Kibble-Zurek scaling in diffusion dynamics

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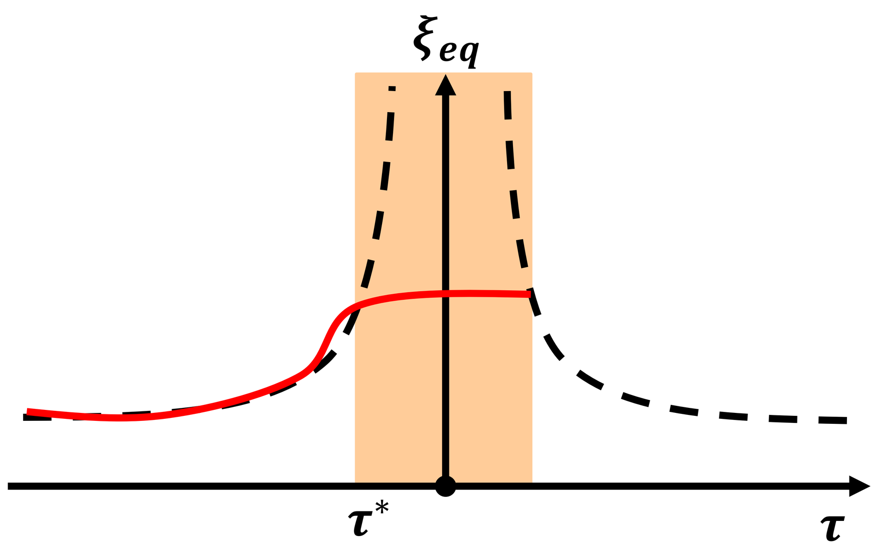
Introduction

- The non-equilibrium fluctuations near the QCD critical point is non-universal, depending on various free parameters, such as the relaxation time and the trajectory of evolving fireball on phase diagram. The constructed universal variables may be a strong indication of the critical point.
- In the framework of Kibble-Zurek Mechanism (KZM), the universal functions of order parameter field has been studied which are insensitive to the relaxation time and evolving trajectory [1,2]. We will investigate the critical universal scaling of the conserved charge within the framework of stochastic diffusion equation [3].

Kibble-Zurek Mechanism

- Relaxation time τ_{rel}: time scale of the system relaxes to equilibrium.
- Quench time τ_{quench}: time scale of the system driven by external field.
- $\tau_{rel} < \tau_{quench}$: the system have enough time to equilibrate;
- $\tau_{rel} > \tau_{quench}$: the system becomes out-of-equilibrium.



- For the system cools down to the critical point, the relaxation time τ_{rel} diverges due to the critical slowing down. After the point τ^* where $\tau_{rel} \simeq \tau_{quench}$, the system becomes out-of-equilibrium and the typical scales fixed:

$$\tau_{KZ} = \tau_{rel}(\tau^*) = \tau_{quench}(\tau^*), \qquad l_{KZ} = \xi_{eq}(\tau^*)$$

- Variables in this region have universal behavior, such as the correlation function :

$$C(r,\tau) = l_{KZ}^{-2\Delta}\tilde{C}\left(\frac{r}{l_{KZ}},\frac{\tau}{\tau_{KZ}}\right)$$

where Δ denotes the critical exponent.

Stochastic diffusion equation

The 1+1-dimensional evolution of the conserved charge density $n(y,\tau)$ follows the stochastic diffusion equation [4]:

$$\frac{\partial}{\partial\tau}\delta n(y,\tau) = D_y(y)\frac{\partial^2}{\partial y^2}\delta n(y,\tau) + \frac{\partial}{\partial y}\zeta(y,\tau),$$

where the noise $\zeta(y,\tau)$ satisfies the fluctuation-dissipation theorem:

$$\langle\zeta(y,\tau)\rangle = 0,$$
$$\langle\zeta(y_1,\tau_1)\zeta(y_2,\tau_2)\rangle = 2\chi_y(\tau)D_y(\tau)\delta(y_1-y_2)\delta(\tau_1-\tau_2)$$

Here, χ_y and D_y respectively stand for the susceptibility and diffusion coefficient, which can be parametrized from 3D Ising model. Following the stochastic diffusion equation, we calculated the correlation function $C(y_1,y_2;\tau) \equiv \langle\delta n(y_1,\tau)\delta n(y_2,\tau)\rangle$ and second order cumulant $K(\Delta y,\tau) \equiv \langle\delta Q_{\Delta y}(\tau)^2\rangle/\Delta y$ of charge $Q_{\Delta y}(\tau) \equiv \int_{-\Delta y/2}^{\Delta y/2} dyn(y,\tau)$ deposed within rapidity window Δy.

References

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[2] S.Wu, Z.Wu and H.Song, Phys.Rev.C. 99,064902 (2019).
[3] S.Wu and H.Song, Chin.Phys.C. 43,084103 (2019).
[4] M.Sakaida, M.Asakawa, H.Fujii and M.Kitazawa, Phys.Rev. C 95,064905 (2017).

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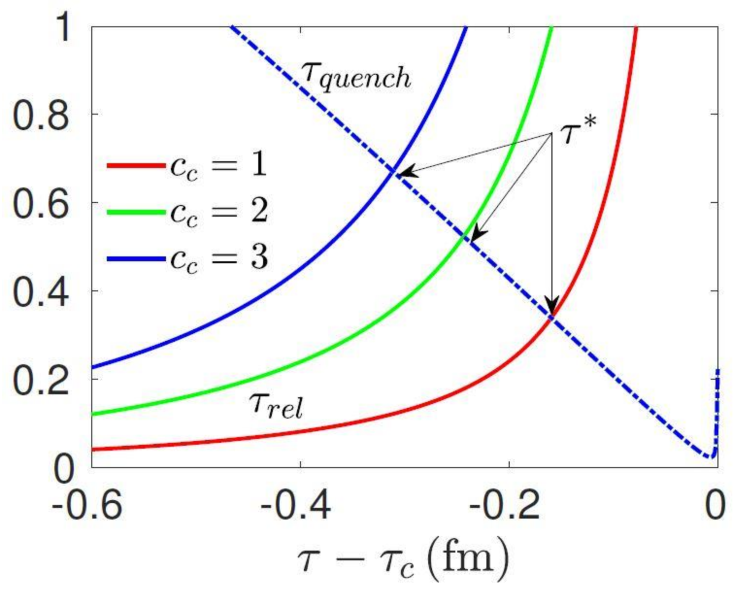
Kibble-Zurek scaling of the conserved charge

- The quench time of the system under the Bjorken expansion:

$$\tau_{quench} = \left|\frac{\xi}{\partial_\tau\xi}\right|$$

- The relaxation time:

$$\tau_{rel} = \frac{\xi^2}{2D_y}$$



- Comparing the quench time and the relaxation time gives the point τ^*:

$$\tau_{KZ} = \tau_{rel}(\tau^*) = \tau_{quench}(\tau^*), \qquad l_{KZ} = \xi_{eq}(\tau^*)$$

- Rescaling variables with characteristic scales τ_{KZ}, l_{KZ}:

$$\tilde{\tau} \equiv \frac{\tau-\tau_c}{\tau_{KZ}}, \quad \tilde{y} \equiv \frac{y}{l_{KZ}}, \quad \tilde{\xi} \equiv \frac{\xi}{l_{KZ}}, \quad \tilde{D}_y \equiv \frac{D_y}{l_{KZ}^{-2+\chi_\eta+\chi_\lambda}}, \quad \tilde{\chi}_y \equiv \frac{\chi_y}{l_{KZ}^{2-\chi_\eta}}$$

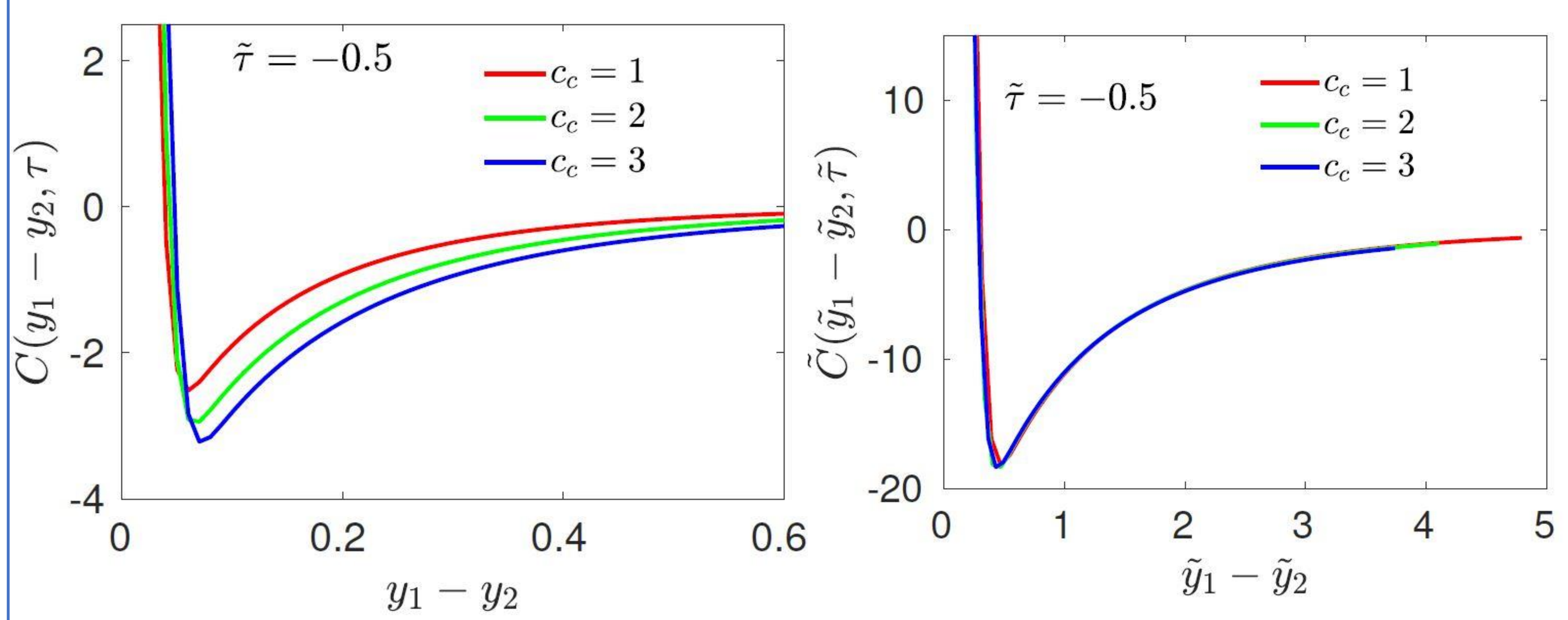
- The rescaled correlation function and rescaled function of cumulant can be constructed as

$$C(y_1-y_2,\tau) \equiv l_{KZ}^{1-\chi_\eta}\tilde{C}(\tilde{y}_1-\tilde{y}_2,\tilde{\tau})$$
$$K(\Delta y,\tau) \equiv l_{KZ}^{2-\chi_\eta}\tilde{K}(\Delta y/l_{KZ},\tilde{\tau})$$

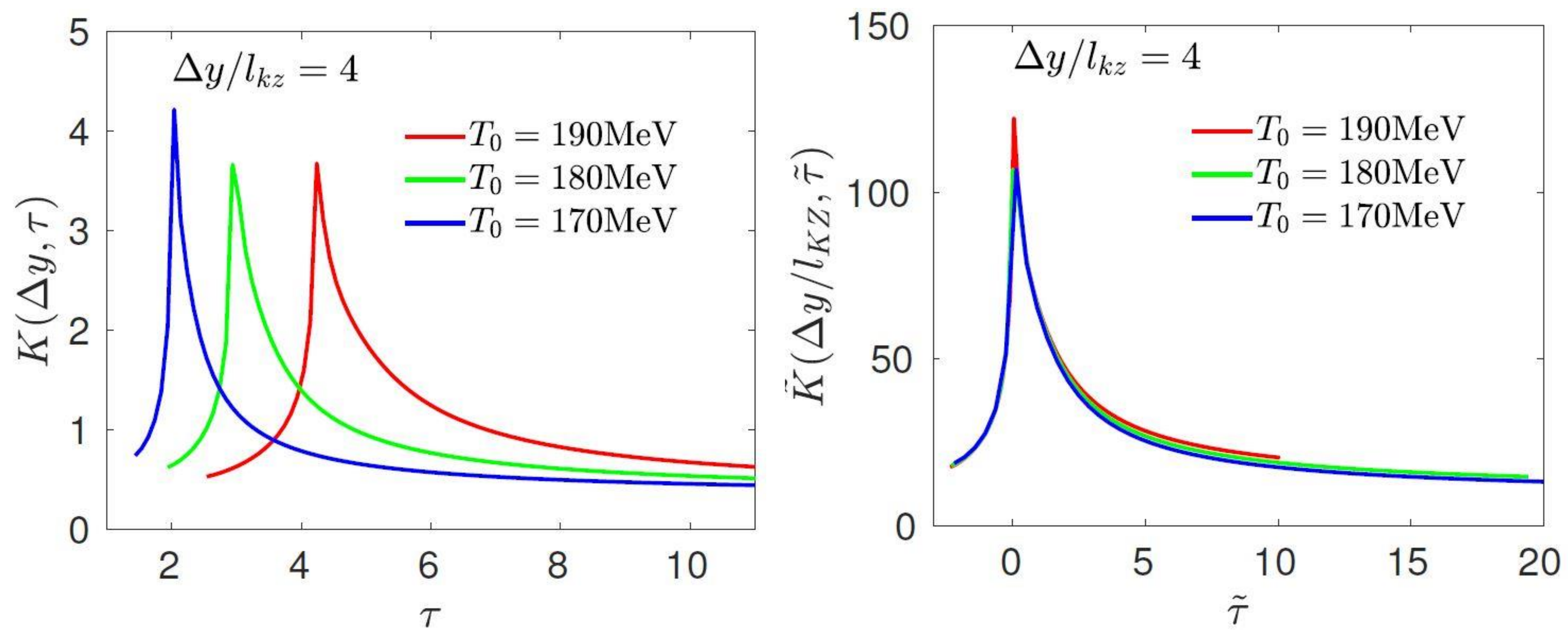
where χ_η, χ_λ are critical exponents.

Results

- Scaling of correlation function with different value of susceptibility strength c_c



- Scaling of cumulant with different value of initial temperature T_0



Conclusions

- For the system cools down to the critical point, the typical scales are fixed at point where the system fall out-of-equilibrium.
- By rescaling variables with the typical scales, universal functions that are insensitive to the free parameters can be constructed. In this work, we constructed the universal functions of two-point correlation function and second order cumulant for conserved charge, which insensitive to the strength of susceptibility or the initial temperature.