Multiparticle correlations from the direct calculation of cumulants using particle azimuthal angles

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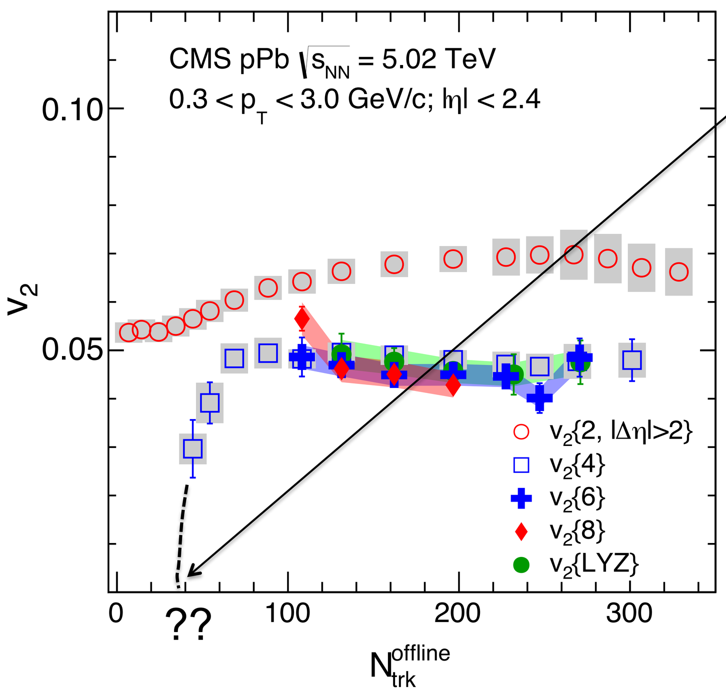
Motivation

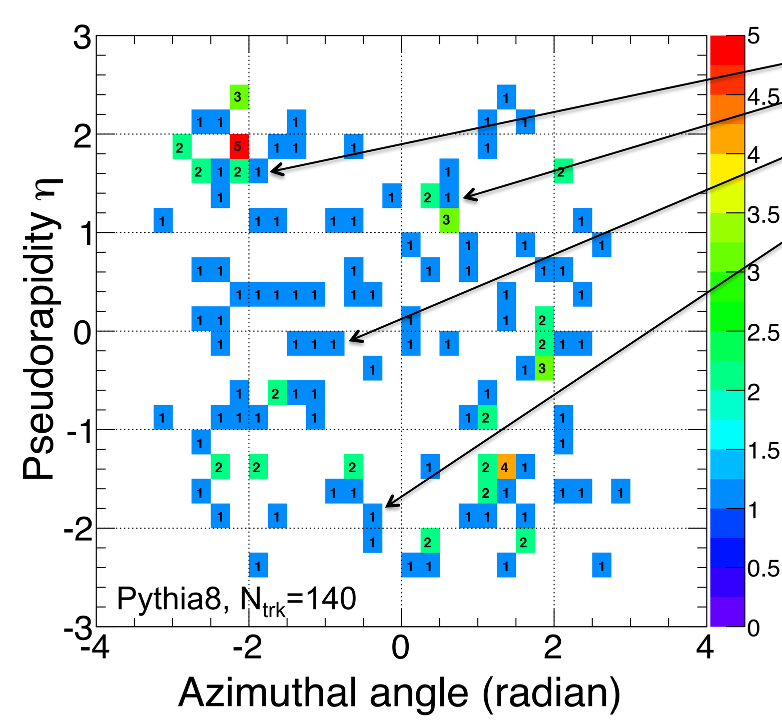
What is the smallest possible droplet of QGP?

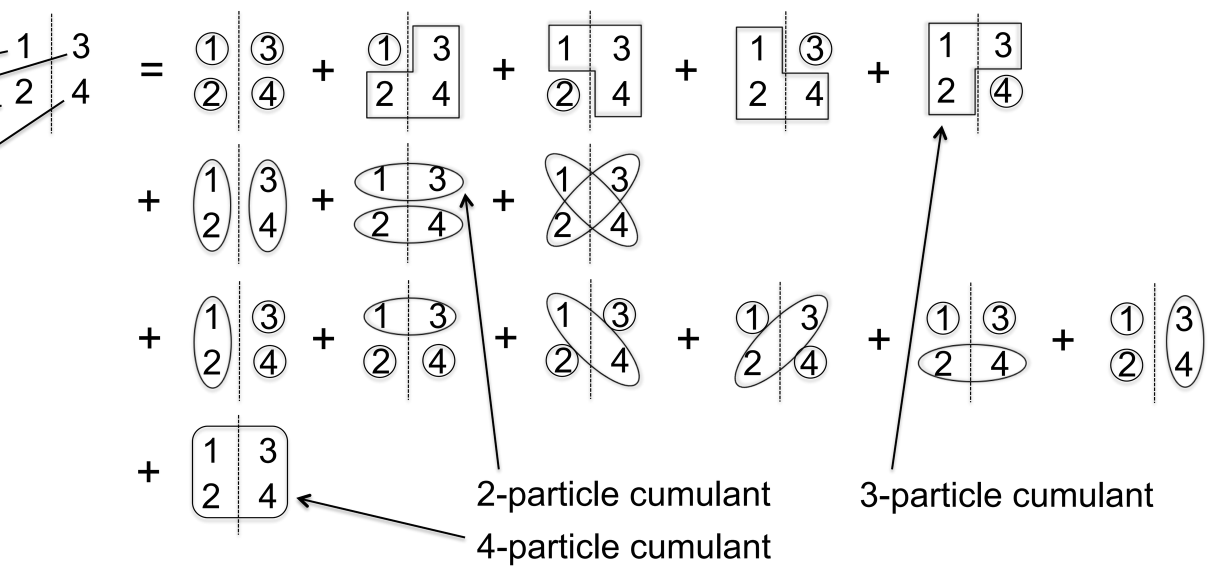


- What is the smallest N_{trk}?
- How the nonflow affect it?
- How to measure 6 and more particle correlations at N_{trk}<100?

To understand how the nonflow affects multiparticle correlations, we propose calculating cumulants directly using particle azimuthal angles (Looping) in small systems.

Multiparticle correlations





$$\langle 4\rangle_{n,n|n,n} \equiv \frac{1}{\binom{M}{4}4!}\sum_{\substack{i,j,k,l=1\\(i\neq j\neq k\neq l)}}^{M} e^{in(\phi_i+\phi_j-\phi_k-\phi_l)}$$

Standard cumulant method

To avoid using nested loops, introduce the Q-vector [1]:

$$Q_n \equiv \sum_{i=1}^{M} e^{in\phi_i} \qquad \langle 2\rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

$$|Q_n|^4 = Q_nQ_nQ_n^*Q_n^* = \sum_{i,j,k,l=1}^{M} e^{in(\phi_i+\phi_j-\phi_k-\phi_l)}$$

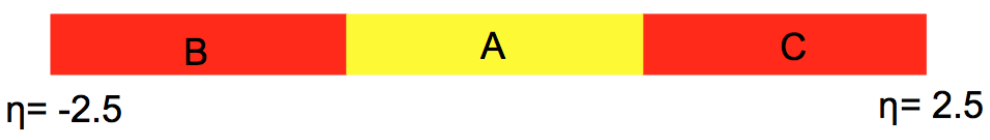
$$\langle 4\rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2\cdot\mathfrak{Re}\left[Q_{2n}Q_n^*Q_n^*\right]}{M(M-1)(M-2)(M-3)} - 2\,\frac{2(M-2)\cdot|Q_n|^2 - M(M-3)}{M(M-1)(M-2)(M-3)}\,.$$

$$c_n\{4\} = \langle\langle 4\rangle\rangle - 2\cdot\langle\langle 2\rangle\rangle^2 \qquad v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$

Subevent cumulant method

Based on the standard method, introducing subevents [2]:

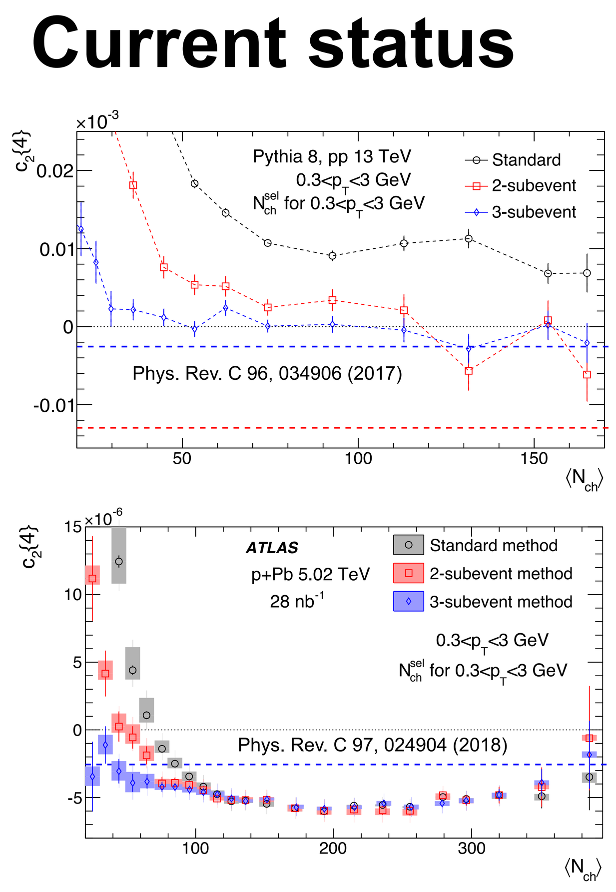
(a) Three subevents

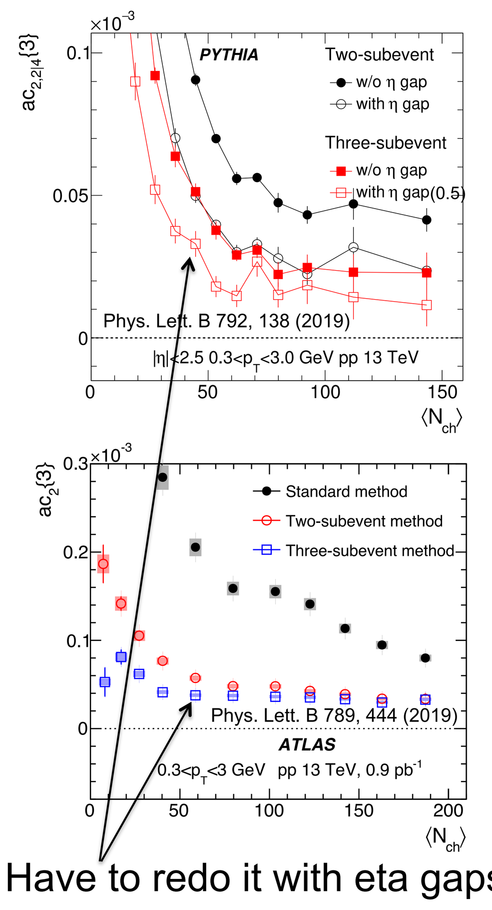


(b) Two subevents

Calculate Q-vectors in different subevents, then estimate cumulants and flow

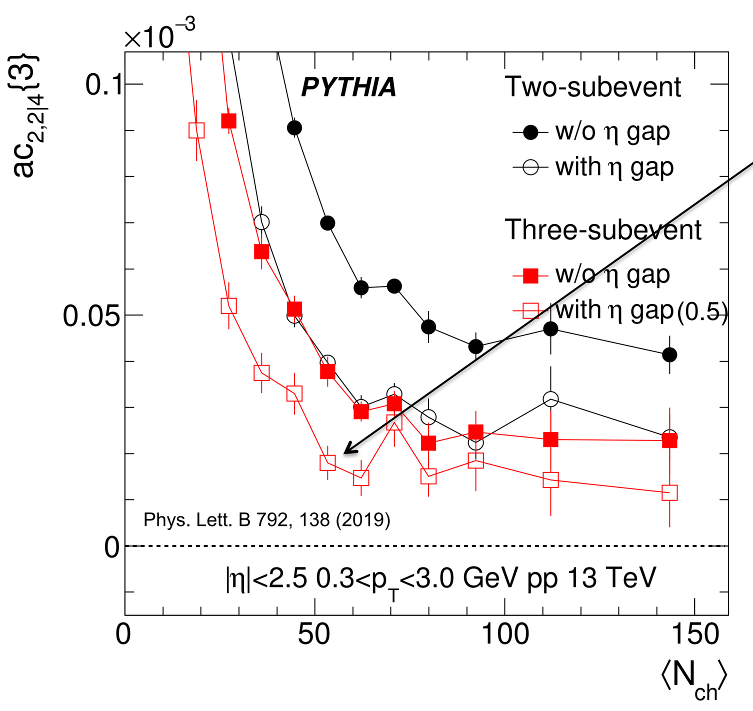
Current status





Have to redo it with eta gaps

Introducing the Looping method



- Need to study correlations vs. η gap
- Run out of statistics quickly
- Any solutions?
- Recall that the idea behind Q-cumulant method is to avoid using nested loops

- If we just loop over particle azimuthal angles, it is very easy to study the η gap dependence
- Also with much better statistics since it keeps all possible combinations

$$\langle 4\rangle_{n,n|n,n} \equiv \frac{1}{\binom{M}{4}4!}\sum_{\substack{i,j,k,l=1\\(i\neq j\neq k\neq l)}}^{M} e^{in(\phi_i+\phi_j-\phi_k-\phi_l)}$$

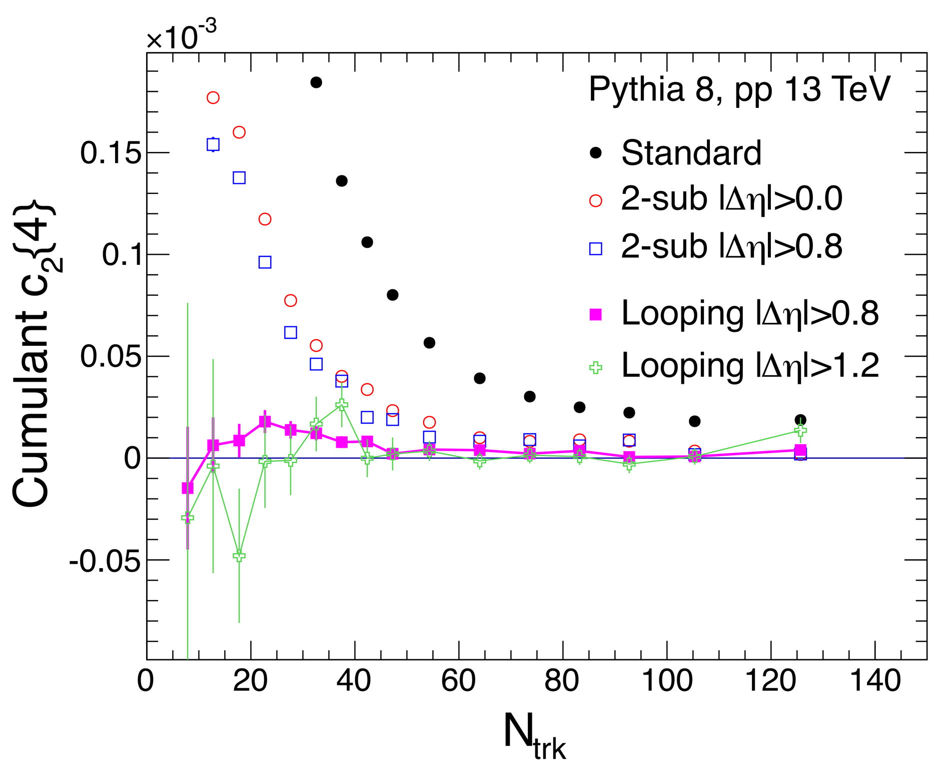
Time complexity:

- O(N_{trk}^n), for n particle cumulant with N_{trk} total number of particles per event
- The 8 particle cumulant in an event with 1000 particles will take ~1 billion years
- However, our interest is in the small system with N_{trk} less than 100
- It takes a few seconds to calculate the 4 particle cumulant with N_{trk}=100
- It could be much faster after applying η gaps between particles

Results

Cumulants

$$c_n\{4\} = \langle\langle 4\rangle\rangle - 2\cdot\langle\langle 2\rangle\rangle^2$$

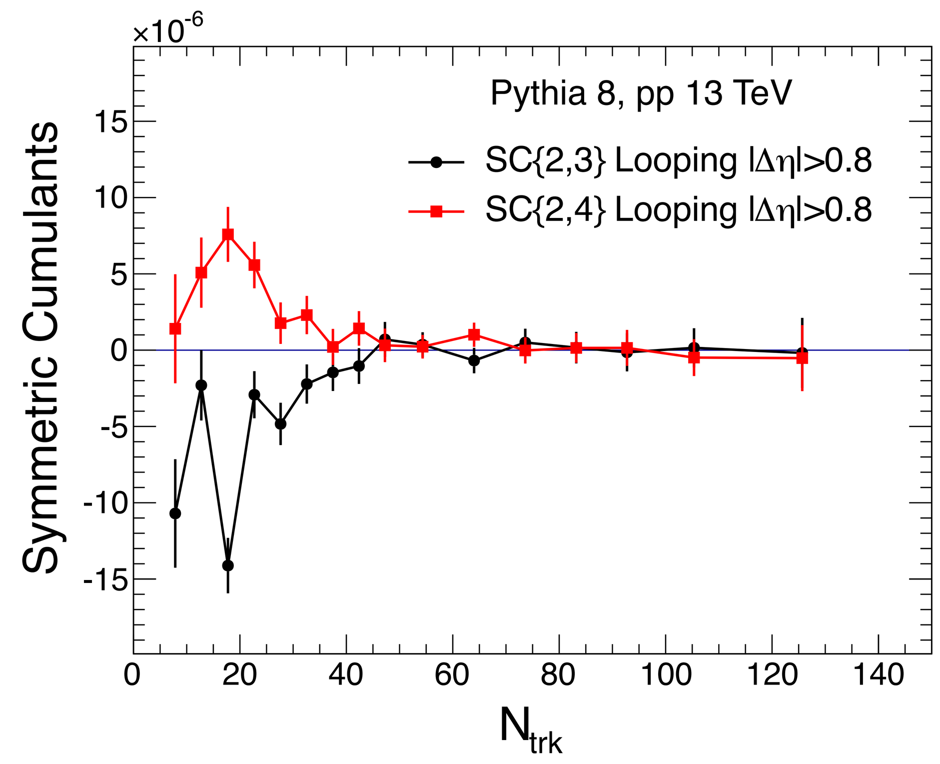


- Introducing subevents makes a huge difference
- Looping method removes much more nonflow

Symmetric Cumulants

$$SC(n,m) = \langle\langle\{4\}_{n,m}\rangle\rangle - \langle\langle\{2\}_n\rangle\rangle\langle\langle\{2\}_m\rangle\rangle = \langle\langle e^{in(\phi_1-\phi_2)+im(\phi_3-\phi_4)}\rangle\rangle - \langle\langle e^{in(\phi_1-\phi_2)}\rangle\rangle\langle\langle e^{im(\phi_1-\phi_2)}\rangle\rangle$$

$$SC(n,m)_{\rm flow} = \langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle$$



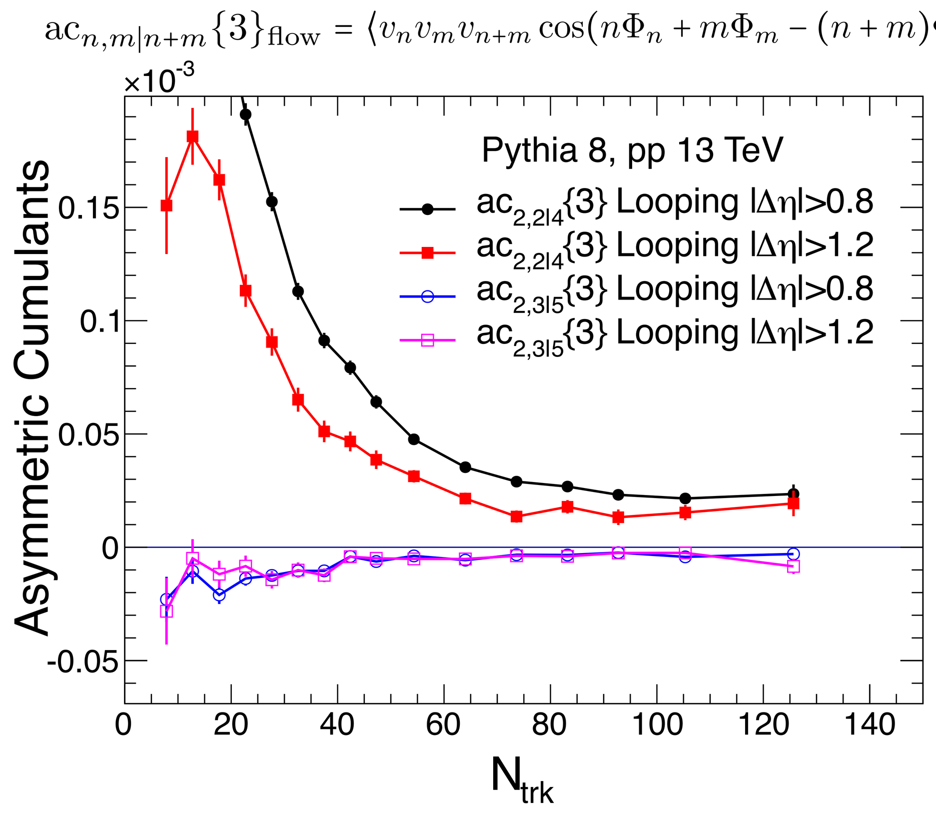
- Looping method suppresses the nonflow for symmetric cumulants

Asymmetric Cumulants

$$\langle 3\rangle_{n,m|n+m} = \langle e^{i(n\phi_1+m\phi_2-(n+m)\phi_3)}\rangle$$

$$ac_{n,m|n+m}\{3\} = \langle\langle 3\rangle\rangle_{n,m|n+m}$$

$$ac_{n,m|n+m}\{3\}_{\rm flow} = \langle v_n v_m v_{n+m}\cos(n\Phi_n + m\Phi_m - (n+m)\Phi_{n+m})\rangle$$



- Not enough for $ac_{2,2|4}\{3\}$ with an eta gap > 0.8

Conclusions

- Turn-on of flow is important for understanding the smallest possible droplet
- The looping method is introduced to suppress nonflow in small systems
- The method could also be used for Symmetric/Asymmetric Cumulants

References

[1] A. Bilandzic, R. Snellings, and S. Voloshin, Phys. Rev. C 83, 044913 (2011)
[2] J. Jia, M. Zhou, and A. Trzupek, Phys. Rev. C 96, 034906 (2017)