Viscous fluid dynamics for neutron star mergers

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How does a lump of baryon rich QCD matter behave under strong gravitational fields?

Neutron Star Mergers

High Density QCD Matter in the Sky

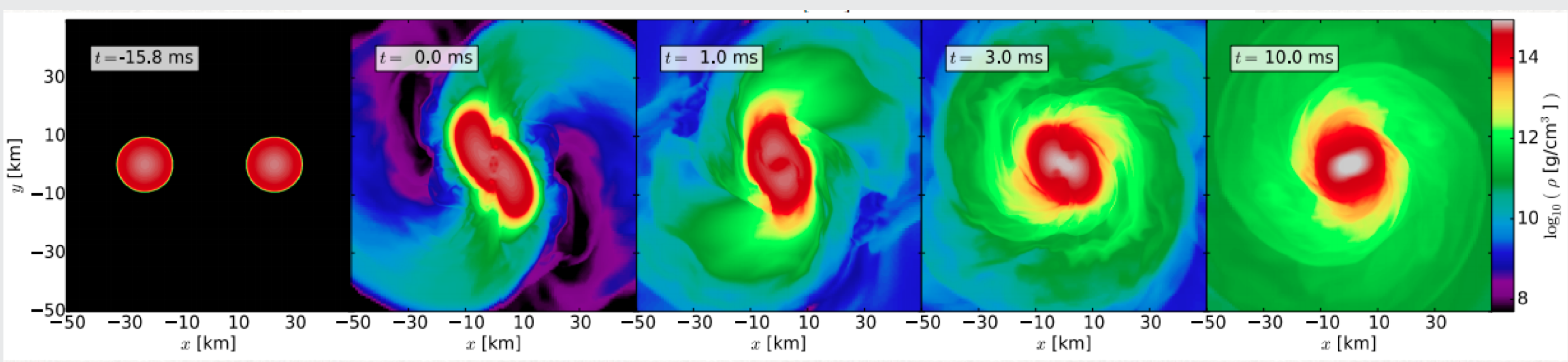

Fig. by L. Rezzolla

Viscous fluid dynamics + strong gravitational fields?

- An open problem in physics and mathematics (since 1940)
  Relativistic Navier-Stokes equations are acausal and unstable
- Viscous effects in neutron star mergers?
  Duez et al PRD (2004), Shibata et al. PRD (2017)
- Bulk viscosity should be the leading viscous effect in mergers.
  Alford, Bovard, Hanauske, Rezzolla, and K. Schwenzer, PRL (2018)

Einstein-Israel-Stewart Theory

- We consider a general theory with bulk viscosity (no shear or particle diffusion), fully embedded in general relativity, that can describe viscous phenomena in neutron star mergers.
- The energy-momentum tensor of the fluid is:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + (P+\Pi)\Delta^{\mu\nu} \qquad \Delta_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

ε is the energy density, u^μ is the flow velocity ($u^\mu u_\mu = -1$), $P = P(\varepsilon, n)$ is the equilibrium pressure, n is the baryon density, $g_{\mu\nu}$ is the space-time metric, and Π is the bulk scalar.

The equations of motion of the theory are:

- Energy-momentum conservation: $\nabla_\mu T^{\mu\nu} = 0$
- Baryon current conservation: $\nabla_\mu J^\mu = 0$ with $J^\mu = n u^\mu$
- Einstein's equations: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = (8\pi G) T_{\mu\nu}$
- Generalized Israel-Stewart-like equations [1] for bulk viscosity:

$$\tau_\Pi u^\alpha \nabla_\alpha \Pi + \zeta \nabla_\alpha u^\alpha + f(\varepsilon, n, \Pi, \tau_\Pi, \zeta) = 0$$

The function f can be an arbitrary.

The bulk viscosity, ζ, and relaxation time, τ_Π, can depend on ε, n, and Π as well.

Fundamental questions

- Does causality hold in this system in the full nonlinear regime? Is the mathematical problem well-posed?
- What are the new nonlinear phenomena that emerge in this system?
- Can neutron star mergers also be used to constrain the out-of-equilibrium dissipative properties of ultradense QCD matter?

General Results

In Ref. [2] it was *rigorously* proven for the first time that the following statements hold for these Einstein-Israel-Stewart equations under completely general conditions:

- Causality holds in the full nonlinear regime.
- The system of equations is first-order symmetric hyperbolic (FOSH), which implies local existence and uniqueness of solutions.
- There exists a global hyperbolic development for the initial data.

These statements hold when the following condition is satisfied:

$$\left[\frac{\zeta}{\tau_\Pi} + n\left(\frac{\partial P}{\partial n}\right)_\varepsilon\right]\frac{1}{\varepsilon+P+\Pi} \le 1 - \left(\frac{\partial P}{\partial \varepsilon}\right)_n$$

In the limit of zero net baryon density (relevant to high-energy heavy ion collisions), the nonlinear causality condition can be written as:

$$\frac{\Pi}{\varepsilon+P} \ge \frac{\zeta}{\tau_\Pi(\varepsilon+P)(1-c_s^2)} - 1$$

where $c_s = \sqrt{dP/d\varepsilon}$ is the speed of sound.

Conclusions/Outlook

- Our results in [2] establish causality, existence, and uniqueness of solutions of viscous relativistic hydrodynamics in Einstein-Israel-Stewart theory for the first time.
- This implies that bulk viscous phenomena in neutron star mergers [3] can be described in a way fully compatible with general relativity.
- Our analysis provides the necessary first step towards future state-of-the-art simulations of neutron star mergers that will constrain not only the equilibrium properties but also the viscous properties of ultradense QCD matter.
- No issues appear at all when the total pressure is negative, as long as

$$\varepsilon + P + \Pi > 0$$

This should be kept in mind when studying "cavitation" in heavy-ion collisions.

- The nonlinear causality condition can be important to constrain the allowed values of dissipative stresses in the initial conditions, especially in small collision systems (such as pA and pp collisions).
- Inclusion of shear and baryon diffusion effects is needed to fully understand the nonlinear interplay between causality and the initial conditions of relativistic viscous fluids in heavy ion collisions and also in neutron star mergers.

References

[1] W. Israel and J. M. Stewart, Ann. Phys. 118, 341 (1979).
[2] F. S. Bemfica, M. M. Disconzi and J. Noronha, PRL 122, no. 22, 221602 (2019).
[3] M. G. Alford, L. Bovard, M. Hanauske, L. Rezzolla, and K. Schwenzer, PRL 120, 041101 (2018).