Chiral magnetic response to arbitrary axial imbalance

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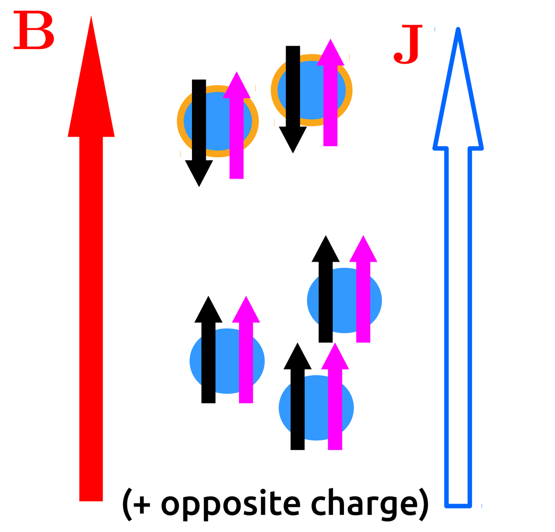
Phenomenology & introduction

We analyze the response of chiral fermions to time and space dependent axial imbalance & constant magnetic field: By analyzing the axial-vector–vector–vector three-point function in real-time QFT at finite temperature in the linear response approximation. The chiral magnetic conductivity is given analytically for non-interacting fermions. It is pointed out that local charge conservation plays an important role when the axial imbalance is inhomogeneous. Proper regularization is needed which makes the constant axial imbalance limit delicate: for static chiral charge the chiral magnetic effect (CME) vanishes. In the homogeneous (but possible time-dependent) limit of the axial imbalance the CME current is determined solely by the chiral anomaly. As a phenomenological consequence, the observability of the charge asymmetry caused by the CME turns out to be a matter of interplay between various scales of the system. Possible plasma instabilities resulted from the gradient corrections to the CME current are also pointed out.



$n_R \neq n_L \quad \langle \mathbf{s} \rangle \sim \mathbf{B}$

$\langle \mathbf{p} \rangle \sim (n_R - n_L)\langle \mathbf{s} \rangle$

$$\mathbf{J} = \frac{e^2}{2\pi^2} A_{5,0}\mathbf{B}$$

Simple cartoon of the CME

- chiral fermions prefer to align their spin parallel to the magnetic field
- fermions move along the direction of B according to their chirality
- in case there is an imbalance of the number of the two chiral species, it results in a charge sensitive electric current J
- *The real, dynamical origin of the CME is the change of momentum space topology in magnetic field (Berry-curvature)*

See:
D. E. Kharzeev *et.al.*, Prog. Part. Nucl. Phys **88**, 1 (2016)
K. Landsteiner, Acta Phys. Pol. B **47**, 2617 (2016)
D. Kharzeev (edited by) *et.al.*, Lec. Notes in Phys., Volume **871** (2013)
A. Bzdak *et.al.*, arXiv:1906.00936

In order to model the phenomenology of the axial imbalance in a fermionic system we take quantum electrodynamics in its chiral limit and supplement it by adding an axial-vector coupling:

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\gamma^\mu i\partial_\mu\psi - eA_\mu\overline{\psi}\gamma^\mu\psi - A_{5,\mu}\overline{\psi}\gamma^\mu\gamma^5\psi$$

vector current $J^\mu = \overline{\psi}\gamma^\mu\psi$ axial-vector current $J_5^\mu = \overline{\psi}\gamma^\mu\gamma^5\psi$

The U(1) axial anomaly prohibits the simultaneous conservation of vector and axial-vector charges. In order to keep the vector charge conservation intact, regularization is needed.

$$\partial_\mu J^\mu = 0 \quad \text{(consistent anomaly!)}$$

$$\partial_\mu J_5^\mu = \frac{1}{2\pi^2}\mathbf{E}\cdot\mathbf{B} + \frac{1}{6\pi^2}\mathbf{E}_5\cdot\mathbf{B}_5$$

See for example: Landsteiner, arXiv: *1610.04413 (2016)*

The introduction of axial coupling causes transport phenomena which have fundamentally different nature compared to the usual electric transport, since they behave differently with respect to parity-inversion and time-reversal transformations.

CME

$$\langle J^\mu \rangle = \langle J^\mu J^\nu \rangle\big|_{A, A_5=0} A_\nu + \langle J^\mu J^\nu J_5^\rho \rangle\big|_{A, A_5=0} A_\nu A_{5,\rho} + \ldots$$

electric current ~ axial imbalance × magnetic field

$$\langle J_5^\mu \rangle = \langle J_5^\mu J_5^\nu \rangle\big|_{A, A_5=0} A_{5,\nu} + \tfrac{1}{2}\langle J_5^\mu J^\nu J^\rho \rangle\big|_{A, A_5=0} A_\nu A_\rho + \tfrac{1}{2}\langle J_5^\mu J_5^\nu J_5^\rho \rangle\big|_{A, A_5=0} A_{5,\nu} A_{5,\rho} + \ldots$$

When the axial and vector fields are dynamical, the transport relations couple together leading to collective excitations like the chiral magnetic wave.

AVV response function

Electric current induced by magnetic field and scalar axial imbalance (in special cases it is analogous to (-1) times the chiral chemical potential): in the linear response approximation controlled by the axial-vector –vector--vector vertex (AVV) function

$$\langle J^i J^j J_5^0 \rangle \equiv \Gamma^{0ij}_{AVV}: \text{AVV vertex}$$

$$J^i(x) = \int \mathrm{d}^4 q_1 \int \mathrm{d}^4 q_2 \widetilde{A}_j(q_1)\widetilde{A}_{5,0}(q_2)\widetilde{\Gamma}^{0ij}_{AVV}(q_1, q_2) e^{ix\cdot(q_1+q_2)} = \int_{-\infty}^{\infty} \mathrm{d}t' \int \mathrm{d}^3\mathbf{q} \widetilde{A}_{5,0}(t', \mathbf{q}) \overline{\sigma}^i_A(t'-t, \mathbf{q}) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

In this approximation there are no axial currents and no net electric charge density:

axial-vector potential $A_5 = (A_{5,0}, \mathbf{0})$

vector potential $A = (0, \mathbf{A})$

As a consequence of local vector charge conservation, the AVV vertex fulfils the following identities (Ward-Takahasi). The third equation reflects the anomalous nonconservation of the axial-vector charge.

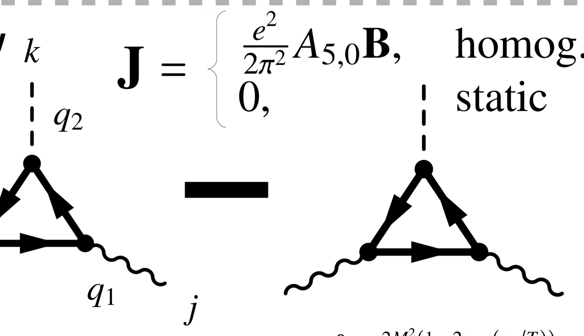
$$(q_1 + q_2)_\mu \widetilde{\Gamma}^{\rho\mu\nu}_{AVV} = q_{1,\nu}\widetilde{\Gamma}^{\rho\mu\nu}_{AVV} = 0,$$

$$q_{2,\rho}\widetilde{\Gamma}^{\rho\mu\nu}_{AVV} = i\epsilon^{\mu\nu\alpha\beta} q_{1,\alpha} q_{2,\beta} \cdot \frac{e^2}{2\pi^2}.$$

Static conductivity

The constant source limit is subtle. Taking the homogeneity limit first only the UV sector contributes. This is cancelled by an other term when the static limit is taken first, leaving zero response current.

$$\mathbf{J} = \begin{cases} \frac{e^2}{2\pi^2} A_{5,0}\mathbf{B}, & \text{homog.} \\ 0, & \text{static} \end{cases}$$



$$\frac{e^2}{2\pi^2}\epsilon^{ijk} \leftarrow \int \mathrm{d}^4 p \frac{n'_{FD}(p_0/T)}{T} \mathrm{tr}\left\{\Gamma^5 G_A^{-1}\partial^i G_A G_A^{-1}\partial^j G_A G_A^{-1}\partial^k G_A\right\} \qquad \epsilon^{ijk}\int \mathrm{d}^4 p \frac{2M^2(1-2n_{FD}(p_0/T))}{[(p_0 - i0^+)^2 - \mathbf{p}^2 - M^2]^3}$$

topologically protected for *m=0* — beyond the scales of any interactions

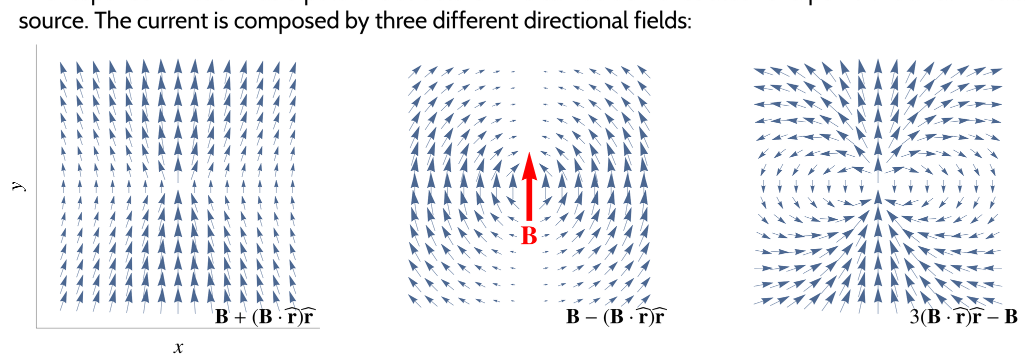
Constant magnetic field & arbitrary axial imbalance

In the weak coupling limit the conductivity can be given analytically. There are finite temperature contributions, which are absent in the charge density. For T=0 there are contributions result of the retardation, but there is an instantaneous response as well.

$$\overline{\sigma}^i_A(t, \mathbf{q}) = \frac{e^2}{2\pi^2}\left\{B^i\delta(t) + \frac{\theta(-t)}{2}\left[q\sin(qt)\left(B^i + \widehat{q}^i(\mathbf{B}\cdot\widehat{\mathbf{q}})\right) - \frac{\partial}{\partial t}\left(\frac{\sin(qt)}{qt}\right) f(tT)\left(B^i - \widehat{q}^i(\mathbf{B}\cdot\widehat{\mathbf{q}})\right)\right]\right\}$$

Example of point-like source

The response function has dipole-like structure. This can be demonstrated for a point-like axial imbalance source. The current is composed by three different directional fields:

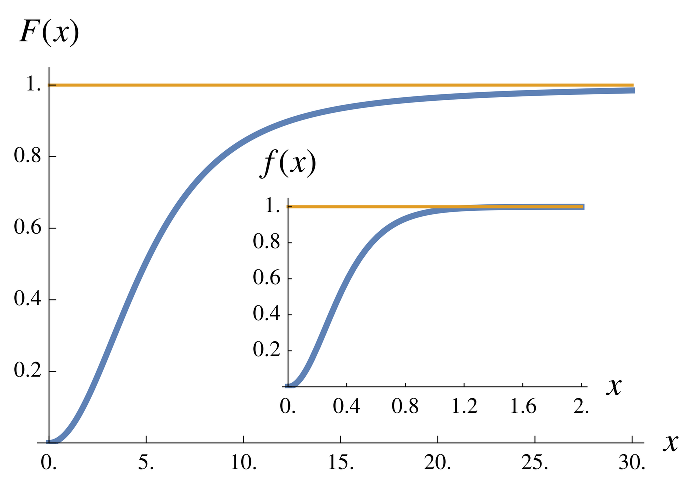


$$\mathbf{J}(t, \mathbf{r}) = \frac{e^2}{2\pi^2}\left\{\frac{1}{3}\mathbf{B}A_{5,0}(t)\delta^{(3)}(\mathbf{r}) + \frac{1}{2}\left[\frac{A''_{5,0}(t-r)}{r}(\mathbf{B} + (\mathbf{B}\cdot\widehat{\mathbf{r}})\widehat{\mathbf{r}}) - \left(\frac{A'_{5,0}(t-r)}{r^2} + \frac{A_{5,0}(t-r) - A_{5,0}(t)}{r^3}\right)(\mathbf{B} - 3(\mathbf{B}\cdot\widehat{\mathbf{r}})\widehat{\mathbf{r}}) + \frac{A'_{5,0}(t-r)f(rT) - A_{5,0}(t-r)Tf'(rT)}{r^2}(\mathbf{B} - (\mathbf{B}\cdot\widehat{\mathbf{r}})\widehat{\mathbf{r}})\right]\right\}.$$

Transported charge

After the sudden onset of axial imbalance (*quench*) the long-time form of the current is this:

$$\mathbf{J}(t, \mathbf{q}) \xrightarrow{t\to\infty} \frac{e^2}{2\pi^2}\frac{\widetilde{A}_{5,0}(\mathbf{q})}{2}\left(\mathbf{B} - \frac{\mathbf{q}(\mathbf{B}\cdot\mathbf{q})}{q^2}\right) F(q/T)$$



It has a dipole-like structure, which proves to be important for the charge conservation to hold.

$$\mathbf{q}\cdot\mathbf{J}(t\to\infty, \mathbf{q}) \equiv 0$$

For *T=0* the current is described by a dipole field. As a consequence, this current transports zero net charge through the plane perpendicular to the direction of the magnetic field.

$$\mathbf{J}(t\to\infty, \mathbf{r})\big|_{T=0} = -\frac{e^2}{16\pi^3}\nabla_\mathbf{r}\times(\mathbf{B}\times\nabla_\mathbf{r})\int \mathrm{d}^3\mathbf{r}'\frac{A_{5,0}(\mathbf{r}-\mathbf{r}')}{r'}$$

Assuming a localized source, we can define an average current over the region which contains the axial imbalance source. The current is suppressed farther than *R ~ 1/T*.

$$\overline{\mathbf{J}} := \frac{1}{V}\int_{S_R} \mathrm{d}^3\mathbf{r}\mathbf{J}(t\to\infty, \mathbf{r}) \overset{\widetilde{A}_{5,0}(\mathbf{q}) = V\cdot A_{5,0}}{=} \frac{e^2}{2\pi^2}A_{5,0}\mathbf{B}\frac{1 - f(RT)}{3}$$

Vanishing long-time charge asymmetry

$$\Delta Q = \int_{-\infty}^{\infty}\mathrm{d}t\int_S \mathrm{d}^2\mathbf{r}\widehat{\mathbf{B}}\cdot\mathbf{J}(t, \mathbf{r}) = \int \mathrm{d}^3\mathbf{q}\int_S \mathrm{d}^2\mathbf{r}\widehat{B}_i\overline{\sigma}^i_A(q_0 = 0, \mathbf{q})\widetilde{A}_{5,0}(q_0 = 0, \mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{r}} =$$

$$= \frac{e^2 B}{4\pi^2}\int \mathrm{d}^3\mathbf{q}\int_S \mathrm{d}^2\mathbf{r}\left(1 - (\widehat{\mathbf{B}}\cdot\widehat{\mathbf{q}})^2\right)F(q/T)\widetilde{A}_{5,0}(0, \mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{r}} \longrightarrow 0!$$

We have already seen the long-time behavior of the conductivity: it is dipolar in space.

It is essential to assume that the observation time is long enough (t→∞, compared to the time-scales of the sources), and that we take into account all the current through a large enough surface (area of S→∞) !

With the *long-time/large S* assumption the vanishing of the transported charge is robust (i.e. no corrections from interactions)

$$\Delta Q = \int \mathrm{d}^4 q\int_{-\infty}^{\infty}\mathrm{d}q_\parallel \widehat{B}_i\widetilde{\Gamma}^{0ij}_{AVV}(q, q')\widetilde{A}_j(q)\widetilde{A}_{5,0}(q')$$

$q' = (-q_0, -\mathbf{q} + q_\parallel\widehat{\mathbf{B}})$

$q = (q_0, \mathbf{q})$

Writing ΔQ down in terms of the vertex function, we can see that it is essentially the consequence of the local charge conservation.

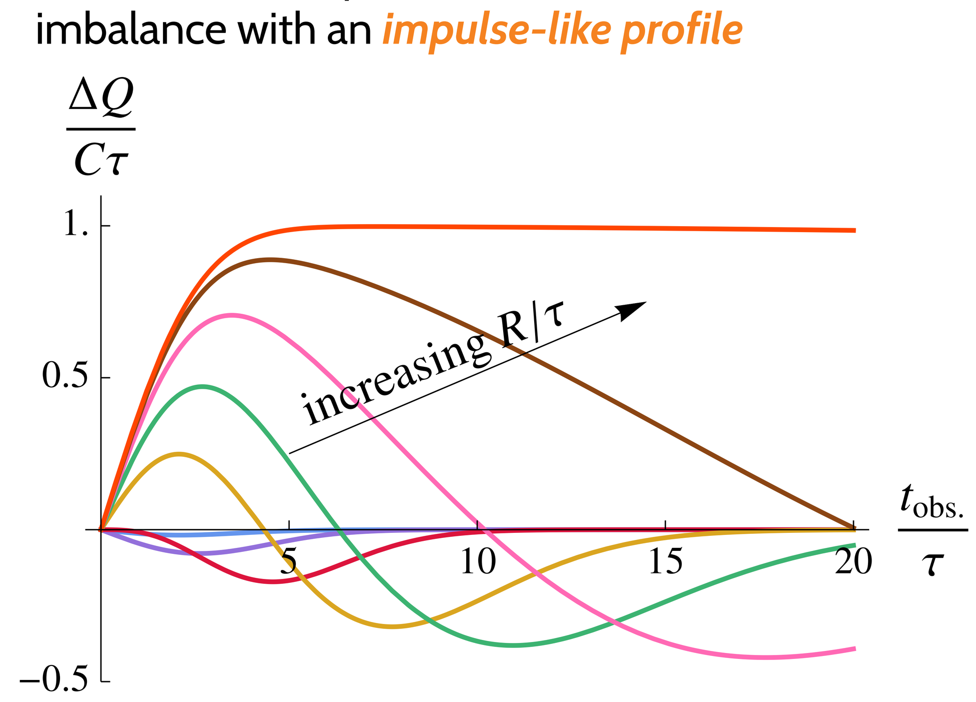
$$q_\parallel\widehat{B}_i\widetilde{\Gamma}^{0ij}_{AVV}(q, q') = (q + q')_\mu\widetilde{\Gamma}^{0\mu j}_{AVV}(q, q') \equiv 0$$

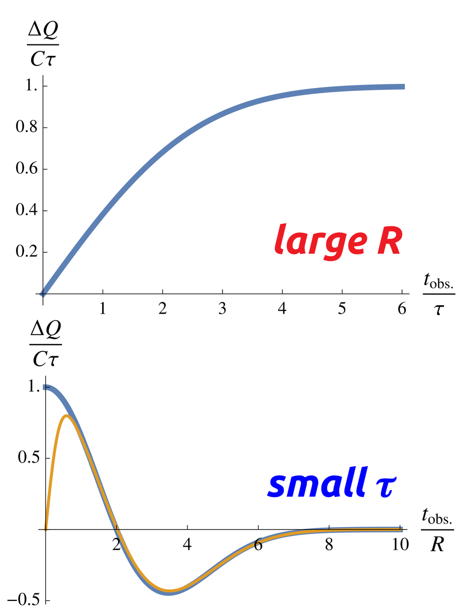
Let us explore the charge separation but only in a finite time window $t_{obs.}$!

Furthermore we parametrize the axial imbalance with an *impulse-like profile*

$$\overline{A}_{5,0}(t, q) \propto \exp\left(-\frac{q^2R^2}{2} - \frac{t^2}{2\tau^2}\right)$$

Interplay of many scales: $(R, \tau, t_{obs.})$





$$\frac{\Delta Q(t_{obs.}, \tau, R)}{C\tau} \longrightarrow \begin{cases} \mathrm{erf}\left(\frac{t_{obs.}}{2\sqrt{2}\tau}\right) & \textit{large R} \\ e^{-\frac{t_{obs.}^2}{8R^2}}\left(1 - \frac{t_{obs.}^2}{4R^2}\right) & \textit{small } \tau \\ \approx \frac{1}{\sqrt{2\pi}}\frac{(R/\tau)^2 - 1}{((R/\tau)^2 + 1)^2}\frac{R^2}{\tau^2}\frac{t_{obs.}}{\tau} + \mathscr{O}((t_{obs.}/\tau)^2) \end{cases}$$

Large enough source behaves like the naive CME expectation for intermediate times (homogeneity limit).

For smaller size or shorter pulse the system quickly reaches the vanishing of ΔQ within the observation time window.

Possible chiral instabilities

Expanding the current for *q→0* up to the first non-trivial contribution:

$$\delta\mathbf{J}(t, \mathbf{q}) \equiv \mathbf{J}(t, \mathbf{q}) - \frac{e^2}{2\pi^2}\overline{A}_{5,0}(t, \mathbf{q})\mathbf{B} \approx \frac{e^2}{4\pi^2}\int_{-\infty}^{0}\mathrm{d}\tau\overline{A}_{5,0}(t+\tau, \mathbf{q})q^2\tau\left[\left(1 + \tfrac{1}{3}f(\tau T)\right)\mathbf{B}\left(1 - \tfrac{1}{3}f(\tau T)\right)(\mathbf{B}\cdot\widehat{\mathbf{q}})\widehat{\mathbf{q}}\right] + \mathscr{O}(q^4)$$

To get a simpler for, we have to assume further separations of scales. Suppose the characteristic timescale of the source is τ_r. There are two liming cases in τ_r/T where the deviation from the homogeneous current can be given in a differential form:

$$\partial_t^2\delta\mathbf{J}(t, \mathbf{r}) = \frac{e^2}{4\pi^2}\left(C_1\mathbf{B}\nabla^2_\mathbf{r} + C_2(\mathbf{B}\cdot\nabla_\mathbf{r})\nabla_\mathbf{r}\right)A_{5,0}(t, \mathbf{r}) = -\frac{e^2}{4\pi^2}\left(C_1\mathbf{B}(\nabla\cdot\mathbf{E}_5) + C_2(\mathbf{B}\cdot\nabla)\mathbf{E}_5\right)$$

	C_1	C_2
$\tau_r \gg T$	1	1
$T \gg \tau_r$	$\frac{4}{3}$	$\frac{2}{3}$

The gradient correction can couple to vorticity, therefore it can lead to vortex formation: $\partial_t\nabla\times\delta\mathbf{J}(t, \mathbf{r}) \neq 0$

Future plans

- *full linear response analysis with axial currents*
- *small-q expansion with non-homogeneous EM & axial fields*
- *possible implementation into simulation frameworks – to describe relaxation dynamics*
- *hydrodynamic simulation with gradient terms taken into account*
- *plasma modes with vorticity: detailed analysis in dynamical situations*

Take-home message

Local charge conservation greatly affects the real-time chiral magnetic response.

Charge separation can be suppressed depending on the scales of the system.

Paper online

Questions and comments are welcome!

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