Hydrodynamic Attractor in Viscous Hubble Flow

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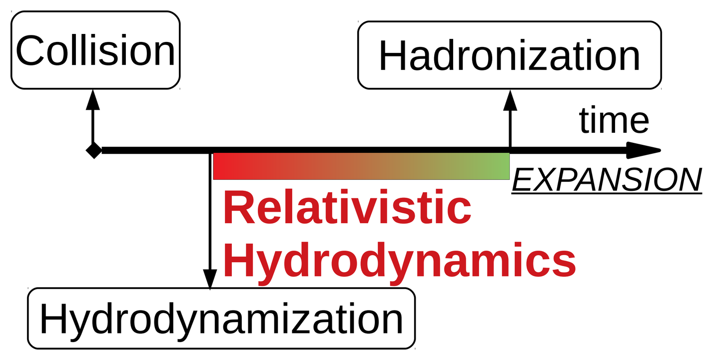
1 Introduction

1.1 Resummed Hydrodynamics: an appropriate generalization to off-equilibrium cases

Relativistic hydrodynamics is successful in simulating the evolution of near-equilibrium many-body systems, e.g. nuclear matter produced by HIC.



In viscous Bjorken flow[3][4] and Gubser flow[1][2], resummed hydrodynamics shows that

- the **trans-series nature** of the attractor solution invalidates gradient expansion.
- systems finally relax to the **numerically determined hydrodynamic attractor**.
- the hydro attractor can be recovered with **resurgence method**.

1.2 Research Objective

	dimension	curvature
Bjorken flow	1+1 dimensional	flat
Gubser flow	2+1 dimensional	de-Sitter
Hubble flow	3+1 dimensional	de-Sitter

Is resurgence method applicable in Hubble flow ?

To apply resurgence method to viscous Hubble flow for the first time.

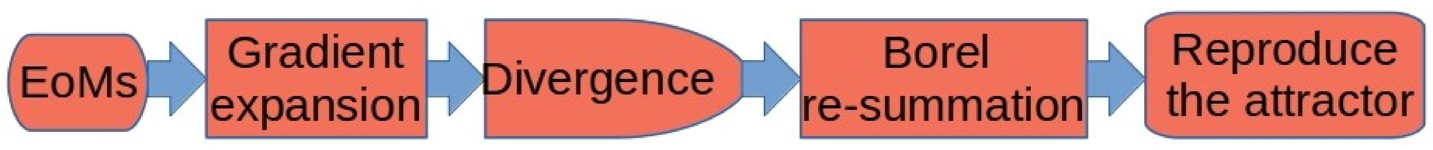


Figure 1: Steps for resummed hydro.

2 Theory

2.1 Equations of Motion

- Robertson-Walker metric $g_{\mu\nu} = \mathrm{diag}(1, -a^2, -a^2, -a^2)$
- Energy-momentum conservation & Israel-Stewart eq. for bulk viscosity
- Expansion factor $a(t)$ **set as** $a(t) = t^\alpha$

$$\frac{3\dot{a}(P+\Pi+E)}{a} + \dot{E} = 0, \quad (1)$$

$$\Pi + \frac{3\dot{a}}{a}\zeta + \tau_\Pi \dot{\Pi} = 0. \quad (2)$$

2.2 Remarks on the Model

1. **FLAT** Hubble space (vanishing curvature on the spatial manifold)
2. **WITHOUT** shear viscosity η due to spatial isotropy
3. **CONSTANT** viscosity coefficient ζ

2.3 Gradient Expansion Leads to FACTORIAL DIVERGENCE

Treat the term $\tau_\Pi \Pi$ as a correction.

$$\dot{E} + 3\tfrac{\alpha}{t}(c_s^2+1)E = -3\tfrac{\alpha}{t}\Pi$$
$$\epsilon\tau_\Pi\dot{\Pi} + \Pi = -3\zeta\tfrac{\alpha}{t}$$

Expand $\Pi = \sum_0^\infty \Pi_k \epsilon^k$. Obtain the perturbative solutions.

$$\Pi_0 = -\tfrac{3\alpha}{t}\zeta$$
$$\Pi_k = -3\alpha\zeta\tfrac{\tau_\Pi^k k!}{t^{k+1}}(\sim \Gamma(k+1)),$$
$$k = 1, 2, 3, \ldots$$

$$E_k = -\tfrac{9\alpha^2\zeta\tau_\Pi^k k!}{k+1-3\alpha(c_s^2+1)} t^{-k-1} (\sim \Gamma(k))$$

Borel transform

$$\mathcal{B}[\Pi](w) = \sum_{k=0}^{\infty} -3\alpha\zeta\tfrac{\tau_\Pi^k}{t^{k+1}} w^k$$
$$\mathcal{B}[E](w) = -\sum_{k=0}^{\infty} \tfrac{9\alpha^2\zeta\tau_\Pi^k w^k}{(k+1-3\alpha(c_s^2+1))t^{k+1}}$$

REMARK: Coefficients in both perturbative series are **factorially divergent but Borel summable**.

2.4 Borel Transform Recovers HYDRO-ATTRACTOR

Resummed viscosity Π (Ei -exponential integral function)

$$\mathcal{B}[\Pi](w) = \frac{3\alpha\zeta}{w\tau_\Pi - t}. \quad (3)$$

$$\tilde{\Pi} = -\frac{3\zeta\alpha}{\tau_\Pi} e^{-\frac{t}{\tau_\Pi}} \mathrm{Ei}\left(\frac{t}{\tau_\Pi}\right), \quad (4)$$

Resummed energy density

$$\mathcal{B}[E](w) = \frac{9\alpha^2\zeta}{(3\alpha(c_s^2+1)-1)t}\, {}_2F_1\left(\begin{matrix}1-3\alpha(c_s^2+1), 1\\ 2-3\alpha(c_s^2+1)\end{matrix}; \frac{w\tau_\Pi}{t}\right) \quad (5)$$

$$\tilde{E} = \int_0^{+\infty e^{i0+}} \mathcal{B}[E](w) e^{-w} dw \quad (6)$$

*The inverse Borel transformation(Eq.6) is numerically calculated in this project.

- $\mathcal{B}[\Pi]$ has a **POLE** at $\frac{t}{\tau_\Pi}$ $\Longrightarrow$ Non-hydro contribution $\sim e^{-\frac{t}{\tau_\Pi}}$
- $\mathcal{B}[E]$ has a **BRANCH CUT** starting from $\frac{t}{\tau_\Pi}$

3 Resummation Result

3.1 Resummed Π and E

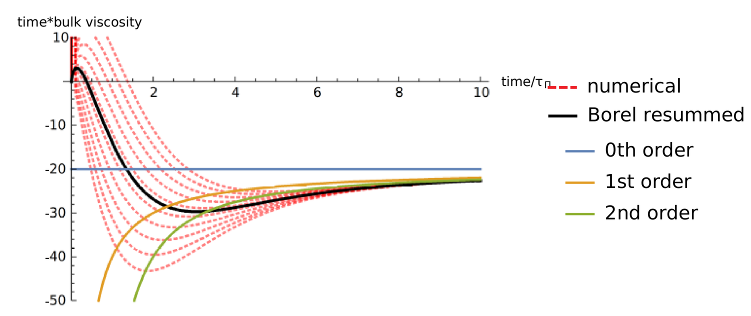


Figure 2: Viscosity

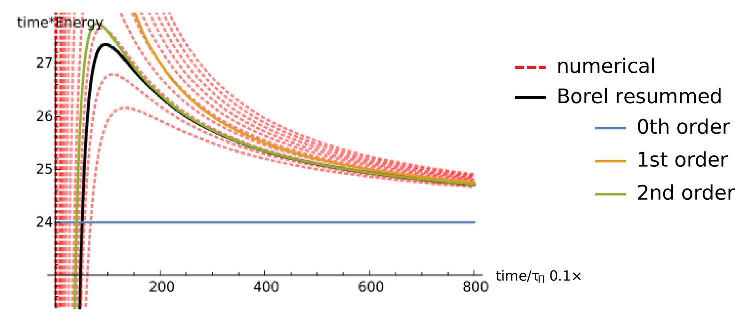


Figure 3: Energy density

The comparison between the perturbative solution and the resurgence result with constants $\alpha = 2/3, \zeta = 10, \tau_\Pi = 1, c_s^2 = 1/3$

- Hydrodynamic attractor
  - Numerical solutions with initial conditions set as various values converge **RAPIDLY** to an attractor solution
  - If α is larger, which indicates a more violent expansion, the convergence to the attractor will be **FASTER**.
- Resummed hydrodynamics
  - Resummation reproduces the hydro attractor and works better than perturbative method.

3.2 Resummed Viscosity: ζ_B in $\Pi = -\frac{3\zeta_B\alpha}{t}$

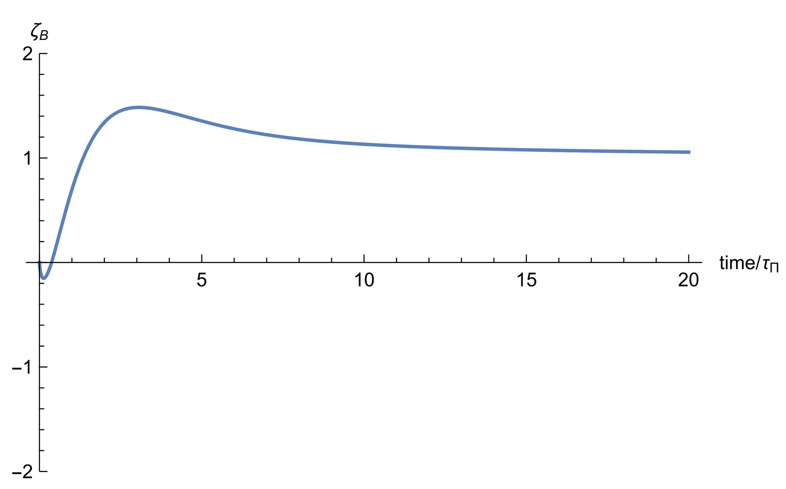


Figure 4: Resummed viscosity as a function of time

$$\zeta_B = \frac{t}{\tau_\Pi} e^{-\frac{t}{\tau_\Pi}} Ei\left(\frac{t}{\tau_\Pi}\right)$$

- resummed viscosity is defined as ζ_B in $\Pi = -\frac{3\zeta_B\alpha}{t}$, in a similar form as in first-order hydro.
- late-time (t large, weak gradients, $\frac{\tau_\Pi}{t}$ small) $\to$ ζ_B equals ζ
- strong gradients ($\frac{\tau_\Pi}{t}$ large) $\to$ ζ_B negative value, off-equilibrium behaviour

4 Summary: resummed Hubble flow for the first time

- The numerical solutions converge quickly to the attractor solution, as those in Bjorken/Gubser flow.
- The trans-series nature of hydro attractorsis related to the divergence of the perturbation series, as is found in Bjorken/Gubser flow.
- The effective viscosity is negative in strong gradient region, but negative values do not appear in Bjorken flow[4].
- The next step will be to find out the dynamic evolution of viscous Hubble flow with kinetic theory.

References

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