Curing the numerical Cherenkov instability in 3+1D Glasma simulations

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Particles and Interactions

1 Introduction

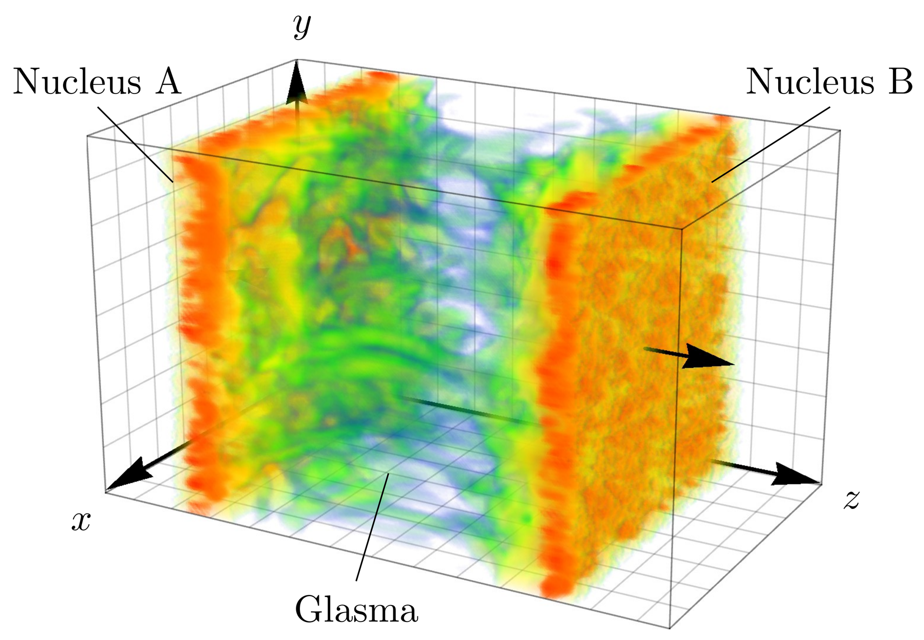

Figure 1: Plot of energy density of color fields in a 3+1D collision from [1].

- Collision of two nuclei in the Color Glass Condensate (CGC) framework
- Creation of the **Glasma**:
  - Intermediate state between CGC and quark-gluon plasma (transition $\tau \lesssim 1\,\mathrm{fm}/c$)
  - Pre-equilibrium stage (before hydrodynamic stage)

2 Simulations in 3+1D

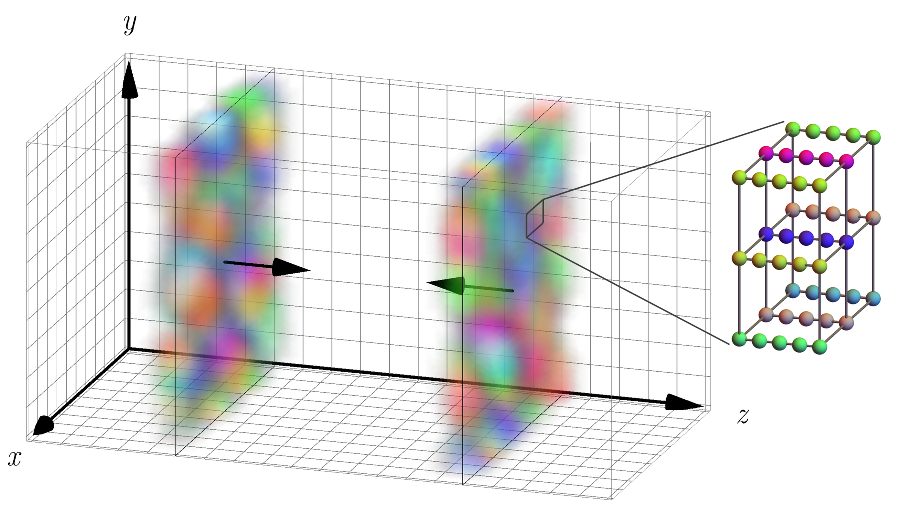

Figure 2: Colored Particle-In-Cell (CPIC) simulation in the laboratory frame [2].

- Collisions at finite collision energy $\sqrt{s_{\mathrm{NN}}}$ with finite thickness of nuclei along beam axis $\propto R/\gamma$
- Colored particle-in-cell (CPIC) simulation contains hard particles and soft fields

Explicitly broken boost invariance

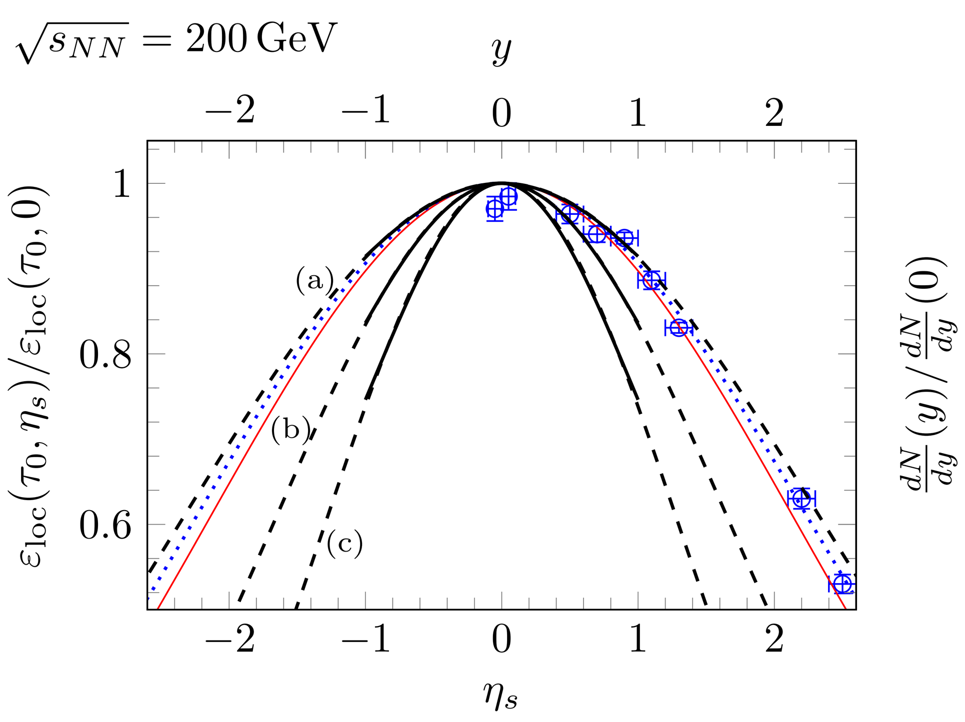

Figure 3: Rapidity profile of local rest frame energy density for $\sqrt{s_{\mathrm{NN}}} = 200\,\mathrm{GeV}$ at $\tau = 1\,\mathrm{fm}/c$ from [1]. Solid black lines: simulation data; (a), (b), (c): different values of infrared regulator. Dashed lines: Gaussian fits. Blue dots and curve: measured pion multiplicities at RHIC. Red solid line: Landau model.

- Rapidity dependence due to classical time evolution: leading order result

3 Variational integrators

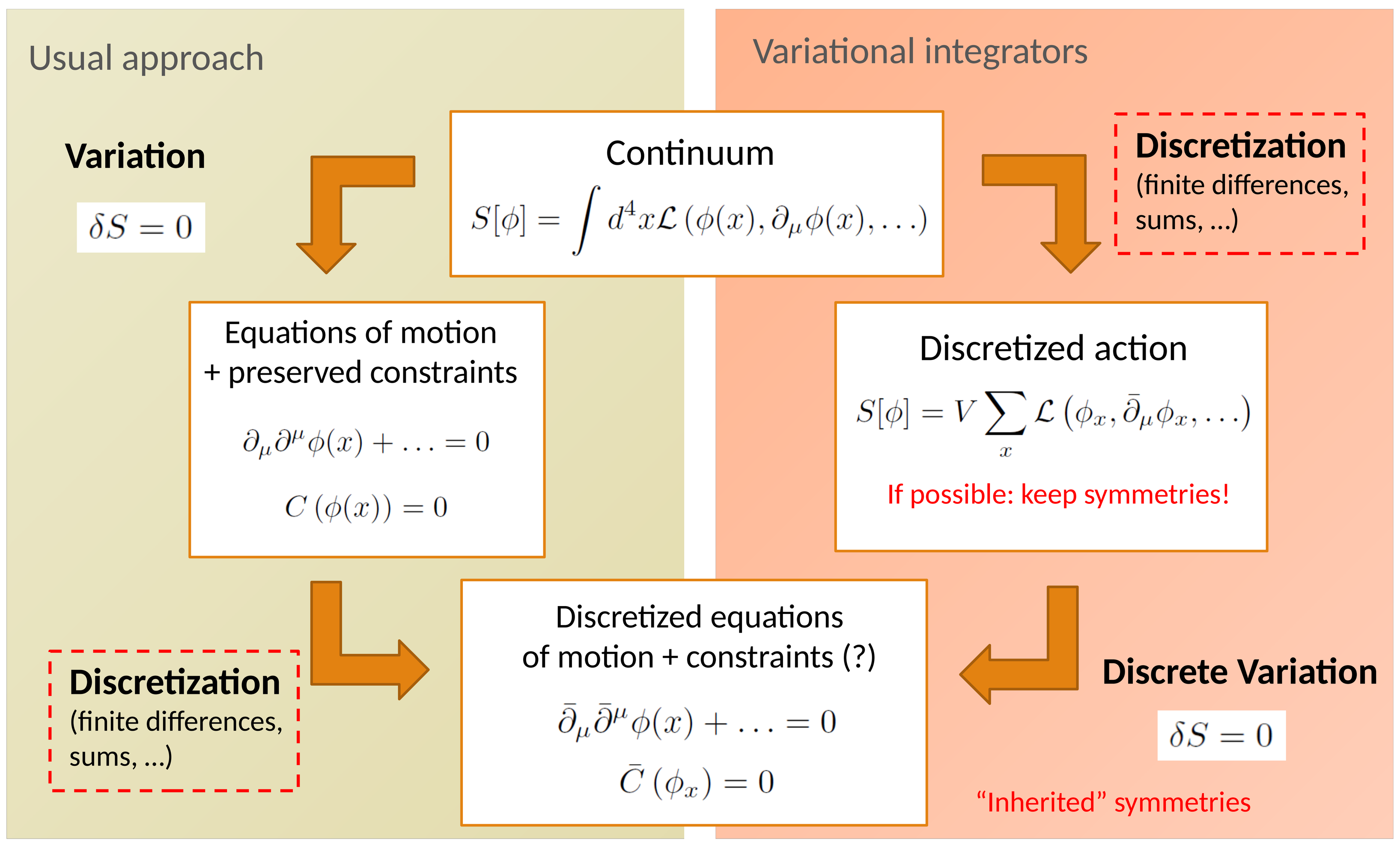

Figure 4: The strategy behind variational integrators: first discretize the action S, then demand $\delta S = 0$.

4 Semi-implicit solver for real-time lattice gauge theory

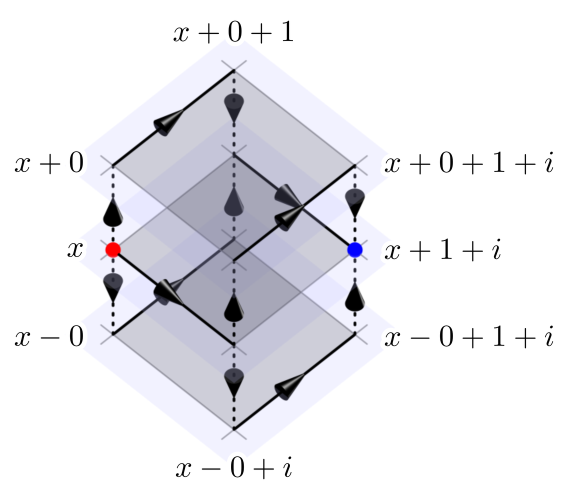

Figure 5: Wilson lines used in the semi-implicit scheme [3].

- Standard **Wilson action**:

$$S[U] = \frac{V}{g^2} \sum_x \left(\sum_i \frac{1}{(a^0 a^i)^2} \mathrm{tr}\left(2 - U_{x,0i} - U^\dagger_{x,0i}\right) - \frac{1}{2} \sum_{i,j} \frac{1}{(a^i a^j)^2} \mathrm{tr}\left(2 - U_{x,ij} - U^\dagger_{x,ij}\right) \right)$$

- Discretized action for **semi-implicit scheme**:

$$S[U] = \frac{V}{g^2} \sum_x \left(\frac{1}{(a^0 a^1)^2} \mathrm{tr}\left(C_{x,01} C^\dagger_{x,01}\right) + \sum_i \frac{1}{(a^0 a^i)^2} \mathrm{tr}\left(C_{x,0i} C^\dagger_{x,0i}\right) \underbrace{- \frac{1}{4} \sum_{i,|j|} \frac{1}{(a^i a^j)^2} \mathrm{tr}\left(C_{x,ij} M^\dagger_{x,ij}\right)}_{\text{implicit part}} \underbrace{- \frac{1}{4} \sum_{|j|} \frac{1}{(a^1 a^j)^2} \mathrm{tr}\left(C_{x,1j} W^\dagger_{x,1j} + \mathrm{h.c.}\right)}_{\text{semi-implicit part}} \right)$$

5 Curing the numerical Cherenkov instability

Numerical Cherenkov instability

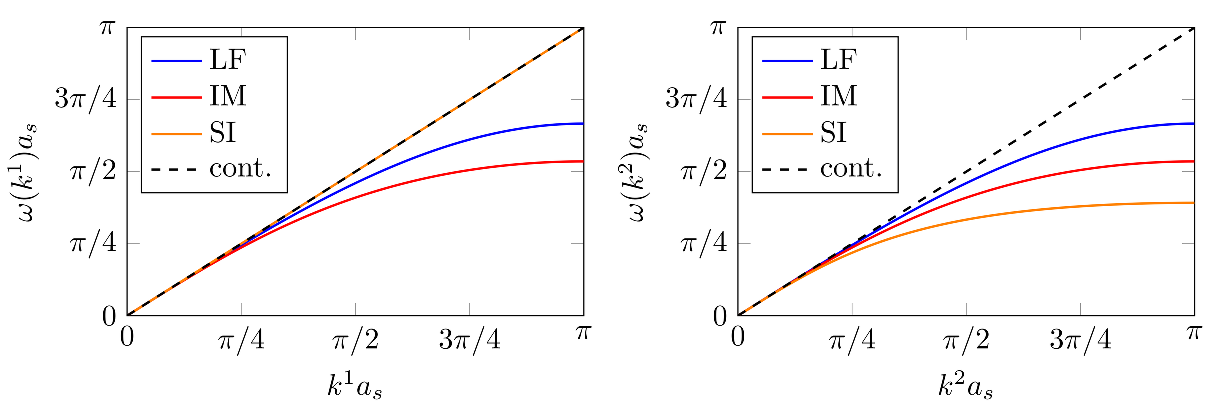

Figure 6: Lattice dispersion for leapfrog (LF), implicit (IM) and semi-implicit (SI) schemes, along propagation direction x^1 and transverse to it x^2 [4].

- High momentum modes propagate slower than the speed of light due to **numerical dispersion**
- Mismatch between particles and fields leads to unphysical Cherenkov radiation of color charges

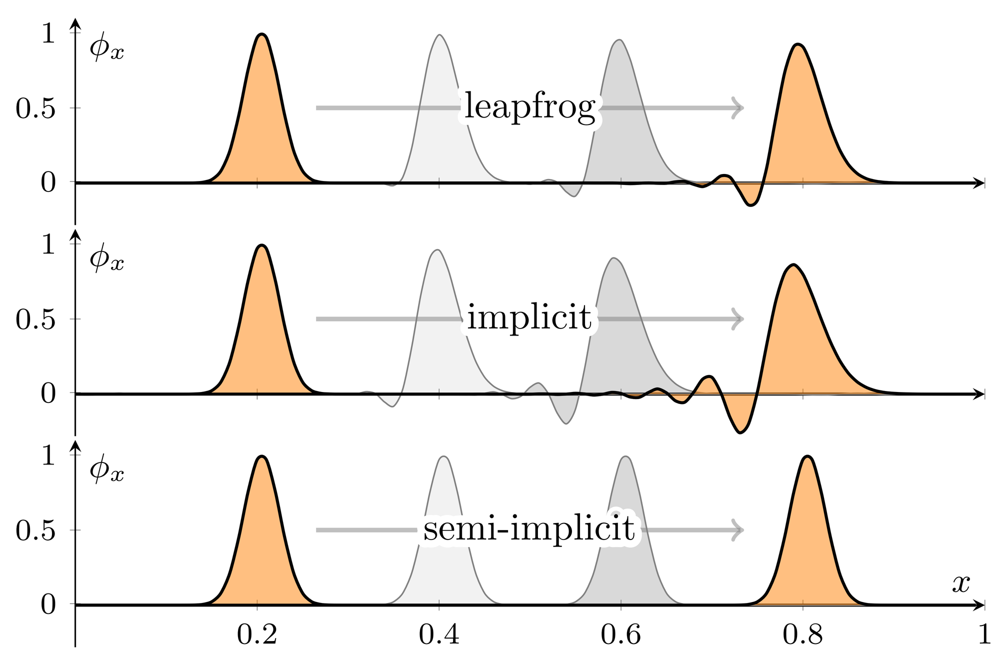

Figure 7: Comparison of numerical dispersion in various schemes [3]: wave pulses disperse over time due to non-linear dispersion relation. New semi-implicit scheme is free of dispersion along propagation direction and preserves pulse shape. Analogous phenomenon present in lattice gauge theory, where this drives a numerical instability. The semi-implicit scheme eliminates this problem entirely.

6 Summary & References

- **3+1D setup** for studying collisions at finite collision energy within CGC framework
- **Explicit breaking of boost invariance** from classical time evolution (leading order)
- **New semi-implicit scheme** to study complicated initial conditions at higher energies

[1] A. Ipp and D. Müller, PLB **771**, 74 (2017) [arXiv:1703.00017]
[2] D. Gelfand, A. Ipp and D. Müller, PRD **94**, no. 1, 014020 (2016) [arXiv:1605.07184]
[3] A. Ipp and D. Müller, EPJC **78**, no. 11, 884 (2018) [arXiv:1804.01995]
[4] D. Müller, PhD thesis (2019) [arXiv:1904.04267]

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