Soft symmetry breaking effects in effective theories of the QCD phase diagram

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Introduction

QCD with massless quarks in vacuum is symmetric under $SU(2)_L \times SU(2)_R$ (chiral symmetry), CP and CPT. At finite T there is a special frame, implying space and time components should be treated differently, but the system is rotationally symmetric. These symmetries constrain effective theories (EFT's) of QCD.

A subgroup of flavor, $SU(2)_A$ is explicitly broken at finite quark mass m_0 by

$$m_0 \overline{\psi}\psi \, . \tag{1}$$

A finite chemical potential breaks CP symmetry of QCD via

$$-\mu \overline{\psi}\gamma^4 \psi \, . \tag{2}$$

Both Eq. 1 and Eq. 2 are soft terms.

EFT's of QCD in principle obtained by integrating out modes above some UV scale will feature operators of dimension greater than (or equal to) 4 giving rise to hard symmetry breaking terms for each explicitly broken symmetry.

In previous work [1] we studied an EFT at finite T, μ not including hard symmetry breaking terms, considering operators up to dimension 6. (The dimension 6 terms considered were symmetric under $SU(2)_A$ and CP.)

One of the observables studied was the curve of critical transitions with $m_0 = 0$. For small μ, the curve can be written as

$$T_c(\mu) = T_c(0) - \frac{1}{2}\kappa \frac{\mu_B^2}{T_c(0)} - \frac{1}{4!}\kappa_4 \frac{\mu_B^4}{T_c(0)^3} + \mathcal{O}(\mu_B^6). \tag{3}$$

where $\mu_B = 3\mu$.

The coefficient κ was computed and was found to be [1],

$$\kappa = 1/(3\pi^2) \ , \tag{4}$$

which, while consistent with [2], is above other lattice estimates [3, 4, 5].

In this work we explore the effect of adding hard symmetry breaking operators and the CP breaking number condensate on these results.

Effective lagrangian at finite T, μ

We are in particular interested in the region near the QCD crossover and the dimensional scale for the theory is set by T_0 [1], which we take to be the critical temperature (T_c) at $\mu = 0$ in the chiral limit.

The general structure of the Euclidean lagrangian is,

$$L = L^3 + L^4 + L^6 + L^6_A + L^6_B \, . \tag{5}$$

where superscripts label the dimensions of the operators.

L^3 is a sum of the soft symmetry breaking terms Eq. 1, 2. We define the dimensionless variable d^3 by $m_0 = d^3 T_0$. In the kinetic term

$$L^4 = \bar{\psi}[\partial\!\!\!/_4 + d^4 \partial\!\!\!/_i]\psi \, . \tag{6}$$

spatial gradients (∂_i) appear with an extra factor d^4 compared to ∂_4. L_6, L^A_6 and L^B_6 are sums of dimension 6 operators with specific symmetry structures.

L_6 is a sum of the 10 possible dimension 6 operators that are invariant under global chiral transformations and space-time symmetries of a finite temperature Euclidean theory [1]. All terms are also CP symmetric.

One new term we add is L^A_6 which is a sum of all operators that break $SU(2)_A$. It features 4 terms with the structure,

$$\frac{d_A^{6j}}{T_0^2}\left[(\overline{\psi}\Gamma^j\psi)^2 - (\overline{\psi}i\gamma_5\Gamma^j\tau^a\psi)^2\right] \ , \tag{7}$$

with $\Gamma^j \in \{1, i\gamma^5, S^{4i}, S^{ij}\}$. It features 2 terms with the form,

$$\frac{d_A^{6k}}{T_0^2}\left[(\overline{\psi}\Gamma^k\psi)^2 - (\overline{\psi}\gamma_5\Gamma^k\tau^a\psi)^2\right] \ , \tag{8}$$

with $\Gamma^k \in \{\gamma^4, i\gamma^i\}$. d_A^{6j}, d_A^{6k} are non-zero only if $m_0 \neq 0$.

The second new term we add is L^B_6 which has terms that break chiral symmetry and CP symmetry. There are only two such terms,

$$L^B_6 = \frac{d_B^{61}}{T_0^2}\left[(\overline{\psi}\psi)(\overline{\psi}\gamma_4\psi)\right] + \frac{d_B^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)(\overline{\psi}\gamma_4\tau^a\psi)\right] \, . \tag{9}$$

d_B^{6i} are non-zero only if $m_0 \neq 0$ and $\mu \neq 0$.

Mean field

We introduce two condensates. The chiral condensate $\langle\bar{\psi}\psi\rangle$ and the number density $\langle\bar{\psi}\gamma^4\psi\rangle$ in the QCD phase diagram. The corresponding mean fields are,

$$\Sigma = \frac{\alpha\langle\bar{\psi}\psi\rangle}{T_0^2} \quad \text{and} \quad \Gamma = -\frac{\alpha'\langle\bar{\psi}\gamma^4\psi\rangle}{T_0^2} \tag{10}$$

where the low energy constants (LEC's) α, α' are specific linear combinations of the coupling constants in L_6 and L^A_6.

In the chiral limit, α, α' and d^4 are the only constants needed to determine the free energy. Taking T_0 to be the T_c at $\mu = 0$ fixes $\alpha = 24(d^4)^3$ [1].

The mean field lagrangian is,

$$L = d^3 T_0 \overline{\psi}\psi + \overline{\psi}\partial\!\!\!/_4\psi - \mu\overline{\psi}\gamma_4\psi + d^4\overline{\psi}\partial\!\!\!/_i\psi + V + \bar{\psi}\psi(\Sigma - \beta\frac{\Gamma}{\alpha'}) + \bar{\psi}\gamma^4\psi(\Gamma + \beta\frac{\Sigma}{\alpha}) \, . \tag{11}$$

where,

$$V = 4N_cN_fT_0^2\left(\frac{\Gamma^2}{2\alpha'} - \frac{\Sigma^2}{2\alpha} + \beta\frac{\Gamma\,\Sigma}{\alpha'\alpha}\right) \ , \tag{12}$$

$$\beta = 2(4N_c - 1)d_B^{61} - 6d_B^{62} \tag{13}$$

As is clear from V, in the absence of β, the two condensates are connected by mean field renormalizations of the mass and the chemical potential. Hard breaking induces a direct coupling between the two condensates.

Away from the chiral limit there are two additional LEC's. β and d^3. From the requirement that β vanish for $\mu, d^3 = 0$ and that LEC's are analytic in μ, d^3, one finds $\beta \propto d^3\mu$.

We define $\tilde{\beta}$ by the equation,

$$\beta = \tilde{\beta} d^3 \frac{\mu}{T_0} \, . \tag{14}$$

Chiral limit

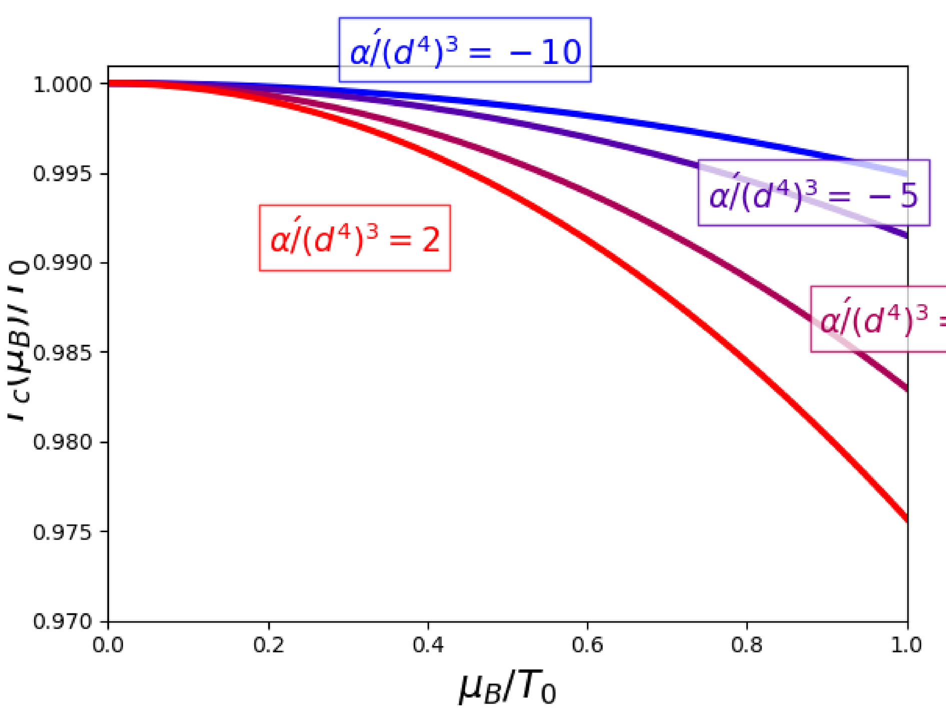

Figure 1: Chiral critical temperature curve for small μ. Note that $\alpha' < 0$ reduces the curvature compared to Eq. 4

The combination $\frac{\alpha'}{(d^4)^3}$ determines the curve of T_c in the $T - \mu$ plane (Fig. 1).

In mean field with two condensates,

$$\kappa = \frac{1}{3\pi^2}\frac{1}{[1 - \frac{\alpha'}{12(d_4)^3}]^2} \ , \kappa_4 = \frac{1}{108\pi^4}\frac{36 - 19\frac{\alpha'}{(d_4)^3}}{[1 - \frac{\alpha'}{12(d_4)^3}]^5} \, . \tag{15}$$

Note that κ and κ_4 feature α' and d^4 in the same combination, $\alpha'/(d^4)^3$. Therefore, once κ is known, κ_4 is a prediction in our theory.

It is convenient to define the combination

$$\bar{\kappa}_4 = \kappa_4 - 3(\kappa)^2 \ , \tag{16}$$

which tells us about the deviation of the critical line from the circular shape, $[T_c(\mu)]^2 = [T_c(0)]^2 - \kappa\ \mu^2$. For $\alpha' = 0$ the curve is circular and $\bar{\kappa}_4 = 0$ but this is a highly fine-tuned case.

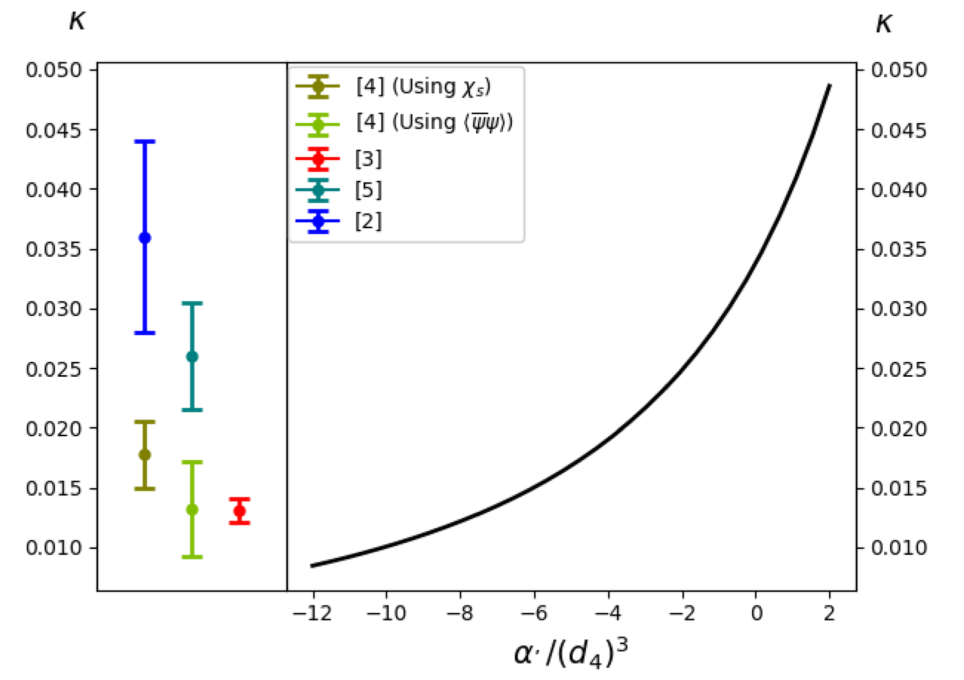

Figure 2: Comparison of κ as a function of $\alpha'/(d^4)^3$ with lattice measurements [2, 3, 4, 5]

Results away from the chiral limit

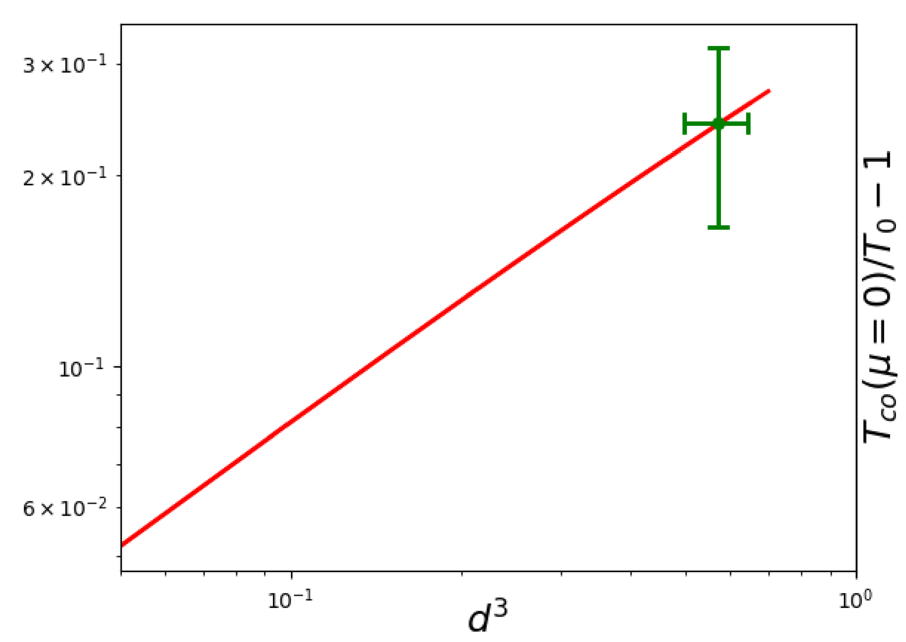

Figure 3: The dependence of the crossover temperature at $\mu = 0$ on d^3. The green point shows the value of $T_{co}/T_c - 1$ from [6] at the d^3 evaluated in [1] to match the π screening mass in [6].

There are two direct effects of a finite d^3. First, the critical curve changes to a crossover curve. Second, the crossover temperature is larger than the critical temperature for the same α', d^4. d^3 was fixed in the EFT [1] using π correlators calculated on the lattice in [6]. $T_{co}(\mu = 0)$ (Fig. 3) does not depend on $\tilde{\beta}$.

At finite μ, $\tilde{\beta}$ affects observables. In Fig. 4 we see the effect of $\tilde{\beta}$ on the curvature κ of the crossover curve.

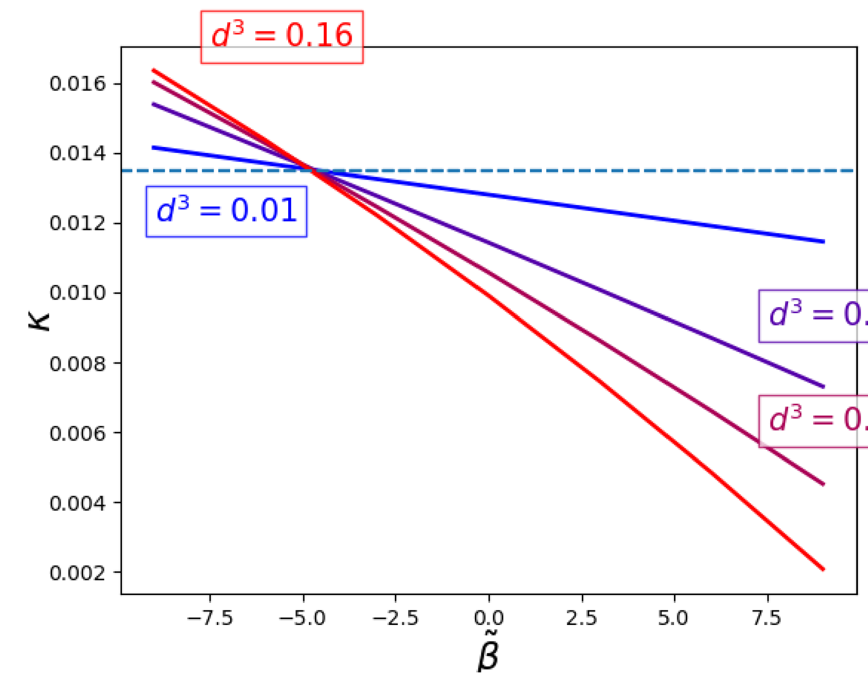

Figure 4: Effect of $\tilde{\beta}$ on κ for a fixed value of $\alpha'/(d^4)^3 = -7$ motivated by Fig. 4. As d^3 decreases, κ tends to the chiral value (Eq. 15) shown by the dashed line.

References

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