\tau} \underbrace{\frac{\partial}{\partial p_z} f(p_z, p_\perp; \tau)}_{-\frac{1}{\tau} (\dots) F_\epsilon} - \hat{C}[f] \right)$$

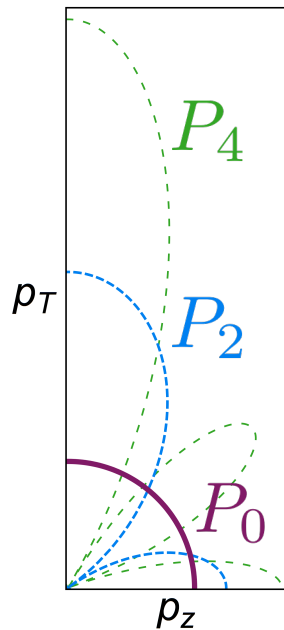
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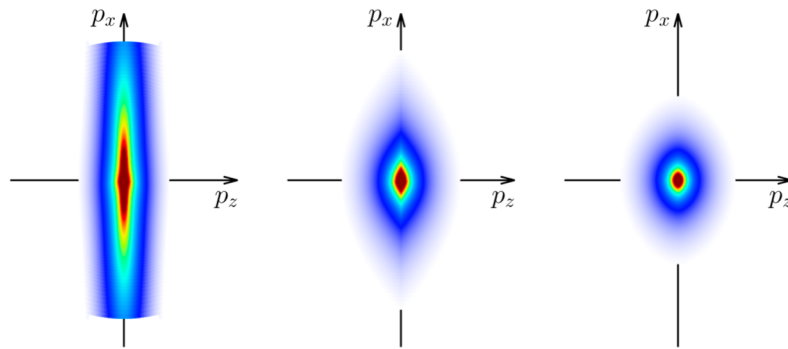
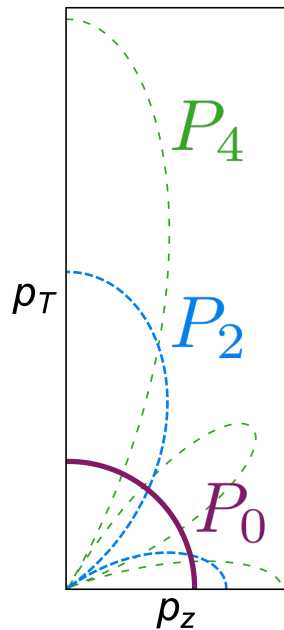
For some $\hat{C}[f]$ $\tau \frac{\partial}{\partial \tau} F_\epsilon = (\dots) F_\epsilon \iff \partial_y \psi = -\mathcal{H}(y) \psi \quad y = \log \left(\frac{\tau}{\tau_I} \right)$

Schrodinger-type evolution of
generalized Hamiltonian \mathcal{H}

$$F_\epsilon(\cos \theta; \tau) = \epsilon(\tau) + \sum_{n=1} \frac{4n+1}{2} \mathcal{L}_n(\tau) P_{2n}(\cos \theta) \iff \psi = (\epsilon, \mathcal{L}_1, \mathcal{L}_2, \dots)$$



$$F_\epsilon(\cos \theta; \tau) = \epsilon(\tau) + \sum_{n=1} \frac{4n+1}{2} \mathcal{L}_n(\tau) P_{2n}(\cos \theta) \longleftrightarrow \psi = (\epsilon, \mathcal{L}_1, \mathcal{L}_2, \dots)$$

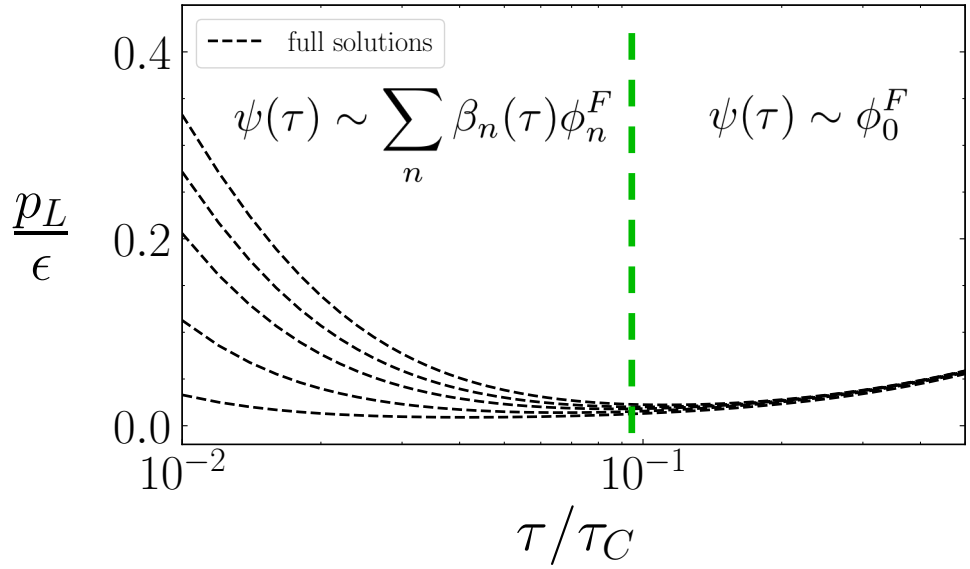


$$(\epsilon, \mathcal{L}_1, \mathcal{L}_2, \dots) \longrightarrow (\epsilon, 0, 0, \dots)$$

hydrodynamization

Is the system prepared in the ground state?

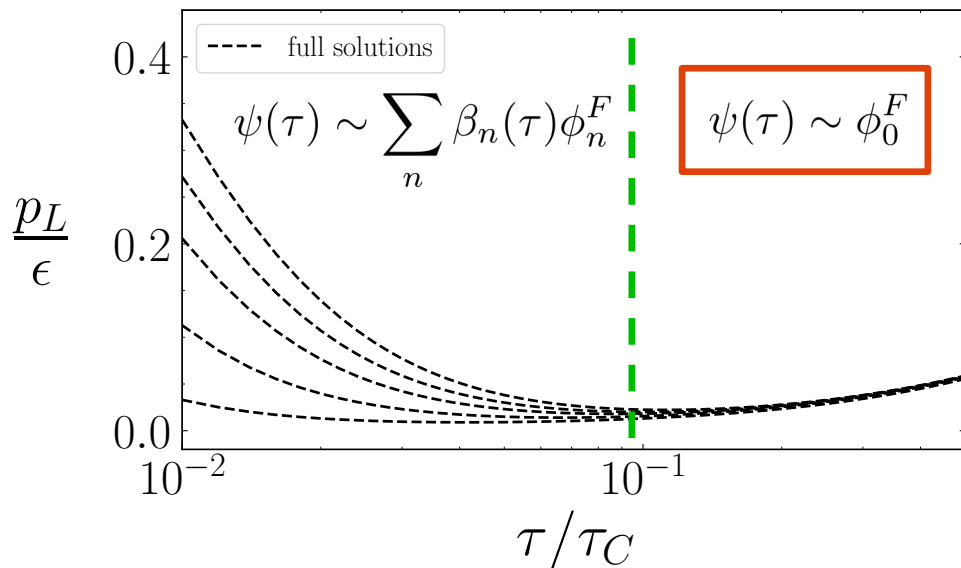
At early times $\hat{C}[f] = 0 \longrightarrow \partial_y \psi = -\mathcal{H}_F \psi$



$\beta_n(\tau) \sim \beta_n(\tau_I) e^{-\mathcal{E}_n^F \tau}$ gives time scale for decay of initial conditions

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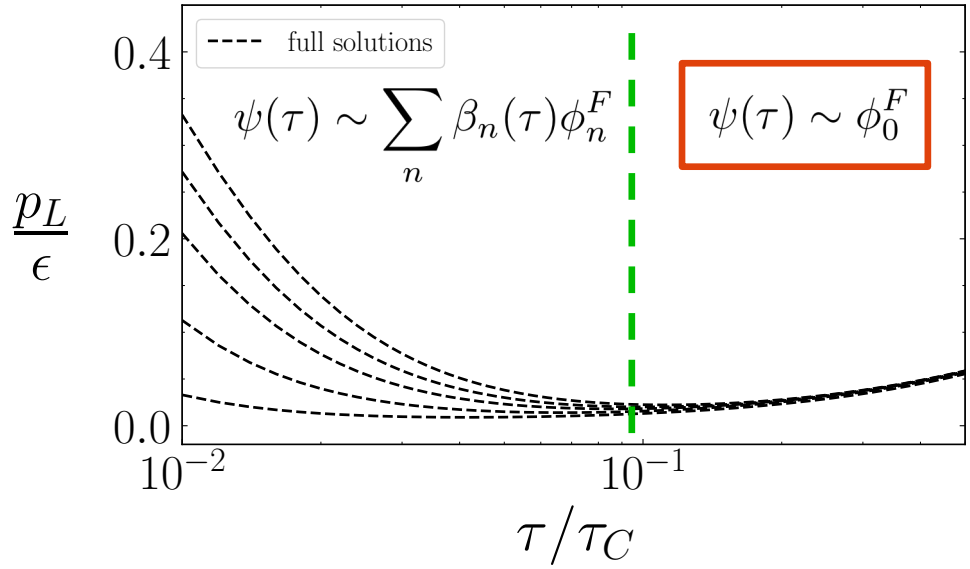


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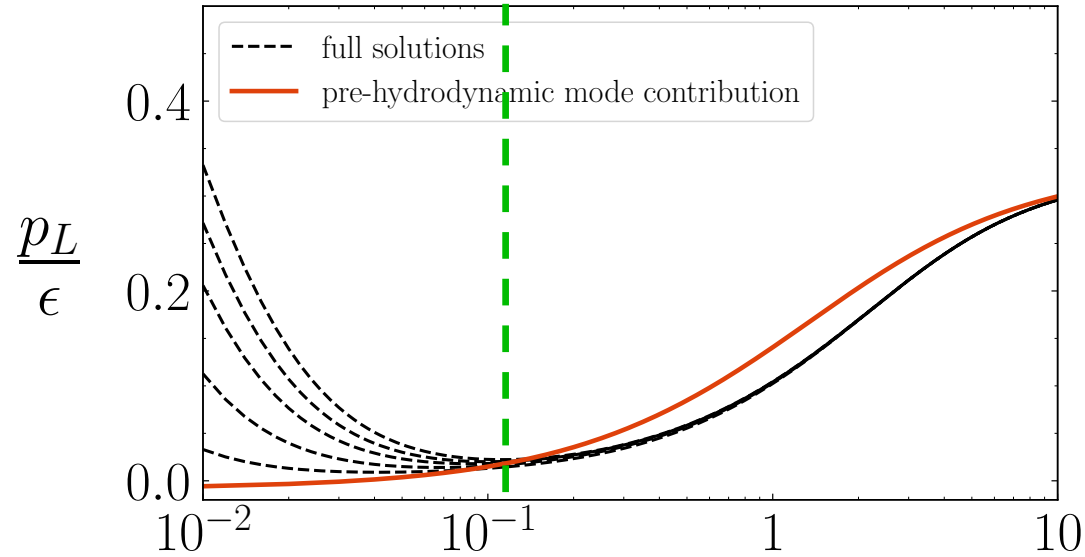
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Attractor behavior doesn't imply collectivity



System prepared in τ/τ_C
 ground state

Is the subsequent evolution adiabatic?

Definition of adiabatic hydrodynamization:

System evolution determined by
the instantaneous ground state

$$\psi(y) \sim \phi_0(y)$$

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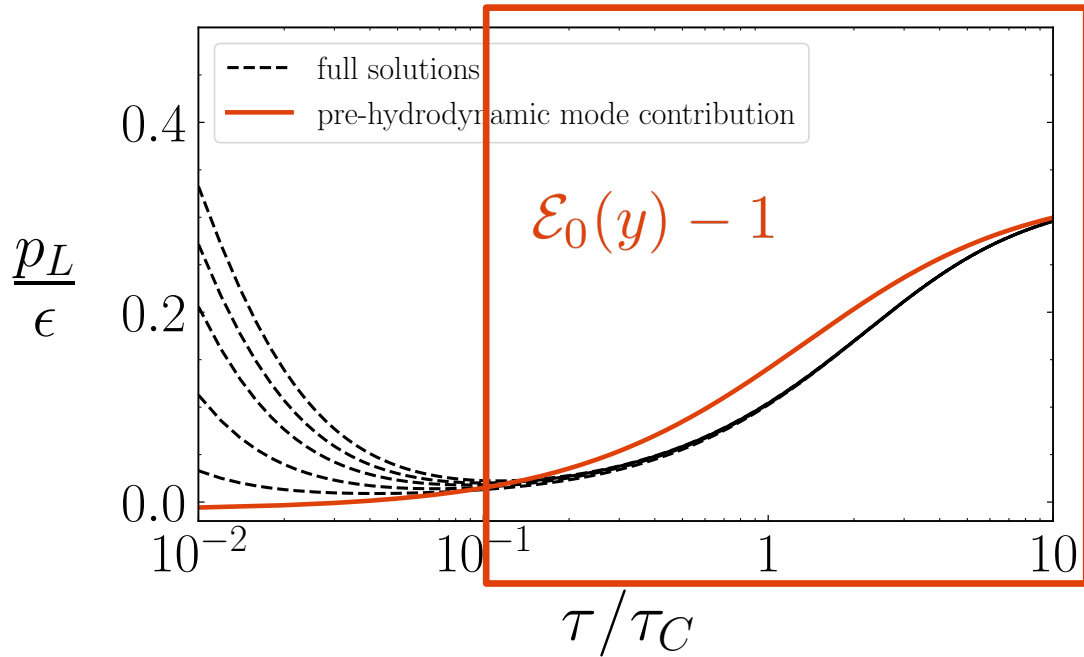
Predicts non-trivial relations between components of ψ

$$\partial_y \psi = -\mathcal{H}(y)\psi \quad \longrightarrow \quad \partial_y \phi_0(y) = -\mathcal{E}_0(y)\phi_0(y)$$

e.g. $g(y) \equiv 1 + p_L/\epsilon = \mathcal{E}_0(y)$

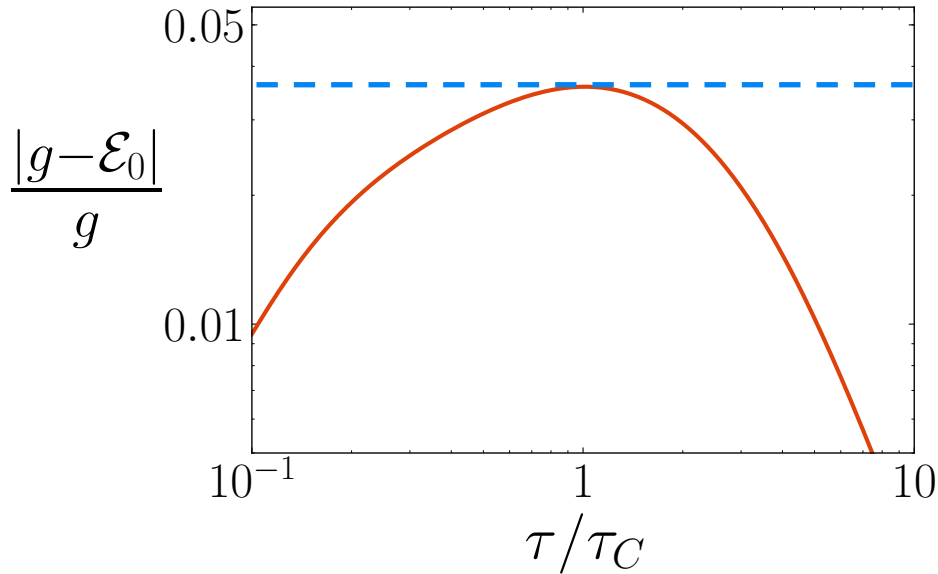
Test extent to which these relations are satisfied!

Hydrodynamization in RTA kinetic theory is adiabatic!



Far-from-equilibrium evolution dominated just by evolution of instantaneous ground state (“pre-hydrodynamic”) mode!

Implies presence of a small “adiabatic” parameter that suppresses contributions from other modes



> 95% of g described
by instantaneous
ground state mode

Why would the rapidly-expanding QGP be adiabatic?

Suppression of
excited states:

$$\delta_A \sim \frac{\partial_\tau \log \lambda}{\Delta E_n} \langle 0(\tau) | H | n(\tau) \rangle$$

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Hamiltonian evolves slowly
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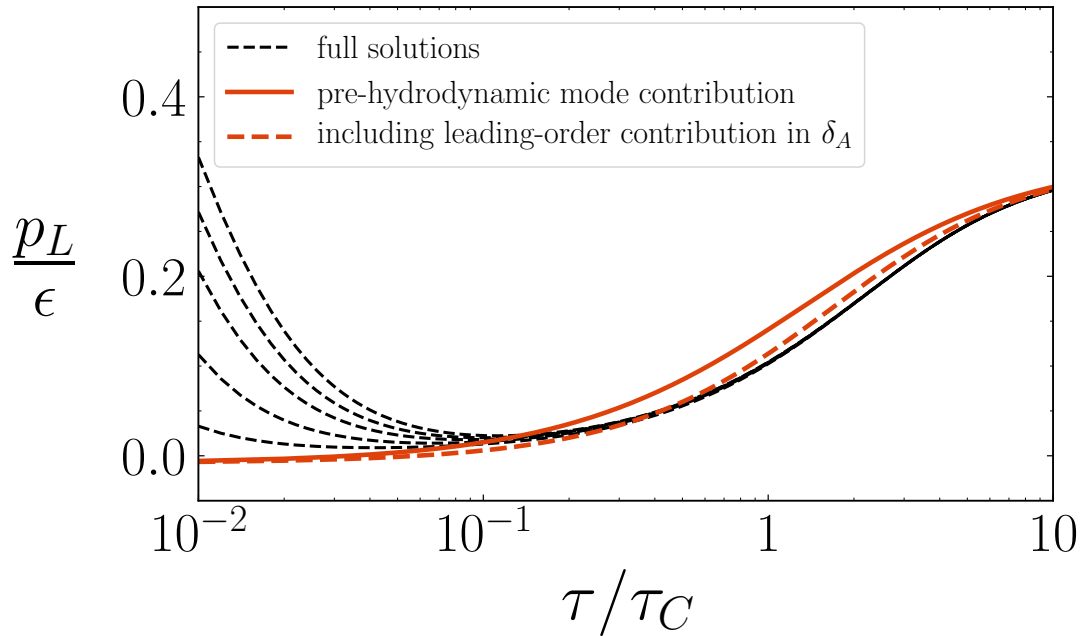
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Bottom-up thermalization: parametrically narrow window when δ_A can be large

A new perturbative expansion



Including contributions from the excited states to the evolution at $\mathcal{O}(\delta_A)$ shows explicitly that they are a small correction

