Phenomenological formulation of relativistic spin hydrodynamics

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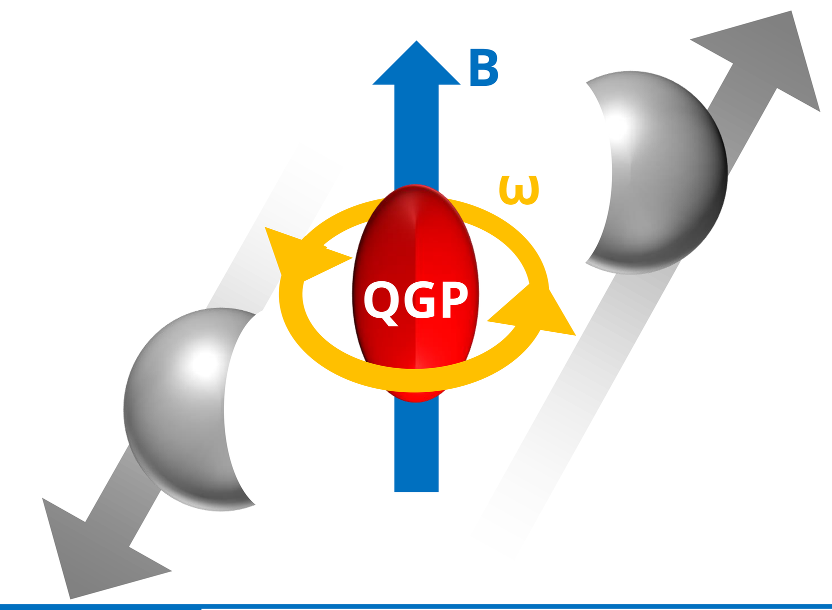
Ref: Hattori, Hongo, Huang, Matsuo, HT, Phys. Lett. B795, 100 (2019) [arXiv:1901.06615]

1. Introduction

1-1. Spin-polarization in HIC

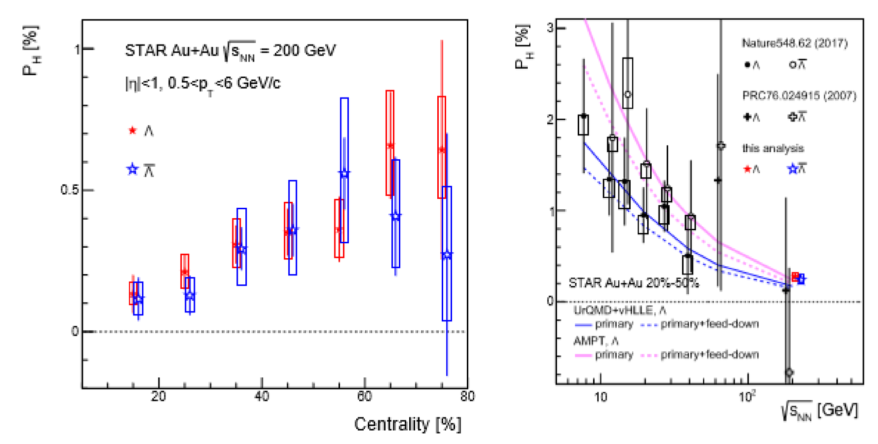
✓ **huge vorticity ω magnetic field B in non-central collisions**



✓ **spin-polarization by ω, B**

e.g. Barnett effect, Zeeman splitting

✓ **experimentally confirmed already !**



[STAR, PRC 98, 014910 (2018)]

1-2. Theoretical status of spin hydro

Hydrodynamics is a powerful framework to describe QGP. However, it is far from complete for spin-polarized QGP

✓ **"hydro simulations" exist, but...**
- ordinary hydro (i.e., hydro w/o spin) is solved
- thermal vorticity $\tilde{\omega}^{\mu\nu} \equiv 2\partial^{[\mu}(\beta u^{\nu]})$ is converted into spin via Cooper-Frye

✓ **relativistic spin hydro is still under construction**
- established for non-relativistic case, but **not for relativistic case** ☹
  e.g. micropolar fluid: Eringen (1998); Lukaszewicz (1999)
- some trials exist, but **only at the "ideal" level (no dissipative corr.)** ☹
- **controversy exists** ☹ on the conservation of spin
  ⇒ some relativistic people claim spin is conserved, while it is established in non-relativistic spin hydro that **spin must NOT be conserved**

Aim
(1) Formulate relativistic spin hydro including 1st order dissipative corrections for the first time
(2) Clarify that spin must be a dissipative quantity

2. Phenomenological formulation based on entropy-current analysis

2-1. hydro without spin (review)

step 1 **Write down the conservation law**

Conservation of energy-momentum $0 = \partial_\mu T^{\mu\nu}$

✓ 4 equations

step 2 **Express $T^{\mu\nu}$ i.t.o hydro variables (constitutive relation)**

[2-1] Define hydro variables

Slow modes that (you expect to) survive in IR limit: $\{\beta, u_\mu\}$ w/ $u^2 = -1$

✓ 1 + 3 = 4 DoGs = # of equations

[2-2] Write down all the possible tensor structures of $T^{\mu\nu}$

$$T^{\mu\nu} = f_1(\beta)g^{\mu\nu} + f_2(\beta)u^\mu u^\nu + f_3(\beta)\epsilon^{\mu\nu\rho\sigma}\partial_\rho u_\sigma + f_4(\beta)\partial^\mu u^\nu + f_5(\beta)\partial^\nu u^\mu + f_6(\beta)g^{\mu\nu}\partial^\rho u_\rho + f_7(\beta)u^\mu u^\nu \partial^\rho u_\rho + f_8(\beta)u^\mu \partial_\mu u^\nu + \cdots + O(\partial^2)$$

[2-3] Simplify the tensor structures with some assumptions

(1) symmetry
(2) power counting (gradient expansion)
(3) thermodynamics (entropy-current analysis)

Define energy e & pressure p by (matching condition)

$$T^{\mu\nu} \equiv T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + O(\partial^2) \equiv eu^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu) + T^{\mu\nu}_{(1)} + O(\partial^2)$$

1st law of thermodynamics: $\mathrm{d}s = \beta \mathrm{d}e, s = \beta(e+p)$
2nd law of thermodynamics: $0 \le \partial_\mu s^\mu = -T^{\mu\nu}_{(1)}\partial_\mu(\beta u_\nu) + O(\partial^3)$

2nd law is satisfied if **RHS is a semi-positive bilinear**, i.e.,

$$-T^{\mu\nu}_{(1)}\partial_\mu(\beta u_\nu) = \sum_{X_i \in T_{(1)}} \lambda_i X_i^{\mu\nu} X_{i\,\nu\mu} \ge 0 \text{ with } \lambda_i \ge 0$$

✓ give **strong** constraints on $T^{\mu\nu}$

ex) heat current: $2h^{(\mu}u^{\nu)} \equiv h^\mu u^\nu + h^\nu u^\mu \in T^{\mu\nu}_{(1)}$ $(u_\mu h^\mu = 0)$
$\Rightarrow T^{\mu\nu}_{(1)}\partial_\mu(\beta u_\nu) = -\beta h^\mu(\beta\partial_{\perp\mu}\beta^{-1} + u^\nu\partial_\nu u^\mu) \ge 0$
$\Rightarrow h^\mu = -\kappa(\beta\partial_{\perp\mu}\beta^{-1} + u^\nu\partial_\nu u^\mu)$ with $\kappa \ge 0$

$$\boldsymbol{T^{\mu\nu} = eu^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu) -2\kappa\left(Du^{(\mu} + \beta\partial_\perp^{(\mu}\beta^{-1}\right)u^{\nu)} - 2\eta\partial_\perp^{<\mu}u^{\nu>} - \zeta(\partial_\mu u^\mu)\Delta^{\mu\nu}}$$

heat current — shear viscosity — bulk viscosity

Goal **Hydro eq. = conservation law + constitutive relation**

2-2. hydro with spin

step 1 **Write down the conservation laws**

Conservation of EM $0 = \partial_\mu T^{\mu\nu}$ and total angular mom. $0 = \partial_\mu M^{\mu,\alpha\beta}$

✓ 4 + 6 equations

✓ Define spin tensor by $\Sigma^{\mu,\alpha\beta} \equiv M^{\mu,\alpha\beta} - J^{\mu,\alpha\beta} \equiv M^{\mu,\alpha\beta} - (x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha})$ [$J^{\mu,\alpha\beta}$: (canonical) orbital angular momentum]
$\Rightarrow 0 = \partial_u M^{\mu,\alpha\beta} \Leftrightarrow \partial_u \Sigma^{\mu,\alpha\beta} = T^{\alpha\beta} - T^{\beta\alpha} \equiv 2T^{\alpha\beta}_{(\mathrm{a})}$
$\Rightarrow$ spin is not conserved if $T^{\mu\nu}_{(\mathrm{a})} \ne 0$

✓ There is no reason $T^{\mu\nu}$ must be symmetric
$\Rightarrow$ spin must not be a hydro mode in a strict sense
$\Rightarrow$ nevertheless, it behaves *like* a hydro mode if $T^{\mu\nu}_{(\mathrm{a})} \ll 1$, for which **inclusion of dissipation is crucially important**

step 2 **Express $T^{\mu\nu}$, $\Sigma^{\mu,\alpha\beta}$ i.t.o hydro variables (constitutive relations)**

[2-1] Define hydro variables

Assume spin is slow and introduce spin chemical potential $\omega_{\mu\nu}$ as a new hydro variable: $\{\beta, u_\mu, \boldsymbol{\omega_{\mu\nu}}\}$ w/ $\omega_{\mu\nu} = -\omega_{\nu\mu}$

✓ 4 + 6 = 10 DoGs = # of equations

✓ in general, $\omega_{\mu\nu} \ne 2\partial_{[\mu}(\beta u_{\nu]}) = (\text{thermal vorticity})$

[2-2] Write down all the possible tensor structures of $T^{\mu\nu}$, $\Sigma^{\mu,\alpha\beta}$

$T^{\mu\nu} = (\text{too many terms})$, $\Sigma^{\mu,\alpha\beta} = (\text{too many terms})$

[2-3] Simplify the tensor structures with some assumptions

(1) symmetry
(2) power counting (gradient expansion)
(3) thermodynamics (entropy-current analysis)

Define spin density $\sigma^{\mu\nu}$ by $\Sigma^{\mu,\alpha\beta} \equiv \Sigma^{\mu,\alpha\beta}_{(0)} + O(\partial^1) \equiv u^\mu\sigma^{\alpha\beta} + O(\partial^1)$

1st law w/ spin: $\mathrm{d}s = \beta(\mathrm{d}e - \omega_{\mu\nu}\mathrm{d}\sigma^{\mu\nu}), s = \beta(e + p - \omega_{\mu\nu}\sigma^{\mu\nu})$
2nd law w/ spin: $0 \le \partial_\mu s^\mu = -T^{\mu\nu}_{(1\mathrm{s})}\partial_\mu(\beta u_\nu) - T^{\mu\nu}_{(1\mathrm{a})}\{\partial_{[\mu}(\beta u_{\nu]}) - 2\beta\omega_{\mu\nu}\} + O(\partial^3)$

✓ RHS is a semi-positive bilinear ⇒ constraints on $T^{\mu\nu}$, $\Sigma^{\mu,\alpha\beta}$
✓ in global equilibrium, $\omega_{\mu\nu} = 2\partial_{[\mu}(\beta u_{\nu]}) = (\text{thermal vorticity})$ holds

$$\boldsymbol{\Sigma^{\mu,\alpha\beta} = u^\mu\sigma^{\alpha\beta}}$$

$$\boldsymbol{T^{\mu\nu} = eu^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu) -2\kappa\left(Du^{(\mu} + \beta\partial_\perp^{(\mu}\beta^{-1}\right)u^{\nu)} - 2\eta\partial_\perp^{<\mu}u^{\nu>} - \zeta(\partial_\mu u^\mu)\Delta^{\mu\nu} -2\lambda\left(-Du^{[\mu} + \beta\partial_\perp^{[\mu}\beta^{-1} + 4u_\rho\omega^{\rho[\mu}\right)u^{\nu]} - 2\gamma\left(\partial_\perp^{[\mu}u^{\nu]} - 2\Delta^\mu_\rho\Delta^\nu_\lambda\omega^{\rho\lambda}\right)}$$

NEW !! boost heat current — rotational (spinning) viscosity

✓ a generalization of the non-relativistic micropolar fluid

Goal **Hydro eqs. = conservation laws + constitutive relations**

3. Linear mode analysis

2-1. setup

Consider perturbations on top of a global static thermal equil. config.

$$\beta = \beta_0 + \delta\beta$$
$$u^\mu = (1, \mathbf{0}) + \delta u^\mu$$
$$\omega^{\mu\nu} = 0 + \delta\omega^{\mu\nu}$$

2-2. Linearized hydro eq.

Expand hydro eqs. i.t.o. the pert. δ

$$i\partial_t \begin{pmatrix} \delta\beta \\ \delta u^\mu \\ \delta\omega^{\mu\nu} \end{pmatrix} = M \begin{pmatrix} \delta\beta \\ \delta u^\mu \\ \delta\omega^{\mu\nu} \end{pmatrix} + O(\delta^2)$$

✓ Eigenvalues ω of M determine how the perturbations evolve in time:
$\mathrm{Im}\,\omega = 0$: propagate ($\delta \sim \text{const}$)
$\mathrm{Im}\,\omega < 0$: diffusive ($\delta \to 0$)

2-3. Eigenvalues of M

Usual 4 gapless modes + 6 diffusive gapped modes appear

2 sound modes: $\omega = \pm c_s k + O(k^2)$
2 shear modes: $\omega = -i\eta k^2/(e+p) + O(k^2)$

usual 4 gapless modes
✓ completely the same as hydro w/o spin

3 "spin" modes: $\omega = -2i\tau_{\mathrm{b}}^{-1} + O(k^2)$
3 "boost" modes: $\omega = -2i\tau_{\mathrm{s}}^{-1} + O(k^2)$

where $\tau_{\mathrm{s}} \equiv \frac{\partial\sigma^{ij}/\partial\omega^{ij}}{4\gamma}, \tau_{\mathrm{b}} \equiv \frac{\partial\sigma^{i0}/\partial\omega^{i0}}{4\lambda}$

6 gapped modes associated w/ spin DoG
✓ $\mathrm{Im}\,\omega < 0 \Rightarrow$ spin must be **diffusive**
✓ time-scale is controlled by the new viscous constants γ, λ

4. Summary

✓ **Relativistic spin hydrodynamics with 1st order dissipative corrections is formulated for the first time based on the phenomenological entropy-current analysis**

✓ **Spin must be dissipative because of the mutual conversion b/w orbital angular momentum and spin**

✓ **Linear mode analysis of the spin hydro equation found 6 new diffusive modes, which means that spin is dissipative, and time-scale of the dissipation is controlled by the new viscous constants γ, λ**

✓ **Outlook:**
extension to 2nd order, Kubo formula, MHD, application to cond-mat, numerical simulations