Spin Polarizations in a Covariant Angular Momentum Conserved Chiral Transport Model

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Based on the work: Liu, Sun, and Ko arXiv:1910.06774, Ref[1]

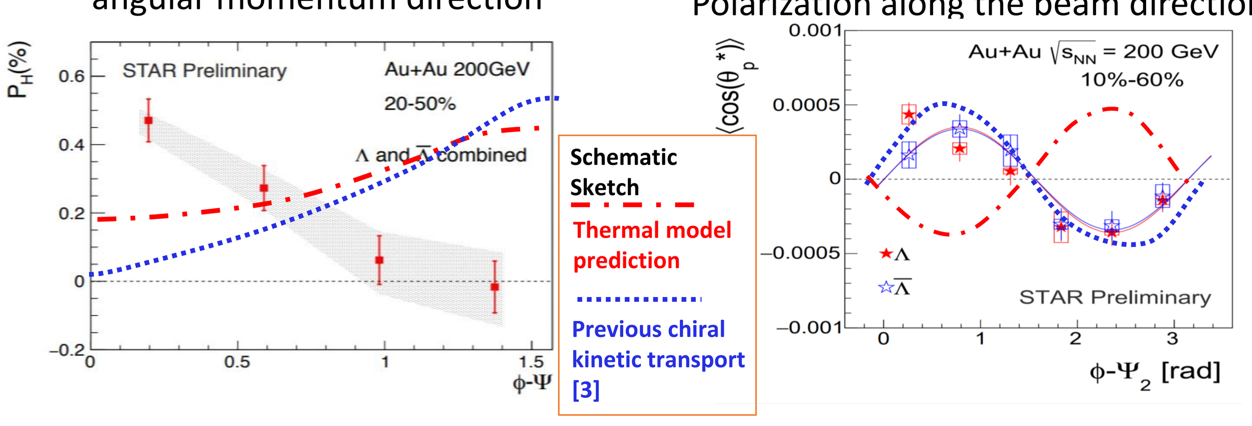
Abstract: Using a covariant and total angular momentum conserved chiral transport model, which takes into account the spin-orbit interactions of chiral fermions in their scatterings via the anomalous side jump effect, we study the quark spin polarizations in quark matter. For a system of rotating and unpolarized massless quarks in an expanding box, we find that the side jump effect can dynamically polarize the quark spin with the final quark spin polarization consistent with that of thermally equilibrated massless quarks in a self-consistent vorticity field. For the quark matter produced in non-central relativistic heavy ion collisions, we find that both the quark local spin polarizations in the direction perpendicular to the reaction plane and along the longitudinal beam direction show an azimuthal angle dependence in the transverse plane similar to those observed in experiments for Lambda hyperons.

Motivation:

1. Spin Puzzles: Disagreements between Theory and Experiments

$-\mathcal{P}_y$: Polarization along the orbital angular momentum direction

$\mathcal{P}_z = \langle\cos(\theta)\rangle/[\alpha_H\langle\cos^2(\theta)\rangle]$
Polarization along the beam direction



- Some physics beyond the thermal model?
- Ambiguity in local thermal equilibrium involving spin and vorticities? [2]
- Previous chiral kinetic transport indicates some new features, but failed for $\mathcal{P}_y$
- Several conceptually important questions in the chiral kinetic framework remain to be solved

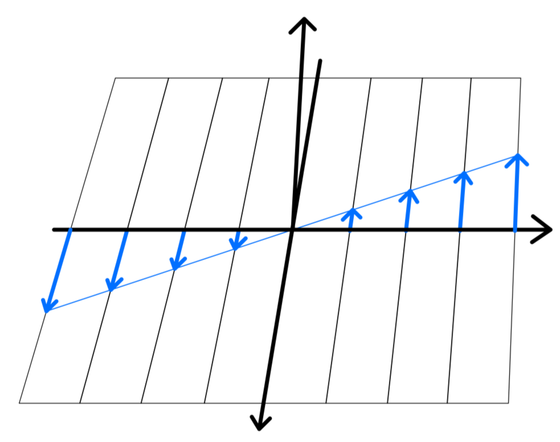
2. Paradox: Chiral Kinetic Equations or Newton's First Law?

- Chiral kinetic equation in previous work

$$\dot{\mathbf{r}}' = \frac{\hat{\mathbf{p}}' + 2\lambda p'(\hat{\mathbf{p}}'\cdot\mathbf{b}')\omega}{1+2\lambda p'(\omega\cdot\mathbf{b}')}$$

  - $v_z \neq 0$ due to anomalous velocity along $\boldsymbol{\omega}$
- Newton's First Law:
  - $v_z = 0$ since particle should move along a straight line

- A thought experiment
  - Preparing a system of particles with vorticity
  - **Turn off all interactions!**
  - Inject a test particle with **v** along x into the system
  - **How should the particle move?**



Contradiction!

What is the theoretical formalism for CVE in consistent with newton's first law?

3.Challenge: total angular momentum conservation

- Power of total angular momentum conservation
  - Chiral kinetic equation by $\mathbf{J} = \mathbf{L} + \mathbf{S}$ conservation in external force:

$$\frac{d\mathbf{J}}{dt} = \frac{d\left(\mathbf{r}\times\mathbf{p} \pm \frac{\hat{\mathbf{p}}}{2}\right)}{dt} = \mathbf{r}\times\mathbf{F}$$

$$\dot{\mathbf{r}} = \hat{\mathbf{p}} \pm \dot{\mathbf{p}} \times \frac{\mathbf{p}}{2p^3} = \hat{\mathbf{p}} + \dot{\mathbf{p}} \times \mathbf{b}$$

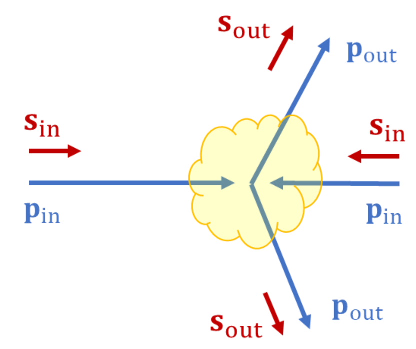
Sun, Ko, Li, 2016

  - Statistical mechanics from conserved quantity:

$$\exp[-(E - \mathbf{v}\cdot\mathbf{P} - \boldsymbol{\omega}\cdot(\mathbf{L}+\mathbf{S})]/T]$$

Polarization: $\langle S\rangle = \omega/(4T)$

- Interactions between partons in the QGP: scattering process
- Does it conserve **J**?



$\mathbf{L}_{\text{out}} + \mathcal{S}_{\text{out}} \neq 0$, Since $\mathcal{S}_{\text{out}} \neq 0$

Not J conserved!

Solution:

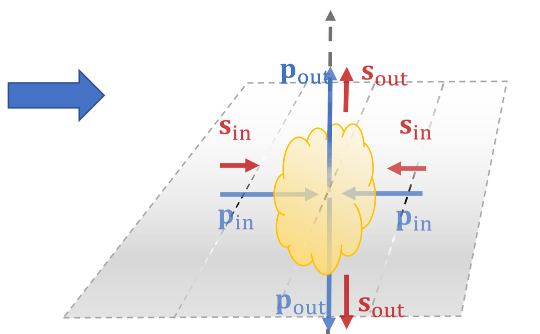
4. J Conservation By Side-Jump

- Conservation of **J** in CM frame

  Relatively Simple

- How to boost this result to the general Lab frame?

  Non-trivial for chiral fermion



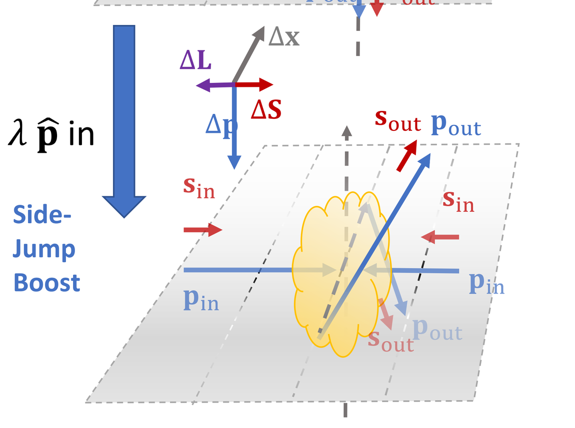
- Requiring the $J' = \Lambda^T J \Lambda$ covariant and $s = \lambda\,\hat{\mathbf{p}}$ in all frames leads to **side-jump boost:**

$$x'^{\mu} = \Lambda^{\mu}{}_{\alpha}x^{\alpha} + \Delta^{\mu}_{\tilde{n}n}$$

- A side-jump term $\Delta_{\tilde{n}n'}$ appear:

$$\Delta^{\mu}_{\tilde{n}n'} = \lambda\frac{\epsilon^{\mu\alpha\beta\gamma}p'_{\alpha}\tilde{n}_{\beta}n'_{\gamma}}{(p'\cdot\tilde{n})(p'\cdot n')},$$

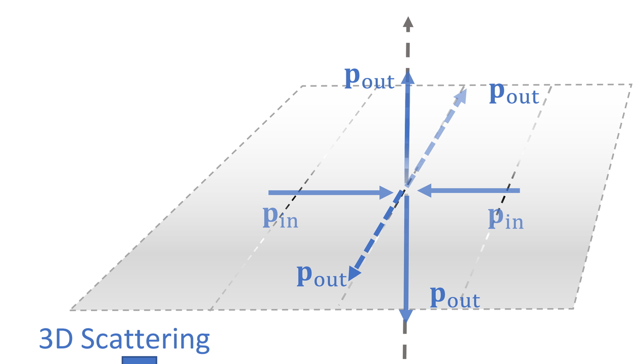
- $\Delta_{\tilde{n}n'} \perp \mathbf{p}'$, and $\Delta_{\tilde{n}n'} \perp \mathbf{P}_t'$ in the lab frame



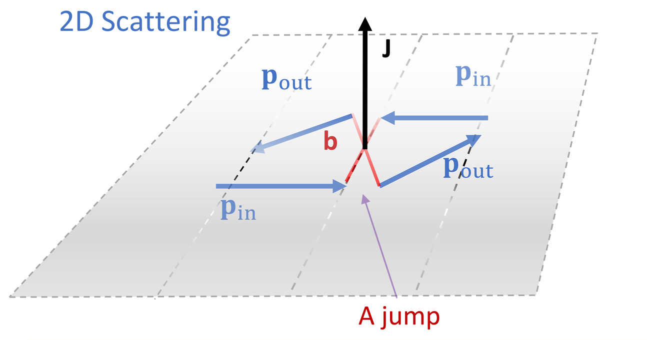
- As shown, $\Delta\mathbf{J} = \Delta\mathbf{L} + \Delta\mathbf{S} = \mathbf{0}$
- **J** is conserved by a side-jump

5. Generalization Required for Transport Simulation

- Idealized zero impact parameter [4],[5]
  - Permittable phase space for $\mathbf{p}_{out}$ is a 3D sphere
  - Non-jump in position in CM frame
- Collision can happen between partons with the same helicity



- Finite impact parameter **b**, [1]
  - Permittable phase space for $\mathbf{p}_{out}$ is a 2D circle in the plane perpendicular to **J**.
  - Jump in position in CM frame
  - Other new features, still numerical feasible
- Collision can happen between partons with same or different helicity
- Using the same side-jump boost to obtain results in the LAB frame.



- $J^{\mu\nu} = x^{\mu}p^{\nu} - x^{\nu}p^{\mu} + S^{\mu\nu}$
  conserved in realistic simulation!

6. How We Address the Paradox and Go Beyond

- The kinetic equation in vortical flows in our approach (No B field):

$$\dot{\mathbf{r}}' = \frac{\hat{\mathbf{p}}' + 2\lambda p'(\hat{\mathbf{p}}'\cdot\mathbf{b}')\omega}{1+2\lambda p'(\omega\cdot\mathbf{b}')} \Rightarrow \dot{\mathbf{r}} = \hat{\mathbf{p}}$$

Usual cross section $\Rightarrow$ Generalized side-jump J conserved collision

- Without interaction (no collision), all particles move in straight line, newton's first law recovered
- With collisions, side-jump collisions will transport the axial charge along the ω direction and the anomalous currents can reproduce chiral vortical effects.

[4], [5]

The paradox is solved by J conserved scattering

Simulation:

7. A Box Calculation as a Benchmark

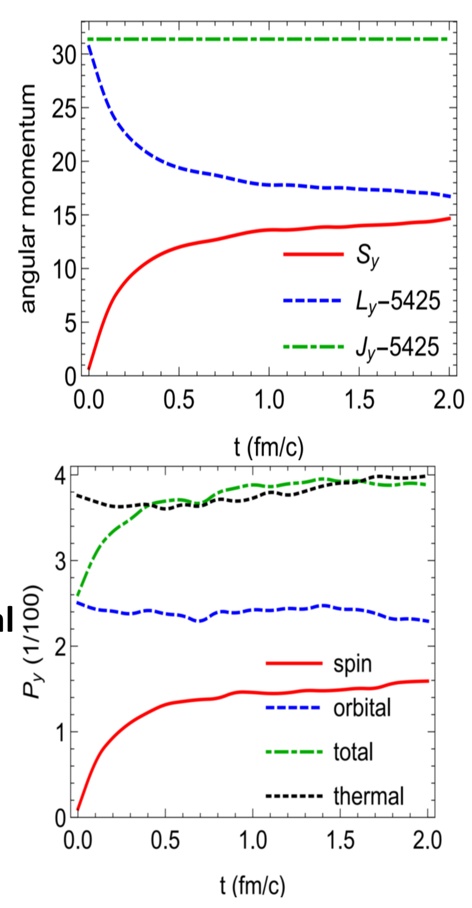
- Box initally at $5\times5\times5$ fm, $\omega = 0.012$/fm (z direction), T = 0.3 GeV, then, free expand
- Check conservation angular $J = \sum_i r_i \times p_i + \lambda_i\hat{p}_i$
- Define covariant current as: [4],[5]

$$j^{\mu}_{R/L}(x) = \int\frac{d^3\mathbf{p}}{(2\pi)^3 p}\left(\boxed{p^{\mu}f_{R/L}} + \boxed{S^{\mu\nu}\partial_{\nu}f_{R/L}}\right).$$

- In addition to the normal spin term, there is an additional magnetization term (orbital term) required by the covariance of the $j^{\mu}_{L/R}$ of chiral fermion.
- The space component of the $j^{\mu}_{R/L}$ is defined as the **total "spin"** so that polarization can be related to $\mathbf{j}_5$ as

$$\mathcal{P} = \int d^3x\,\mathbf{j}_5(x) \Big/ \int d^3x\,n(x)$$

- Recover the thermal benchmark, well defined Lorentz transformation

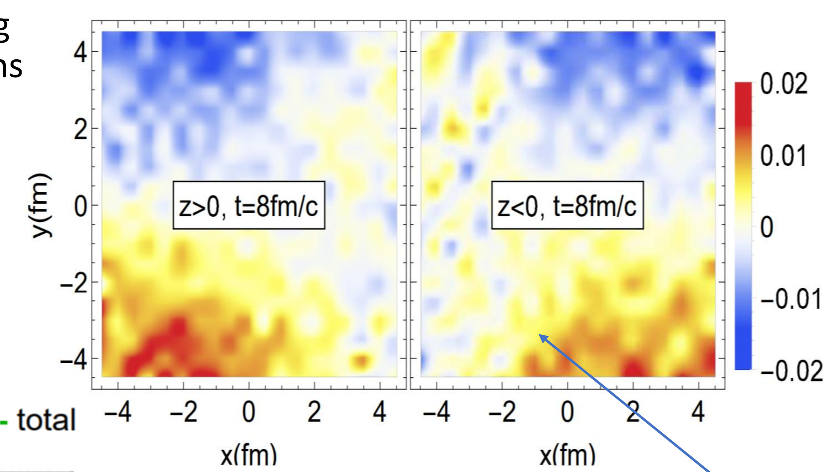


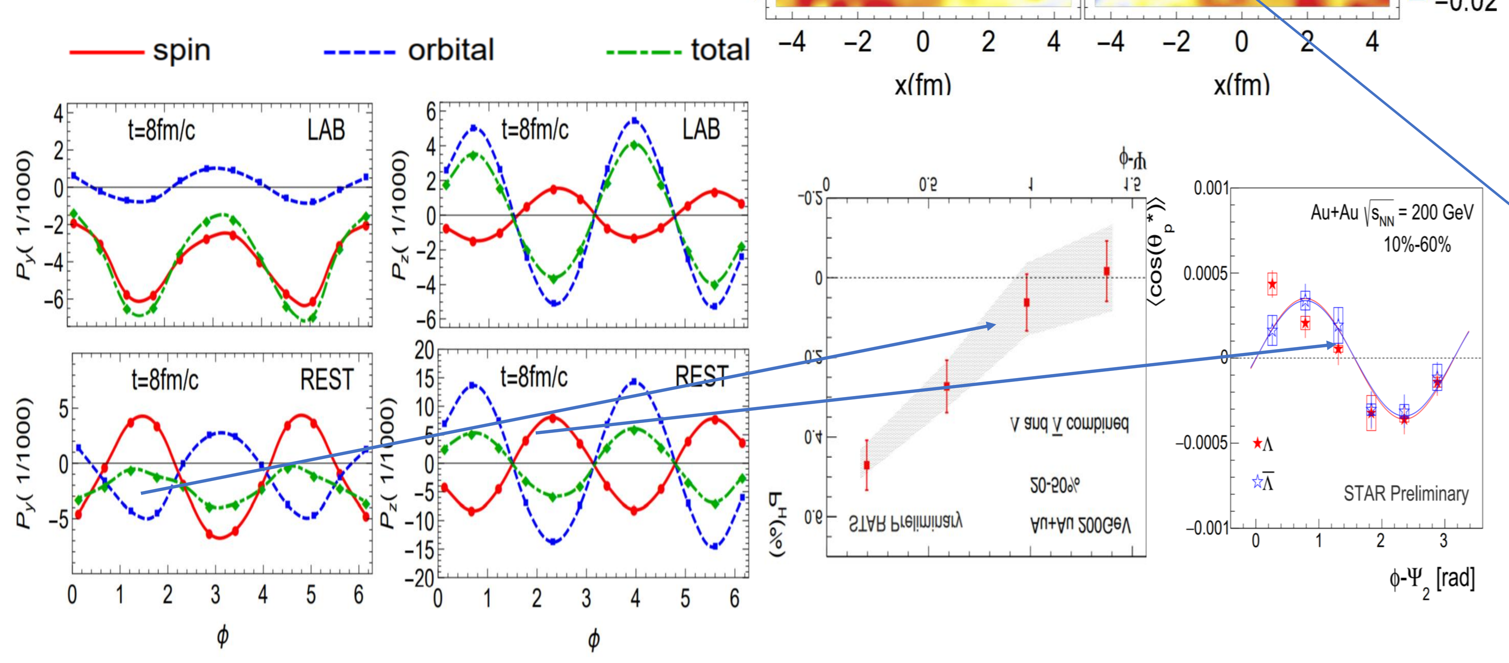
Spin in proton also has an orbital contribution

J conservation dynamically leads to polarization

8. Transport Simulation for Heavy-ion Collision

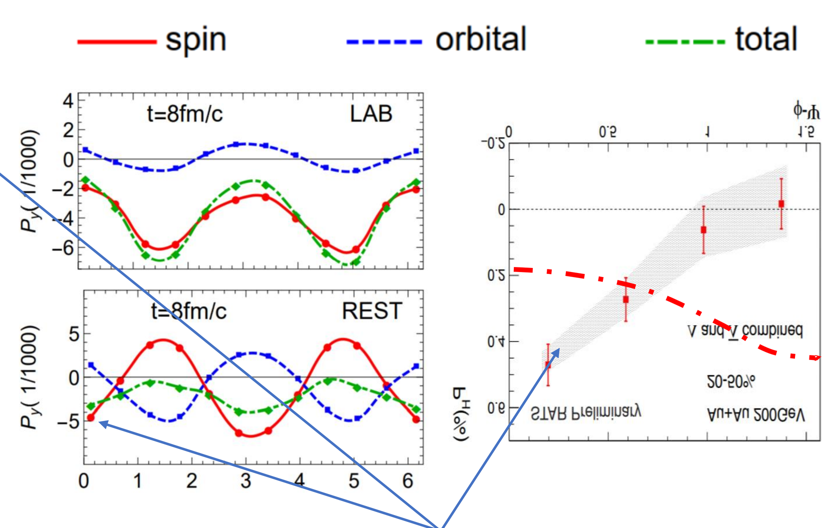
- Large axial charge redistribution according to the vorticity through side-jump collisions
- Both spin part and orbital part are important for total polarization
- Boost affects the result





9. How Axial Charge Redistribution and Boost Affect Polarization?

- Polarization is:
- $j^{\mu}_5 = (n_5, \mathbf{j}_5)$ is a well defined four-vector with the time component
- $(\mathbf{j}_5)_{||} = \gamma\left((\mathbf{j}_5)_{||} - v\,n_5\right)$, $(\mathbf{j}_5)_{\perp} = (\mathbf{j}_5)_{\perp}$
- With the nontrivial distribution of n_5, it affects the space part of $\mathbf{j}_5$, thus the polarization



- Without n_5, we do not get this trend.
- Does this trend provide us indications for the **axial charge redistribution** ?

Reference:

[1] S. Y.F. Liu, Y. Sun, and C. M. Ko, arXiv:1910.06774
[2] Y. Sun, C. M. Ko, Phys.Rev. C99 (2019) no.1, 011903
[3] F. Becattini, W. Florkowski, E. Speranza, Phys.Lett. B789 (2019) 419
[4] J. Y. Chen, D. T. Son, M. A. Stephanov, Phys.Rev.Lett. 115 (2015) no.2, 021601
[5] J. Y. Chen, D. T. Son, M. Stephanov, H.U. Yee, Y. Yin, Phys.Rev.Lett. 113 (2014) no.18, 182302