Dualities of QCD phase diagram

Khunjua Tamaz[1,4], **Klimenko Konstantin**[2], **Zhokhov Roman**[1,2,3]

[1] Department of Theoretical Physics, Faculty of Physics, Lomonosov Moscow State University

[2] Logunov Institute for High Energy Physics, NRC "Kurchatov Institute", Protvino, Moscow Region, Russia

[3] Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radiowave Propagation (IZMIRAN), Troitsk, Moscow, Russia

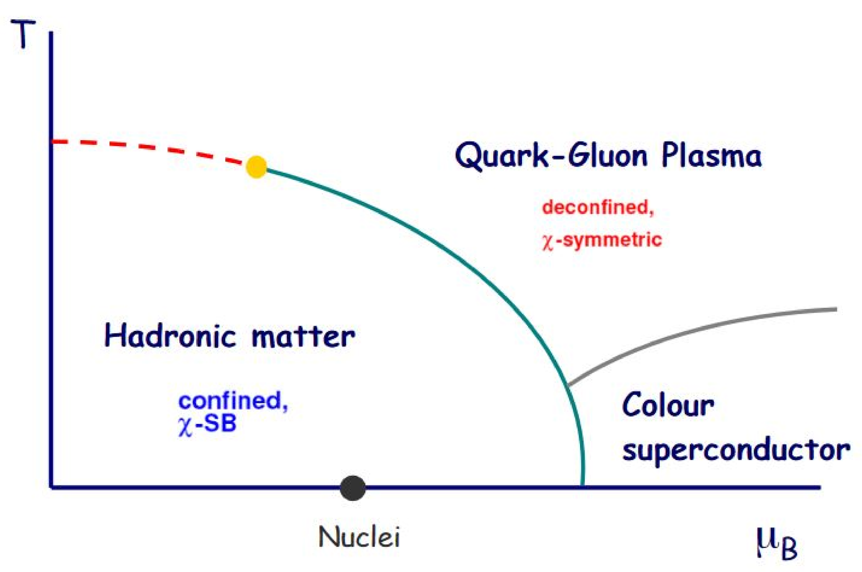
[4] University of Georgia, Georgia

ИФВЭ IHEP · ИЗМИРАН · RSF Russian Science Foundation · БАЗИС Фонд развития теоретической физики и математики

Quark Matter 2019 - the XXVIIIth International Conference on Ultra-relativistic Nucleus-Nucleus Collisions
Max Planck Institute for Nuclear Physics (MPIK) - Heidelberg - Germany

QCD phase diagram

Let us consider the QCD phase diagram (μ_B, T)



QCD is hard to be dealt with,
strong coupling
it is possible to study the phase structure at finite temperature on lattice,
Lattice QCD
but at finite μ lattice Monte-Carlo simulations are impossible.
In the region of $\mu \neq 0$ one can use effective models.
One of the most widely used one is
Nambu–Jona-Lasinio (NJL) model

The model and its thermodynamical potential

We study two-flavour chiral ($\mu_5 \neq 0$ and $\mu_{I5} \neq 0$ and isospin ($\mu_I \neq 0$) imbalanced dense ($\mu \neq 0$) quark matter in the framework of (3+1)-dimensional NJL model. Its Lagrangian has the form:

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \mu_5\gamma^0\gamma^5 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5\Big]q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

- μ_B is baryon chemical potential,
- μ_I is taken into account to introduce the non-zero imbalance between u and d quarks,
- μ_{I5} and μ_5 accounts for chiral isospin and chiral imbalances.

From here we use notations $\mu \equiv \mu_B/3$, $\nu = \mu_I/2$ and $\nu_5 = \mu_{I5}/2$.

One uses a semibosonized version of the Lagrangian, which contains composite bosonic fields $\sigma(x)$ and $\pi_a(x)$ $(a = 1, 2, 3)$

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q).$$

The ground state expectation values $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ of the composite bosonic fields are

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \Delta, \quad \langle\pi_2(x)\rangle = \langle\pi_3(x)\rangle = 0,$$

Dualities of the phase diagram

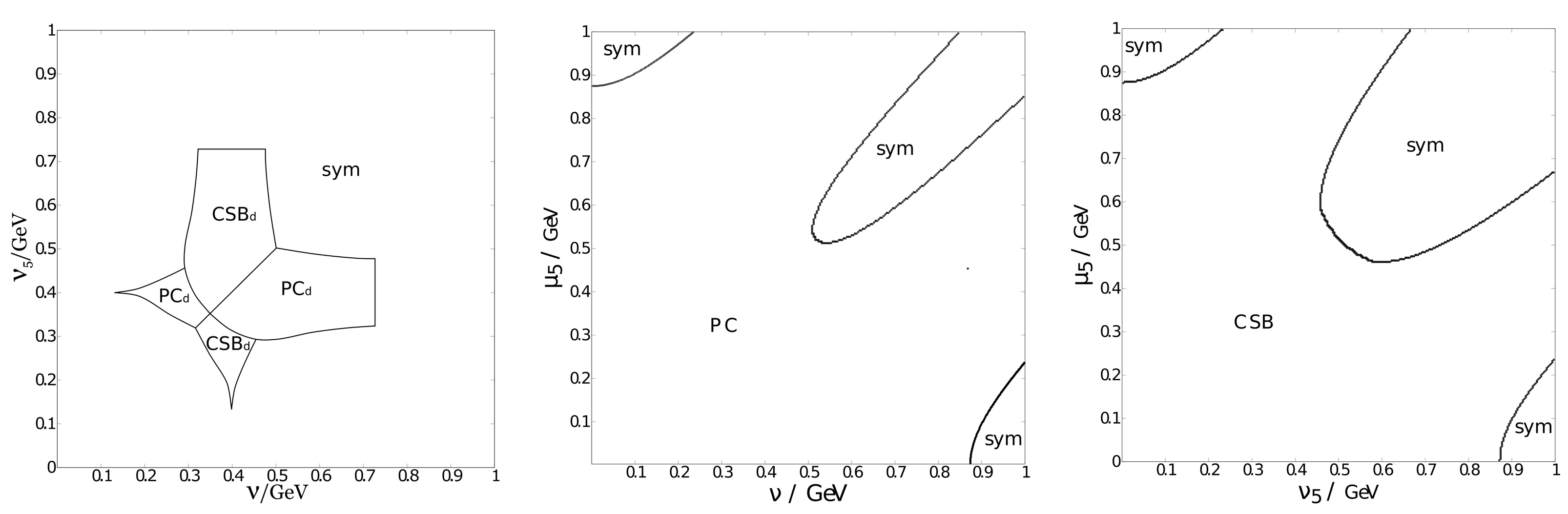
It is possible to demonstrate that the TDP is invariant with respect to the so-called duality transformation

$$\mathcal{D}_H: \quad M \longleftrightarrow \Delta, \ \ \nu \longleftrightarrow \nu_5.$$

Moreover, there are two other dualities.

$$\mathcal{D}_{HM}: \quad \mu_5 \longleftrightarrow \nu_5 \quad \text{if} \quad \Delta = 0, \qquad and \qquad \mathcal{D}_{H\Delta}: \quad \mu_5 \longleftrightarrow \nu \quad \text{if} \quad M = 0$$

Phase portrait of the model



- The chiral imbalance leads to the generation of charged pion condensation in dense quark matter, so called PC_d phase.

Duality in lattice QCD

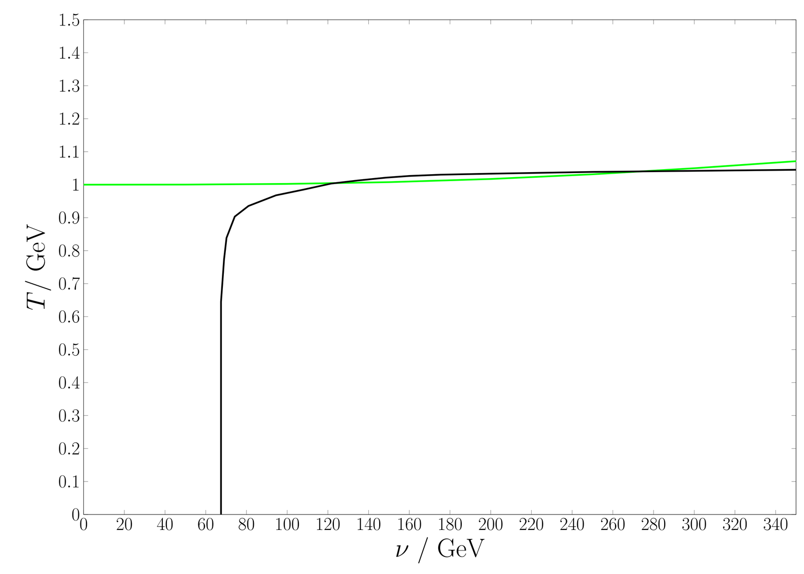


Figure 1: Critical temeperature T_c as a function of ν at $\mu_5 = 0$ and as a function of μ_5 at $\nu = 0$.

NJL model works very well in describing QCD phase diagram, but it is effective model for QCD and not QCD itself. The duality was shown only in the large N_c approximation (or mean field approximation). It is also exact only in the chiral limit. So it is interesting to investigate whether duality can be seen in lattice QCD simulations results or not. Lattice QCD simulations are almost impossible at $\mu \neq 0$, but one can search for the fingerprints of the duality in the particular cases, for example, for $\mu = 0$.

Lattice studies:

- non-zero μ_I: B. Brandt, G. Endrodi et al, Phys. Rev. D 97 (2018) no.5, 054514; PoS LATTICE 2016 (2016) 039
- non-zero μ_5: V. V. Braguta et al

One can see from lattice QCD results that at ν and μ_5 greater that the half of the pion mass m_π the duality is a very good approximation.

Inhomogeneous condensates

In vacuum, at $\mu = \nu = \mu_5 = 0$ and ν_5, $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on space coordinate x. However, in a dense medium $\mu \neq 0$, they might depend on spatial coordinates. Though the chiral limit is an excellent approximation to QCD, one knows that in reality the current quark masses are nonzero. If current quark mass $m_0 \neq 0$ (physical point), we use the following the ansatz

$$\langle\sigma(x)\rangle = M\cos(2kx^1) - m_0, \quad \langle\pi_3(x)\rangle = M\sin(2kx^1),$$
$$\langle\pi_1(x)\rangle = \Delta\cos(2k'x^1), \quad \langle\pi_2(x)\rangle = \Delta\sin(2k'x^1), \tag{1}$$

We showed that the TDP is invariant with respect to the duality that in inhomogeneous case has the form

$$\mathcal{D}_I: \quad M \longleftrightarrow \Delta, \ \ \nu \longleftrightarrow \nu_5, \ \ k \longleftrightarrow k'. \tag{2}$$

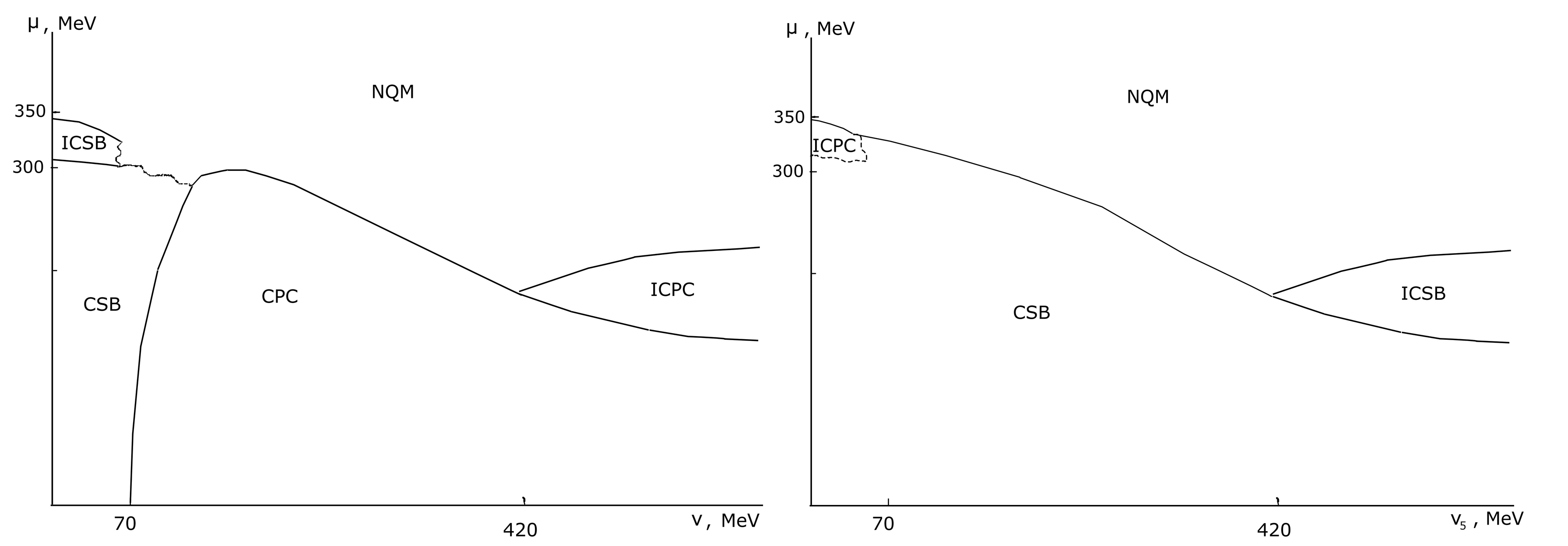


Figure 1. (Left). Combined schematic (ν, μ)-phase diagram at $\mu_5 = \nu_5 = 0$. (Rigth). (ν_5, μ)-phase diagram at $\nu = \mu_5 = 0$. This plot is duality conjugated to the left one.

Conclusions

- There exist several dualities of the phase diagram
- Chiral imbalance generate charged pion condensation in dense quark matter
- Obtained phase portraits are in qualitative accordance with the recent lattice simulations
- Duality holds with very good accuracy in lattice QCD results
- Inhomogeneous phase structure of dense quark matter with chiral imbalance is obtained using the duality notion

References

[1] T. G. Khunjua, K. G. Klimenko, R. N. Zhokhov and V. C. Zhukovsky, Phys. Rev. D **95**, 105010 (2017).

[2] D. Nowakowski, M. Buballa, S. Carignano and J. Wambach, arXiv:1506.04260 [hep-ph].

[3] D. Nowakowski, Ph.D. Thesis, Technische Univ. Darmstadt, 2017.

[4] T. G. Khunjua, K. G. Klimenko and R. N. Zhokhov, Phys. Rev. D **97**, 054036 (2018).

[5] T. G. Khunjua, K. G. Klimenko and R. N. Zhokhov, Phys. Rev. D **98**, 054030 (2018).

[6] V. V. Braguta, E. M. Ilgenfritz, A. Y. Kotov et al. Phys. Rev. D **93** (2016) no.3, 034509

[7] B. B. Brandt, G. Endrodi and S. Schmalzbauer, Phys. Rev. D **97** (2018) no.5, 054514

[8] C.f Mu, L.y. He and Y.x. Liu, Phys. Rev. D **82**, 056006 (2010).

[9] T. G. Khunjua, K. G. Klimenko and R. N. Zhokhov, Eur. Phys. J. C **79** (2019) no.2, 151,

[10] T. G. Khunjua, K. G. Klimenko and R. N. Zhokhov, JHEP 06(2019); 006