Soft symmetry breaking effects in effective theories of the QCD phase diagram
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Introduction

QCD with massless quarks in vacuum is symmetric under SU(2)_L x SU(2)_R (chiral symmetry), CP and CPT. At finite T there is a special frame, implying space and time components should be treated differently, but the system is rotationally symmetric. These symmetries constrain effective theories (EFT's) of QCD. A subgroup of flavor, SU(2)_A is explicitly broken at finite quark mass m_0 by
\[ m_0 \bar{\psi} \psi. \]  
(1)

A finite chemical potential breaks CP symmetry of QCD via
\[ -\mu \bar{\psi} \gamma \cdot \tau \psi. \]  
(2)

Both Eq. 1 and Eq. 2 are soft terms. EFT's of QCD in principle obtained by integrating out modes above some UV scale will feature operators of dimension greater than (or equal to) 4 giving rise to hard symmetry breaking terms for each explicitly broken symmetry.

In previous work [1] we studied an EFT at finite T, µ not including hard symmetry breaking terms, considering operators up to dimension 6. (The dimension 6 terms considered were symmetric under SU(2)_A and CP.)

One of the observables studied was the curve of critical transitions with m_0 = 0. For small µ, the curve can be written as
\[ T_c(\mu) = T_c(0) \left( 1 - \frac{\mu^2}{2T_c(0)} + O(\mu^4) \right), \]  
(3)

where µ = 3µ. The coefficient κ was computed and was found to be [1],
\[ \kappa = 1/(3\pi^2), \]  
(4)

which, while consistent with [2], is above other lattice estimates [3, 4, 5].

In this work we explore the effect of adding hard symmetry breaking operators and the CP breaking number condensate on these results.

Effective lagrangian at finite T, µ

We are in particular interested in the region near the QCD crossover and the dimension scale for the theory is set by T_c [1], which we take to be the critical temperature \( T_c(\mu) \) at µ = 0 in the chiral limit.

The general structure of the Euclidean lagrangian is,
\[ L = L^0 + \bar{\psi} \gamma_5 \psi \frac{d^2}{dT^2} + \bar{\psi} \gamma_\tau \psi \frac{d^4}{dT^4} + \bar{\psi} \gamma_\tau \gamma_\tau \psi \frac{d^6}{dT^6} + \bar{\psi} \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \psi \frac{d^8}{dT^8}, \]  
(5)

where superscripts label the dimensions of the operators. \( L^0 \) is a sum of the soft symmetry breaking terms Eq. 1, 2. We define the dimensionless variable \( d^4 \) by \( m_0 = d^4 T^4 \). In the kinetic term
\[ L^4 = \bar{\psi} \frac{d^4}{dT^4} \psi; \]  
(6)

spatial gradients (\( \partial^\mu \)) appear with an extra factor \( d^4 \) compared to \( \partial \). \( L^0, L^4 \) and \( L^6 \) are sums of dimension 6 operators with specific symmetry structures. \( L_0 \) is a sum of the 10 possible dimension 6 operators that are invariant under global chiral transformations and space-time symmetries of a finite temperature Euclidean theory [1]. All terms are also CP symmetric.

One new term we add is \( L^8 \) which is a sum of all operators that break SU(2)_A. It features 4 terms with the structure,
\[ \frac{d^8}{dT^8} \left[ \bar{\psi} \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \psi \right]^2, \]  
(7)

with \( \gamma^I \in \{ \gamma_1, \gamma_2, \gamma_3 \} \). It features 2 terms with the form,
\[ \frac{d^6}{dT^6} \left[ \bar{\psi} \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \psi \right]^2, \]  
(8)

with \( I^4 \in \{ \gamma_1, \gamma_2 \} \). \( d^4, d^6 \) are non-zero only if \( m_0 \neq 0 \).

The second new term we add is \( L^9 \) which has terms that break chiral symmetry and CP breaking symmetry. There are only two such terms,
\[ \frac{d^9}{dT^9} \left[ \bar{\psi} \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \psi \right]^2 + \frac{d^9}{dT^9} \left[ \bar{\psi} \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \psi \right]^2. \]  
(9)

Both \( d^9 \) are non-zero only if \( m_0 \neq 0 \).

Mean field

We introduce two condensates. The chiral condensate \( \langle \bar{\psi} \gamma_\tau \psi \rangle \) and the number density \( \langle \bar{\psi} \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \psi \rangle \) in the QCD phase diagram. The corresponding mean fields are,
\[ \Sigma = \frac{\alpha \langle \bar{\psi} \gamma_\tau \psi \rangle}{4}, \]  
(10)

where the low energy constants (LEC’s) \( \alpha, \beta \) are specific linear combinations of the coupling constants in \( L_0 \) and \( L^9 \). In the chiral limit, \( \alpha, \beta \) and \( d^4 \) are the only constants needed to determine the free energy. Taking \( T_c \) to be the T, \( \mu = 0 \) fixes \( \alpha = 24 T_c^4 \) [1].

The mean field Lagrangian is,
\[ L = d^4 T_c^4 \bar{\psi} \gamma_\tau \gamma_\tau \gamma_\tau \gamma_\tau \psi \frac{d^4}{dT^4} \psi + d^4 T_c^4 V(\Sigma, \beta \gamma \Sigma T_c^4), \]  
(11)

where,
\[ V = 4N_c N_f T_c^4 \left( \frac{\alpha^2}{24} - \frac{\alpha \beta}{2} \Sigma \right), \]  
(12)

\[ \beta = 2 \left( 4N_c N_f - 1 \right) \frac{d^4}{d^4} \frac{d^4}{d^4} \]  
(13)

As is clear from \( V \), in the absence of \( \beta \), the two condensates are connected by mean field renormalizations of the mass and the chemical potential. Hard breaking induces a direct coupling between the two condensates.

Away from the chiral limit there are two additional LEC’s, \( \gamma, \delta \). From the requirement that \( \beta \) vanish for \( \mu = 0, d^4 = 0 \) and that LEC’s are analytic in \( \mu, d^4 \), one finds \( \beta \propto \mu \). We define \( \beta \) by the equation,
\[ \beta = \beta_0 \frac{d^4}{d^4} \]  
(14)

Chiral limit

It is convenient to define the combination
\[ \tilde{\kappa}_i = \kappa_i - 3\alpha \]  
(16)

which tells us about the deviation of the critical line from the circular shape. \( |T_c| \mu^2 = |T_0|^2 - \kappa \mu^2 \). For \( \alpha = 0 \), the curve is circular and \( \kappa_i = 0 \) but this is a highly fine-tuned case.

References