Relaxation to the CKW state in a transport calculation

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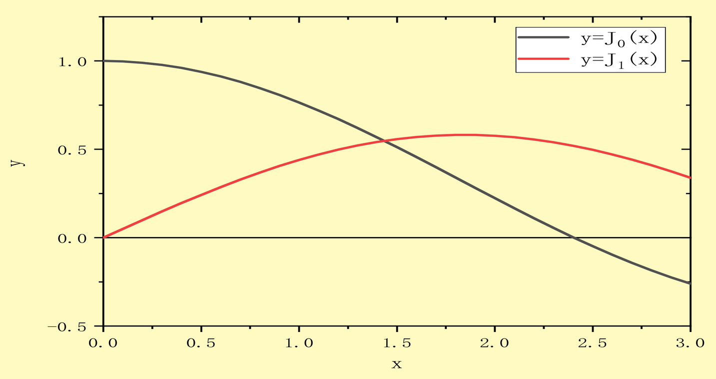
Contribution

Using a transport model based on BAMPS, we investigate the evolution of magnetic field in a system of charged particles, which is embedded in a cylinder with the periodic boundary condition. We find that when the ratio of the helicity to the magnetic energy goes to a maximum, the system relaxes towards the CKW state with $\nabla \times \boldsymbol{B} \parallel \boldsymbol{B}$, which explained the reversal of the magnetic field in pinch experiments by Budin in 1970s and provided the feasibility of BAMPS for the research of magnetic field in quark-gluon plasma.

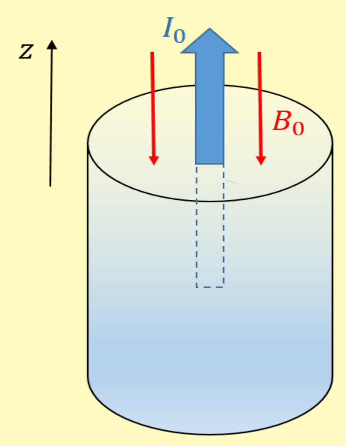
CKW state

The CKW state was proposed by Chandrasekhar, Kendall and Woltjer as a force-free state when the internal force $\boldsymbol{F} = q\boldsymbol{v}\times\boldsymbol{B} = 0$. In another word, ignoring the electric field, $\nabla \times \boldsymbol{B} \parallel \boldsymbol{B}$. The solution in cylindrical symmetric system is so called Bessel function.



What's more, the plasma can evolve to the CKW state only if the helicity $H = \int_V \boldsymbol{A}\cdot\boldsymbol{B}\,\mathrm{d}V$ is constant and meanwhile the magnetic energy $W = \frac{1}{2}\int_V \boldsymbol{H}\cdot\boldsymbol{B}\,\mathrm{d}V$ gets to minimum.

Model



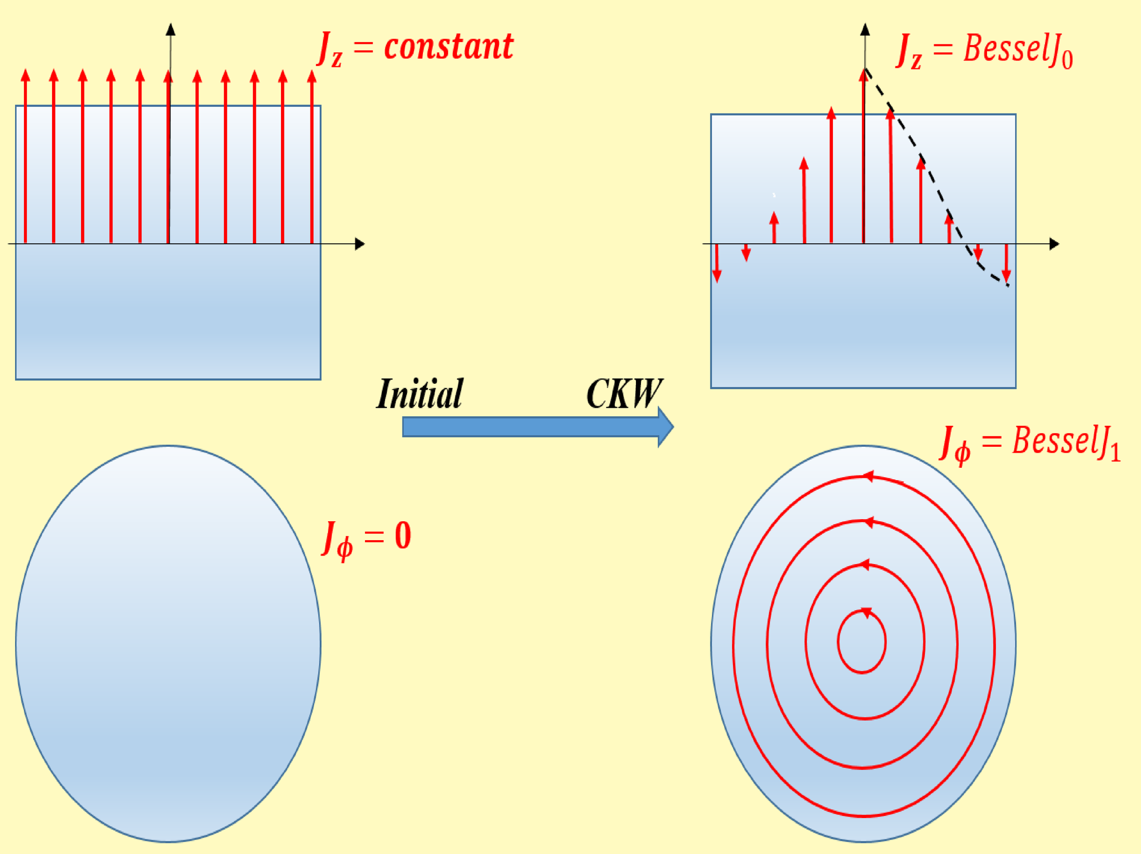
We use Bolzmann equation to describe the transport process of plasma, that is

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{m}\cdot\nabla f + \boldsymbol{F}\cdot\frac{\partial f}{\partial \boldsymbol{p}} = \left(\frac{\partial f}{\partial t}\right)_{coll}, \quad (1)$$

where $\boldsymbol{F} = q\boldsymbol{v}\times\boldsymbol{B}$ is just the Lorentz force. We solve the Bolzmann equation with BAMPS(Boltzmann Approach of Multi Parton Scatterings)

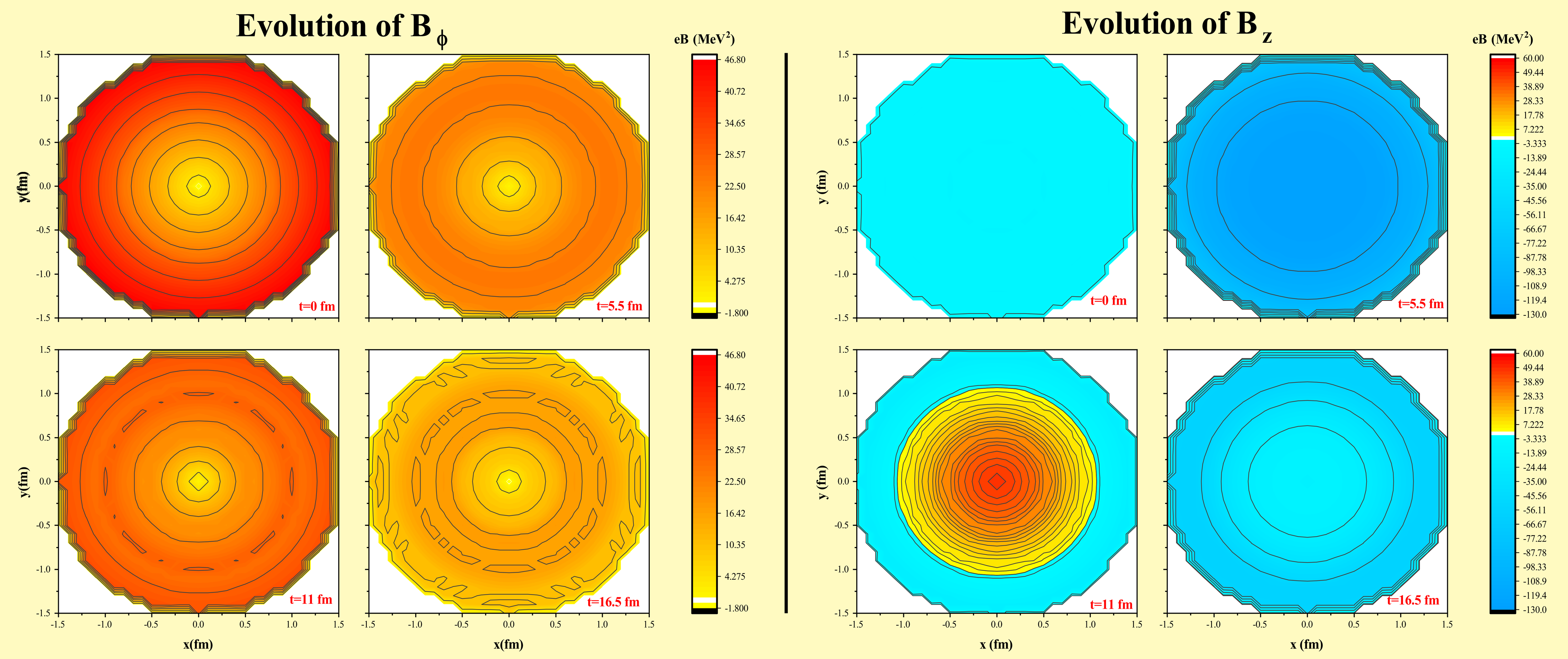
Magnetic field(Liénard-Wiechert potential):

$$e\boldsymbol{B}(\boldsymbol{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\boldsymbol{v}_n\times\boldsymbol{R}_n}{(R_n - \boldsymbol{R}_n\cdot\boldsymbol{v}_n)^3}(1-v_n^2), \quad (2)$$

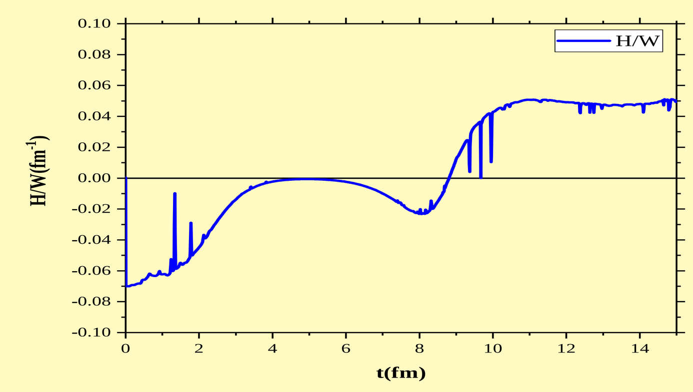


Results of electron plasma

We investigate the evolution of magnetic field in electron plasma and sketch out as following

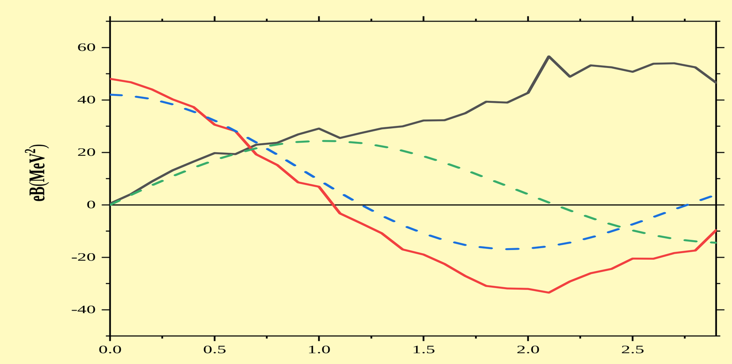


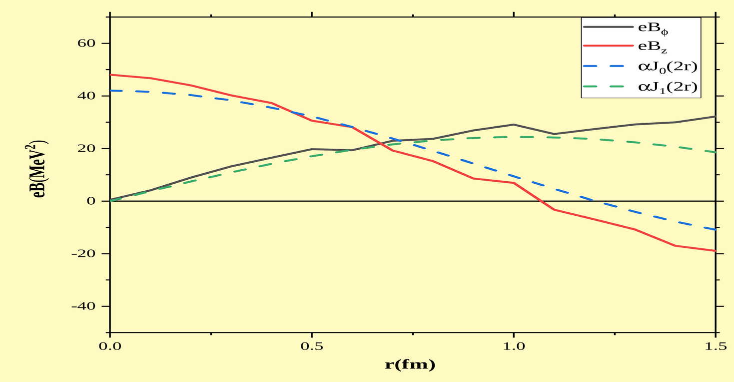
According to the CKW theorem, the plasma evolove to the CKW state when the ratio H/W gets to the maximum.

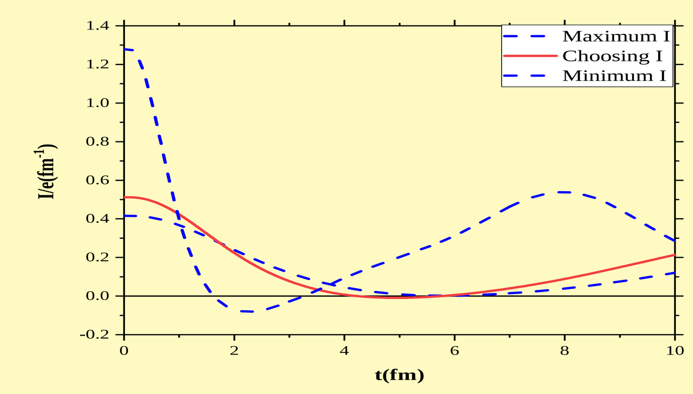


Then we picture out the evolution of H/W and show the distribution of magnetic field at $t = 11\,\mathrm{fm}$. We find the plasma has evolved to the CKW state partially then.

Additionally, we find the boundary initial condition and only under the the plasma can evolve to the CKW state. In another word, the initial current cannot be too large or too small.

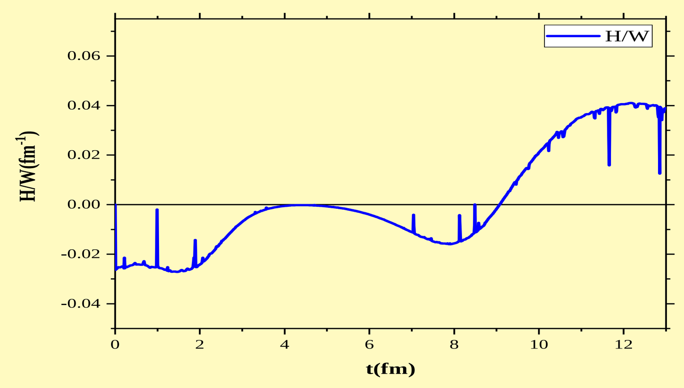




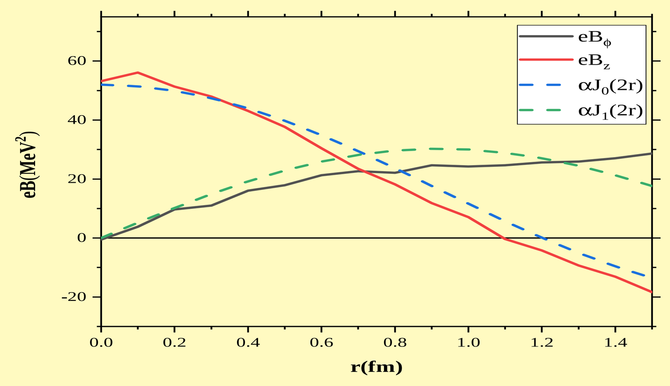


Results of $e - p$ plasma

The evolution of H/W in electron-proton plasma and $N_e : N_p = 1 : 1$, in another word, it's an electroneutral plasma.

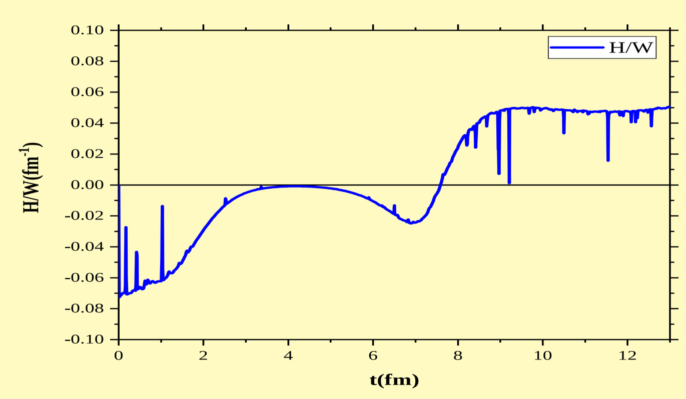


The following figure is the distribution of magnetic field at $t = 11\,\mathrm{fm}$.

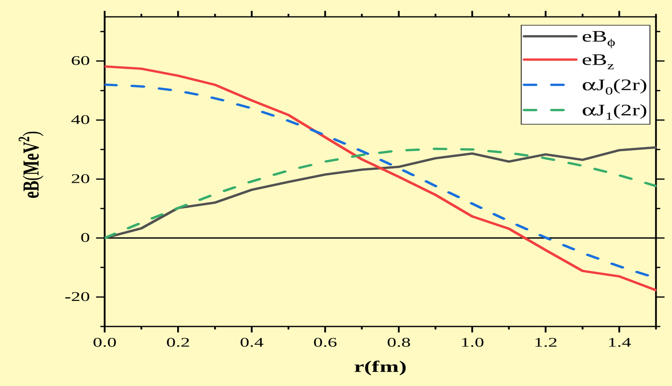


Results of $e - e^+$ plasma

The evolution of H/W in electron-positron plasma and $N_e : N_{e+} = 1 : 1$, also, it's an electroneutral plasma.



The following figure is the distribution of magnetic field at $t = 11\,\mathrm{fm}$.



References

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