Gluon emission from heavy quarks in dense nuclear matter

Le Zhang[1,2], De-Fu Hou[2] , Guang-You Qin[2]

[1] College of Physics and Electronic Science, Hubei Normal University

[2] Instituted of Particle Physics & Key Laboratory of Quark and Lepton Physics (MOE), Central China Normal University

Abstract: We study the medium-induced gluon emission process experienced by a hard jet parton propagating through the dense nuclear matter in the framework of deep inelastic scattering off a large nucleus. We work beyond the collinear rescattering expansion and the soft-gluon emission limit, and derive a closed formula for the medium-induced single gluon emission spectrum from a heavy or light quark jet interacting with the dense nuclear medium via transverse and longitudinal scatterings. Without performing the collinear rescattering expansion, the medium-induced gluon emission spectrum is controlled by the full distribution of the differential elastic scattering rates between the propagating partons and the medium constituents. We further show that if one utilizes heavy static scattering centers for the traversed nuclear matter and takes the soft-gluon emission limit, our result can reduce to the first order in opacity Djordjevic-Gyulassy-Levai-Vitev formula.

I. Motivation

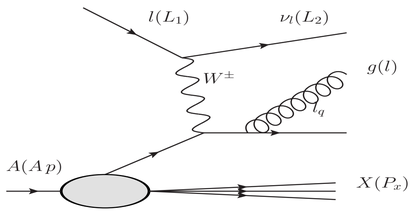
Jets interact with the medium constituents via elastic and inelastic collisions. The interaction causes the energy loss of hard partons. There exist a few theoretical schemes on radiative parton energy loss based on the pQCD framework, such as BDMPS-Z, GLV ,ASW, AMY and HT formalisms.

1. **Previous pQCD-based jet quenching formalisms have made various approximations in the calculation of medium-induced radiative energy loss:**
   - Soft gluon emission approximation (ASW, GLV)
   - Collinear rescattering expansion (BDMPS-Z, HT)
2. **Recent developments:**
   - Non-eikonal corrections for medium-induced emission within the path integral formalism [1]
   - Relaxing soft-gluon emission approximation for GLV(DGLV) formalism [2, 3]
   - Beyond collinear expansion and soft-gluon emission limit in DIS framework [4, 5, 6]
3. **The main purpose of our work [5]:**
   - Extend our previous study [4] and study the medium-induced gluon emission from a massive quark jet within the framework of DIS.
   - Work beyond the collinear rescattering expansion and soft-gluon emission approximation, and derive a closed formula for the medium-induced gluon emission spectrum from a heavy quark jet interacting with medium constituents via both transverse and longitudinal scattering.
   - Without performing the collinear expansion, our medium-induced gluon emission spectrum is controlled by the full distribution of the differential elastic scattering rates between the propagating partons and medium constituents.

II. Framework

1. Deep inelastic scattering (DIS)

In the framework of deep inelastic scattering (DIS), a lepton and a nucleon in the medium exchange the W boson, then get a heavy quark and a neutrino,



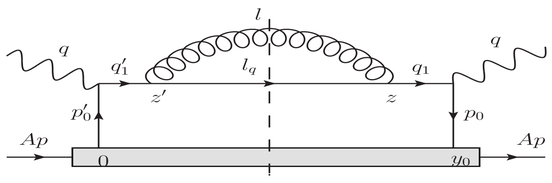
Differential cross section for DIS process

$$E_{L_2}\frac{d\sigma_{DIS}}{d^3\mathbf{L}_2}=\frac{G_F^2}{(4\pi)^3 s}\left\{\frac{1}{2}\mathrm{Tr}[\not{L}_1(1+\gamma^5)\gamma_\mu \not{L}_2\gamma_\nu(1-\gamma^5)]\right\}W^{\mu\nu}, \tag{1}$$

where $W^{\mu\nu}$ is the semi-inclusive hadronic tensor,

$$W^{\mu\nu}=\frac{1}{2}\sum_X(2\pi)^4\delta^4(q+Ap-P_X-l_q)\langle A|J_\mu(0)|X\rangle\langle X|J_\nu{}^\dagger(0)|A\rangle.$$

2. Vacuum radiation



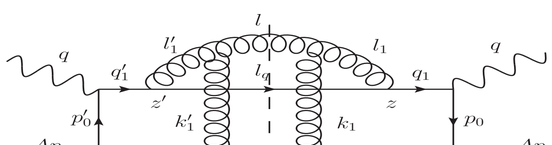
The gluon emission spectrum from heavy quark in vacuum:

$$\frac{dN_g^{vac}}{dl_\perp^2 dy}=C_F\frac{\alpha_s}{2\pi}P(y)\frac{\left(l_\perp^2+\frac{y^4}{1+(1-y)^2}M^2\right)}{(l_\perp^2+y^2M^2)^2}, \tag{2}$$

here, $P(y)=[1+(1-y)^2]/y$ is the leading order light quark to gluon (photon) splitting function.

3. Medium-induced radiation

There are 21 diagrams contributing to the gluon emission process from a heavy quark jet via single rescattering in the DIS framework [4, 5]. Now we present one diagram, and the other 20 diagrams are completely analogous and have been listed in Ref [5].



The medium-induced single gluon emission spectrum for the above diagram,

$$\frac{dN_{g,(1)}^{med}}{dyd^2\mathbf{l}_\perp}=\frac{\alpha_s}{2\pi^2}P(y)\int dZ_1^-\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\mathcal{D}(k_1^-,\mathbf{k}_{1\perp})\left\{2-2\cos\left(\frac{y(1-y)}{(y-\lambda_1^-)(1+\lambda_1^--y)}\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2}{l_\perp^2+y^2M^2}\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right\}$$
$$\times\ C_A\left[\frac{1+(1+\lambda_1^--y)^2}{1+(1-y)^2}\left(\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-}\right)^2\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+\frac{(y-\lambda_1^-)^4M^2}{1+(1+\lambda_1^--y)^2}}{\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2\right]^2}\right]^2. \tag{3}$$

Here, $\tilde{\tau}_{\rm form}^-$ is the formation time for gluon radiation from a heavy quark jet, and $\lambda_1^-=k_1^-/q^-$ is the forward momentum fraction.

In the above formula, we have defined the distribution function $\mathcal{D}(k_1^-,\mathbf{k}_{1\perp})$,

$$\mathcal{D}(k_1^-,\mathbf{k}_{1\perp})=\int d\delta z_1^-\int d^3\delta\mathbf{z}_1 e^{-i\mathbf{k}_1\cdot\delta\mathbf{z}_1}\left(g^2\frac{C_FC_2(R)}{N_c^2-1}\right)\langle A|A^+(\delta z_1^-,\delta\mathbf{z}_1)A^+(0)|A\rangle, \tag{4}$$

which is, up to a constant factor, the full differential elastic scattering rate [5],

$$\mathcal{D}(k_1^-,\mathbf{k}_{1\perp})=(2\pi)^3\frac{dP_{\rm el}}{dk_1^-d^2\mathbf{k}_{1\perp}dZ_1^-}. \tag{5}$$

4. The final formula

After summing up the contributions from all 21 diagrams, the closed formula for the medium-induced single gluon emission spectrum can been written as ,

$$\frac{dN_g^{med}}{dyd^2\mathbf{l}_\perp}=\frac{\alpha_s}{2\pi^2}P(y)\int dZ_1^-\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\mathcal{D}(k_1^-,\mathbf{k}_{1\perp})\left\{\left[2-2\cos\left(\frac{y(1-y)}{(y-\lambda_1^-)(1+\lambda_1^--y)}\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2}{l_\perp^2+y^2M^2}\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right]\right.$$
$$\times C_A\left[\frac{1+(1+\lambda_1^--y)^2}{1+(1-y)^2}\left(\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-}\right)^2\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+\frac{(y-\lambda_1^-)^4M^2}{1+(1+\lambda_1^--y)^2}}{\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2\right]^2}\right.$$
$$-\frac{1+(1+\lambda_1^--y)(1-y)}{2[1+(1-y)^2]}\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-}\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-\mathbf{k}_{1\perp})+\frac{y^2(y-\lambda_1^-)^2}{1+(1+\lambda_1^--y)(1-y)}M^2}{[l_\perp^2+y^2M^2]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2\right]}$$
$$\left.-\frac{1+(1+\lambda_1^--y)\left(1-\frac{y}{1+\lambda_1^-}\right)}{2[1+(1-y)^2]}\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-}\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})\cdot\left(\mathbf{l}_\perp-\frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)+\frac{\left(\frac{y}{1+\lambda_1^-}\right)^2(y-\lambda_1^-)^2}{1+(1+\lambda_1^--y)\left(1-\frac{y}{1+\lambda_1^-}\right)}M^2}{\left[\left(\mathbf{l}_\perp-\frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)^2+\left(\frac{y}{1+\lambda_1^-}\right)^2M^2\right]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2\right]}\right]$$
$$+\left[2-2\cos\left(\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right]\left[C_F\frac{l_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]^2}-\frac{C_A}{2}\frac{\left(y-\frac{\lambda_1^-}{2}\right)^2}{y(y-\lambda_1^-)}\frac{l_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]^2}\right.$$
$$\left.+\left(\frac{C_A}{2}-C_F\right)\frac{1+(1-y)\left(1-\frac{y}{1+\lambda_1^-}\right)}{1+(1-y)^2}\frac{\mathbf{l}_\perp\cdot\left(\mathbf{l}_\perp-\frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)+\frac{y^2\left(\frac{y}{1+\lambda_1^-}\right)^2M^2}{1+(1-y)\left(1-\frac{y}{1+\lambda_1^-}\right)}}{[l_\perp^2+y^2M^2]\left[\left(\mathbf{l}_\perp-\frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)^2+\left(\frac{y}{1+\lambda_1^-}\right)^2M^2\right]}\right]$$
$$\left.+C_F\left[\frac{1+\left(1-\frac{y}{1+\lambda_1^-}\right)^2}{1+(1-y)^2}\frac{\left(\mathbf{l}_\perp-\frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)^2+\frac{\left(\frac{y}{1+\lambda_1^-}\right)^4}{1+\left(1-\frac{y}{1+\lambda_1^-}\right)^2}M^2}{\left[\left(\mathbf{l}_\perp-\frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)^2+\left(\frac{y}{1+\lambda_1^-}\right)^2M^2\right]^2}-\frac{l_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]^2}\right]\right\}. \tag{6}$$

Since we work ***beyond the collinear rescattering expansion***, the medium-induced emission spectrum is ***controlled by the full distribution of the differential elastic scattering rate*** between the propagating hard partons and medium constituents. In addition, the contributions from ***both transverse and longitudinal momentum*** exchanges with the medium constituents are included.

5. Static scattering center approximation

If we take the static screened potential for the exchanged gluon field,

$$A^\mu(\mathbf{p})=g^{\mu-}(2\pi)\delta(p^-)\frac{-g}{\mathbf{p}_\perp^2+\mu^2}. \tag{7}$$

Then by performing the Fourier transformation, the gluon field correlation may be obtained as:

$$\langle A|A^\mu(\delta\mathbf{z}_1)A^\nu(0)|A\rangle\ =\ \delta_+^\mu\delta_+^\nu\rho^-\delta(\delta z_1^-)\int\frac{d^2\mathbf{p}_\perp}{(2\pi)^2}e^{i\mathbf{p}_\perp\cdot\delta\mathbf{z}_{1\perp}}\frac{g^2}{(\mathbf{p}_\perp^2+\mu^2)^2}, \tag{8}$$

where ρ^- is the light-cone density of the medium constituents (scattering centers).

The distribution function $\mathcal{D}(k_1^-,\mathbf{k}_{1\perp})$ with static scattering centers is then obtained as,

$$\mathcal{D}(k_1^-,\mathbf{k}_{1\perp})=(2\pi)\delta(k_1^-)(2\pi)^2\rho^-\frac{C_FC_2(R)}{N_c^2-1}\frac{4\alpha_s^2}{(\mathbf{k}_{1\perp}^2+\mu^2)^2}=(2\pi)\delta(k_1^-)(2\pi)^2\rho^-\frac{d\sigma_{\rm el}}{d^2\mathbf{k}_{1\perp}}. \tag{9}$$

The medium-induced single gluon emission spectrum then becomes:

$$\frac{dN_g^{\rm med}}{dyd^2\mathbf{l}_\perp}=\frac{\alpha_s}{2\pi^2}P(y)\int dZ_1^-\int d^2\mathbf{k}_{1\perp}\frac{dP_{\rm el}}{d^2\mathbf{k}_{1\perp}dZ_1^-}\times\left\{C_F\left[\frac{(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]^2}-\frac{l_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]^2}\right]\right.$$
$$+C_A\left[2-2\cos\left(\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2}{l_\perp^2+y^2M^2}\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right]\times\left[\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]^2}\right.$$
$$-\frac{1}{2}\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]}-\frac{1}{2}\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})\cdot(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]}\Big]$$
$$\left.+\left(\frac{C_A}{2}-C_F\right)\left[2-2\cos\left(\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right]\left[\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]}-\frac{l_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]^2}\right]\right\}. \tag{10}$$

6. Soft-gluon emission limit

Further taking the soft-gluon emission limit ($y=l^-/q^-\ll 1$)

$$\frac{dN_g^{\rm med}}{dyd^2\mathbf{l}_\perp}=\frac{\alpha_s}{2\pi^2}P(y)\int dZ_1^-\int d^2\mathbf{k}_{1\perp}\frac{dP_{\rm el}}{d^2\mathbf{k}_{1\perp}dZ_1^-}\times C_A\left[2-2\cos\left(\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2}{l_\perp^2+y^2M^2}\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right] \tag{11}$$
$$\times\left[\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2}{\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]^2}-\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-\mathbf{k}_{1\perp})}{[l_\perp^2+y^2M^2]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]}\right].$$

Our result reduces to the first order in opacity static DGLV formula (with zero thermal mass for radiated gluon) [7].

III.Summary

- We derive a closed formula for medium-induced single gluon emission from a heavy quark jet interact with a dense nuclear medium ***via transverse and longitudinal scatterings***.
- Our study is a generalization of both HT and DGLV one-rescattering-one-emission formula,
  beyond collinear rescattering expansion used in HT,
  beyond soft gluon emission limit used in static DGLV.
- Outlook: Phenomenological studies of parton energy loss and jet quenching in relativistic heavy-ion collisions.

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