Zilch currents and chiral kinetic theory for vector particles

Xu-Guang Huang[1], Pavel Mitkin[2], Andrey Sadofyev[3] and **Enrico Speranza**[4]

[1]*Physics Department and Center for Particle Physics and Field Theory, Fudan University, Shanghai, China*
[2]*Institute of Theoretical and Experimental Physics, Moscow, Russia*
[3]*Theoretical Division, T-2, Los Alamos National Laboratory, Los Alamos, USA*
[4]*Institute for Theoretical Physics, Goethe University, Frankfurt am Main, Germany*

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC-TR 211

Bundesministerium für Bildung und Forschung

Introduction

The chiral vortical effect (CVE) is believed to be intrinsically related to the anomalies of the axial current and the topological properties of the system. It was suggested that the CVE can be generalized to systems of higher-spin particles and, particularly, to photons [1, 2, 3]. However, there is no local gauge invariant definition of photonic helicity current. This problem can be overcome with an appropriate choice of the polarization measure. Recently, it was shown that there is a vortical effect in photonic Zilch current (ZVE), which can play the role of a local gauge invariant helicity separation measure [4]. In this work we study the Zilch current in terms of chiral kinetic theory and show that the ZVE can be related to the non trivial topological properties of the system in momentum space manifested through the Berry phase. We also show how the ZVE arises in terms of the Wigner-function formalism for vector particles.

Chiral Vortical Effects

- **Fermions** [5]

$$J^\mu_{R/L} = \pm\left(\frac{T^2}{12} + \frac{\mu^2_{R/L}}{4\pi^2}\right)\omega^\mu$$

- **Photons** [1, 2]

$$J^\mu_{R/L} = \pm\frac{T^2}{6}\omega^\mu$$

In [1] helicity current $K^\mu = \epsilon^{\mu\nu\alpha\beta}A_\nu\partial_\alpha A_\beta$ was used

Connection to Berry curvature [2]

Is there any gauge-invariant measure of helicity current?

Zilch currents

- Extra conservation law for Maxwell's equation **independent** of energy and momentum conservation [6]

$$\partial_\mu Z^\mu = 0$$

Zilch charge and current:

$$Z^0 = \mathbf{E}\cdot(\nabla\times\mathbf{E}) + \mathbf{B}\cdot(\nabla\times\mathbf{B}), \qquad \mathbf{Z} = \mathbf{E}\times\dot{\mathbf{E}} + \mathbf{B}\times\dot{\mathbf{B}}$$

- Zilch is actually a component of a tensor [7]

$$\partial^\rho Z_{\mu\nu\rho} = 0$$

$$Z_{\mu\nu\rho} = \tilde{F}^\lambda{}_\mu\partial_\rho F_{\nu\lambda} - F^\lambda{}_\mu\partial_\rho\tilde{F}_{\nu\lambda} = \tilde{F}^\lambda{}_\mu \overleftrightarrow{\partial}_\rho F_{\nu\lambda} + \tilde{F}^\lambda{}_\nu \overleftrightarrow{\partial}_\rho F_{\mu\lambda}$$

with $Z_{\mu 00} = Z_\mu$, $\overleftrightarrow{\partial} = \frac{1}{2}(\overrightarrow{\partial} - \overleftarrow{\partial})$

- Infinite set of conserved quantities $Z^{(s)}_{\alpha_1\cdots\alpha_s}$ [7]
- It is convenient to define a **symmetrized Zilch**

$$\bar{Z}^{(s)}_{\alpha_1\cdots\alpha_s} = \tilde{F}^\lambda{}_{\{\alpha_1}\overleftrightarrow{\partial}_{\alpha_2}\cdots\overleftrightarrow{\partial}_{\alpha_{s-1}}F_{\alpha_s\}\lambda}$$

$\bar{Z}^{(s)}_{\alpha_1\cdots\alpha_s}$ and $Z^{(s)}_{\alpha_1\cdots\alpha_s}$ have the same charge and both are conserved

Chiral kinetic theory

- Construction of current in chiral kinetic theory (CKT)

(i) Vectors at our disposal: p^μ and $j^\mu = p^\mu f + S^{\mu\nu}\partial_\nu f$

(ii) Dimensionality

- Case $s=3$, $[\bar{Z}] = [E]^5$, thus

$$\bar{Z}_{i00} = \int\frac{d^4p}{(2\pi)^3}\delta(p^2)\,p_{\{0}\,p_0\,j_{i\}}$$

- **Global equilibrium**, first order in vorticity $\omega \Longrightarrow$ **Zilch Vortical Effect**

$$\boxed{\bar{Z}_{i00} = \frac{2\pi^2T^4}{27}\omega_i}$$

- Case $s>3$

$$\bar{Z}_{i0\cdots0} = \omega_i\frac{1}{2\pi^2}\frac{(s+1)(s+2)}{3s}\int_0^{+\infty}d|\mathbf{p}||\mathbf{p}|^s f^{(0)}$$

- Relation to the **Berry curvature** through $S^{\mu\nu}$!

Wigner function for spin-one particles

- **Gauge-dependent** Wigner function [8]

$$W^{\mu\nu}(x,p) = \int\frac{d^4y}{(2\pi\hbar)^4}e^{-\frac{i}{\hbar}p\cdot y}\langle : A^\mu\left(x+\frac{y}{2}\right)A^\nu\left(x-\frac{y}{2}\right):\rangle$$

- Equations of motion (we choose $\partial_\mu A^\mu = 0$)

$$\left(p_\alpha - i\frac{\hbar}{2}\partial_\alpha\right)W^{\alpha\mu}(x,p) = \left(p_\alpha + i\frac{\hbar}{2}\partial_\alpha\right)W^{\mu\alpha}(x,p) = 0$$

$$\left(p^2 - m^2 - \frac{\hbar^2}{4}\partial_\alpha\partial^\alpha\right)W^{\mu\nu}(x,p) = 0, \qquad \hbar\,p_\alpha\partial^\alpha W^{\mu\nu}(x,p) = 0$$

Mass m is introduced to regularize

- Solution using semiclassical expansion and truncating at first order

$$W^{\mu\nu} = W^{(0)\mu\nu} + \hbar W^{(1)\mu\nu} + \ldots$$

- **Global equilibrium:**

$$W^{\mu\nu} = \left[P^{\mu\nu}(f^{(0)} + \hbar f^{(1)}) + i\hbar\,\omega^{\mu\nu}f^{(0)\prime}\right]\delta(p^2-m^2)$$

Thermal vorticity $-\,\omega^{\mu\nu} = \frac{1}{2}(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu)$, $\ f^{(0)} = (2\pi)^{-3}[\exp(\beta\cdot p)-1]^{-1}$

- **Gauge-invariant** Wigner function [8]

$$Y^{\sigma\delta\nu\alpha} = \int\frac{d^4y}{(2\pi\hbar)^4}e^{-\frac{i}{\hbar}p\cdot y}\langle : F^{\sigma\delta}\left(x+\frac{y}{2}\right)F^{\nu\alpha}\left(x-\frac{y}{2}\right):\rangle$$

$$Y^{\sigma\delta\nu\alpha} = k^{\sigma*}k^\nu W^{\delta\alpha} - k^{\sigma*}k^\alpha W^{\delta\nu} - k^{\delta*}k^\nu W^{\sigma\alpha} + k^{\delta*}k^\alpha W^{\sigma\nu}$$

with $k^\mu = p^\mu + i\frac{\hbar}{2}\partial^\mu$

- **Chiral Vortical Effect**

$$K^\mu = -i\epsilon^{\mu\nu\alpha\beta}\int d^4p\,k_\alpha W_{\nu\beta} = \frac{T^2}{6}\omega^\mu$$

with $k^\mu = p^\mu + i\frac{\hbar}{2}\partial^\mu$

- **Zilch Vortical Effect**

Typical terms are of the form

$$\langle : \tilde{F}^\lambda{}_\mu\partial_\rho F_{\nu\lambda} :\rangle = \frac{1}{2}\int d^4p\,k_\rho\epsilon^\lambda{}_\mu{}^{\sigma\delta}Y_{\sigma\delta\nu\lambda}$$

$$\boxed{\bar{Z}^{\mu00} = \frac{2\pi^2T^4}{27}\omega^\mu}$$

in agreement with CKT!

- $\bar{Z}^{(s)}_{\alpha_1\cdots\alpha_s}$ from Wigner function also agrees with CKT

Outlook

- Origin of Chiral Vortical Effects
- Transport theory for vector particles
- Polarization effects in heavy-ion collisions

References

[1] A. Avkhadiev and A. V. Sadofyev, Phys. Rev. D **96**, no. 4, 045015 (2017).
[2] N. Yamamoto, Phys. Rev. D **96**, no. 5, 051902 (2017).
[3] X. G. Huang and A. V. Sadofyev, JHEP **1903**, 084 (2019).
[4] M. N. Chernodub, A. Cortijo and K. Landsteiner, Phys. Rev. D **98**, no. 6, 065016 (2018).
[5] D. E. Kharzeev, J. Liao, S. A. Voloshin and G. Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016).
[6] H. Lipkin, J. Math. Phys. **5**, 696 (1964).
[7] T. W. B. Kibble, J. Math. Phys. **6**, 1022 (1965).
[8] D. Vasak, M. Gyulassy and H. T. Elze, Annals Phys. **173**, 462 (1987).