Hydrodynamic calculations for multiplicity fluctuations in heavy-ion collisions

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ABSTRACT

It is understood that multiplicity fluctuations serve as one of important observables for the study of the critical end point in the beam energy scan program of RHIC. As a matter of fact, the problem is rather subtle, and many different factors may potentially affect the observable in question. In this work, we take into consideration the thermal fluctuations, resonance decay, as well as the hydrodynamic expansion of the system. Subsequently, we focus on the noncritical aspects of the topic and investigate the multiplicity fluctuations in heavy-ion collisions by using a hydrodynamic model. In particular, we evaluate the effects of initial state fluctuations and resonance decay. The obtained results are compared to those obtained by the statistical model as well as the experimental data.

1 Introduction

1.1 Particle fluctuations in relativistic heavy ion collisions

An important topic in our understanding of the properties of strongly interacting nuclear matter is its phase structure and the whereabouts of the critical point. In principle, the transition is described by the so-called Quantum Chromodynamics (QCD), associated with chiral symmetry, which is spontaneously broken in vacuum but restored at the extremely hot and/or dense environment. Besides, it serves as a primary motivation of the Beam Energy Scan (BES) [1, 2] program at the Relativistic Heavy-Ion Collider (RHIC). In this regard, multiplicity fluctuations have acquired much attention recently as one of the vital hadronic observables which carry thermal properties of the primordial medium created by the collisions.

The experimentally observed multiplicity fluctuations is an eventual outcome governed by various distinct aspects of the physical system. As a thermodynamical system, a considerable proportion of the measured multiplicity fluctuation is related to the thermal fluctuations of a grand canonical ensemble (GCE) with conserved charges [3]. Meanwhile, resonance decay also plays a part to cause significant deviation from pure statistical distributions [3]. This is, in part, consistent with results from statistical models calculations [4]. On the other hand, near the critical point, various physical quantities become divergent, including particle fluctuation. While a quantitative description of the critical phenomena is provided by the theory of renormalization group, owing to the sophistication of the problem at hand, one usually resorts to phenomenological approaches, such as σ model [5]. In fact, instead of being a stationary state, the system evolves dynamically, while the measurements are carried out at the freeze-out surface after the hadronization takes place. In this context, to understand the effect of the critical endpoint to the dynamical equation is essential. Tentatives following this train of thought [6] eventually leads to models such as chiral fluid dynamics [7, 8] and Hydro+ [9]. Moreover, even in the framework of conventional hydrodynamics, the existence of a critical point may impact the temporal evolution via its modification to the equation of state (EoS). Also, there are many other sources which may affect the resulting multiplicity fluctuations. For instance, experimental uncertainties, cuts, and spurious origin may substantially attenuate the measured signals [10].

The present work involves a hydrodynamic study of the multiplicity fluctuations by taking into consideration thermal fluctuations as well as resonance decay. In our approach, every elementary degree of freedom of the Smoothed Particle Hydrodynamics (SPH) algorithm, namely, a small fluid element denoted by an SPH particle, is treated as a GCE. In comparison with statistical model approaches, besides the fact that system expansion is encoded in terms of freeze-out surface, event-by-event initial conditions (IC) also plays a significant role in the resulting quantities, such as scaled variance.

1.2 Thermodynamical fluctuations and renonance decay

For a GCE, the particle number fluctuations can be measured regarding the variance and covariance of particle numbers. The variance for species i is found to be

$$\langle(\Delta N_i)^2\rangle = T\left(\frac{\partial N_i}{\partial \mu}\right)_T, \tag{1}$$

where

$$N_i = \sum_p \langle n_{p,i}\rangle, \tag{2}$$

$$\langle n_{p,i}\rangle = \frac{1}{\exp\left[\left(\sqrt{p^2+m_i^2}-\mu_i\right)/T\right]-\gamma_i}. \tag{3}$$

(4)

Since the covariance between different particle species vanishes, we have

$$\langle\Delta N_i \Delta N_j\rangle = \sum_{p,k}\langle\Delta n_{p,i}\Delta n_{k,j}\rangle = \delta_{ij}\sum_p v_{p,i}^2 \tag{5}$$

as particle distributions are assumed to be independent

$$\langle\Delta n_{p,i}^2\rangle \equiv v_{p,i}^2 = \langle(n_{p,i}-\langle n_{p,i}\rangle)^2\rangle = \langle n_{p,i}\rangle(1+\gamma_i\langle n_{p,i}\rangle), \tag{6}$$

$$\langle\Delta n_{p,i}\Delta n_{k,j}\rangle \equiv v_{p,i}^2\delta_{ij}\delta_{pk}. \tag{7}$$

In order to consider the particle number conservation, one may include a factor [3, 11]

$$\int_{-\infty}^{\infty} d\lambda_q \Pi_{p,i}\exp\left[i\lambda_q q_i \Delta n_{p,i}\right] \tag{8}$$

into the grand partition function for each conserved charge q if the system is overall neutral. Here q_i is the charge number for species i. The resulting system is "canonical" for those charges, and the partition function may be evaluated by using the saddle point approximation. Regarding specific model parameters, the effect of conserved charges can be substantial.

The resonance decay can be considered by introducing the following generting function

$$G \equiv \Pi_R\left(\sum_r b_r^R \Pi_i \lambda_i^{n_{i,r}^R}\right)^{N_R}, \tag{9}$$

where for a given resonance R, each different decay channel is denoted by r with the branching ratio b_r^R. $n_{i,r}^R$ indicates the number of particles i obtained through the decay channel r of the resonance in question. Here λ_i is the source terms which will be taken to be 1 by the end of the calculations. Since r is a dummy variable, the resulting particle number of a specific particle species i essentially corresponds to the operation $\lambda_i\frac{\partial}{\partial\lambda_i}$. As a result

$$\overline{N_i} \equiv \sum_R\langle N_i\rangle = \lambda_i\frac{\partial}{\partial\lambda_i}G = \sum_R N_R\sum_r\sum_r b_r^R n_{i,r}^R \equiv \sum_R N_R\langle n_i\rangle_R, \tag{10}$$

$$\begin{aligned}\overline{N_iN_j} &\equiv \sum_R\langle N_iN_j\rangle_R + \sum_{R\neq R'}\langle N_iN_j\rangle_{R,R'} = \lambda_i\frac{\partial}{\partial\lambda_i}\left(\lambda_i\frac{\partial}{\partial\lambda_i}G\right)\\ &= \sum_R\left[N_R(N_R-1)\langle n_i\rangle_R\langle n_j\rangle_R + N_R\langle n_in_j\rangle_R\right] + \sum_{R\neq R'}N_RN_{R'}\langle n_i\rangle_R\langle n_j\rangle_{R'}.\end{aligned} \tag{11}$$

where $\langle\cdots\rangle_R$ means the average over different decay modes of a given resonance, for instance $\langle n_in_j\rangle_R = \sum_r b_r^R n_{i,r}^R n_{j,r}^R$, and the bar symbol indicates to sum up all the contributions from different resonance.

One observes the contributions may come from decay from the same resonance, two resonances from the same type, and two different resonance, from the last line of Eq.(11). Subsequently, for a given initial resonance distribution, the scaled variance is found to be

$$\omega_R^{i*} \equiv \frac{\langle N_i^2\rangle_R - \langle N_i\rangle_R^2}{\langle N_i\rangle_R} = \frac{\langle n_i^2\rangle_R - \langle n_i\rangle_R^2}{\langle n_i\rangle_R} = \frac{\sum_r b_r^R\left(n_{i,r}^R\right)^2 - \left(\sum_r b_r^R n_{i,r}^R\right)^2}{\sum_r b_r^R n_{i,r}^R}, \tag{12}$$

while the resulting expression when all different resonances are taken into considerations is

$$\overline{\omega_R^{i*}} = \frac{\overline{N_i^2} - \overline{N_i}^2}{\overline{N_i}} = \frac{\sum_R N_R\langle n_i^2\rangle_R - \sum_R N_R\langle n_i\rangle_R^2}{\sum_R N_R\langle n_i\rangle_R}. \tag{13}$$

In practice, resonance yields N_R also fluctuate, and the resultant scaled variance reads

$$\omega^{i*} \equiv \frac{\langle\overline{N_i^2}\rangle_T - \langle\overline{N_i}^2\rangle_T}{\langle\overline{N_i}\rangle_T} = \overline{\omega_R^{i*}} + \sum_R\langle n_i\rangle_R\omega_R, \tag{14}$$

where

$$\omega_R \equiv \frac{\langle N_R^2\rangle_T - \langle N_R\rangle^2_T}{\langle N_R\rangle_T} \tag{15}$$

is the scaled variance of the resonance, which, for instance, can be assumed to possess the form of the thermal fluctuations discussed above.

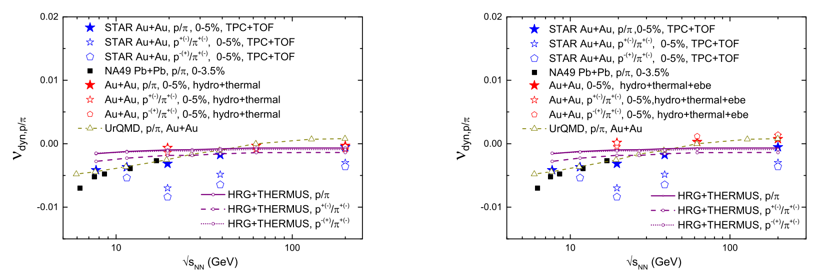
1.2 The hydrodynamic model

In this work, we employ the hydrodynamic code SPheRIO [12] which is based on Smooth Particle Hydrodynamic, an algorithm developed initially in astrophysics and adapted to relativistic heavy ion collisions. The method parameterizes the matter flow in terms of discrete Lagrangian coordinates, known as SPH particles. Those SPH particles are attached to small volumes with some conserved quantities. The hydrodynamic equation is governed by the continuity equations expressing the conservation of energy-momentum, baryon-number, and other conserved charges. In term of SPH degree of freedom, the equation of motion is derived by using the variational principle. Its main advantage is that any complex geometry and violent dynamics such as shock phenomena can be treated without any numerical difficulties, providing that the size of particles is appropriately chosen. We shall neglect in this work any dissipative effects and also assume Cooper-Frye sudden freeze-out take place at constant temperatures where thermal fluctuations are evaluated. It is noting that there is no free parameter in the present simulation since the few existing ones have been fixed in earlier studies of η and p_T distributions.

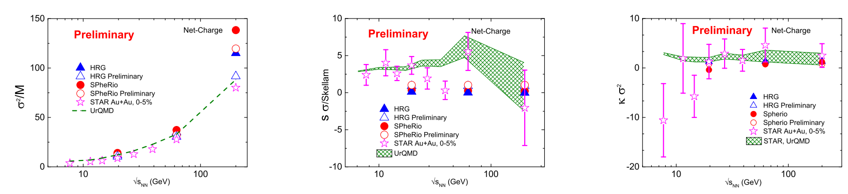
For the present study, we treat every elementary degree of freedom of the system, denoted by an SPH particle, as a GCE. We investigate, in particular, the effect of hydrodynamical evolution, event by event IC, and resonance decay on multiplicity fluctuations.

2 Results and discussions

Here, we first show the calculated fluctuations of particle ratio p/π at different energies. The results are compared with the data from the STAR [13] and NA49 [14] Collaborations. The results from SPheRIO are presented for the case with and without event by event IC fluctuations. Preliminary results show that the effect of IC fluctuations is significant and push the results further away from the data.



In the following plots, we present the calculated moments of net-charge fluctuations at different energies. The STAR data [15] is compared with the statistical model, with the event-by-event IC fluctuations being switched on and off. The work is in progress.



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