Quarkonia - An open quantum system treatment

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Introduction

R_{AA} for Quarkonia for medium-induced dissociation has been traditionally calculated in a classical- rate-equation kind of approach. We implement a quantum-evolution which takes into account the complete color-structure of $Q\bar{Q}$ pair and the transitions between different angular momentum states by means of stochastic-hamiltonian evolution, while assuming that small $\vec{r}$ expansion is justified [Pineda 1998, Brambilla 2000]. We also compare our result with traditionally used rate-equation approaches,
$R_{AA} = e^{-\int_0^t dt' \Gamma(t')}$.

Setup:

We treat quarkonia as an open quantum system, interacting with medium as it evolve in time. The main equation we implement in our calculation is

$$\mathrm{H(t)} = \begin{pmatrix} H_0^S(\vec{r}) & 0 & 0 & \frac{1}{\sqrt{2N_c}}\vec{r}\cdot\vec{\theta}^c(t)\delta_{ac} \\ 0 & H_1^S(\vec{r}) & \frac{1}{\sqrt{2N_c}}\vec{r}\cdot\vec{\theta}^c(t)\delta_{ac} & 0 \\ 0 & \frac{1}{\sqrt{2N_c}}\vec{r}\cdot\vec{\theta}^c(t)\delta_{ac} & H_0^{O,a}(\vec{r}) + f^{abc}\theta^c(t) & \frac{d^{abc}}{2}\vec{r}\cdot\vec{\theta}(t) \\ \frac{1}{\sqrt{2N_c}}\vec{r}\cdot\vec{\theta}^a(t)\delta_{ac} & 0 & \frac{d^{abc}}{2}\vec{r}\cdot\vec{\theta}(t) & H_1^{O,a}(\vec{r}) + f^{abc}\theta^c(t) \end{pmatrix}$$

Where,

- 'S' and 'O' denotes color-state of $Q\bar{Q}$ pair, singlet and octet respectively. The indices 'a, b, c' runs from 1 to $N_C^2 - 1$.
- $l = 0, 1$ denotes angular momentum states S and P respectively.
- $V_S(t) = \frac{-C_F\alpha}{r}e^{-m_D r}$, $V_O(t) = \frac{\alpha}{2N_C r}e^{-m_D r}$
- $T(t) = T_0(\frac{t_0}{t_0+t})^{(1/3)}$, where $T_0 = 0.475\mathrm{GeV}, t_0 = 0.6\mathrm{fm}$

Where the noise '$\theta^a(t)$' is generated using the following correlation functions

$$\langle\vec{\theta}_i^a(t)\rangle = 0$$
$$\langle\vec{\theta}_i^a(t)\vec{\theta}_j^b(t')\rangle = \delta^{ab}\delta_{ij}\delta(t-t')\kappa \quad (1)$$

for decoherence [Akamatsu 2015] and

$$\langle\vec{\theta}_i^a(t)\rangle = 0$$
$$\langle\vec{\theta}_i^a(t)\vec{\theta}_j^b(t')\rangle = \delta^{ab}\delta_{ij}C(t,t') \quad (2)$$

for gluo-dissociation.

Check of small $\vec{r}$ appx.:

To justify the small $\vec{r}$ approximation we do a check for a one-dimensional system, for which results are available in [Akamatsu et al., 2018]. Initial states were chosen to be first three Cornell states, with a Bjorken expanding medium. The noise correlation function and others parameters were kept same as in [Akamatsu et al., 2018].

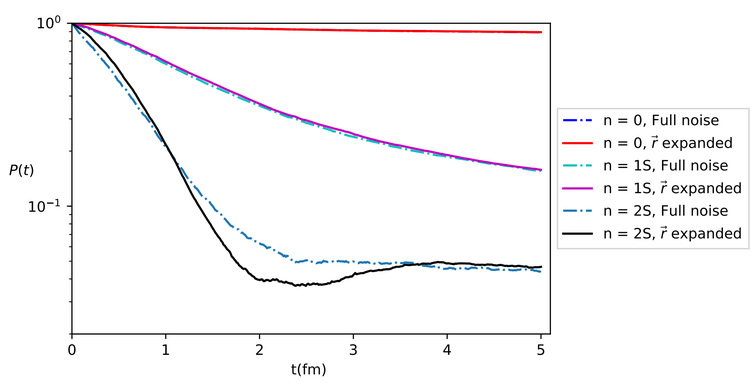

Figure 1: 1D Bjorken, Cornell states

We find that, for wavefunction which satisfies $\langle r\rangle \ll 1/T$, T is temperature, small $\vec{r}$ expansion is reliable.

R_{AA}: Classical vs Quantum

For a rate-equation approach, R_{AA} is given by

$$\frac{dN_\psi(t)}{dt} = -\Gamma_\psi(t)N_\psi(t)$$
$$R_{AA}(t) = e^{-\int_0^t dt'\Gamma(t')} \quad (3)$$

For the two different cases we calculated, the width in leading-order in perturbation theory is

$$\Gamma_{DC}(\psi_0) = \int_{\psi_f} |\langle\psi_f \mid \hat{r} \mid \psi_0\rangle|^2 \kappa(T)$$
$$\Gamma_{GD}(\psi_0) = \int_{\psi_f} |\langle\psi_f \mid \hat{r} \mid \psi_0\rangle|^2 \tilde{F}(\delta E, T) \quad (4)$$
$$\text{where, } \tilde{F}(\omega, T) = \int dt\langle\vec{E}(t)\vec{E}(0)\rangle e^{-i\omega t}$$

'GD'-gluo-dissociation, 'DC'-decoherence and κ is momentum-diffusion cof. at LO in perturbation theory [Huot et al., 2008]. For quantum calculation, it is given by

$$R_{AA}(t) = |\langle\psi_0 \mid \psi_0(t)\rangle|^2$$

At $t = 0\mathrm{fm}$ the value of width for $1S$ states is

$$\Gamma_{DC}(1S, Coulomb) = 0.16\mathrm{GeV} \quad (5)$$
$$\Gamma_{DC}(1S, Cornell) = 0.3\mathrm{GeV} \quad (6)$$
$$\Gamma_{GD}(1S, Coulomb) = 0.12\mathrm{GeV} \quad (7)$$
$$\Gamma_{GD}(1S, Cornell) = 0.33\mathrm{GeV} \quad (8)$$

Results: I

We compare the R_{AA} for a classical calculation vs a quantum calculation, for $1S$ state.

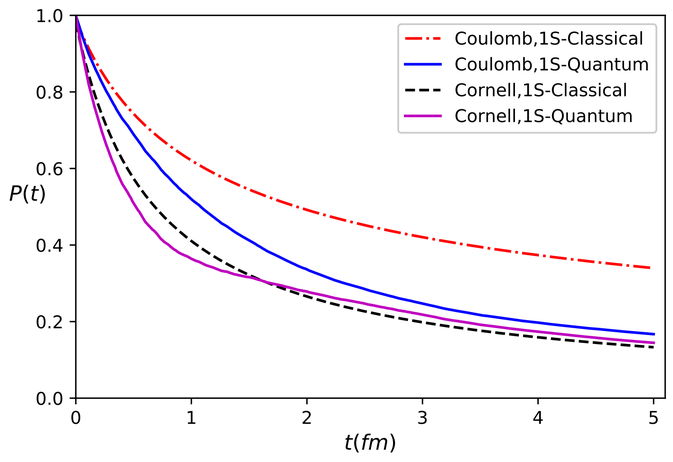

Figure 2: R_{AA} Decoherence, $1S$ Coulomb and Cornell, Quantum vs Classical

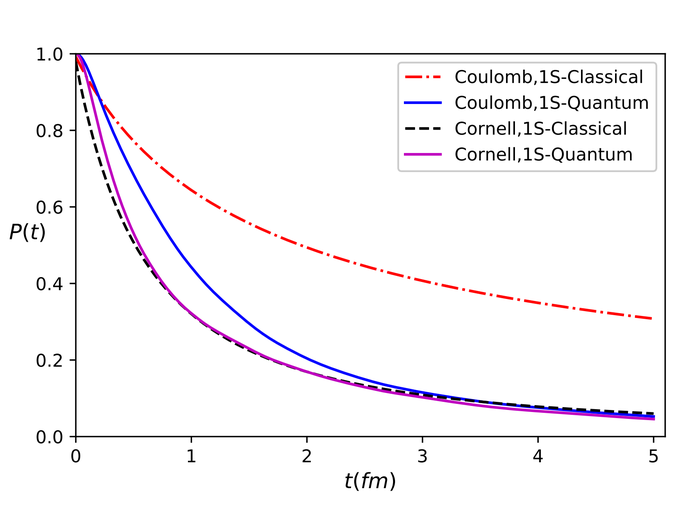

Figure 3: R_{AA} Gluo-dissociation, $1S$ Coulomb and Cornell, Quantum vs Classical

Results:II

Results for both $1S$ and $2S$ states, for comparison.

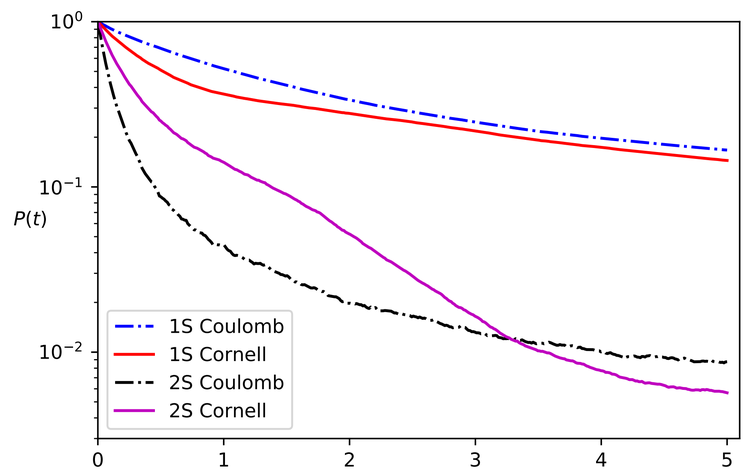

Figure 4: R_{AA} Decoherence, Cornell and Coulomb, $1S$ and $2S$

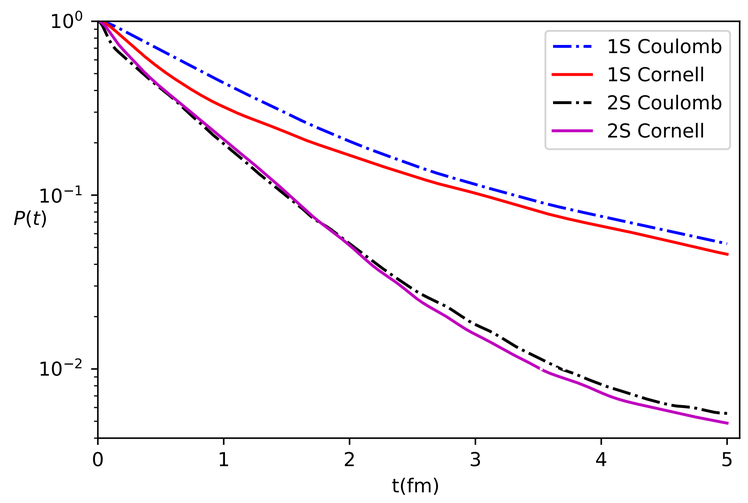

Figure 5: R_{AA} gluo-dissociation, Cornell and Coulomb $1S$ and $2S$

Summary:

- We compare R_{AA} for quarkonia between quantum and classical approaches. Quantum evolution is done for three-dimension case with complete color-structure of $Q\bar{Q}$ pair. Potential depends on time via its temperature dependence.
- Decoherence and gluo-dissociation can be distinguished by finite vs zero energy transfer processes.
- With same initial conditions, gluo-dissociation is a much stronger effect than decoherence. Calculations need to be checked against results from higher-order gluonic-spectral functions and lattice eventually.
- For very narrow states, quantum calculations gives similar results, except for early times where effects of potential change are important.
- Medium induced dissociation has much stronger effects compared to potential effects.
- For $1S$ Cornell states, classical and quantum calculation gives similar results. For $1S$ Coulomb states, they differ significantly.
- $2S$ states are highly suppressed in both cases, both for Coulomb and Cornell case.
- For $2S$ states, we found that the results are not reasonable. It seems that classical and quantum calculations defer in an unpredictable manner.