Quantum Kinetic Theory and spin polarization

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Abstract

We derive quantum kinetic theory for Dirac fermions with the presence of external field and curved spacetime. The resultant framework respects the covariance under the U(1) gauge, local Lorentz, and diffeomorphic transformations. For the chiral system, we study the chiral dynamics in a rotating coordinate and clarify the roles of the Coriolis force and spin-vorticity coupling in generating the chiral vortical effect (CVE). We also show that the CVE is an intrinsic phenomenon of a rotating chiral fluid, and thus independent of observer's frame. For massive fermions, we derive the kinetic equation and the spin evolution equation. Spin polarization induced by the magnetic field and vorticity is investigated from the kinetic theory, and the results are available in non-equilibrium state.

Wigner operator and curved spacetime

- Wigner operator

$$\hat{W}_{\alpha\beta}(x,p) \equiv \int \frac{\sqrt{-g}d^4y}{(2\pi)^4} e^{-ip\cdot y/\hbar}\left[\bar{\psi}(x)e^{1/2y\cdot\overleftarrow{\nabla}}\right]_\beta \left[e^{-1/2y\cdot\nabla}\psi(x)\right]_\alpha. \quad (1)$$

- Horizontal lift (Winter. 1985; Calzetta, Habib, Hu. 1988; Fonarev. 1994)

$$\nabla_\mu \equiv D_\mu - \Gamma^\lambda_{\mu\nu}y^\nu\frac{\partial}{\partial y^\lambda} + \Gamma^\lambda_{\mu\nu}p_\lambda\frac{\partial}{\partial p_\nu} \underbrace{+\Gamma_\mu + \frac{i}{\hbar}A_\mu}_{\text{connection for spinor}}, \quad (2)$$

where D_μ is the usual covariant derivative operator, $\Gamma_\mu \equiv -\frac{i}{4}\omega^{ab}_\mu\sigma_{ab}$ with $\sigma_{ab} = \frac{i}{2}[\gamma_a,\gamma_b]$ and ω^{ab}_μ the vierbein connection. Vierbein: $e^a = e^a_\mu\partial^\mu$.

Solving the Wigner function

- Dynamic equation for the wigner function

$$\left[\gamma^\mu\left(\Pi_\mu + \frac{i\hbar}{2}\Delta_\mu\right) - m\right]W = O(\hbar^2), \quad (3)$$

where $\Pi_\mu = p_\mu + O(\hbar^2)$ and $\Delta_\mu = D_\mu - F_{\mu\lambda}\partial^\lambda_p + O(\hbar)$.

- Decomposition of the Wigner function

$$W = \frac{1}{4}[\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathcal{S}_{\mu\nu}]. \quad (4)$$

- Solutions up to $O(\hbar)$
  - $\mathcal{P}$, $\mathcal{F}$ and $\mathcal{S}^{\mu\nu}$ can be expressed by $\mathcal{V}^\mu$ and $\mathcal{A}^\mu$.
  - In classical limit $\hbar \to 0$

$$\mathcal{V}^\mu_{(0)} = 4\pi p^\mu f^{(0)}\delta(p^2-m^2), \quad (5)$$
$$\mathcal{A}^\mu_{(0)} = 4\pi\mathscr{A}^\mu_{(0)}\delta(p^2-m^2), \quad (6)$$

with $p_\mu\mathscr{A}^\mu_{(0)}\delta(p^2-m^2) = 0$.

  - In $O(\hbar)$

$$\mathcal{V}^\mu_{(1)} = 4\pi\hbar\left\{\left(p^\mu f^{(1)} + \frac{1}{2p\cdot n}\epsilon^{\mu\nu\rho\sigma}n_\nu\Delta_\rho\mathscr{A}^{(0)}_\sigma\right)\delta(p^2-m^2) + \tilde{F}^{\mu\nu}\left(\mathscr{A}^{(0)}_\nu - \frac{p\cdot\mathscr{A}^{(0)}}{p\cdot n}n_\nu\right)\delta'(p^2-m^2)\right\}, \quad (7)$$
$$\mathcal{A}^\mu_{(1)} = 4\pi\hbar\{\mathscr{A}^\mu_{(1)}\delta(p^2-m^2) + \tilde{F}^{\mu\nu}p_\nu f^{(0)}\delta'(p^2-m^2)\}, \quad (8)$$

where n^μ is a unit timelike frame vector and $p_\mu\mathscr{A}^\mu_{(1)}\delta(p^2-m^2) = 0$.

  - rewrite $\mathscr{A}^\mu_{(0)} = \mathscr{A}^\mu_{(0)\perp} + p_\mu f^{(0)}_5$, where $p_\mu\mathscr{A}^\mu_{(0)\perp} = 0$.

Chiral kinetic theory

- Solutions at m=0

$$\mathcal{R}^\mu/\mathcal{L}^\mu = 4\pi\{[p^\mu f_{R/L} \pm \hbar\Sigma^{\mu\nu}_n\Delta_\nu f_{R/L}]\,\delta(p^2) \pm\hbar\tilde{F}^{\mu\nu}p_\nu f_{R/L}\delta'(p^2)\}, \quad (9)$$

where $\mathcal{R}^\mu/\mathcal{L}^\mu \equiv \frac{1}{2}(\mathcal{V}^\mu \pm \mathcal{A}^\mu)$, and $\Sigma^{\mu\nu}_n = \frac{1}{2p\cdot n}\epsilon^{\mu\nu\rho\sigma}p_\rho n_\sigma$ is the spin tensor for chiral fermion.

- Covariant chiral kinetic equation

$$0 = \delta(p^2 \mp \hbar F_{\alpha\beta}\Sigma^{\alpha\beta}_n)\left[p_\mu\Delta^\mu f_{R/L} \pm \frac{\hbar}{p\cdot n}\tilde{F}_{\mu\nu}n^\mu\Delta^\nu f_{R/L} \pm \hbar\Delta^\mu\left(\Sigma^n_{\mu\nu}\Delta^\nu f_{R/L}\right)\right]. \quad (10)$$

Chiral fermions and rotating frame

- External field

$$0 = \left\{\left(1 + \frac{\hbar(\mathbf{B}\cdot\mathbf{p})}{2|\mathbf{p}|^3}\right)\partial_t + \left(\mathbf{v} + \frac{\hbar\mathbf{B}}{2|\mathbf{p}|^2} + \frac{\hbar}{2|\mathbf{p}|^3}[(\mathbf{E}-\nabla\epsilon_p)\times\mathbf{p}]\right)\cdot\nabla + (\mathbf{E}-\nabla\epsilon_p)\cdot\nabla_\mathbf{p} + \mathbf{v}\times\mathbf{B}\cdot\nabla_\mathbf{p} + \frac{\hbar[(\mathbf{E}-\nabla\epsilon_p)\cdot\mathbf{B}]\mathbf{p}}{2|\mathbf{p}|^3}\cdot\nabla_\mathbf{p}\right\}f_r, \quad (11)$$

where $\epsilon_p \equiv p_0 = |\mathbf{p}| - \frac{\hbar\mathbf{B}\cdot\mathbf{p}}{2|\mathbf{p}|^2}$ and $\mathbf{v} \equiv \frac{\partial\epsilon_p}{\partial\mathbf{p}}$. (Son, Yamamoto. 2013; Hidaka, Pu, Yang. 2017; Huang, Shi, Jiang, et al. 2018.)

$$\mathbf{J} = \int d^3\mathbf{p}\left(\mathbf{v} + \frac{\hbar\mathbf{B}}{2|\mathbf{p}|^2} + \frac{\hbar}{2|\mathbf{p}|^3}\mathbf{E}\times\mathbf{p} - \frac{\hbar}{2|\mathbf{p}|^3}\epsilon_\mathbf{p}\mathbf{p}\times\nabla\right)f_r. \quad (12)$$

- Co-rotating fluid in a rotating frame

$$0 = \left\{\left(1 + \hbar\frac{\mathbf{k}\cdot\mathbf{\Omega}}{|\mathbf{k}|^2}\right)\partial_t + \left(1 + \hbar\frac{\mathbf{k}\cdot\mathbf{\Omega}}{|\mathbf{k}|^2}\right)\left[\hat{\mathbf{k}} + \hbar\left(\frac{\mathbf{\Omega}}{2|\mathbf{k}|} - \frac{(\mathbf{k}\cdot\mathbf{\Omega})\mathbf{k}}{2|\mathbf{k}|^3}\right)\right]\cdot\nabla + 2\mathbf{k}\times\mathbf{\Omega}\cdot\frac{\partial}{\partial\mathbf{k}}\right\}f_r. \quad (13)$$

Dispersion relation $\tilde{\epsilon}_\mathbf{k} = |\mathbf{k}| - \frac{\hbar\mathbf{\Omega}\cdot\mathbf{k}}{2|\mathbf{k}|}$

- Two types of the correspondence: $\mathbf{B} \Longleftrightarrow |\mathbf{k}|\mathbf{\Omega}$ in the dispersion relation and $\mathbf{B} \Longleftrightarrow 2|\mathbf{k}|\mathbf{\Omega}$ elsewhere.

Massive kinetic theory

- Removing the frame vector n^μ from the massive kinetic theory

We have $f^{(0)}_5\delta(p^2-m^2) = 0$ and $\mathcal{A}^\mu_{(0)} = 4\pi\mathscr{A}^\mu_{(0)\perp}\delta(p^2-m^2)$.

  - n^μ is removed with the redefinition

$$f^{(1)} \to f^{(1)} + \frac{1}{2m^2 p\cdot n}\epsilon^{\mu\nu\rho\sigma}p_\mu n_\nu\Delta_\rho\mathscr{A}^{(0)}_{\perp\sigma}. \quad (14)$$

  - The redefinition of $f^{(1)}$ in Eq. (14) is equivalent to identifying the frame n^μ as the particle's rest frame $n^\mu = \frac{p^\mu}{m}$.

- Solutions up to $O(\hbar)$

Defining $m\theta^\mu f_A \equiv \mathscr{A}^\mu_{(0)\perp} + \hbar\mathscr{A}^\mu_{(1)\perp}$, with $p^\mu\theta_\mu = 0$, we obtain

$$\mathcal{V}^\mu = 4\pi\left\{p^\mu f\delta(p^2-m^2) + m\hbar\tilde{F}^{\mu\nu}\theta_\nu f_A\delta'(p^2-m^2) + \frac{\hbar}{2m}\epsilon^{\mu\nu\rho\sigma}p_\nu\Delta_\rho\left(\theta_\sigma f_A\right)\delta(p^2-m^2)\right\}, \quad (15)$$
$$\mathcal{A}^\mu = 4\pi\{m\theta^\mu f_A\delta(p^2-m^2) + \hbar\tilde{F}^{\mu\nu}p_\nu f\delta'(p^2-m^2)\}, \quad (16)$$

where $\Sigma^{\mu\nu}_S = \frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\theta_\rho p_\sigma$ is the spin tensor for massive fermion.

- Massive kinetic equation with $f_{\uparrow/\downarrow} \equiv \frac{1}{2}(f \pm f_A)$

$$0 = \delta(p^2 - m^2 \mp \hbar\Sigma^{\alpha\beta}_S F_{\alpha\beta}) \times\left\{\left[p^\mu\Delta_\mu \pm \frac{\hbar}{2}\Sigma^{\mu\nu}_S\left(\nabla_\rho F_{\mu\nu}\partial^\rho_p + [D_\mu, D_\nu]\right)\right]f_{\uparrow/\downarrow} + \frac{\hbar}{2}(f_\uparrow - f_\downarrow)\left(\nabla_\rho F_{\mu\nu}\partial^\rho_p + [D_\mu, D_\nu]\right)\Sigma^{\mu\nu}_S\right\}, \quad (17)$$

$$p\cdot\Delta\theta^\mu\delta(p^2-m^2) = F^{\mu\nu}\theta_\nu\delta(p^2-m^2) - \frac{1}{f_A}\theta^\mu\left(p\cdot\Delta f_A\right)\delta(p^2-m^2) + \frac{\hbar}{2mf_A}\epsilon^{\mu\nu\rho\sigma}p_\sigma\Delta_\nu\Delta_\rho f\delta(p^2-m^2). \quad (18)$$

Spin polarization

The Pauli-Lubanski vector

$$\Lambda^\mu(x) \equiv -\int_p \frac{1}{\hbar(p\cdot n)}\epsilon^{\mu\nu\rho\sigma}p_\nu\mathcal{S}^C_{\rho\sigma}. \quad (19)$$

$$\Lambda^\mu_{m\neq 0} = \pi\int_p \delta(p^2-m^2)\times\left(4m\theta^\mu f_A - \hbar\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\partial^p_\nu f\right),$$
$$\Lambda^\mu_{m=0} = \pi\int_p \delta(p^2)\left[4\left(p^\mu f_5 + \hbar\Sigma^{\mu\nu}_n\Delta_\nu f\right) - \hbar\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\partial^p_\nu f\right]. \quad (20)$$

Equilibrium state (Becattini, Chandra, Zanna, Grossi. 2013)

$$\Lambda^\mu_{eq(m\neq 0)} = -\pi\hbar\int_p \delta(p^2-m^2)f'_{eq}\left(\epsilon^{\mu\nu\rho\sigma}p_\nu\nabla_\rho\beta_\sigma + \epsilon^{\mu\nu\rho\sigma}\beta_\nu F_{\rho\sigma}\right),$$
$$\Lambda^\mu_{eq(m=0)} = \pi\int_p \delta(p^2)f'_{eq}\left[2p^\mu(\alpha_R - \alpha_L) - \hbar\epsilon^{\mu\nu\rho\sigma}p_\nu\nabla_\rho\beta_\sigma - \hbar\epsilon^{\mu\nu\rho\sigma}\beta_\nu F_{\rho\sigma}\right]. \quad (21)$$