Chiral Kinetic Theory from High Density Effective Theory and Reparametrization Invariance

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Abstract

We redevelop the chiral kinetic theory (CKT) by exploiting the effective field theory method and find some disagreements at higher order of $(1/\mu)$ from the earlier results. However, these disagreements are the same which have been pointed out by the off-shell effective theory formalism after identifying a cut-off. We address the discrepancies and by using the reparametrization invariance, we show that these disagreements are in fact expected due the choices of different degrees of freedom in effective theory and field theory. Further, we show that both methods yield the similar dynamics for chiral fermions.

Introduction

- Chiral kinetic theory (CKT) describes the anomalous transport of fermions.
- In CKT formalism, transport equation as well as particle number density is modified by considering the Berry phase and Berry curvature. Due to these modifications, a disssipationless current is generated in the presence of magnetic filed which is known as Chiral Magnetic Effect.
- High density effective (HDET) theory is one of the approaches to derive CKT and it is valid in the vicinity of Fermi surface. HDET is constructed by integrating out heavy modes from the theory which generate the non-local effective Lagrangian.

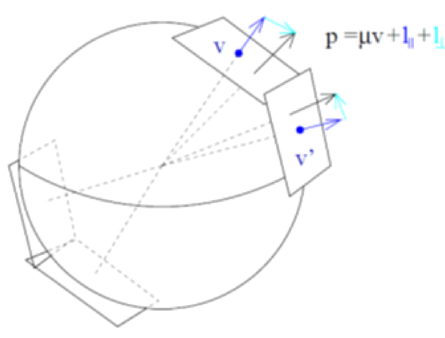
High Density Effective Theory

Lagrangian for right handed chiral fermions

$$\mathcal{L}_0 = \bar\psi(i\gamma^\mu D_\mu)\psi + \mu\bar\psi\gamma^0\psi,$$

where $E_\pm = \mu \pm |\mathbf{p}|$ and $E_+ \sim 0 \implies$ light modes, $E_- \sim -2\mu \implies$ heavy modes

Decomposing the momenta: $p^\mu = \mu v^\mu + l^\mu$ and performing Fourier transformation: $\psi(x) = \sum_{\mathbf{v}} e^{i\mu\mathbf{v}\cdot\mathbf{x}}\left[\psi_{+v}(x) + \psi_{-v}(x)\right]$, results into



$$\mathcal{L}_1 = \psi_{+v}^\dagger iv\cdot D\psi_{+v} + \psi_{-v}^\dagger(2\mu + i\bar v\cdot D)\psi_{-v}$$
$$+\psi_{+v}^\dagger i\not{D}_\perp\psi_{-v} + \psi_{-v}^\dagger i\not{D}_\perp\psi_{+v}$$

where $\not{D}_\perp = \sigma_\perp^\mu D_\mu$ and $\sigma_\perp^\mu = (0, \boldsymbol{\sigma} - \mathbf{v}(\mathbf{v}\cdot\boldsymbol{\sigma}))$.

Integrating out heavy mode ψ_{-v} yields

$$\mathcal{L}_{eff} = \psi_{+v}^\dagger \sum_n D^{(n)}\psi_{+v} = \psi_{+v}^\dagger\left[iv\cdot D + \frac{\not{D}_\perp^2}{2\mu} + \frac{\not{D}_\perp(-i\bar v\cdot D)/D_\perp}{4\mu^2}\right]\psi_{+v}$$

Equations of Motion

The two-point function $G_v(x,y)$ satisfy

$$\mathcal{D}_x G_v(x,y) = 0, \qquad G_v(x,y)\mathcal{D}_y^\dagger = 0,$$
$$P_- G_v(x,y) = 0, \qquad G_v(x,y)P_- = 0 \qquad \text{[Projection Condition]}$$

Now, from the gauge invariant Wigner function:

$$\tilde G_v(X,l) = \int_s e^{il\cdot s}\, G_v(X+s/2, X-s/2)U(X-s/2, X+s/2).$$

with Wilson line $U(y,x) = P\,exp\left[-i\int_x^y dz^\mu A_\mu(z)\right]$, we can construct

$$I_\pm^{(n)} = \int_s e^{il\cdot s}\left(\mathcal{D}_x^{(n)}\,G_v(x,y) \pm G_v(x,y)\,\mathcal{D}_y^{(n)\dagger}\right) \qquad \mathcal{D}^{(n)} = \mathcal{D}^{(0)} + \mathcal{D}_x^{(1)} + \mathcal{D}_x^{(2)}$$

which result into:

$$I_+^{(0)} = 2v\cdot\bar l\,\tilde G_v, \qquad I_-^{(0)} = iv^\mu\Delta_\mu\tilde G_v,$$
$$I_+^{(1)} = \frac{1}{\mu}\left[-\bar l_\perp^2 + \mathbf{B}\cdot\mathbf{v}\right]\tilde G_v, \qquad I_-^{(1)} = \frac{i}{\mu}\bar l_\perp^\mu\Delta_\mu\tilde G_v,$$
$$I_+^{(2)} = \frac{1}{4\mu^2}\left[4\bar l_\parallel\bar l_\perp^2 - 4\bar l_\parallel(\mathbf{B}\cdot\mathbf{v}) + 2\mathbf{B}\cdot\bar{\mathbf{l}}_\perp + 2(\mathbf{E}\times\bar{\mathbf{l}})\cdot\mathbf{v}\right]\tilde G_v$$
$$I_-^{(2)} = -\frac{i}{4\mu^2}\left[\left(4\bar l_\parallel\bar l^\mu - \bar v^\mu(\bar l_\perp^2 - \mathbf{B}\cdot\mathbf{v})\right)\Delta_\mu - \left(\varepsilon^{ijk}v^k\bar v_\mu F^{i\mu}\right)\Delta_j\right]\tilde G_v,$$

$\Delta_\mu = \partial_\mu - F_{\mu\nu}\frac{\partial}{\partial l_\nu}$ and Wigner function from above equations

$$\tilde G_v = 2\pi\,P_+\,\delta\Big(l_0 - l_\parallel - \frac{1}{2\mu}[l_\perp^2 - \mathbf{B}\cdot\mathbf{v}] + \frac{1}{2\mu^2}[l_\parallel(l_\perp^2 - \mathbf{B}\cdot\mathbf{v})] + \frac{1}{4\mu^2}[\mathbf{B}\cdot\mathbf{l}_\perp + (\mathbf{E}\times\mathbf{l})\cdot\mathbf{v}]\Big)n_v(X,l)$$

$n_v(X,l)$: distribution function.

- PUZZLE: dispersion relation depends on **v** and not invariant under Reparametrization!

Reparametrization Invariance

Under reparametrization transformation (RT)

$$\delta\psi_v = i\mu\delta v\cdot x\psi_v - \frac{\delta\!\!\!/ v}{2}\Big(1 - \frac{1}{2\mu + i\bar v\cdot D}i\not{D}_\perp\Big)\psi_v,$$
$$\delta\psi_v^\dagger = -i\mu\delta v\cdot x\psi_v^\dagger - \psi_v^\dagger\Big(1 - i\not{D}_\perp\frac{1}{2\mu + i\bar v\cdot D}\Big)\frac{\delta\!\!\!/ v}{2}.$$

Non-local effective Lagrangian $\mathcal{L}_{eff}$ remains invariant.

$$\text{tr}\,\delta\tilde G_v(X,l) = \tfrac{1}{4\mu}\delta v_j\Delta_i v^k\varepsilon^{ijk}\text{tr}\tilde G_v(X,l) + \tfrac{1}{2\mu}\delta v_j l_i\Delta_{ij}\text{tr}\tilde G_v(X,l) \implies \text{antiparticle contribution}$$

Wigner function is not invariant under RT!

$$\delta I_\pm^{(n)} = \int_s e^{il\cdot s}(\mathcal{D}_x^{(n)}G_v(x,y) \pm G_v(x,y)\mathcal{D}_y^{\dagger(n)}) = 0, \qquad \text{Invariant under RT}$$

- Dispersion relation and CKE depends on **v** and not unique
- Expected: antiparticle contribution is **v** dependent!

Transport Equation

We use a natural scheme by making a choice $\mathbf{l} \parallel \mathbf{v}$,

$$I_+^{(0)} = 2v\cdot l\tilde G_v, \qquad I_-^{(0)} = iv^\mu\Delta_\mu\tilde G_v,$$
$$I_+^{(1)} = \frac{\mathbf{B}\cdot\mathbf{v}}{\mu}\tilde G_v, \qquad I_-^{(1)} = 0,$$
$$I_+^{(2)} = -\frac{\mathbf{B}\cdot\mathbf{v}l}{\mu^2}\tilde G_v, \qquad I_-^{(2)} = \frac{1}{4\mu^2}\big[-i\,\bar v^\mu\,\mathbf{B}\cdot\mathbf{v}\,\Delta_\mu + i\bar v^\nu\,\epsilon^{ijm}\,v^m\,F_{i\nu}\,\Delta_j\big]\tilde G_v.$$

Equations $I_+^{(n)}$

$$I_+^{(0)} + I_+^{(1)} + I_+^{(2)} = \Big[2(l_0 - l) + \frac{\mathbf{B}\cdot\mathbf{v}}{\mu} - \frac{\mathbf{B}\cdot\mathbf{v}\,l}{\mu^2}\Big]\tilde G_v.$$

gives dispersion relation: $\quad l_0 = l + \frac{\mathbf{B}\cdot\mathbf{v}}{\mu} - \frac{\mathbf{B}\cdot\mathbf{v}\,l}{\mu^2}$

$\Downarrow$

In terms of original momentum: $\quad p_0 = p - \frac{\mathbf{B}\cdot\hat{\mathbf{p}}}{2p}$

Chiral kinetic equation from $I_-^{(n)}$ terms is

$$\left[\Delta_0 + \hat{\mathbf{p}}^i\,\left(1 + \frac{\mathbf{B}\cdot\hat{\mathbf{p}}}{2p^2}\right)\,\Delta_i - \frac{\epsilon^{ijk}\,\hat{\mathbf{p}}^j\,\mathbf{E}^k + \mathbf{B}_\perp^i}{4p^2}\,\Delta_i\right]n_v(X,l) = 0$$

On the other hand, CKE from Field Theory Approach

$$\left[\Delta_0 + \hat{\mathbf{p}}^i\,\left(1 + \frac{\mathbf{B}\cdot\hat{\mathbf{p}}}{2p^2}\right)\,\Delta_i - \frac{\epsilon^{ijk}\,\hat{\mathbf{p}}^j\,\mathbf{E}^k}{2p^2}\,\Delta_i\right]n(X,l) = 0$$

Particle number and total current:

$$n = \frac{1}{(2\pi)^4}\int_l\Big(1 + \frac{1}{2\mu^2}[\mathbf{B}\cdot\mathbf{v}]\Big)\,\text{tr}\tilde G_v(X,l),$$
$$j^i = \frac{1}{(2\pi)^4}\int_l\Big[v^i + \frac{1}{2\mu}\Delta_j v^k + \frac{1}{4\mu^2}\Big(-\partial_{Xj}l_\nu\varepsilon^{ijm}v^m\bar v^\nu - 2\mathbf{B}\cdot\mathbf{v}v^i + F_{\nu j}\bar v^\nu v^m\varepsilon^{ijm}\Big)\Big]\text{tr}\tilde G_v(X,l).$$

Equivalence of Chiral Kinetic Equations

Distribution function n and n_v are coefficients of δ-function of $\tilde G$ and $\tilde G_v$

$$\tilde G = \int_s e^{ip\cdot s}\psi(x)\psi^\dagger(y)U(y,x), \quad \tilde G_v = \int_s e^{il\cdot s}\psi_v(x)\psi_v^\dagger(y)U(y,x).$$

using $\psi(x) = e^{i\mu\mathbf{v}\cdot\mathbf{x}}\left(1 + \frac{1}{2\mu}(-i\not{D}_\perp)\right)\psi_v(x) \qquad \psi_v$: Dressed particle

$$\text{tr}\tilde G = \text{tr}\tilde G_v - \tfrac{1}{4\mu^2}l_i\,\Delta_j\,\text{tr}\tilde G_v\,\varepsilon^{ijm}\,v^m \quad \implies \quad n = n_v - \frac{1}{4\mu^2}l_i\,\Delta_j\,n_v\,\varepsilon^{ijm}\,v^m$$

$$\left[\Delta_0 + \hat{\mathbf{p}}^i\,\left(1 + \tfrac{\mathbf{B}\cdot\hat{\mathbf{p}}}{2p^2}\right)\Delta_i - \tfrac{\epsilon^{ijk}\,\hat{\mathbf{p}}^j\,\mathbf{E}^k}{2p^2}\Delta_i\right]n(X,l) = 0 \quad \text{(CKE from Field Theory formalism)}$$

$$\Updownarrow$$

$$\left[\Delta_0 + \hat{\mathbf{p}}^i\,\left(1 + \tfrac{\mathbf{B}\cdot\hat{\mathbf{p}}}{2p^2}\right)\Delta_i - \tfrac{\epsilon^{ijk}\,\hat{\mathbf{p}}^j\,\mathbf{E}^k + \mathbf{B}_\perp^i}{4p^2}\Delta_i\right]n_v(X,l) = 0 \quad \text{(CKE from HDET formalism)}$$

Conclusions

- We revisit CKT and find it differs from its counterpart from field theory approach at higher order of $(1/\mu)$. It agrees with the CKE obtained by OSEFT.
- Despite the disagreement, both CKE obtained from field theory and effective field theory formalisms are equivalent with the difference being choices of degrees of freedom.

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