Shedding new light on photon and dilepton spectral functions

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Motivation

The photo-emission rate of an equilibrated quark-gluon plasma (QGP) at zero chemical potential can be derived from the (contracted) spectral function ρ^{μ}_{μ}, for the photon self-energy. Non-perturbative lattice simulations could provide constraints, via measurements of the imaginary time correlator. The two functions are related by an integral transform:

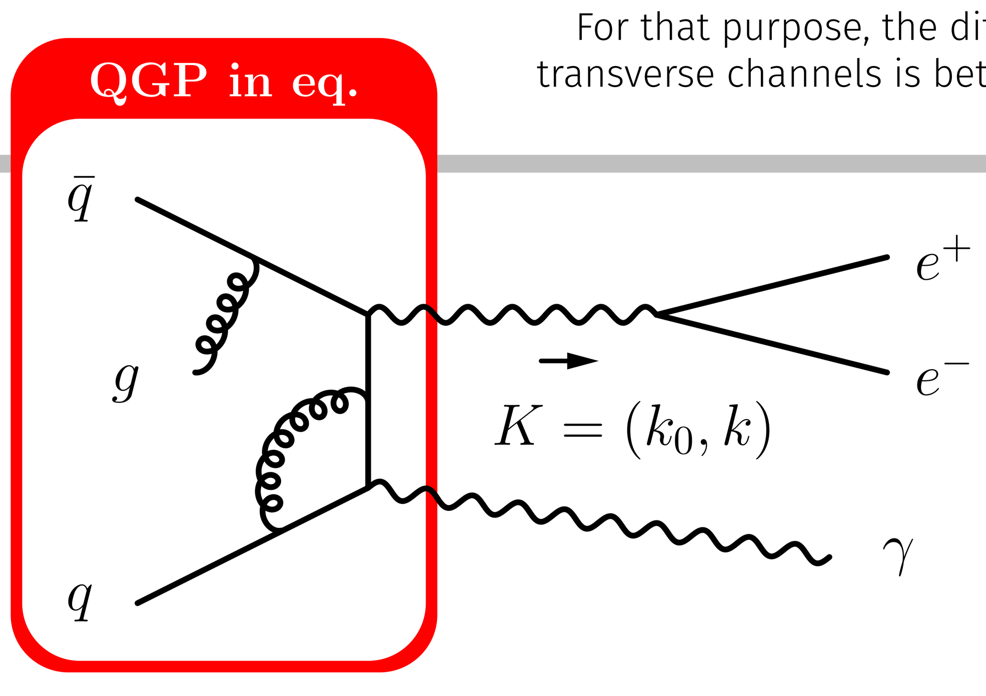
$$G_{\mu\nu}(\tau,k) \;=\; \int_0^\infty \frac{dk_0}{2\pi}\, \rho_{\mu\nu}(k_0,k)\, \frac{\mathrm{ch}[(\frac{1}{2}\beta-\tau)k_0]}{\mathrm{sh}(\frac{1}{2}\beta k_0)}\;, \qquad (1)$$

where $\beta = 1/T$.

The inversion of (1) is ill-posed. Many approaches use a maximum entropy estimator to reconstruct $\rho(k_0,k)$ as a continuous function. However, the resulting $\rho_{\mu\nu}$ (as a function of k_0) is not well constrained. Turning the problem around, we can use Eq. (1) as a means to check perturbative approaches. This would simultaneously provide a means to 'test' candidates for ρ that are extracted from lattice data.

For that purpose, the difference in longitudinal and transverse channels is better suited than ρ^{μ}_{μ} [1].

> At finite temperature, the tensor $\rho_{\mu\nu}$ is specified by transverse (T) and longitudinal (L) scalars. The combination
> $$\rho^{\mu}_{\mu} = 2\rho_{\mathrm{T}} + \rho_{\mathrm{L}}\,,$$
> is needed for the physical rate. Real photons (which have $k_0 = k$) are transverse:
> $$\rho_{\mathrm{L}} = (k^2-k_0^2)\rho_{00}/k^2 = 0\,.$$



> The production rate of photons, per unit of phase space, can be written (the energy $k_0 = k$ is on shell)
> $$k\frac{d\Gamma_\gamma}{d^3k} \;=\; \frac{\alpha_{\mathrm{em}}}{2\pi^2}\, n_{\mathrm{B}}(k) \sum_s Q_s^2\, \rho^{\mu}_{\mu}(k,k)\,.$$
> where n_{B} is the Bose distribution and Q_s is the charge of quark species s in elementary units.
> For dileptons, the invariant mass of the virtual photons must be sufficient: $k_0^2 - k^2 \geq 4m_e^2$. Above the threshold,
> $$\frac{d\Gamma_{e^+e^-}}{dk_0 d^3k} \;=\; \frac{\alpha_{\mathrm{em}}^2}{3\pi^3(k_0^2-k^2)}\, n_{\mathrm{B}}(k_0) \sum_s Q_s^2\, \rho^{\mu}_{\mu}(k_0,k)\,.$$

NLO calculation

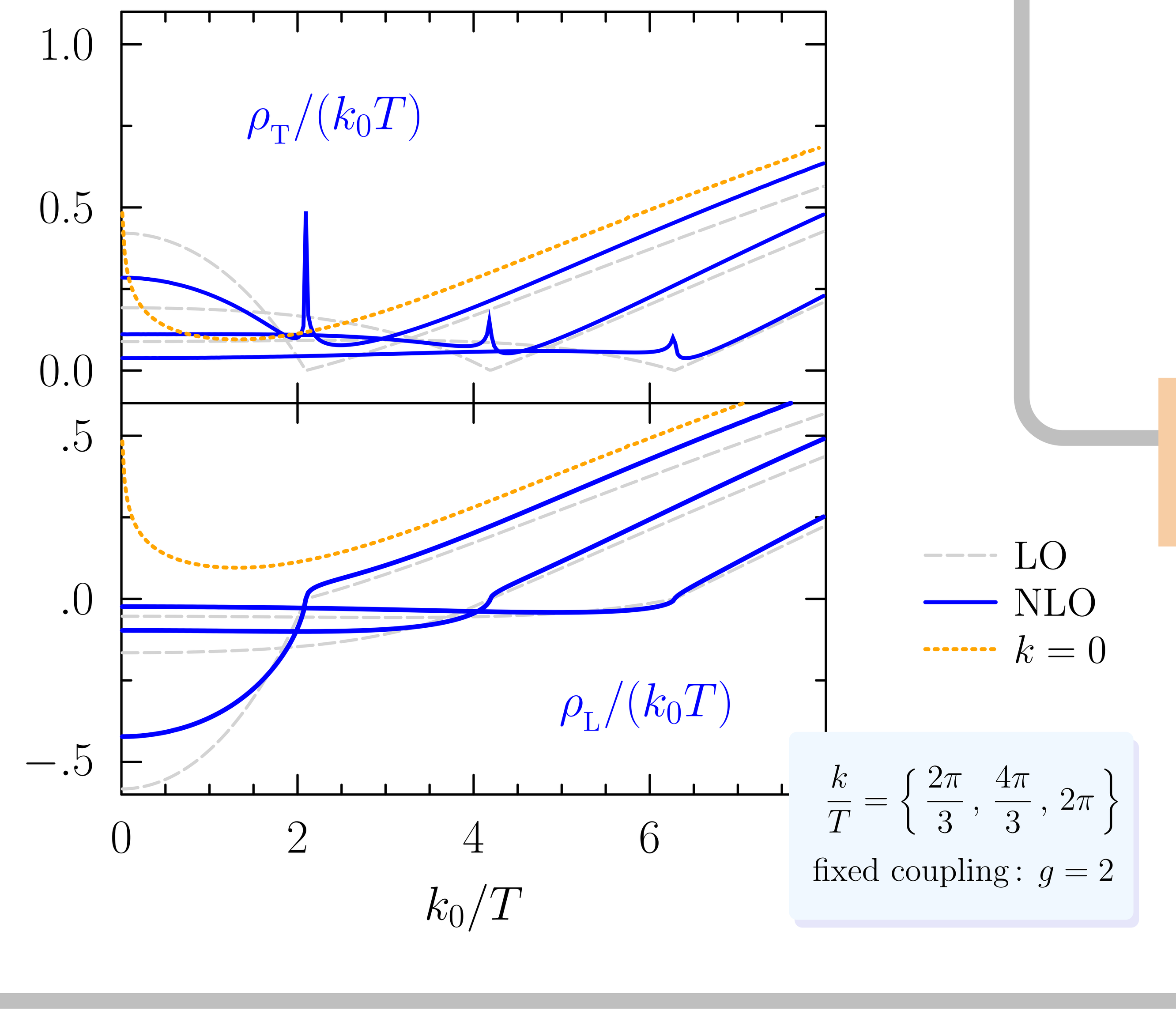
In strict perturbation theory, the spectral function would read

$$\rho_{\mu\nu} \;=\; \rho^{(0)}_{\mu\nu} + g^2 \rho^{(1)}_{\mu\nu} + \ldots\;; \qquad g^2 \equiv 4\pi\alpha_{\mathrm{s}}\,. \qquad (2)$$

The next-to-leading order (NLO) was previously known for ρ^{μ}_{μ} at $k_0 > k$ [2]. We improve the status of perturbative predictions by [3]:

- Moving into the spacelike domain, $k_0 < k$.
- Determining the two polarisations, ρ_{T} and ρ_{L}, separately.

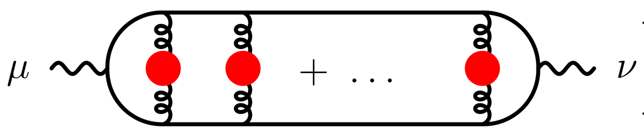
A log-divergence in ρ_{T} for $k_0 \to k$ indicates the breakdown of (2), i.e. where $K^2 \sim g^2T^2$ changes the power counting. The two polarisations coincide, $\rho_{\mathrm{T}} = \rho_{\mathrm{L}}$, for a photon at rest: $k = 0$.



Resummation

For $k_0 \approx k$, thermal screening plays an important role. The LPM (Landau-Pomeranchuk-Migdal) effect means that all ladder diagrams need to be taken into account [4, 5]:

$$\rho_{\mu\nu}(k_0,k) \;=\; \mathrm{Im}\left[\,\mu \quad [\text{ladder diagrams}] + \ldots \quad \nu\,\right].$$



To craft a spectral function that can be used in (1), where all frequencies are required, we combine the two regimes. The re-expanded parts to order g^2 are subtracted from the full LPM results and the remainder is coupled with the full NLO truncation (2),

$$\rho(k_0,k) \;\simeq\; \rho\big|^{\mathrm{NLO}}_{\text{fixed-order}} + \rho\big|^{\mathrm{LPM}}_{\text{resummed}} - \rho\big|_{\text{overcounting}}\,. \qquad (3)$$

The singularity at $k_0 \to k$ that plagues a strict perturbative series is eliminated by LPM resummation. Not only is the result finite there, but it is also continuous.

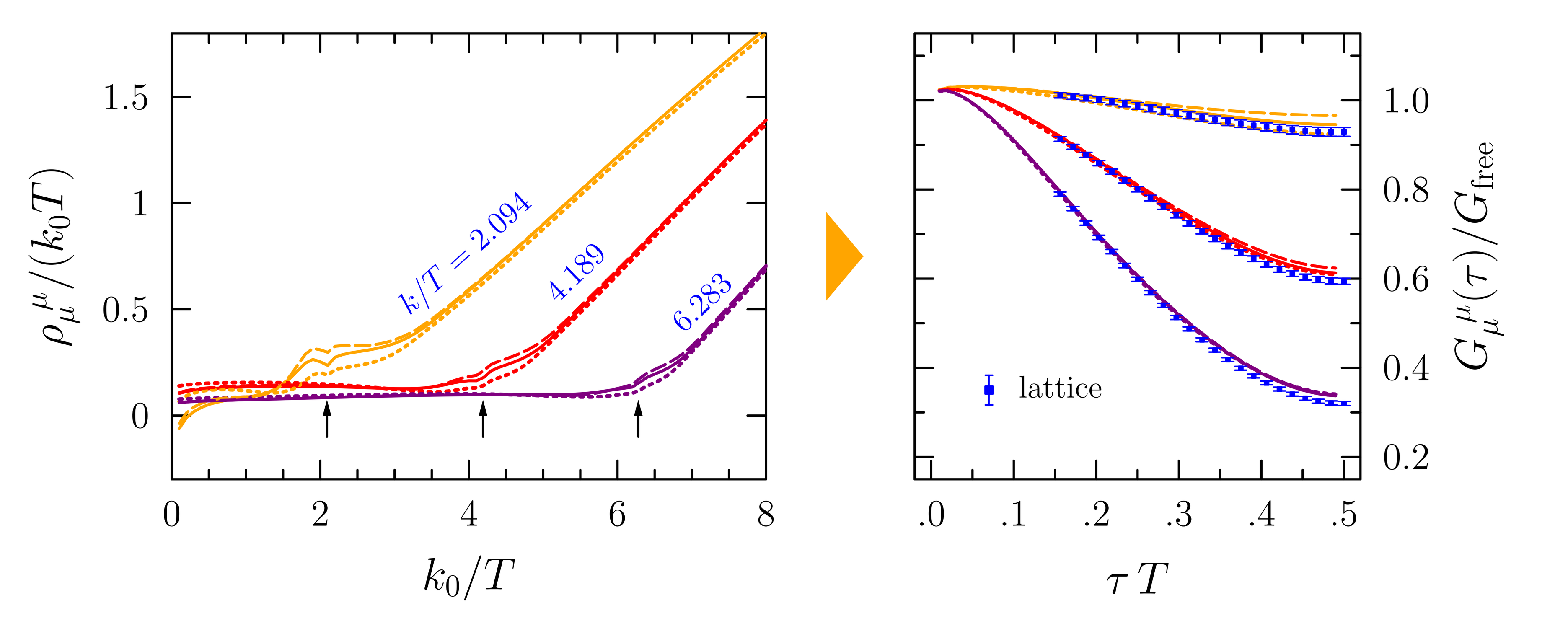
> ### Gauge coupling
> The integral (1) samples k_0 over many scales, hence running coupling is needed. We evaluate the 5-loop $\alpha_{\mathrm{s}}(\mu)$ at an 'optimal' scale
> $$\mu_{\mathrm{opt}} = \sqrt{|K^2| + Q_T^2}\,, \qquad Q_T = \xi \cdot \pi T\,,$$
> where $\xi = 1\,(2)$ for $N_{\mathrm{f}} = 0\,(2)$. Near the light cone, $\mu_{\mathrm{opt}} \approx Q_T$ so that there the interaction strength is set by the thermal scale.

Results

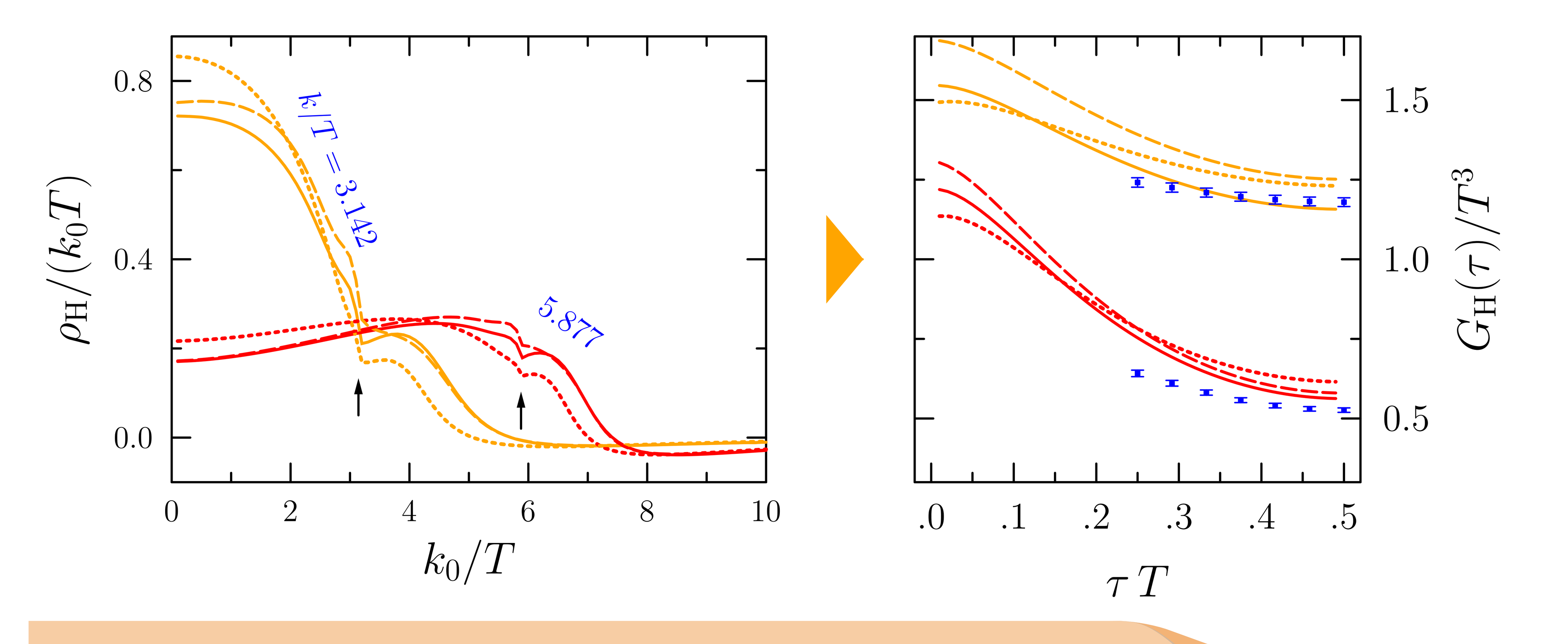
Our fully resummed expression in (3) can be confronted with existing lattice data for $G(\tau,k)$, in the continuum limit. The spectral function is inserted into Eq. (1), for $\tau \in [0, \frac{1}{2}]\beta$. The accuracy of the data is sufficient to constrain the coupling, and thus supports the implementation of physical rates needed in hydrodynamic codes. Sensitivity to the scale choice is judged by varying μ_{opt} by a factor of two. We also show results using both the leading-order (LO) LPM as well as the NLO LPM-resummation from Ref. [6].

$\mu = \mu_{\mathrm{opt}}$, $\mu = 2\mu_{\mathrm{opt}}$ } $\mathrm{LPM^{LO}}$
$\mu = \mu_{\mathrm{opt}}$ $\mathrm{LPM^{NLO}}$

COMPARISON: quenched QCD @ $T = 1.1T_c$, available data for $G^{\mu}_{\mu} = 2G_{\mathrm{T}} + G_{\mathrm{L}}$ [7]



COMPARISON: 2-flavour @ $T = 1.2T_c$, available data for $G_{\mathrm{H}} = 2(G_{\mathrm{T}} - G_{\mathrm{L}})$ [1]



> NLO corrections $\rho^{(1)}_{\mathrm{L}}$ and $\rho^{(1)}_{\mathrm{T}}$ can be broken down into a basis of 'master' integrals. For their evaluation, see https://github.com/gw3g/spectral .

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[1] B. B. Brandt, A. Francis, T. Harris, H. B. Meyer and A. Steinberg, *EPJ Web Conf.* **175** (2018) 07044, [1710.07050].
[2] M. Laine, *JHEP* **05** (2013) 083, [1304.0202].
[3] G. Jackson, *Two-loop thermal spectral functions with general kinematics* 1910.07552.
[4] P. B. Arnold, G. D. Moore and L. G. Yaffe, *JHEP* **12** (2001) 009, [hep-ph/0111107].
[5] P. Aurenche, F. Gelis, G. D. Moore and H. Zaraket, *JHEP* **12** (2002) 006, [hep-ph/0211036].
[6] J. Ghiglieri and G. D. Moore, *JHEP* **12** (2014) 029, [1410.4203].
[7] J. Ghiglieri, O. Kaczmarek, M. Laine and F. Meyer, *Phys. Rev.* **D94** (2016) 016005, [1604.07544].
[8] G. Jackson and M. Laine, *Testing thermal photon and dilepton rates* 1910.09567.