Quantum dissipation in the quarkonium evolution by Lindblad master equation

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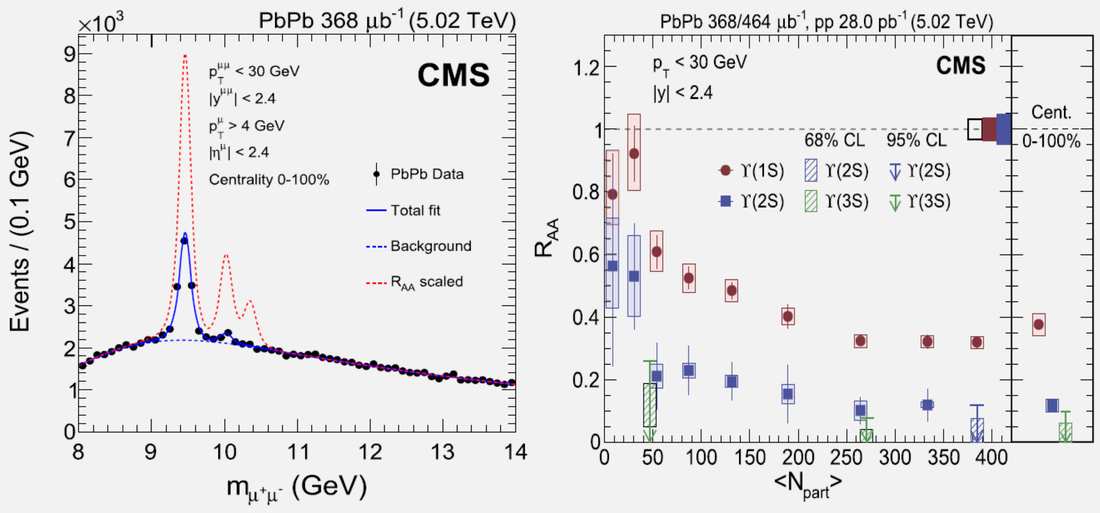
1.Motivation

■ Suppression of quarkonium yields　CMS Collaboration(19)



- Properties of QGP can be diagnosed by studying quarkonium evolution in QGP
- How do we formulate and study this phenomenon?

2.Framework

■ Open quantum system approach

→ Extract quarkonium information by integrating out QGP

quarkonium ---------- QGP　Trace out QGP variables → quarkonium ----------

$$H_{\rm tot} = H_{Q\bar{Q}} + H_{\rm QGP} + H_{\rm int}$$

E.o.M. of total system

$$\partial_t \rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

$$\rho_{Q\bar{Q}} = {\rm Tr}_{\rm QGP}\rho_{\rm tot}$$

all of the information of quarkonium in QGP

$$\partial_t \rho_{Q\bar{Q}} = \quad ?$$

- Lindblad form of master equation　$\partial_t \rho_{Q\bar{Q}} = \mathcal{L}\rho_{Q\bar{Q}}$ (Liouville operator)

$$\partial_t \rho_{Q\bar{Q}} = -i[H'_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \int dk\{2L_k\rho_{Q\bar{Q}}L_k^\dagger - \rho_{Q\bar{Q}}L_k^\dagger L_k - L_k^\dagger L_k \rho_{Q\bar{Q}}\}$$

(Lindblad operator)

✓ Properties
1. $\mathrm{Tr}[\rho_{Q\bar{Q}}] \equiv 1$
2. $\rho_{Q\bar{Q}} = \rho_{Q\bar{Q}}^\dagger$
3. $\langle {}^\forall\alpha | \rho_{Q\bar{Q}} | \alpha \rangle \geq 0$ → probabilistic interpretation

- Kinds of interacting forces

$$\mathcal{L} = (\text{Debye potential}) + (\text{thermal fluctuation = random force}) + (\text{quantum dissipation = friction force})$$

Stochastic Potential model [Kajimoto+ (18)]

How does quantum dissipation affect quarkonium fate?

3.Quantum State Diffusion method

- Lindblad operator for the relative motion

$H'_{Q\bar{Q}} \ni V(r)$

$$L_k^r = \sqrt{\frac{D(k)}{2}}\left[\left\{1 - \frac{k}{4MT}\left(\frac{P_{CM}}{2} + \hat{p}\right)\right\}e^{ik\hat{r}/2} - \left\{1 - \frac{k}{4MT}\left(\frac{P_{CM}}{2} - \hat{p}\right)\right\}e^{-ik\hat{r}/2}\right]$$

momentum transfer frequency

heavy quark sector: thermal fluctuation term

anti heavy quark sector: quantum dissipation heavy quark recoil

- Quantum State Diffusion method

Density matrix in Lindblad master equation can be reconstructed by ensemble average of wave function

$$\rho_{Q\bar{Q}} = M[|\psi\rangle\langle\psi|]$$

- Nonlinear Schroedinger eq. in QSD

$$|d\psi\rangle = -iH'_{Q\bar{Q}}|\psi\rangle dt + \int dk\{2\langle L_k^\dagger\rangle_\psi L_k - L_k^\dagger L_k - \langle L_k^\dagger\rangle_\psi\langle L_k\rangle_\psi\}|\psi\rangle dt + \int dk\{L_k - \langle L_k\rangle_\psi\}|\psi\rangle d\xi_k$$

$$M[d\xi_k d\xi_l^*] = 2\delta_{kl}dt$$

Lindblad master eq. ⟺ (QSD method, equivalent) ⟺ Nonlinear Schroedinger eq.

✓ less numerical cost in vector form than in matrix form

4.Numerical Outline and Results

Initial Wave function → Time evolution event by event → Eigenstate projection → Occupations

$$H = \frac{p^2}{M} - \frac{\alpha}{r}e^{-m_D r} \qquad N_i = \int dxdy\,\phi_i^*(x)\rho(x,y)\phi_i(y)$$

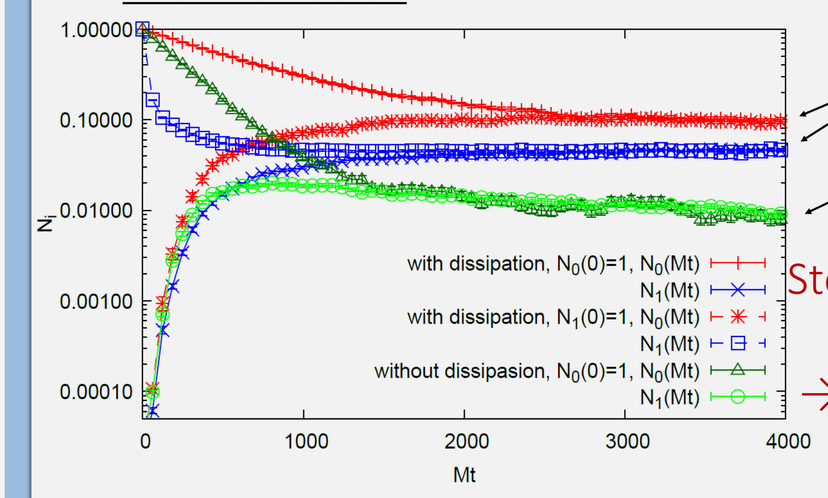
Bjorken expanding QGP → $H = \frac{p^2}{M_b} - \frac{\alpha}{r} + \sigma r$　$\sigma = 0.01M_b^2$　vacuum eigenstate $\phi_i(x)$

setup

Δx	Δt	N_x	γ	$\ell_{\rm corr}$	α	m_D	x_c	$P_{\rm CM}$
$1/M$	$0.1M(\Delta x)^2$	254	T/π	$1/T$	0.3	$2T$	$1/M$	0

- Thermalization

cf. Akamatsu+ (18) for single heavy quark

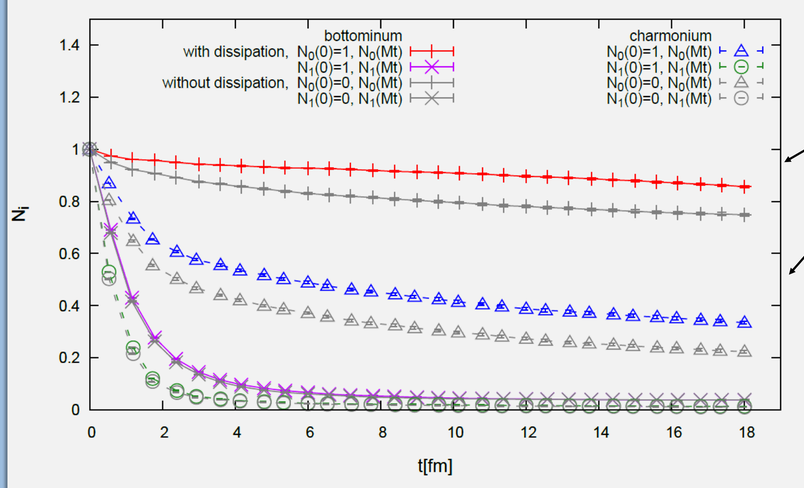


w/ dissipation

w/o dissipation

Steady state is independent of initial conditions → Boltzmann distribution

- Bjorken expanding QGP

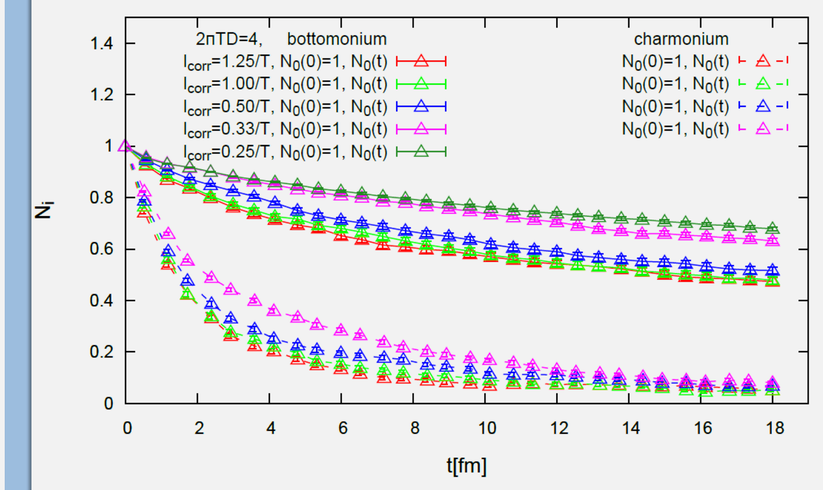


bottomonium

charmonium

Ground state is affected by dissipation in QGP lifetime scale

- With heavy quark diffusion constant fixed [Miura+ in progress]



Heavy quark diffusion constant

$D \sim \frac{4}{2\pi T}$ from lattice QCD

$D/T^2 = l_{\rm corr}^2/\gamma$ is fixed

QGP correlation length can be measured from the suppression!?

5.Summary

- ☐ Dissipative effects on quarkonium relative motion
- ☐ Suppression pattern as a measure of gluon correlation length
- ➢ Future work　SU(3) color effect + 3D simulation

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