Fluctuation-dissipation relation and fluctuation theorem for causal hydrodynamic fluctuations

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Abstract: To implement fluctuating hydrodynamics into dynamical models based on causal dissipative hydrodynamics, the fluctuation-dissipation relation (FDR) in inhomogeneous and non-static background should be considered. FDR is modified depending on the adopted constitutive equations. Without the modification, fluctuation theorem (FT) is violated.

1. Background and Motivation

■ Event-by-event fluctuations
To extract the properties of the matter created in high-energy nuclear collisions, flow observables are important experimental observables to which different kinds of event-by-event fluctuations contribute.

Flow observables, etc.
Other non-flow contributions...

Matter response \( \eta, \zeta, \tau, \lambda, \ldots \)

Initial-state fluctuations
Hydrodynamic fluctuations
Jet-induced medium excitation

For precise determination of matter properties, all kinds of fluctuations have to be implemented and investigated in dynamical models. Also, for the critical point search especially non-trivial behavior of hydrodynamic fluctuations play an important role.

■ Hydrodynamic fluctuations: the thermal fluctuations of hydrodynamics
Hydrodynamic fluctuations can be introduced as a noise term in the constitutive equation (CE) in the framework of fluctuating hydrodynamics. The behavior is determined by fluctuation-dissipation relation (FDR).

Constitutive equation (e.g. Pressure)

\[
P + \Pi = P(\epsilon, n) - \partial \Pi + \delta \Pi
\]

\( \text{Eos+Viscosity} \)

Stochastic differential equation (\( \xi \) is the Langevin equation)

Hydrodynamic fluctuation arises locally in each space-time points

\( (\delta\Pi(\mathbf{x})\delta\Pi(\mathbf{y})) = 2T \delta \zeta(\mathbf{x} - \mathbf{y}) \)

■ Causal viscous hydrodynamics (2nd-order hydro) used in dynamical models

When the system is non-static and inhomogeneous as in high-energy nuclear collisions, we cannot define a single temperature, etc., for the FDR.

FDR becomes non-local. Noise is “colored”

\( \mathbf{P}(x) \)

How the FDR for 2nd-order hydrodynamics should be modified in non-static and inhomogeneous matter?

2. Method: FDR in inhomogeneous systems

■ Step 1. Define (linear-response) CE in inhomogeneous background

\[
\mathbf{L}_x \Gamma(x) = \mathbf{M}_x \kappa(x) F(x) + \xi(x)
\]

\( \Gamma = \eta \kappa \)

\( \kappa: 3^\text{rd} \text{order transport coefficient} \)

\( \xi: \text{noise term} \)

\( \mathbf{L}_x, \mathbf{M}_x: \text{linear operators} \)

\( F: \text{thermodynamic force} \)

\( \eta, \kappa, \xi \)

■ Step 2. Solve CE to obtain the integral form

\[
\Gamma(x) = \int d\mathbf{y} G(x,y) \kappa(y) F(y) + \delta \Gamma(x)
\]

\( \delta \Gamma: \text{integral form noise} \)

\( \xi(x) = L_x \delta \Gamma(x) \)

■ Step 3. Use FDR based on non-equilibrium statistical operator method

"Integral-form FDR"

\[
\langle \delta \Gamma(x) \delta \Gamma(y) \rangle = \mathcal{O}(x^2 - y^2) = G(x-y) T(y) \kappa(y)
\]

Obtain explicit form of memory function \( G(x,y) = L_x^{-1} \mathbf{M}_x \delta \Gamma(y) = \ldots \)

■ Step 4. Obtain noise autocorrelation in original CE

"Differential-form FDR"

\[
\langle \xi(x) \xi(y) \rangle = L_{xy} \langle \delta \Gamma(x) \delta \Gamma(y) \rangle = \ldots
\]

3. Result: e.g. Simplified Israel-Stewart case

■ Input CE: simplified IS eqs.

\[
(1 + \eta \Pi) D = \nabla \cdot F + \zeta \nabla \times F + \kappa \nabla \cdot F^\text{lin} + \xi
\]

\( \kappa = \kappa_0 \Theta(x) \)

Shear stress

Dissipation current

Diffusion current

■ Result (FDR)

Complicated expression due to the tensor structure...

\[
\langle \delta \Gamma(x) \delta \Gamma(y) \rangle = \mathcal{O}(x^2 - y^2) = G(x-y) T(y) \kappa(y)
\]

Essential structure is...

\[
\langle \xi(x) \xi(y) \rangle = L_{xy} \langle \delta \Gamma(x) \delta \Gamma(y) \rangle = \ldots
\]

■ Modify to FDR \( \propto \text{Relaxation time } \tau_0 \) and expansion \( \theta \)

Important in HIC with the short-scale \( \sim \) dynamics rapidly expanding system

4. Fluctuation theorem & FDR modification

■ Fluctuation theorem

Fluctuation theorem (FT), known in the non-equilibrium statistical mechanics, describes the entropy production distribution in non-equilibrium processes. FT contains FDR as a special case, i.e., FT is a more general tool for non-equilibrium processes.

How about FT in causal hydro?

■ E-by-E 0+1D simulations of Bjorken flow (expanding system)

Setup: Dynamical eqs.

Typical structure of FT

\[
\begin{align*}
\text{FDR: Case 1: without modification} & \quad \text{Typical structure of FT} \\
\text{FDR: Case 2: with modification} & \quad \text{Typical structure of FT}
\end{align*}
\]

Check: Steady-state FT (a version of FT)

\[
\Pr(\sigma) = \frac{\sigma}{\tau_0} e^{-\sigma/\tau_0}
\]

\( \Pr(\sigma) = \alpha T \theta^2 / \tau_0 \)

\( \tau_0 : \text{Relaxation time} \)

\( \theta : \text{Expansion} \)

■ Result

FT largely depends on the adopted constitutive eqs.

\[
\begin{align*}
\text{FDR: Case 1: } & \quad \text{FT is broken for non-equilibrium processes} \\
\text{FDR: Case 2: } & \quad \text{FT is satisfied for non-equilibrium processes}
\end{align*}
\]

5. Summary

- FDR has modification for causal (2nd-order) hydrodynamics
- Without FDR modification, FT breaks down
- FDR modification has to be implemented in dynamical models with hydrodynamic fluctuations in causal hydrodynamics