# Search for the QCD phase transition

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Student Lecture
Quark Matter 2019

"A theory is something nobody believes, except the person who made it.
An experiment is something everybody believes, except the person who made it."

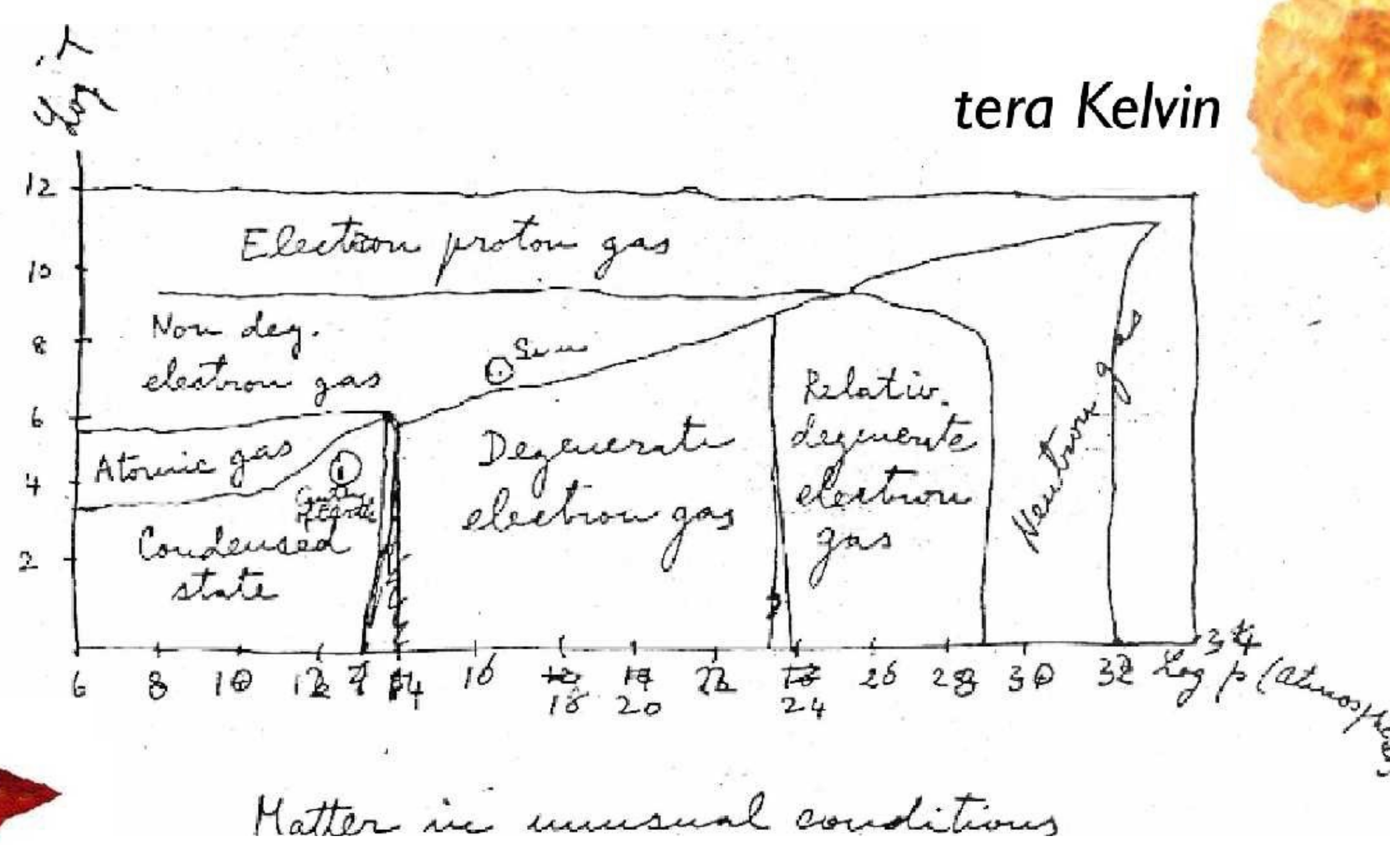
A. Einstein

# Outline

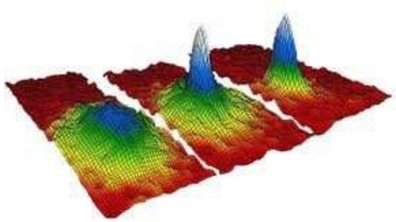
- Phase Transitions
- Cumulants: What are they and why are they useful
- Some preliminary experimental results and what they could mean
  - Some tricky experimental issues
  - Comparing data with Theory
  - Cumulants and correlations
- Spinodal instability
- Summary

# An old question

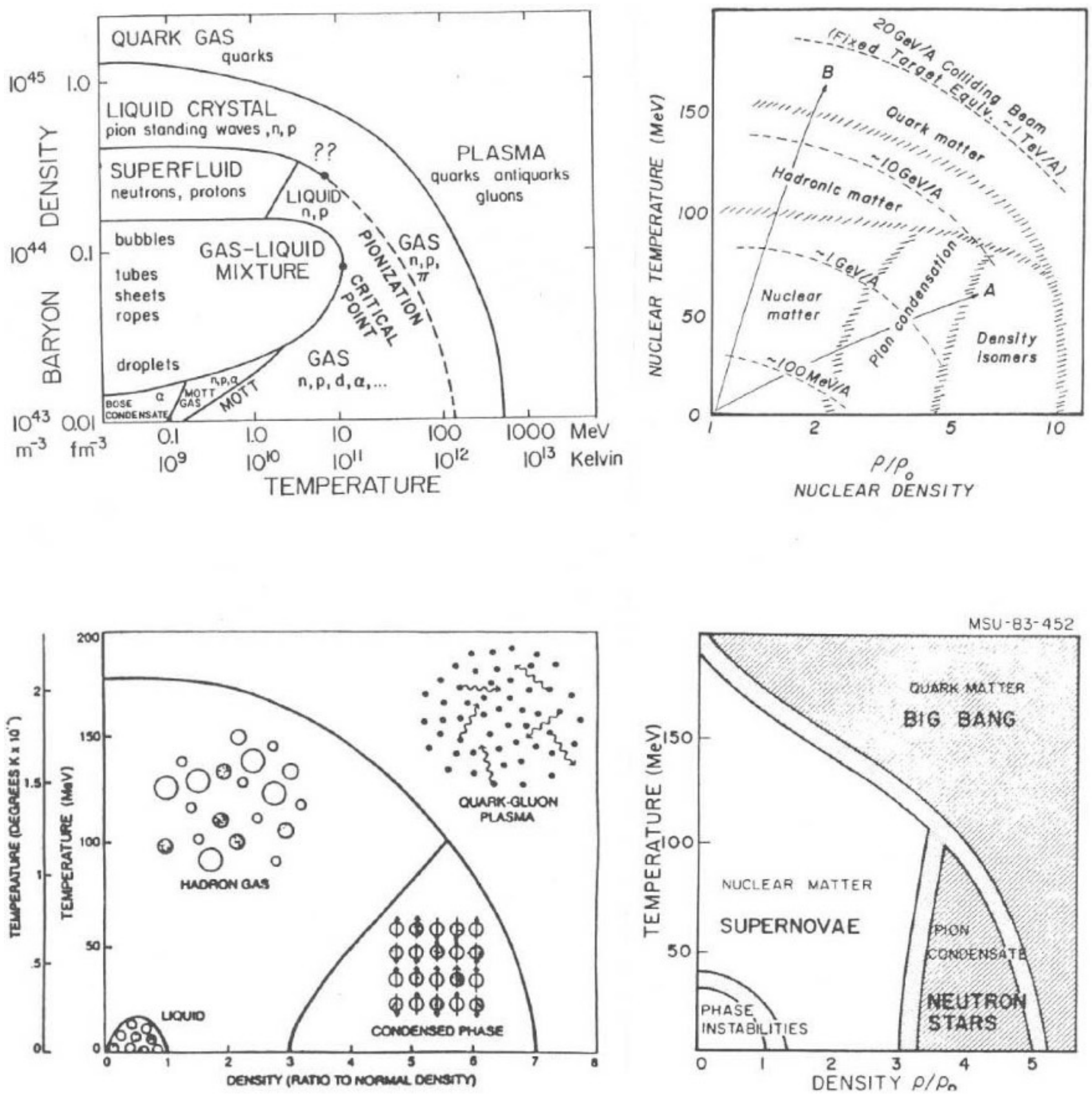
Fermi 1953



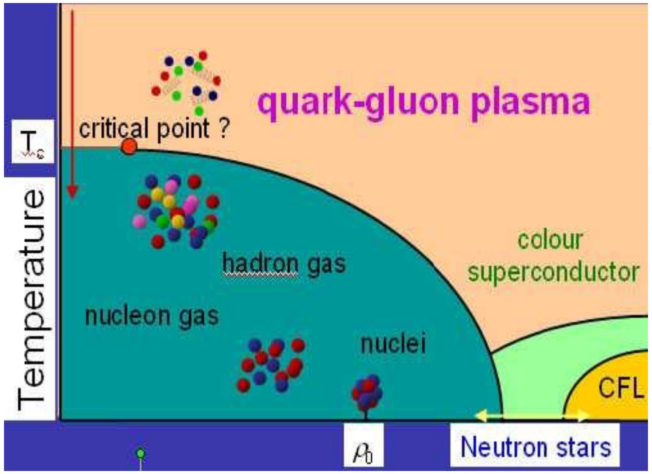

tera Kelvin

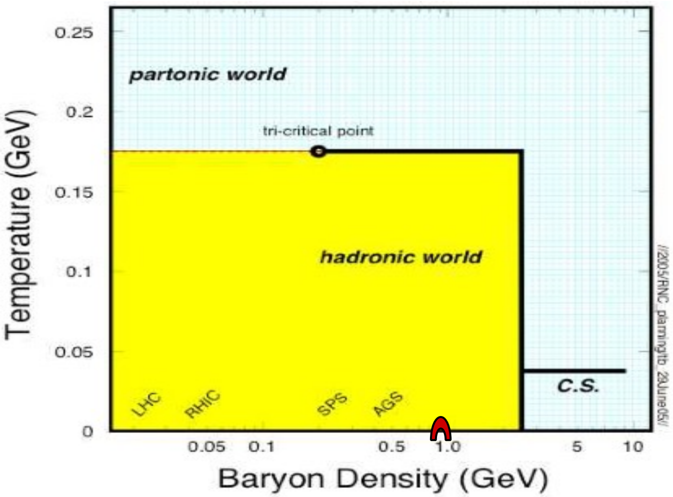


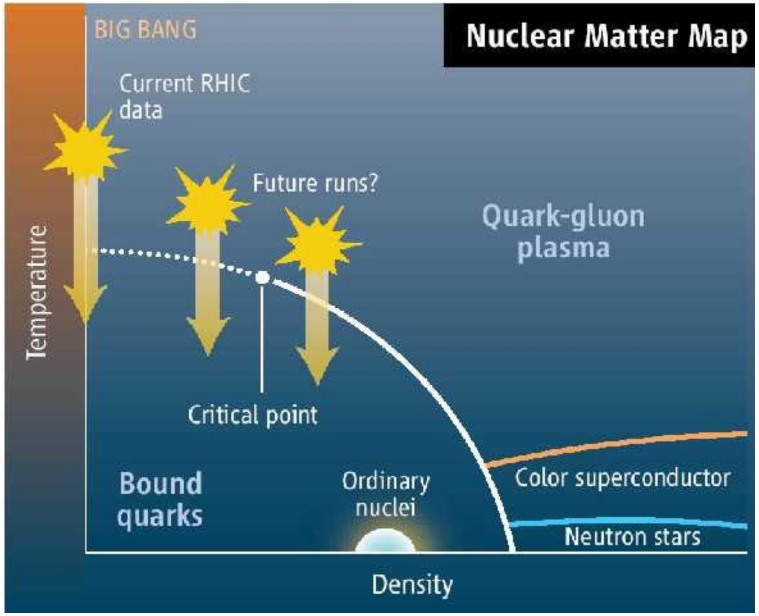
# discussed for many years ....

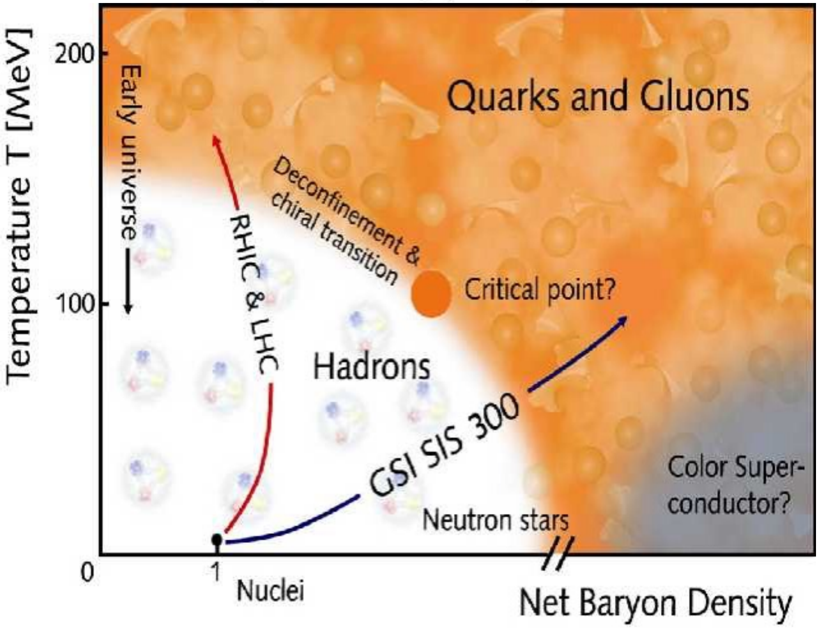


# gets more colorful ...

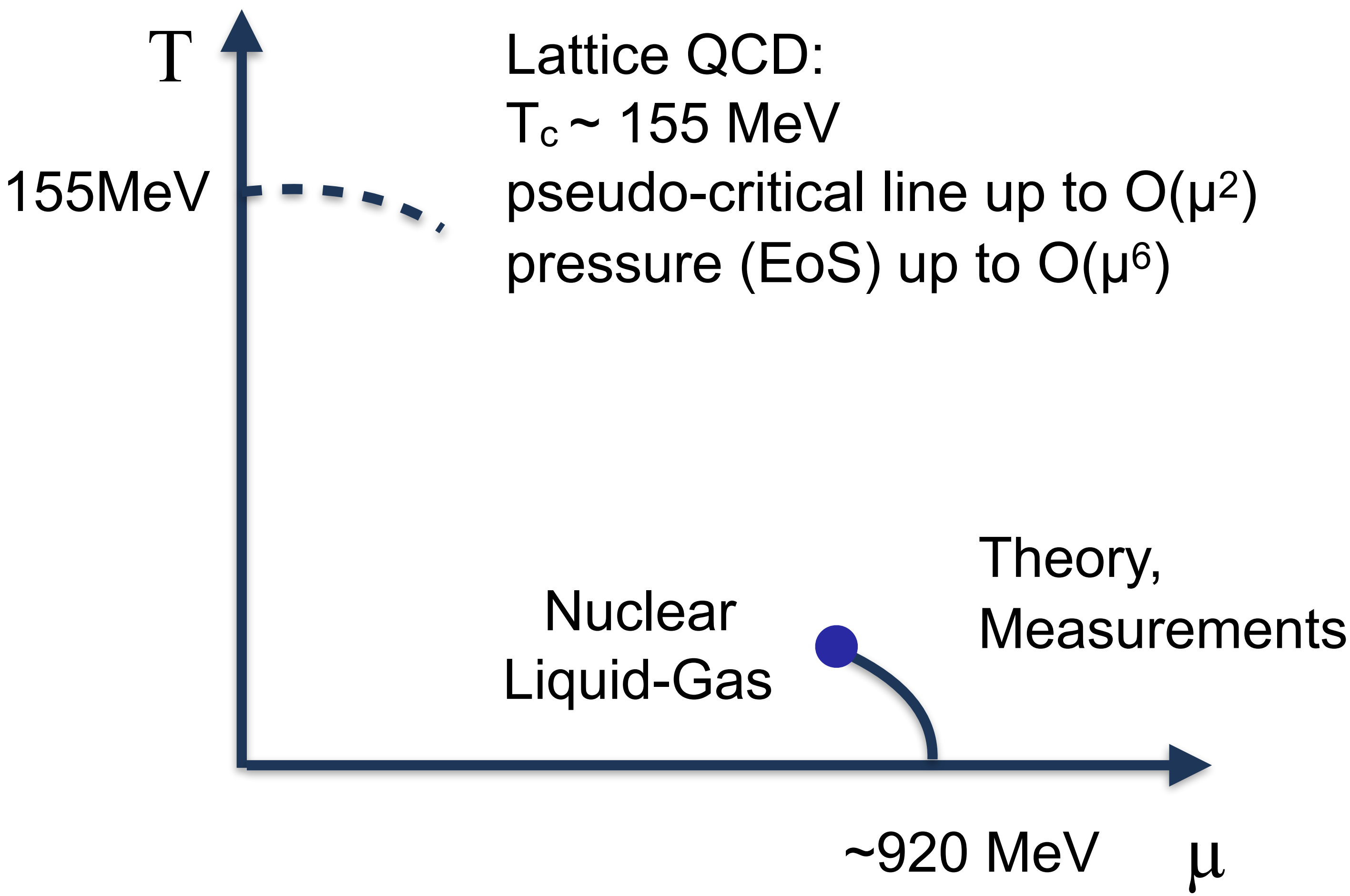


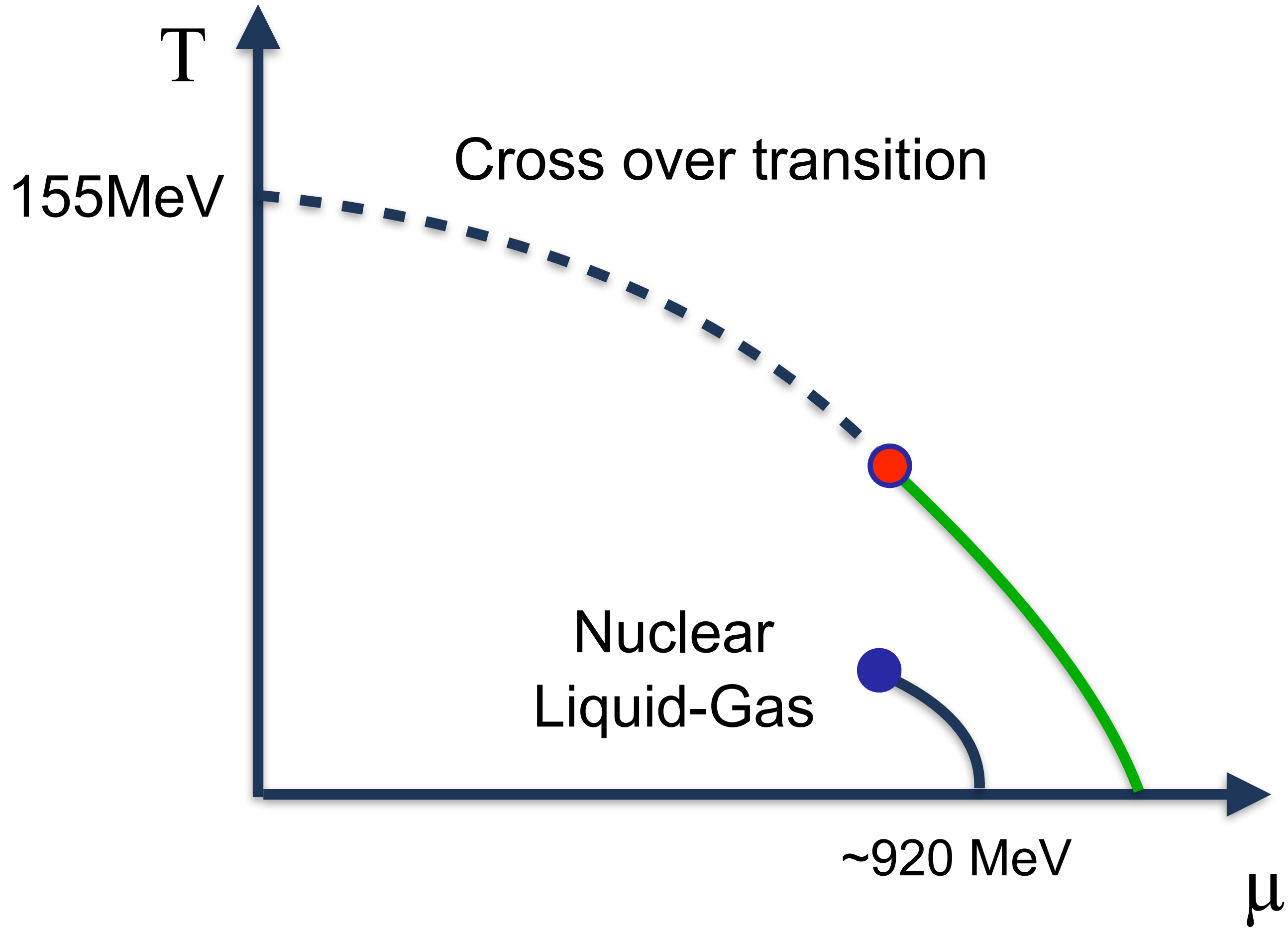



# What we know about the Phase Diagram



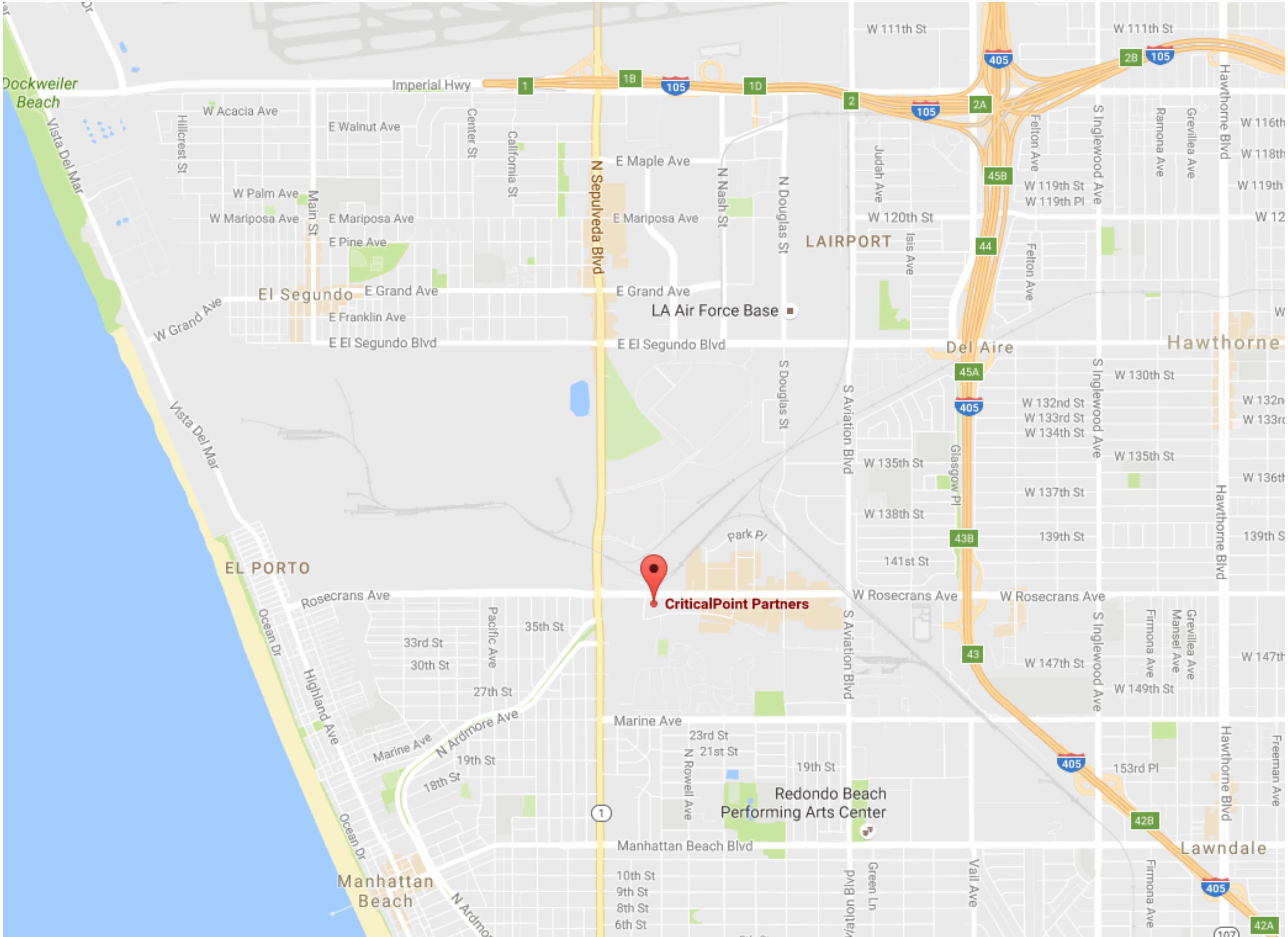
# What we “hope” for




NB: critical point of water is at T=647K and p=22.06 MPa

# Is there a critical point?

# Google finds everything…



# Phase Transitions

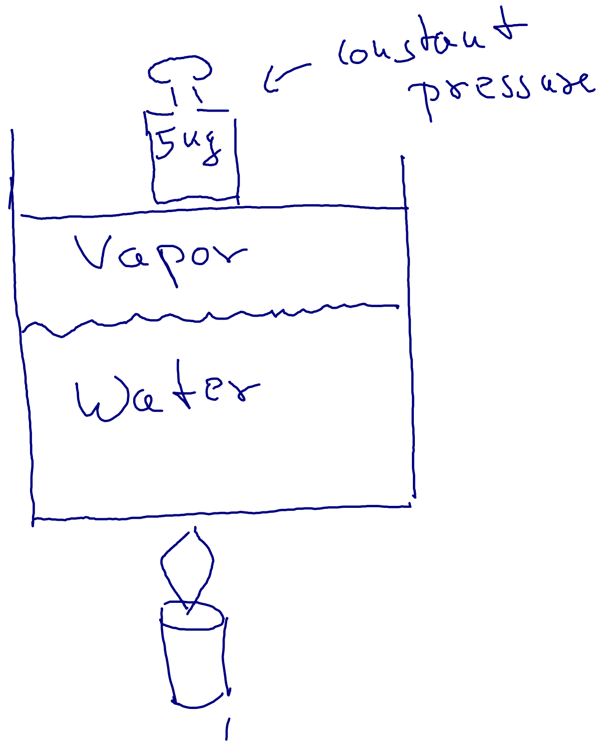
Examples:
Water - vapor  (liquid - gas)
Water - ice
Ferromagnet
….

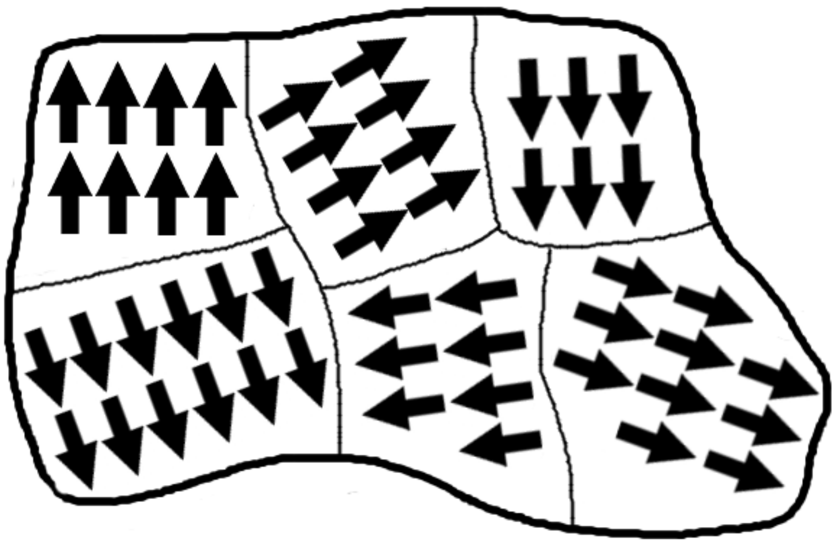
Order parameter: Tells in which phase the system is
Examples ?

Control parameter: Moves system from one phase to another
Examples ?

Phase co-existence: Two or more phases can exist together
Examples ?

# Phase Co-Existence

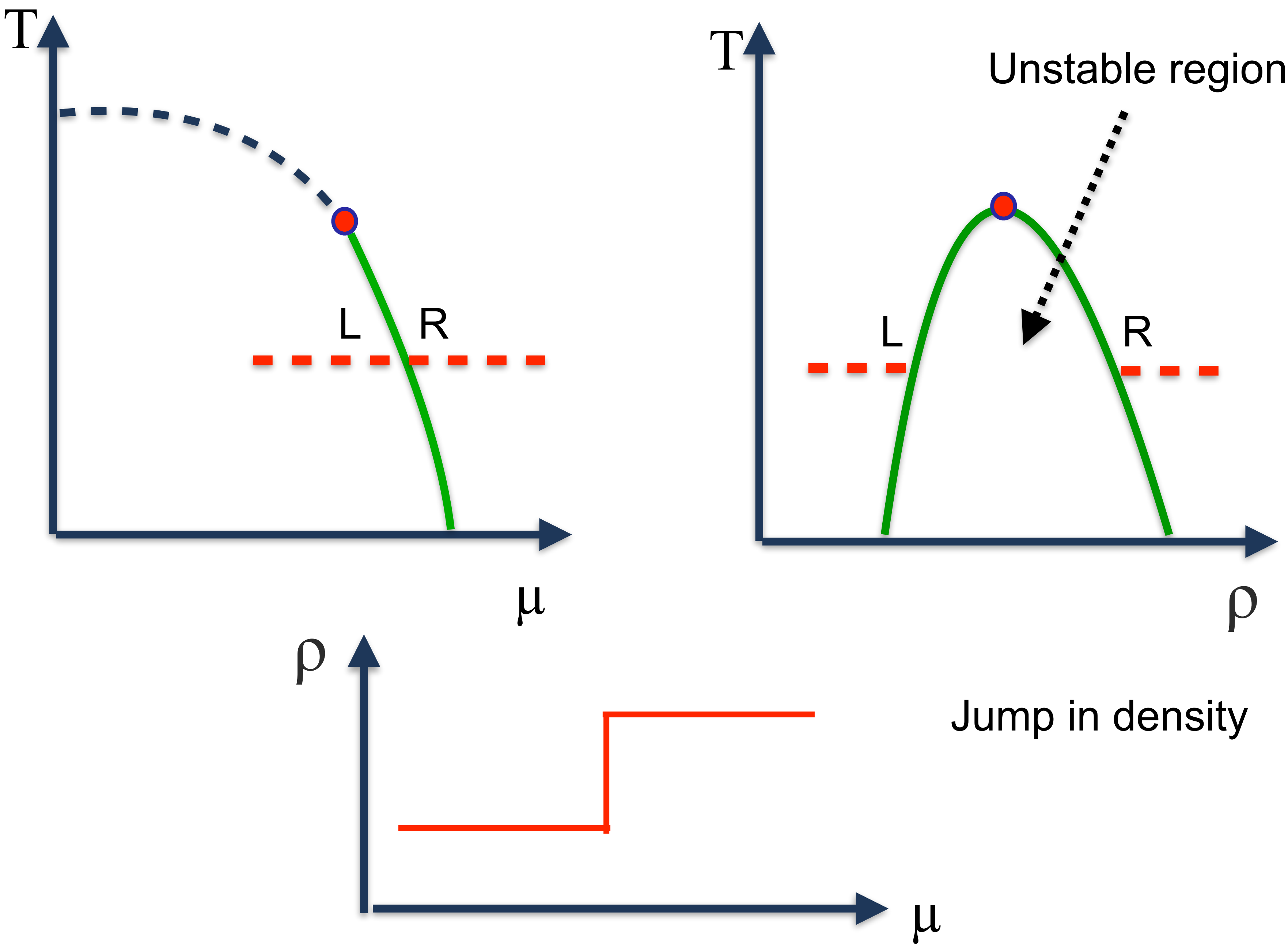


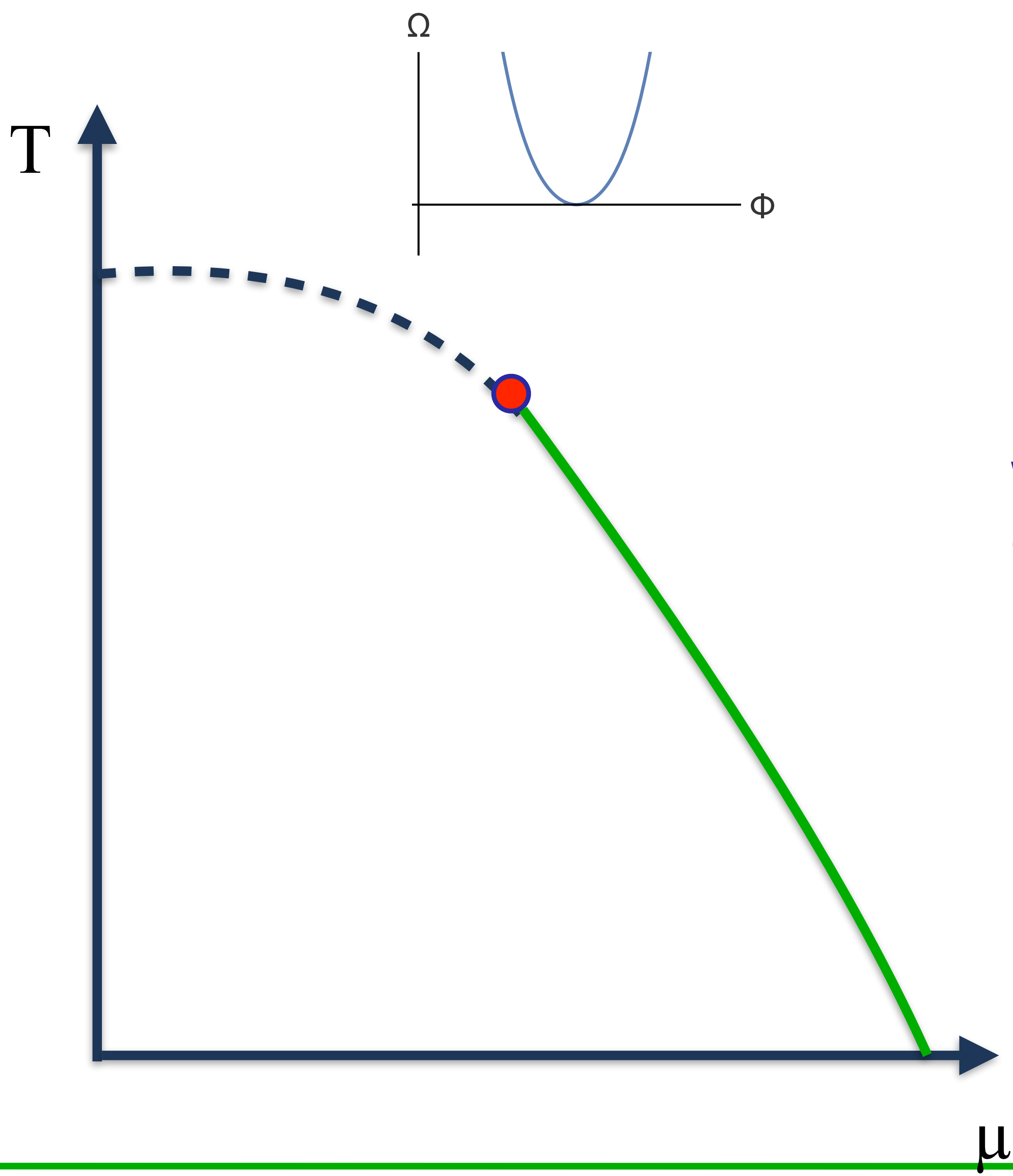
Water-vapor co-existence
a.k.a your water kettle

Ferro-magnet
Weiss domains

# Phase diagrams



# Free Energy



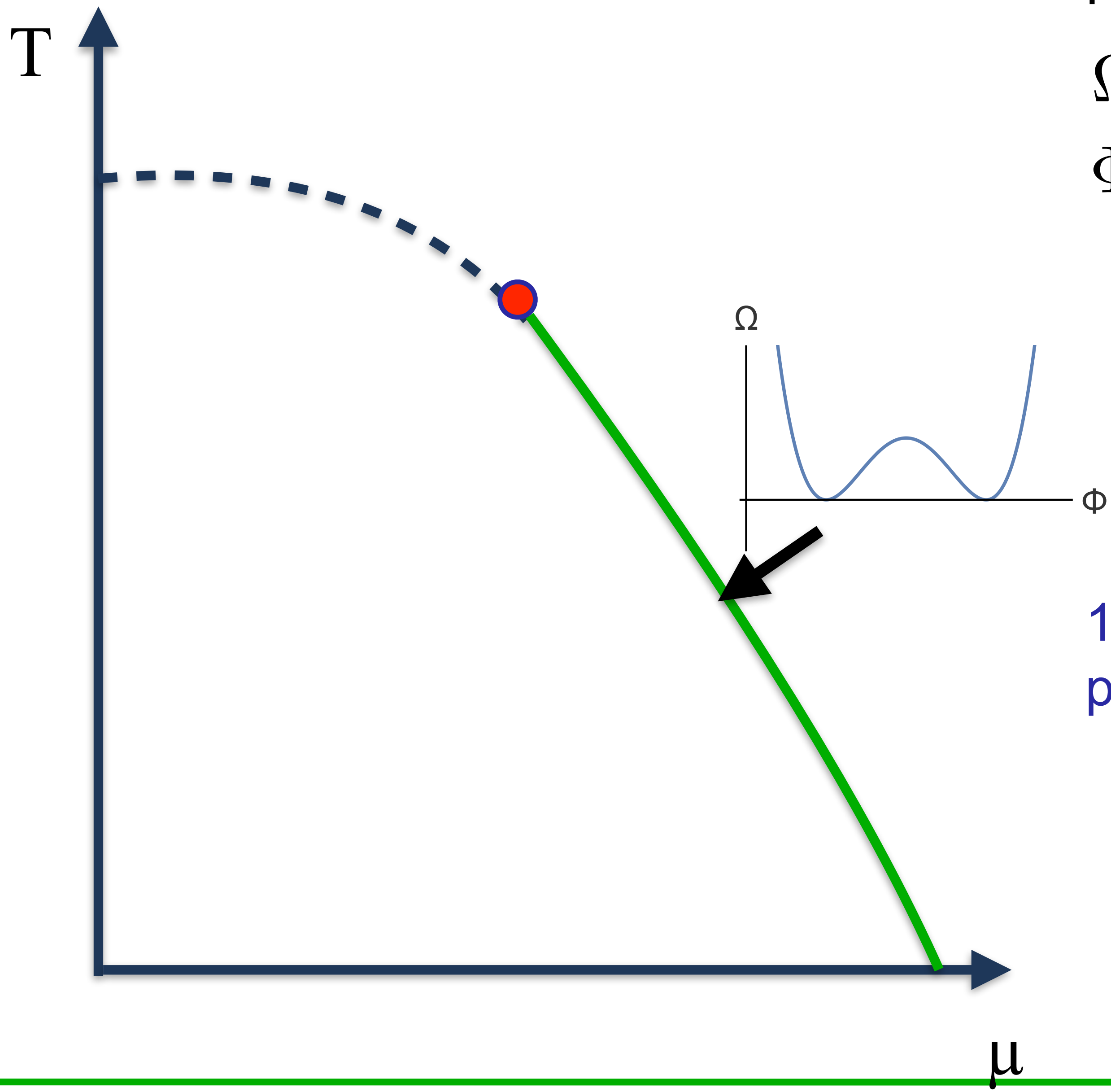

Free Energy:

$$\Omega = \Omega(T, \mu;\ \Phi)$$

$\Phi$: Order parameter

What we are used to:
One minimum

# Free Energy



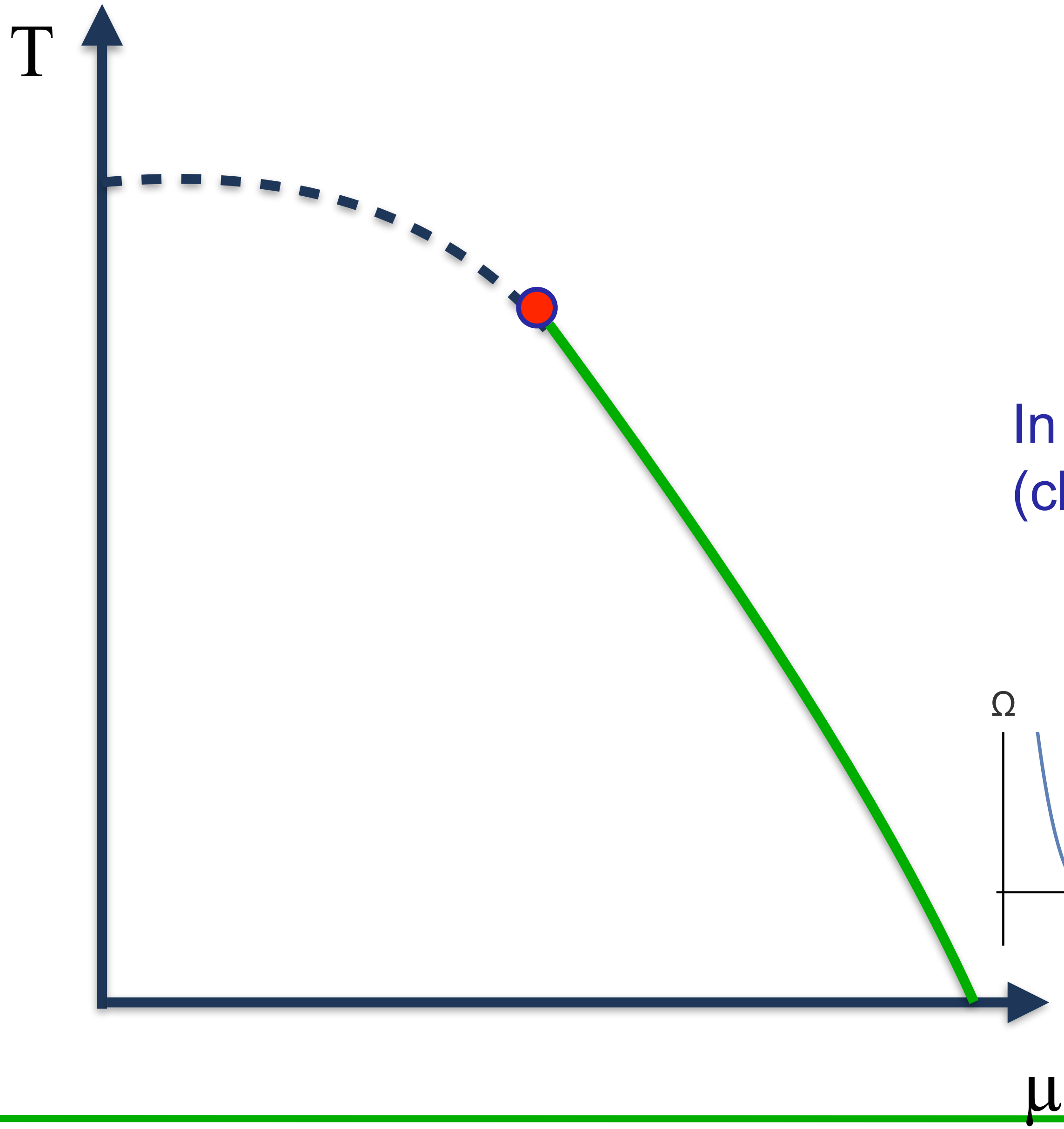

Free Energy:

$$\Omega = \Omega(T, \mu;\ \Phi)$$

$\Phi$: Order parameter

$1^{st}$ order
phase co-existence

# Free Energy

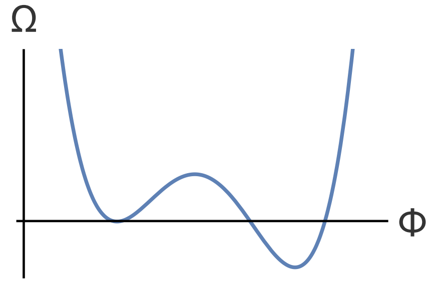



Free Energy:

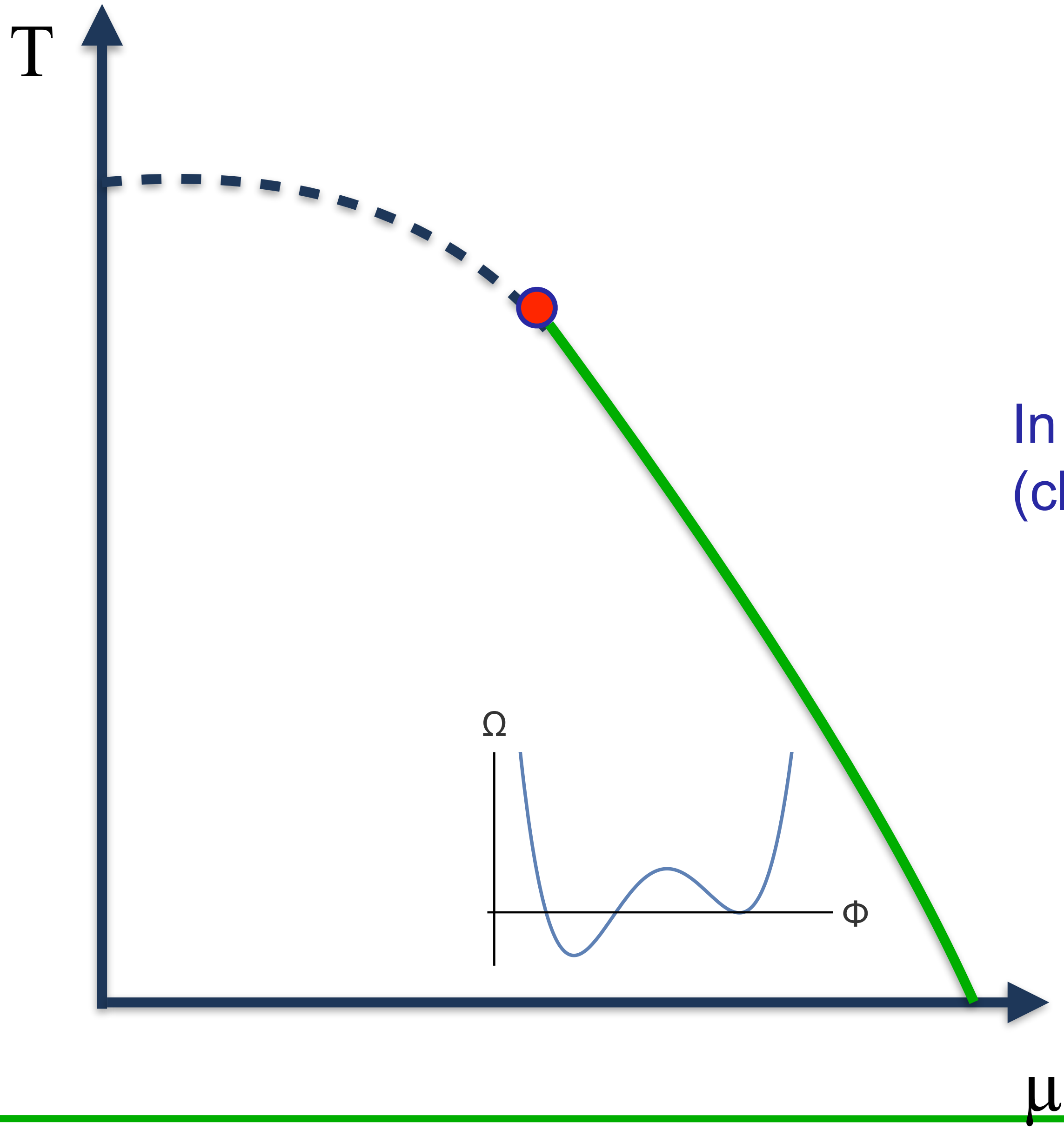
$$\Omega = \Omega(T, \mu;\ \Phi)$$

$\Phi$: Order parameter

In “dense” phase
(close to transition)



# Free Energy



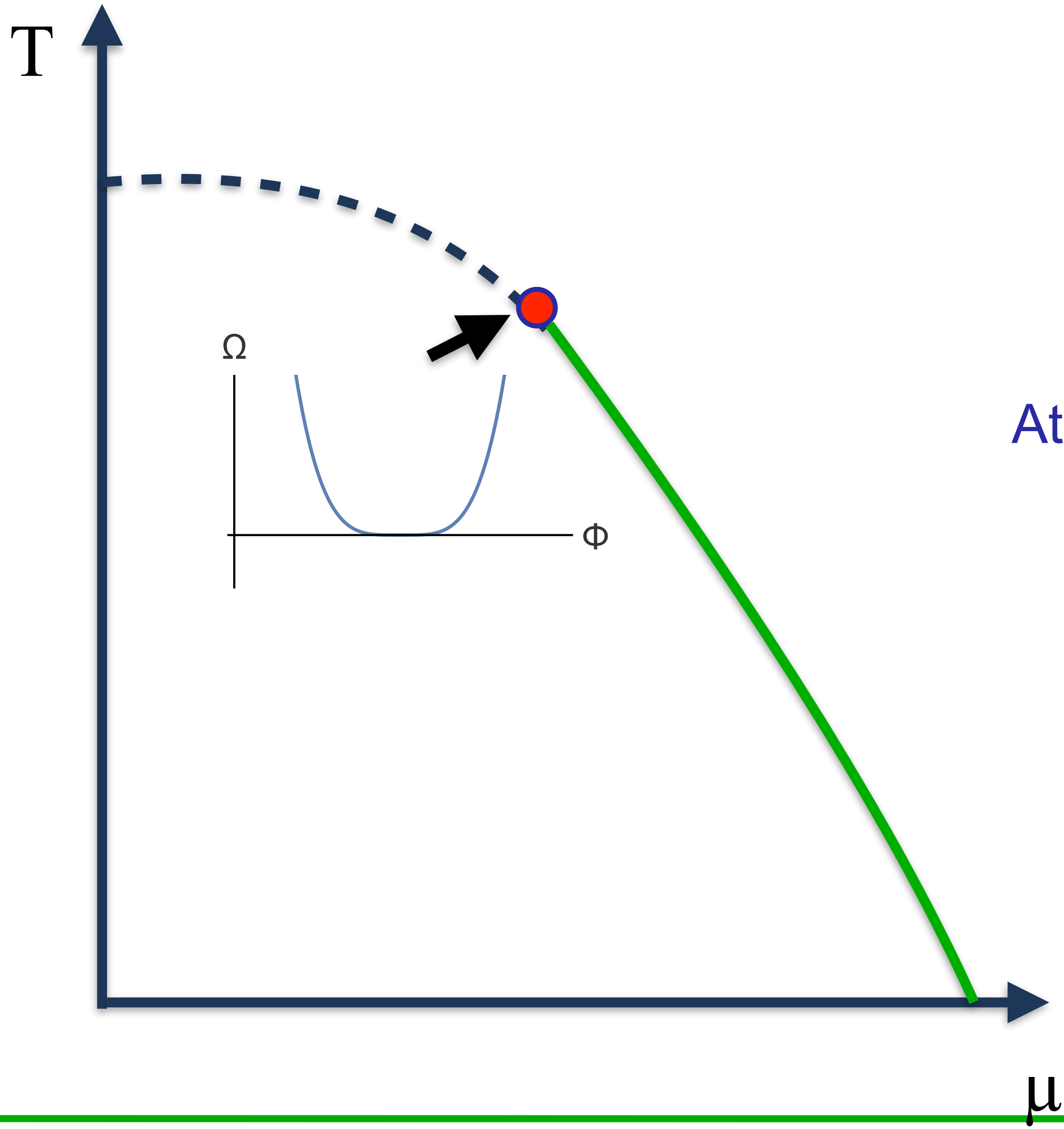

Free Energy:

$$\Omega = \Omega(T, \mu;\ \Phi)$$

$\Phi$: Order parameter

In “dilute” phase
(close to transition)

# Free Energy



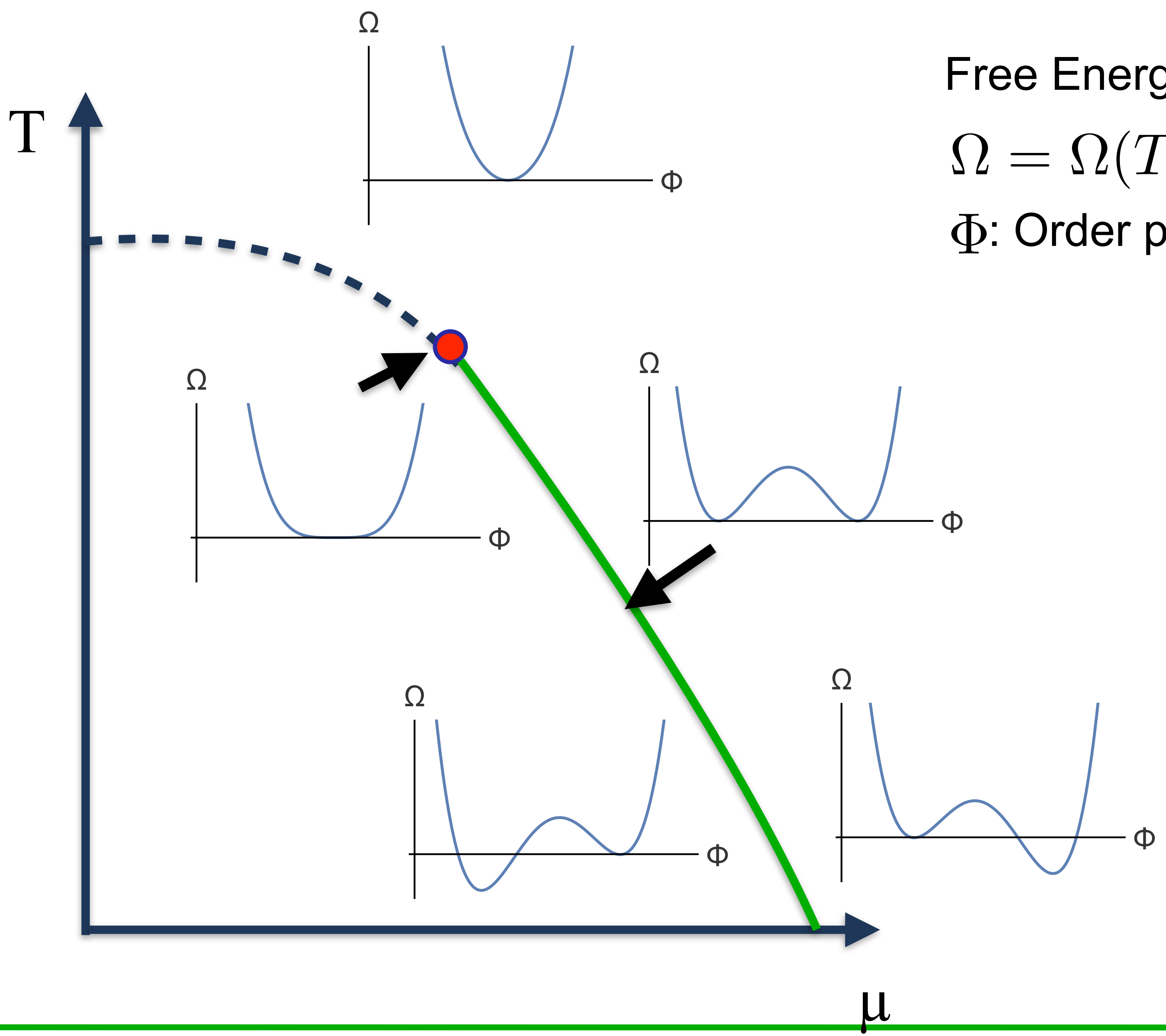

Free Energy:

$$\Omega = \Omega(T, \mu;\ \Phi)$$

$\Phi$: Order parameter

At the critical point

# Free Energy




Free Energy:

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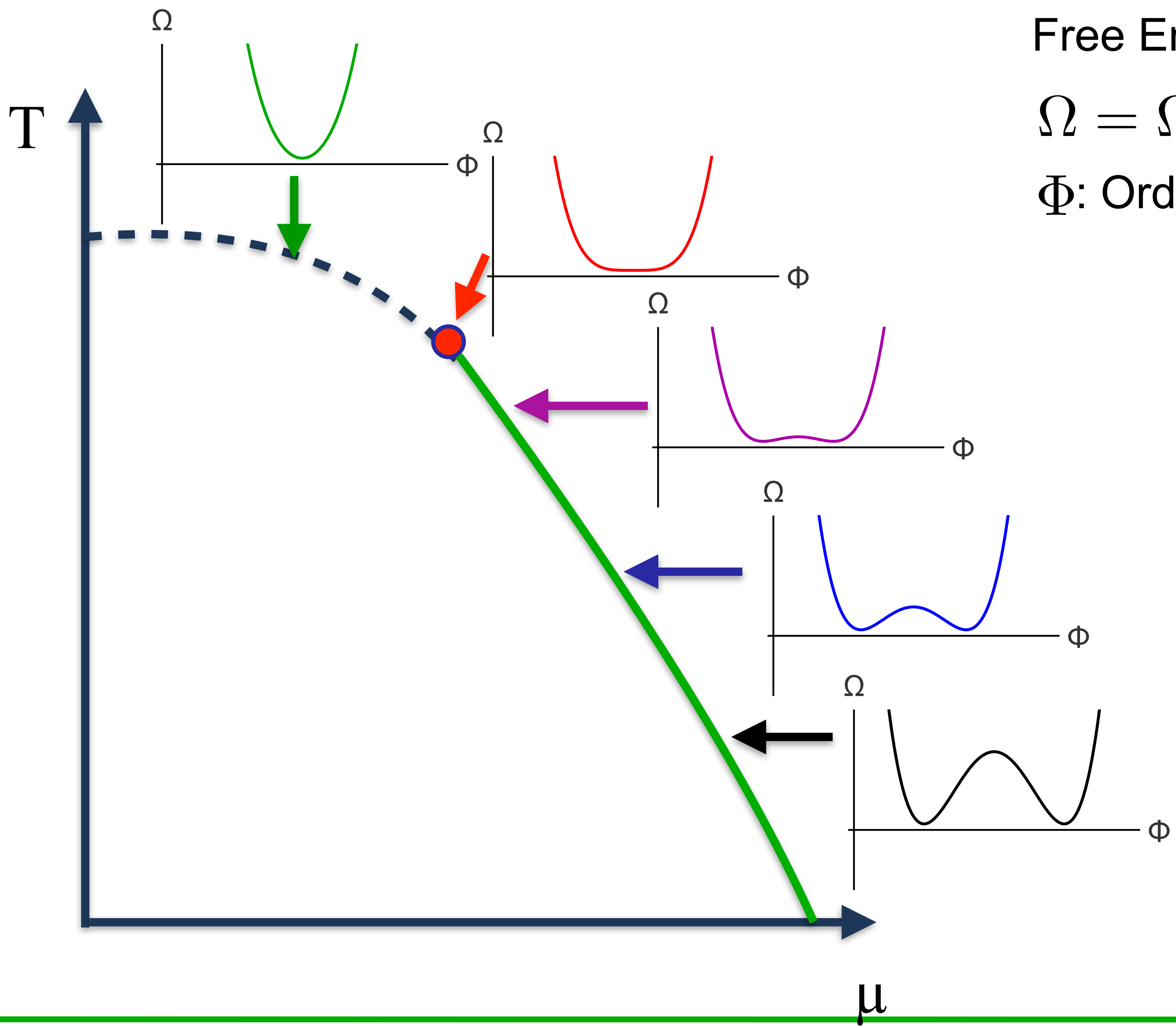
# Free Energy

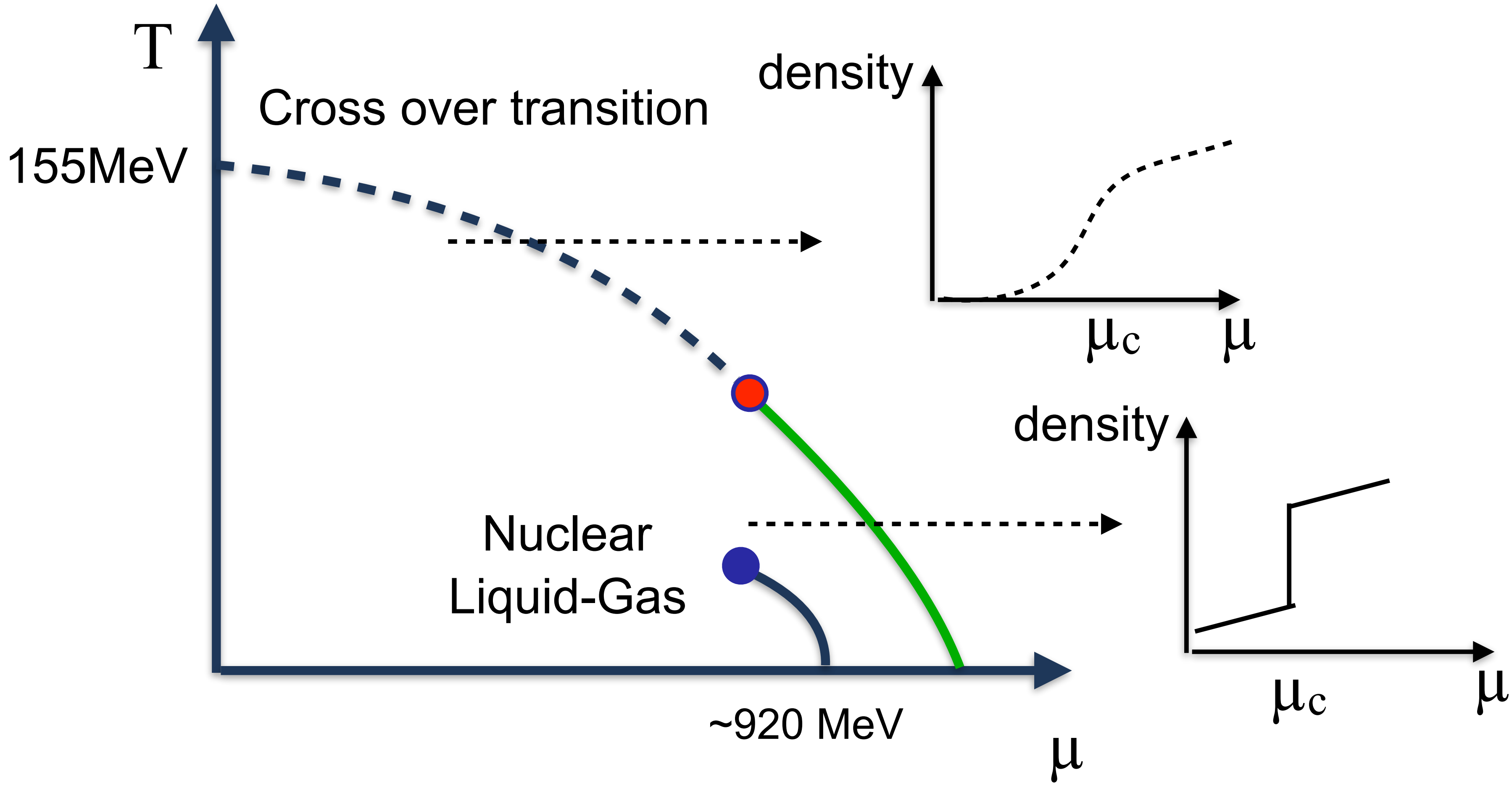



Free Energy:

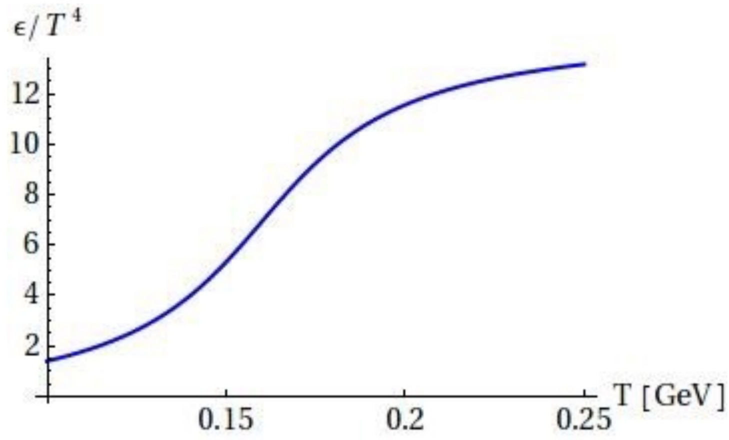
$$\Omega = \Omega(T, \mu;\ \Phi)$$

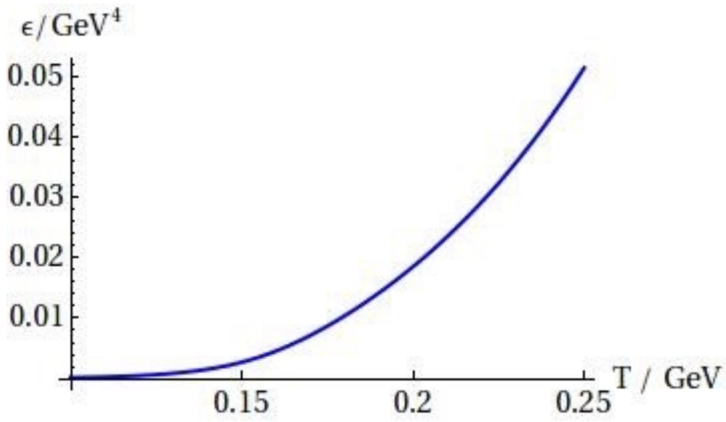
$\Phi$: Order parameter

# Looking for signs of a transition



# Cumulants and phase structure



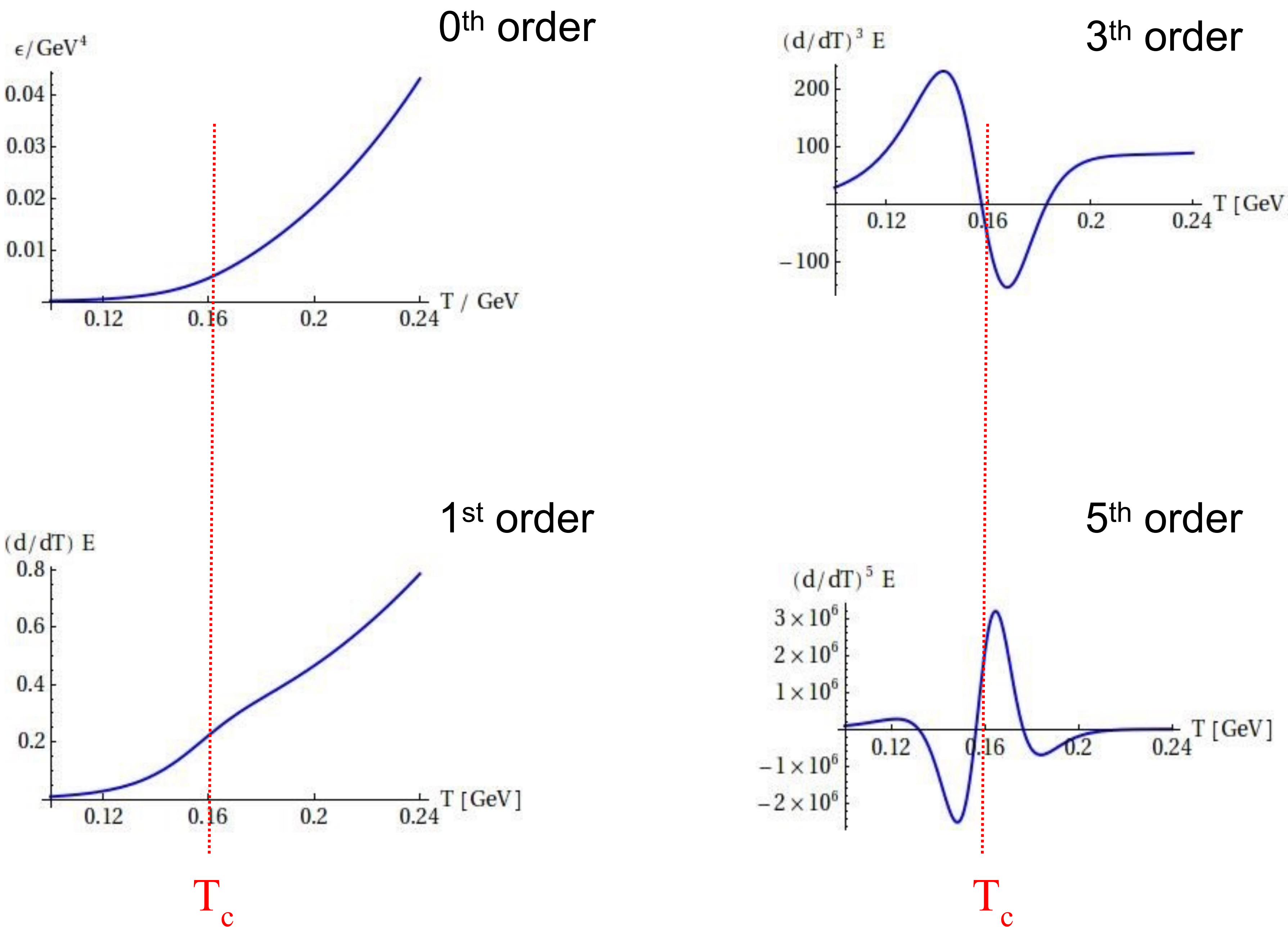




What we always see....

What it really means....

"$T_c$" ~ 160 MeV

# Derivatives



# How to measure derivatives

At μ = 0:

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z}\, tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left( -\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left( -\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left( -\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

# Cumulants of (Baryon) Number

$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

$$K_1 = \langle N \rangle \, , \;\; K_2 = \langle N - \langle N \rangle \rangle^2 \, , \;\; K_3 = \langle N - \langle N \rangle \rangle^3$$

Cumulants scale with volume (extensive): $K_n \sim V$

Volume not well controlled in heavy ion collisions

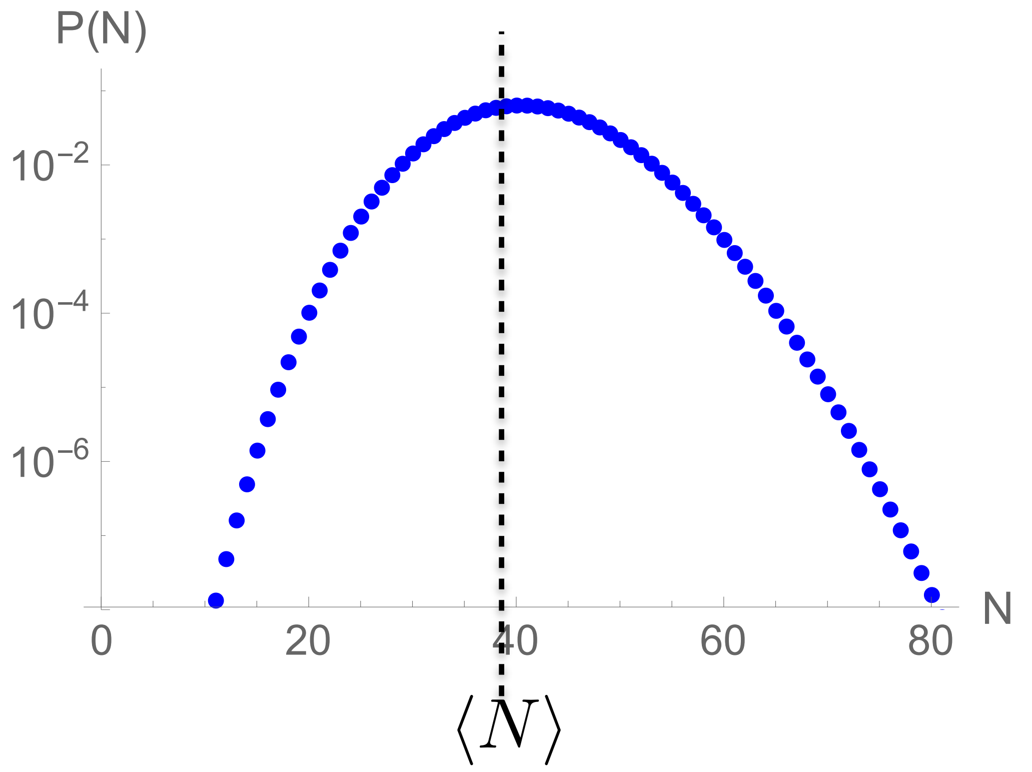
Cumulant Ratios: $\frac{K_2}{\langle N \rangle}, \;\; \frac{K_3}{K_2}, \;\; \frac{K_4}{K_2}$

# Measuring cumulants (derivatives)

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \sum_N P(N)(N - \langle N \rangle)^2$$

$$K_3 = \langle N - \langle N \rangle \rangle^3 = \sum_N P(N)(N - \langle N \rangle)^3$$

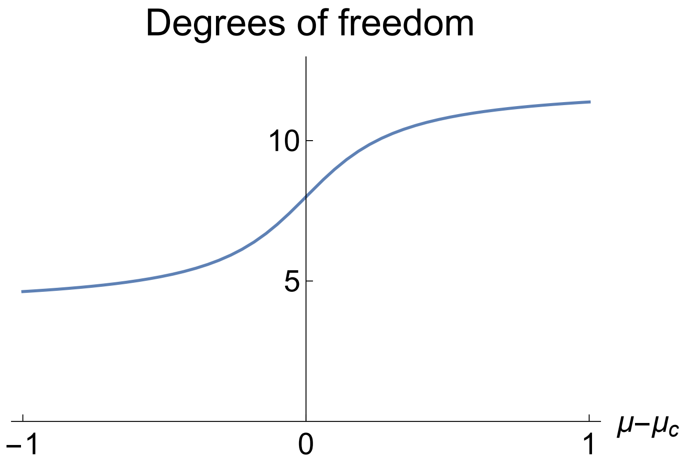
$$P(N) = \frac{N_{events}(N)}{N_{events}(total)}$$

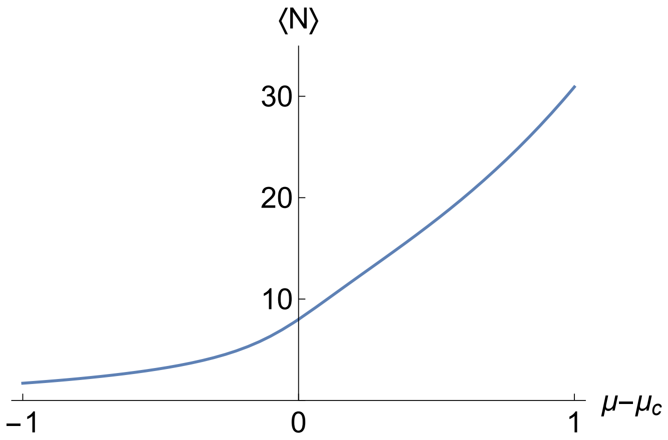


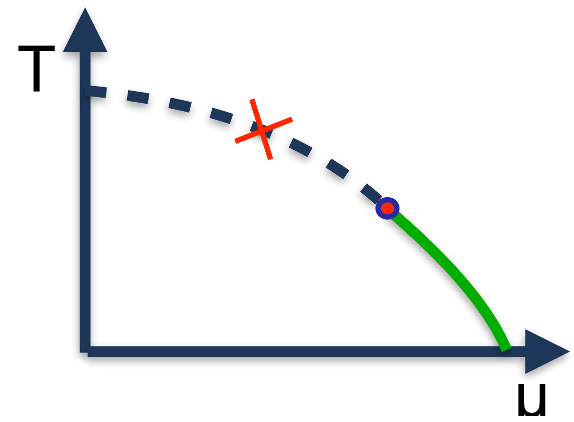
# Simple model

Change degrees of freedom at phase transition

$$\langle N \rangle = dof(\mu)\, e^{\mu/T} \int d^3p e^{-E/T}$$




T
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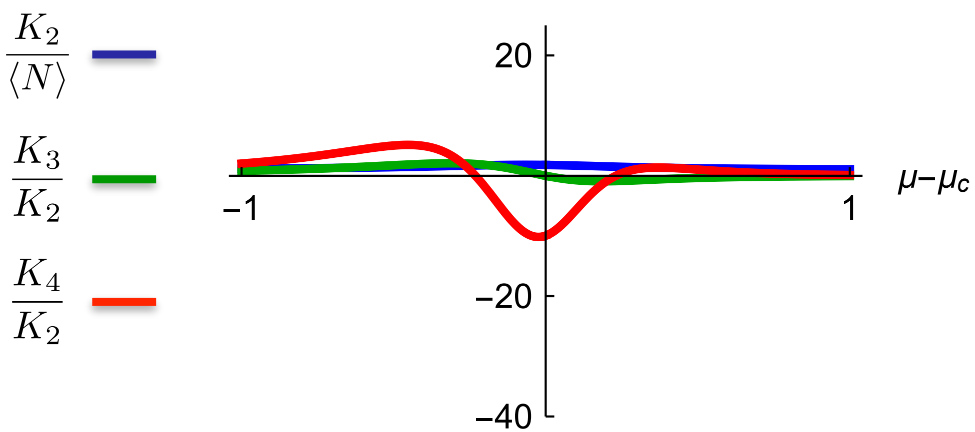
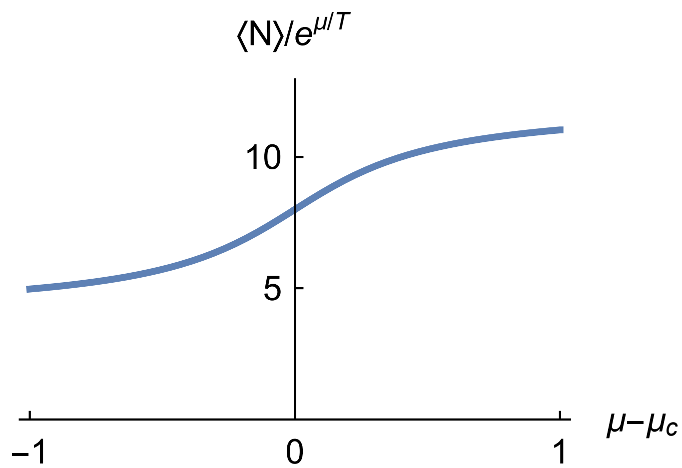
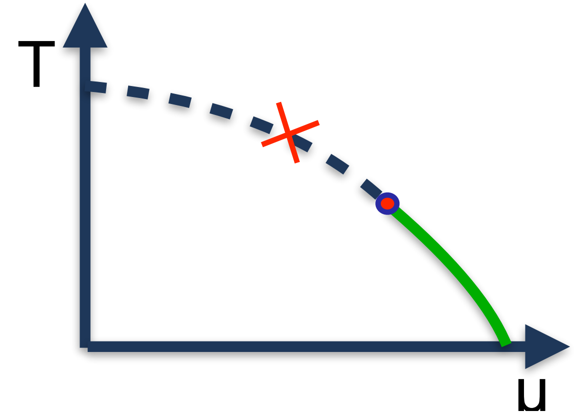

$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
−40
−1
1
$\mu - \mu_c$


$\langle N \rangle / e^{\mu/T}$
10
5
−1
0
1
$\mu - \mu_c$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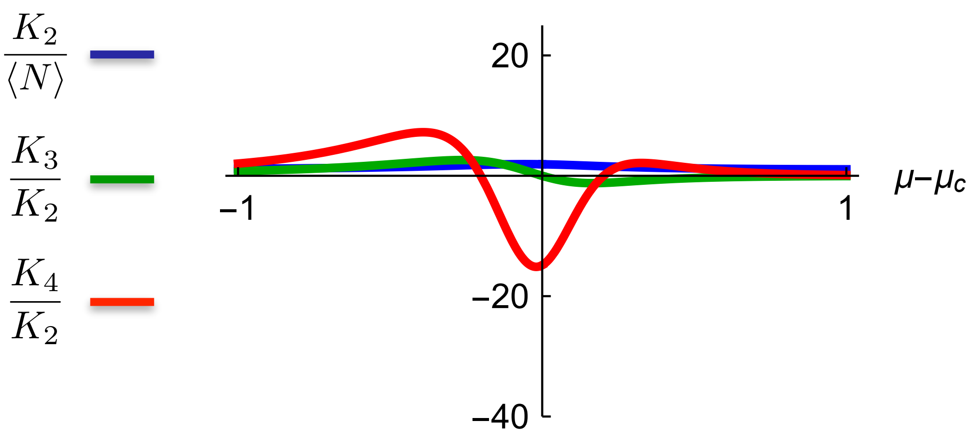
$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
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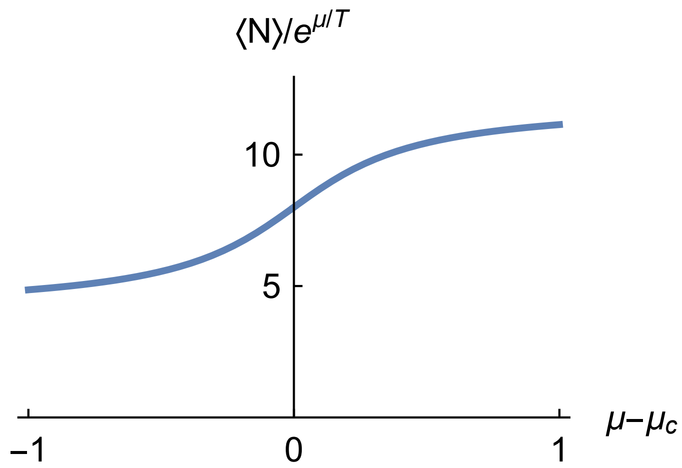
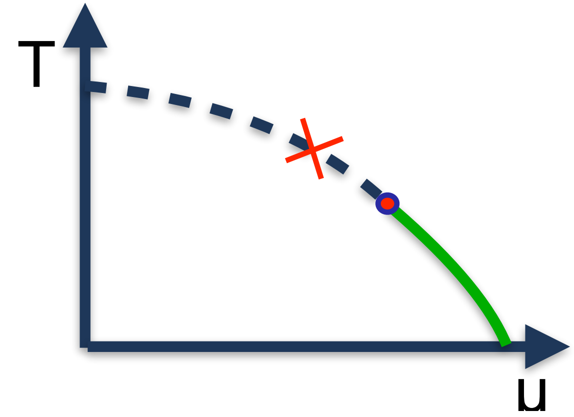
$\mu-\mu_c$


$\langle N \rangle / e^{\mu/T}$
10
5
−1
0
1
$\mu-\mu_c$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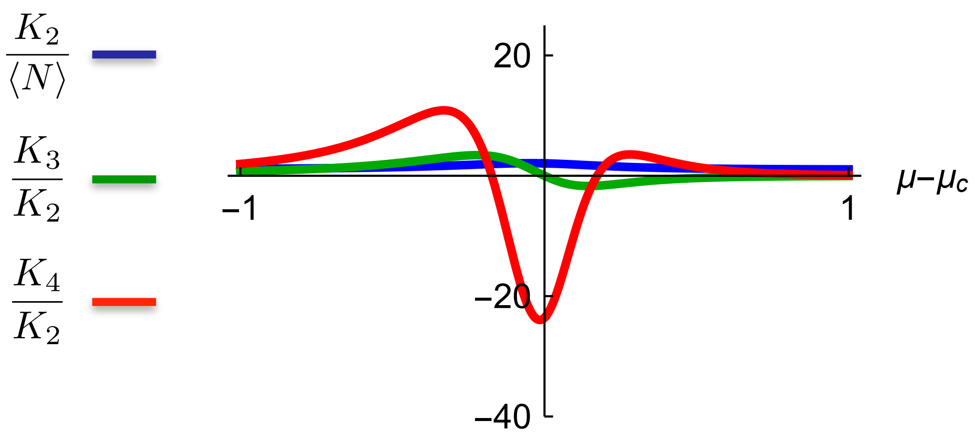
$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
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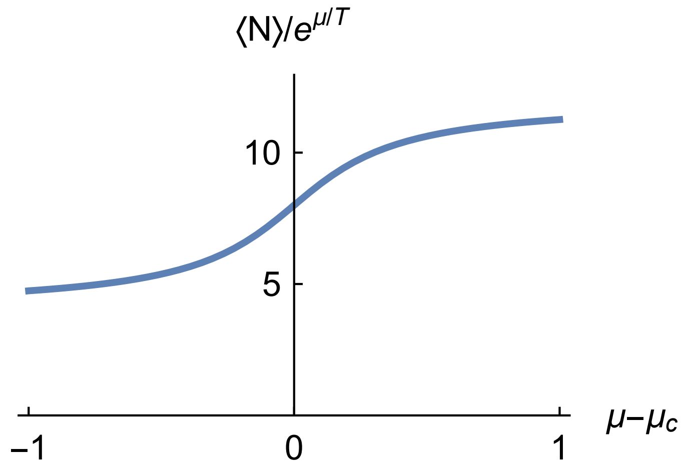
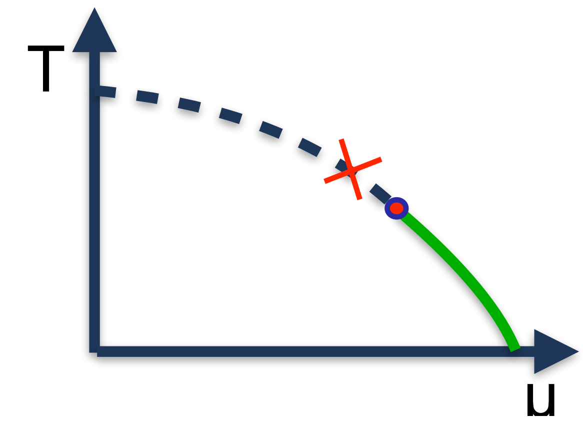
−1
1
μ−μc


⟨N⟩/e^{μ/T}
10
5
−1
0
1
μ−μc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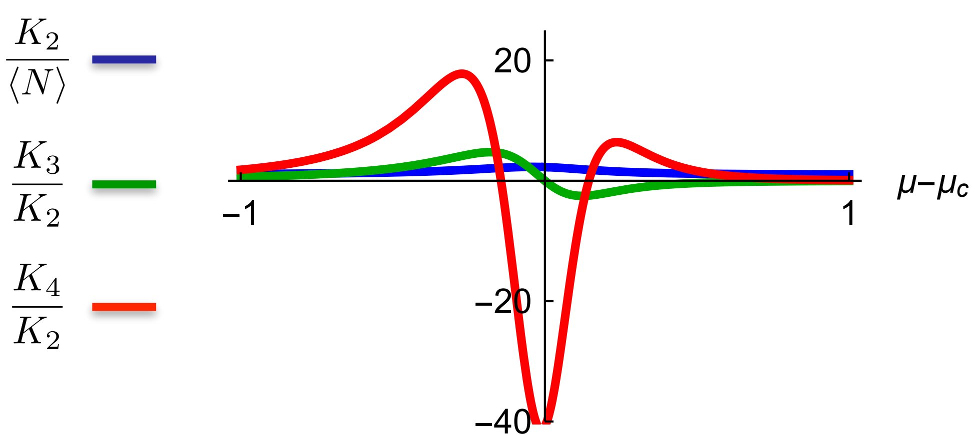
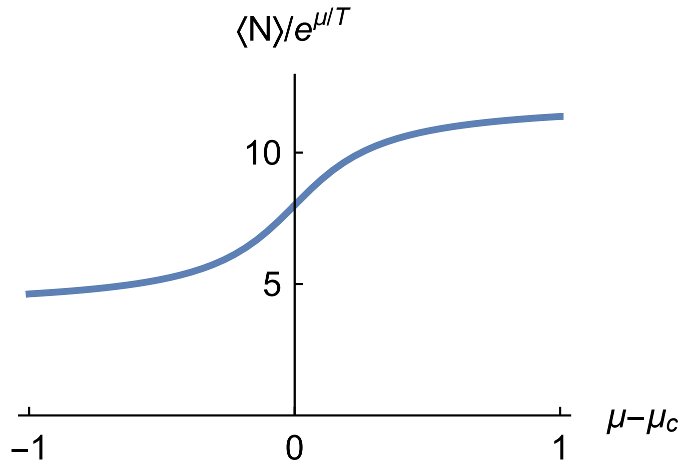
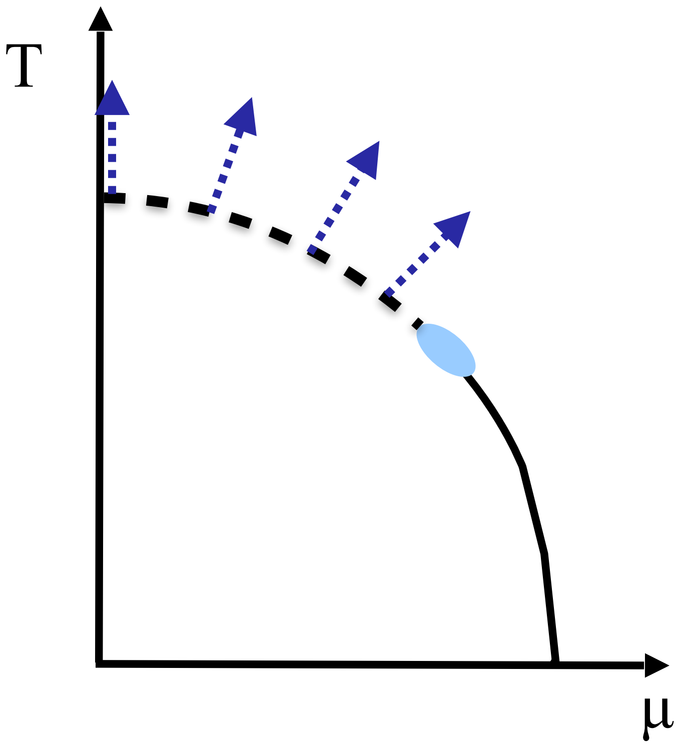

$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
−40
−1
1
μ−μc


⟨N⟩/e^{μ/T}
10
5
−1
0
1
μ−μc

# Close to μ=0

$$F = F(r), \quad r = \sqrt{T^2 + a\mu^2}$$

a ~ curvature of critical line



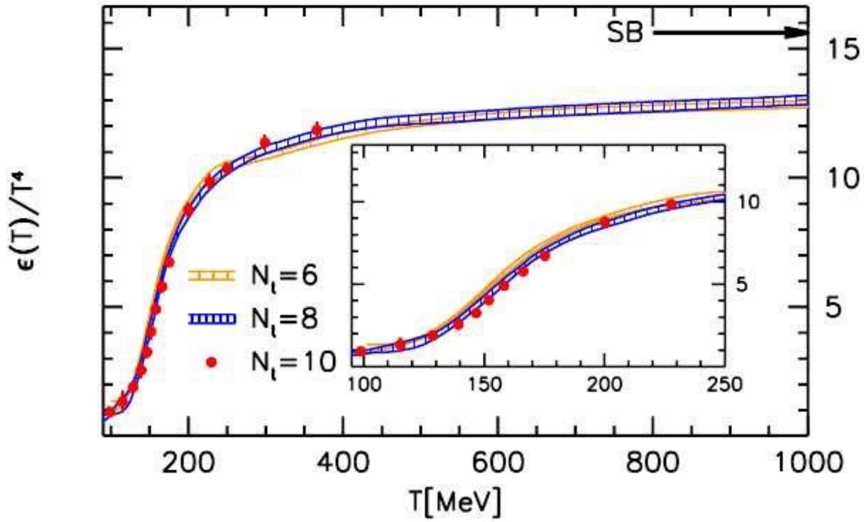

$$\frac{\partial^2}{\partial\mu^2} F(T,\mu)|_{\mu=0} = \frac{a}{T}\frac{\partial}{\partial T} F(T,\mu=0) \sim \langle E \rangle$$

Needs higher order cumulants (derivatives) at μ ~ 0

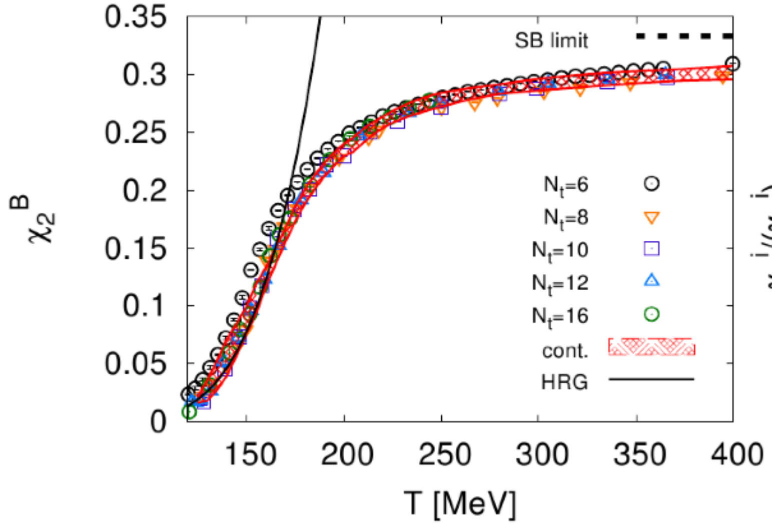
# Lattice at μ=0

Equation of state

S. Borsanyi et al, JHEP 1011 (2010) 077




Second order Cumulant




$$\frac{\partial^2}{\partial \mu^2} F(T,\mu)|_{\mu=0} = \frac{a}{T} \frac{\partial}{\partial T} F(T,\mu=0) \sim \langle E \rangle$$

# Cumulants: a closer look

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

Cumulants are extensive: $K_n \sim V$

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \int d^3x d^3y \, \langle \delta\rho(x)\delta\rho(y) \rangle \, ; \quad \delta\rho(x) = \rho(x) - \bar{\rho}$$

Susceptibility:

$$\chi_{(2)\, i,j} = \frac{1}{VT^3} \int d^3x d^3y \, \langle \delta\rho_i(x)\delta\rho_j(y) \rangle = \frac{1}{T^3} \bar{\rho}_i \delta_{i,j} + \frac{1}{T^3} \int d^3r C_{i,j}(r)$$

Correlation function (in configuration space!):

$$C_{i,j}(\vec{r}) = \langle \delta\rho_i(\vec{r})\, \delta\rho_j(0) \rangle - \bar{\rho}_i \delta_{i,j} \delta(\vec{r}) \sim \frac{\exp\left[-r/\xi_{i,j}\right]}{r}$$

Correlation length (in configuration space!): $\xi_{i,j}$

Relation to cumulant: $K_2 = VT^3 \chi_{(2)\, i,i}$

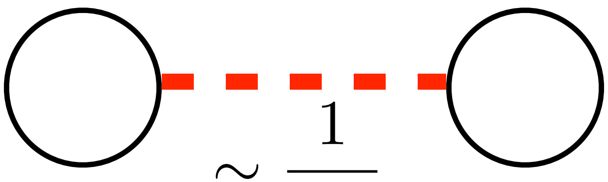
# Correlation length

$$C(r) \sim \frac{\exp[-r/\xi]}{r}$$

Static correlation function;
"Yukawa" potential with mass: $m \sim \frac{1}{\xi}$

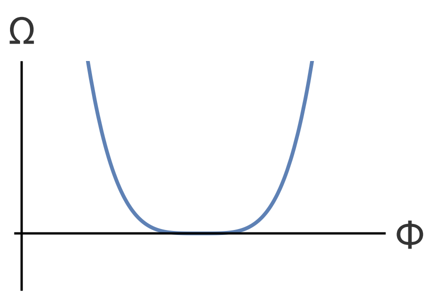
$$\chi \sim \int C(r) d^3 r \sim \xi^2 \sim \frac{1}{m^2}$$

simple "sigma" exchange



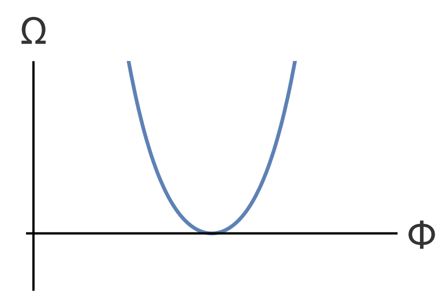
$$\sim \frac{1}{m_\sigma^2}$$

Critical point (second order)




$$m_\sigma \to 0, \quad \xi \to \infty$$

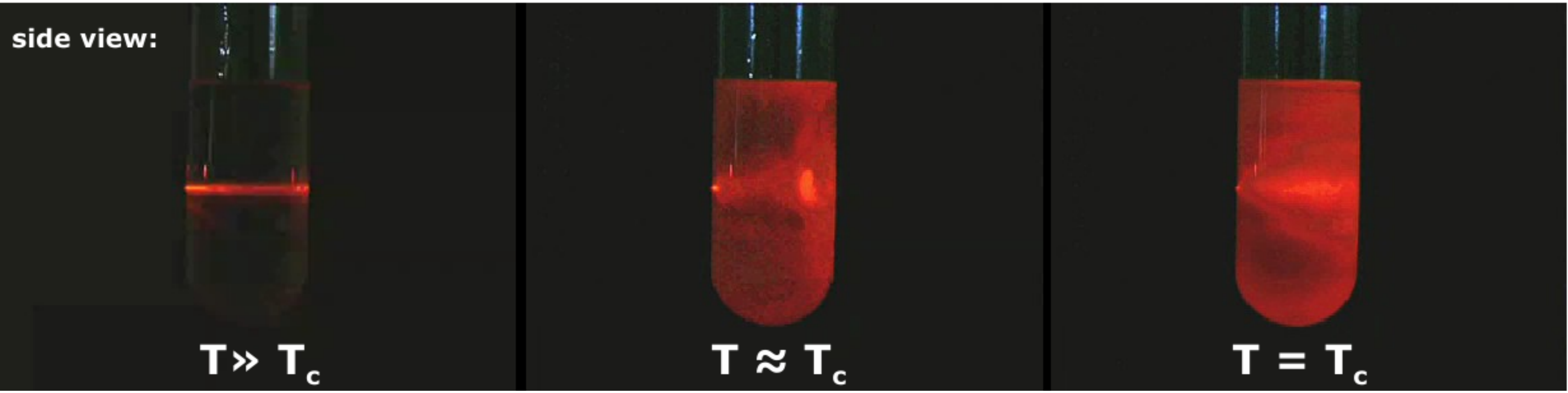
Cross over




$m_\sigma, \xi$ finite

# Critical point

- Second order phase transition
- Fluctuations at all length scales
  - Critical opalescence



# Higher moments (cumulants) and $\xi$

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots\right]. \qquad \Rightarrow \qquad \xi = m_\sigma^{-1}$$

- Moments (connected) of $\boldsymbol{q} = 0$ mode $\sigma_V \equiv \int d^3x\, \sigma(x)$:

First approximation: count σ propagators

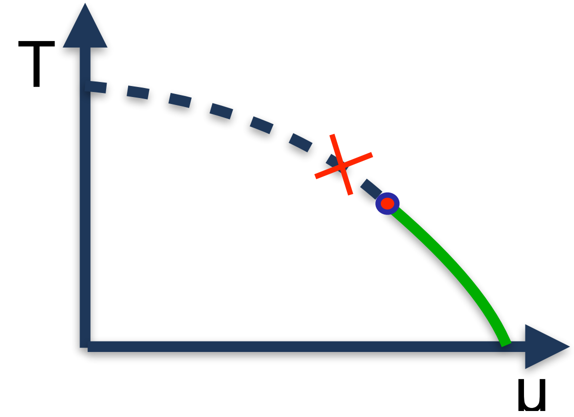
$$\kappa_2 = \langle \sigma_V^2 \rangle = VT\,\xi^2\,; \qquad \kappa_3 = \langle \sigma_V^3 \rangle = 2VT^2\,\lambda_3\,\xi^6\,;$$

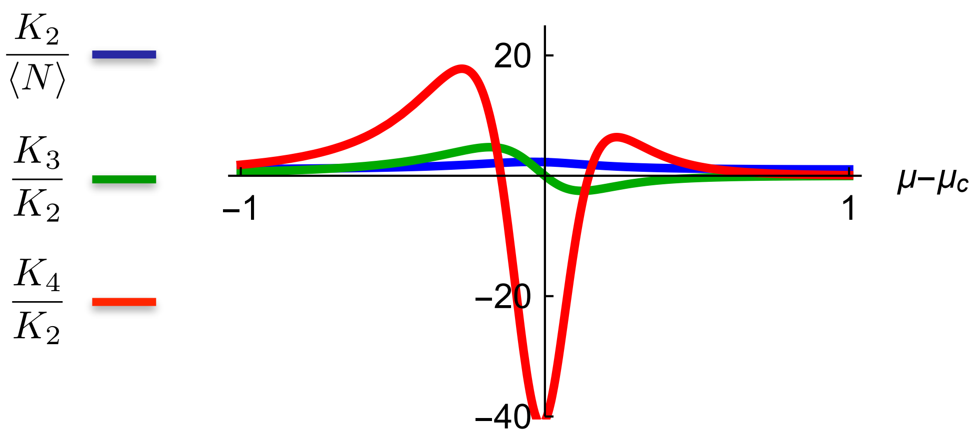
$$\kappa_4 = \langle \sigma_V^4 \rangle_c \equiv \langle \sigma_V^4 \rangle - 3\langle \sigma_V^2 \rangle^2 = 6VT^3\,[\,2(\lambda_3\xi)^2 - \lambda_4\,]\,\xi^8\,.$$

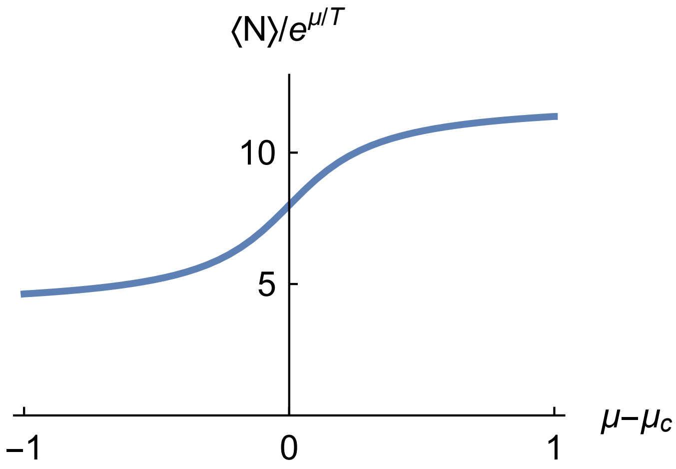
- Tree graphs. Each propagator gives $\xi^2$.


- Scaling requires "running": $\lambda_3 = \tilde\lambda_3 T (T\xi)^{-3/2}$ and $\lambda_4 = \tilde\lambda_4 (T\xi)^{-1}$, i.e.,

$$\kappa_3 = \langle \sigma_V^3 \rangle = 2VT^{3/2}\,\tilde\lambda_3\,\xi^{4.5}\,; \quad \kappa_4 = 6VT^2\,[\,2(\tilde\lambda_3)^2 - \tilde\lambda_4\,]\,\xi^7\,.$$

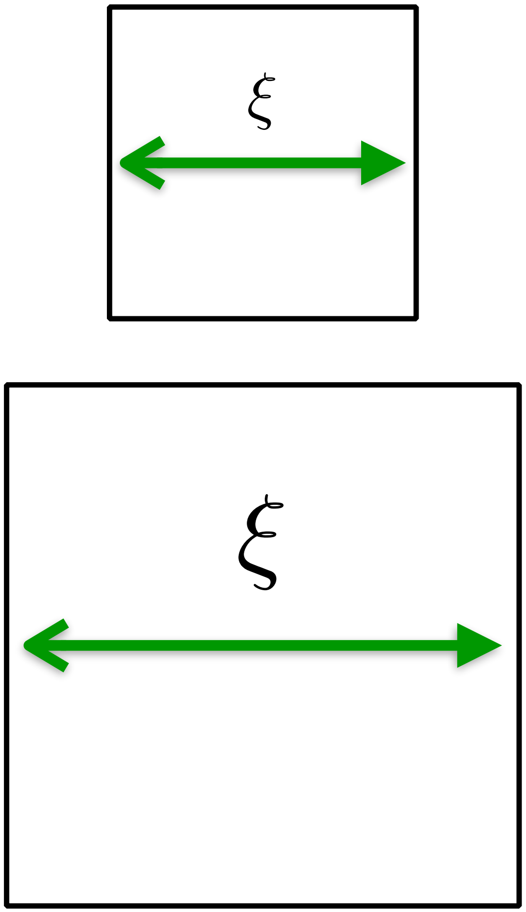

T
μ


$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
−40
−1
1
μ−μc


⟨N⟩/e^{μ/T}
10
5
−1
0
1
μ−μc

# Finite size scaling

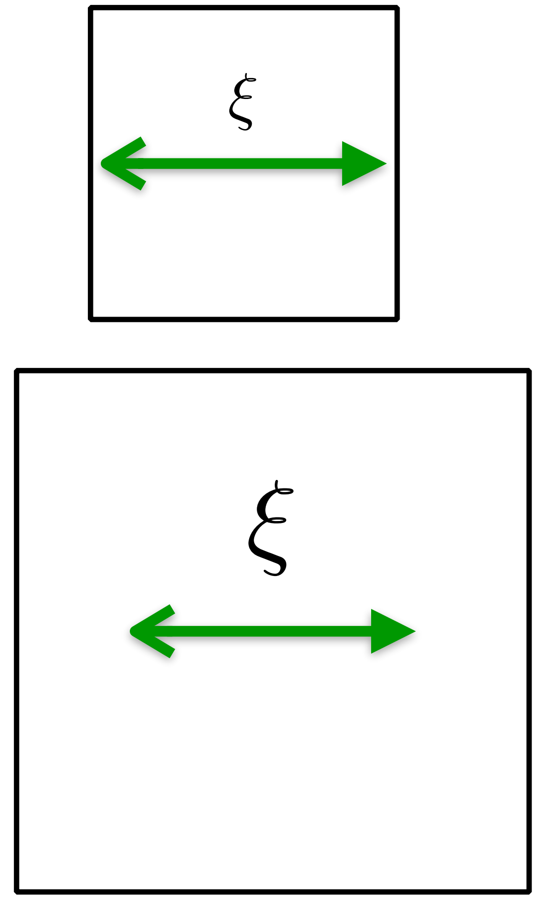
Second order (critical point)




$$\xi \sim V^{2/3}, \ \ \chi \sim V^{4/3}$$

(mean field)

Cross over



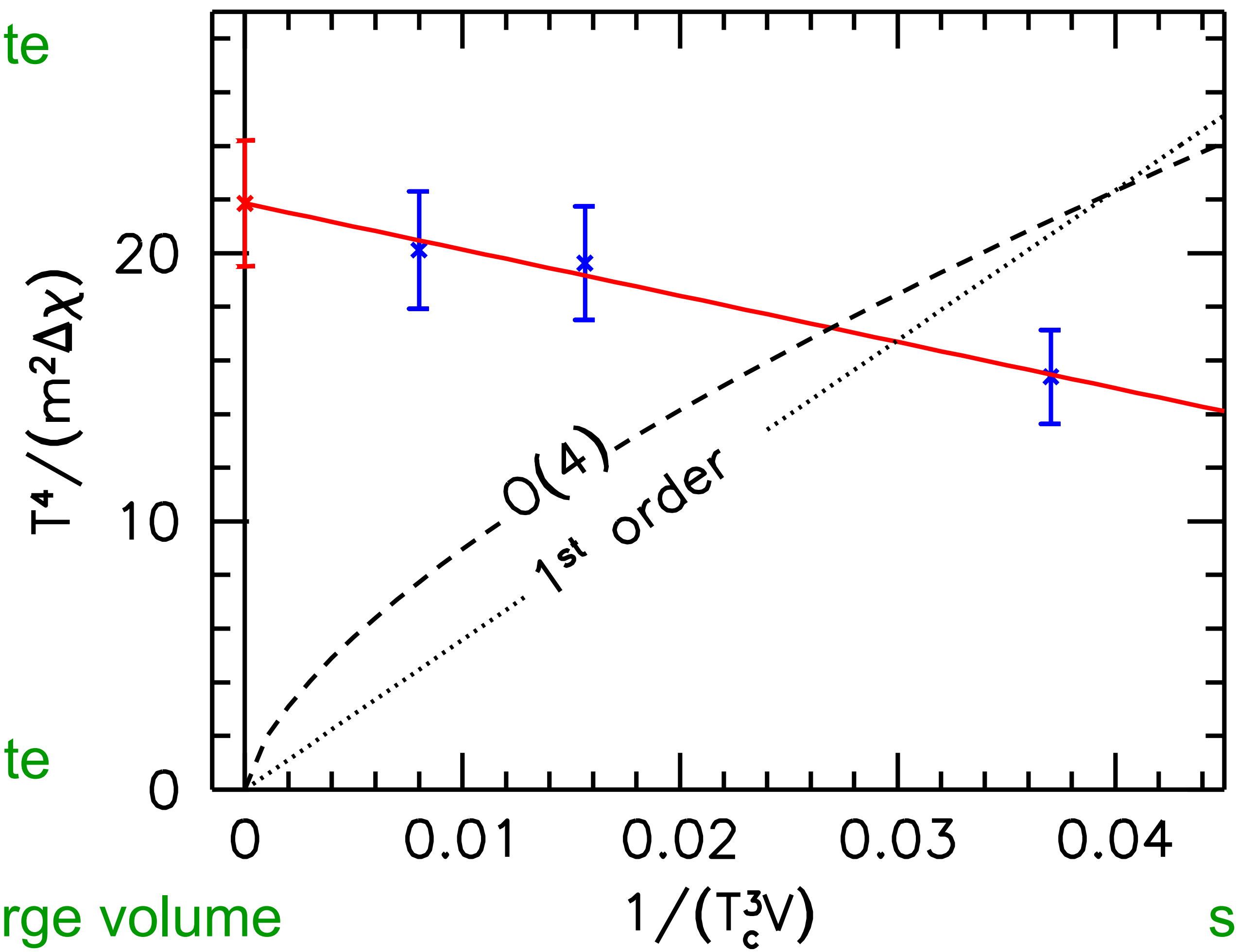

$$\xi = \mathrm{const}, \ \ \chi = \mathrm{const}$$

NB: 1st order: $\chi \sim V$

# QCD at μ=0 is cross-over

Aoki et al, Nature 43:675-678,2006

$\chi$ finite

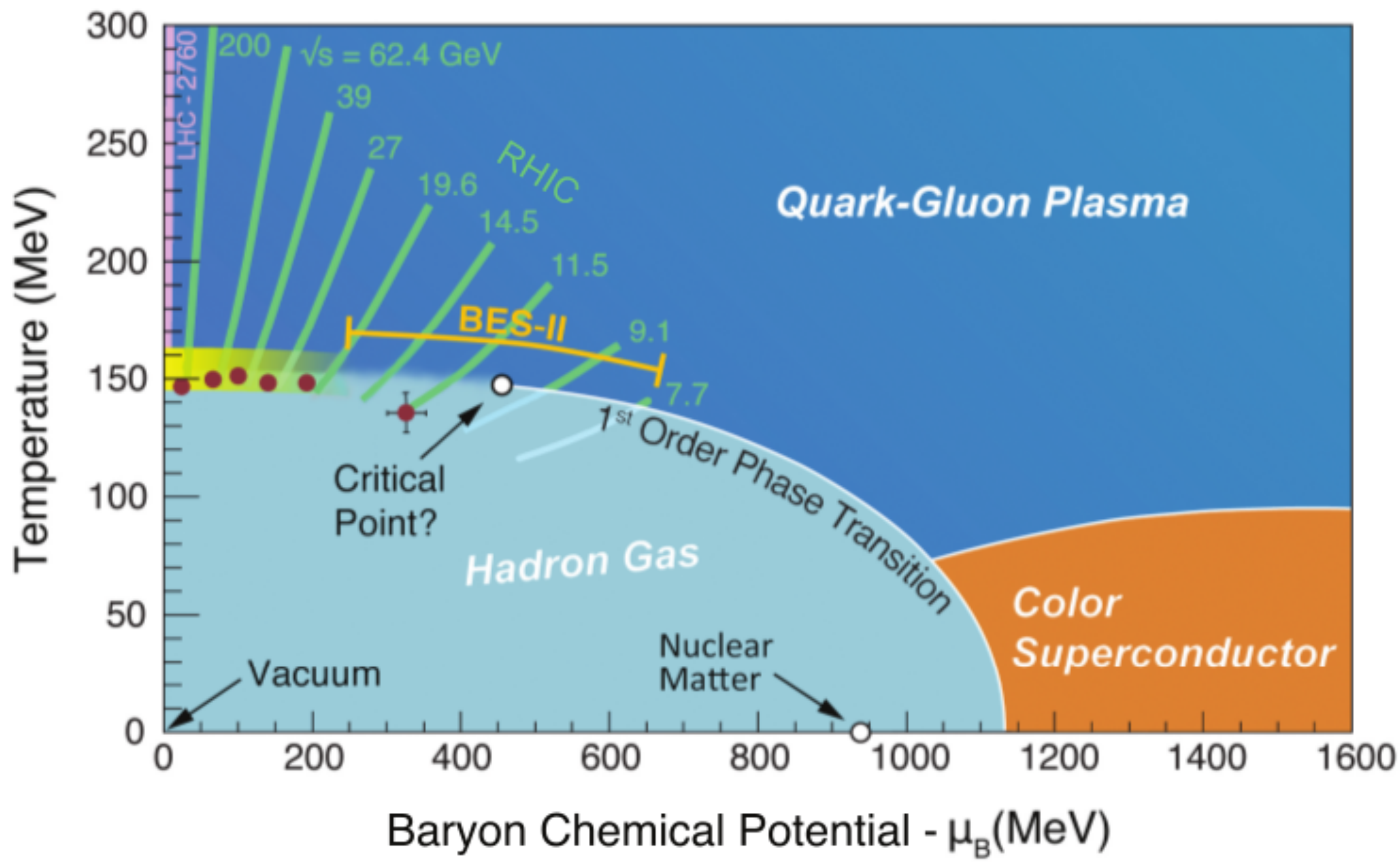



$\chi$ infinite

large volume

small volume

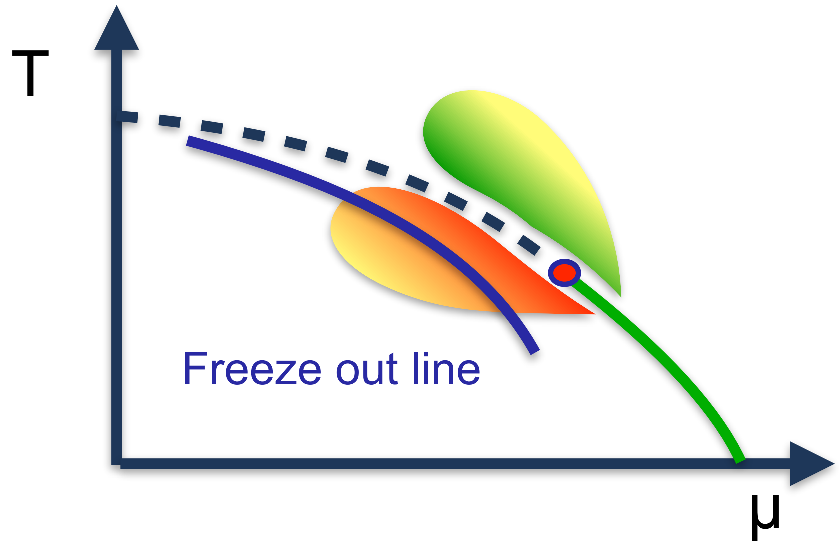
# The phase diagram

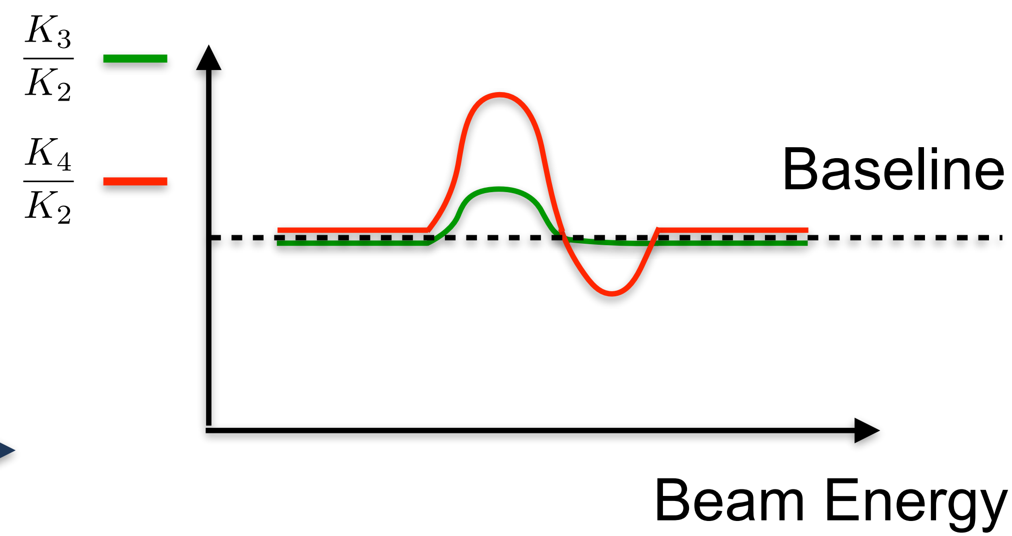


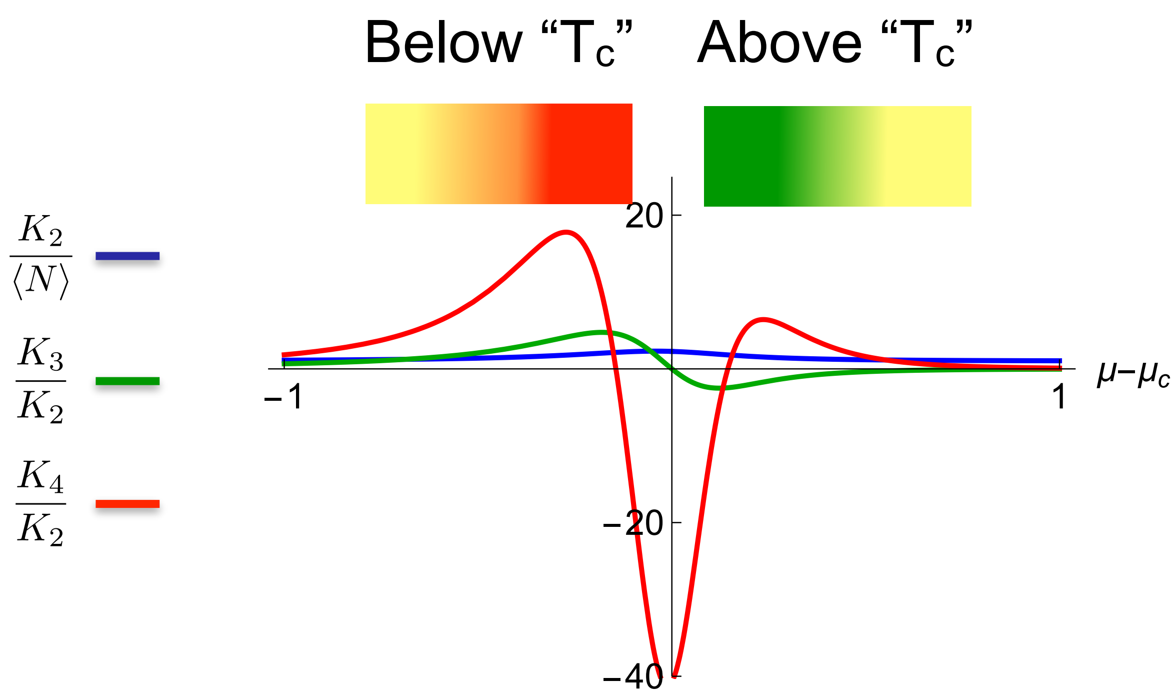

Increase chemical potential by lowering the beam energy

In reality, we add baryons (nucleons) from target and projectile to mid-rapidity

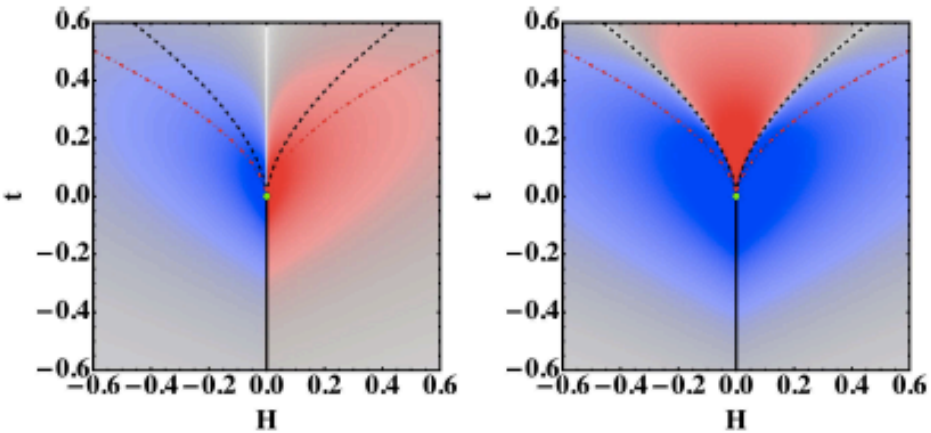
# What to expect from experiment?

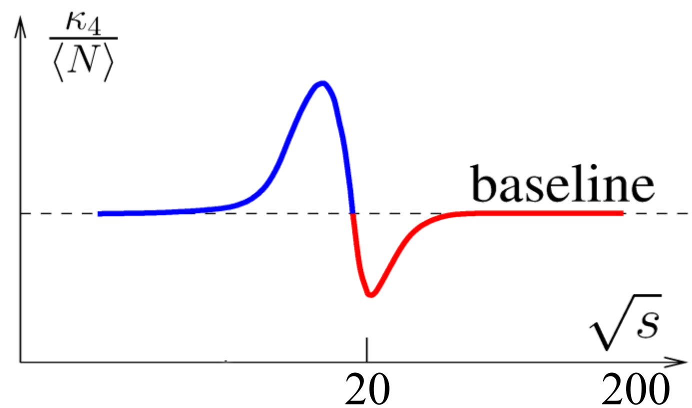


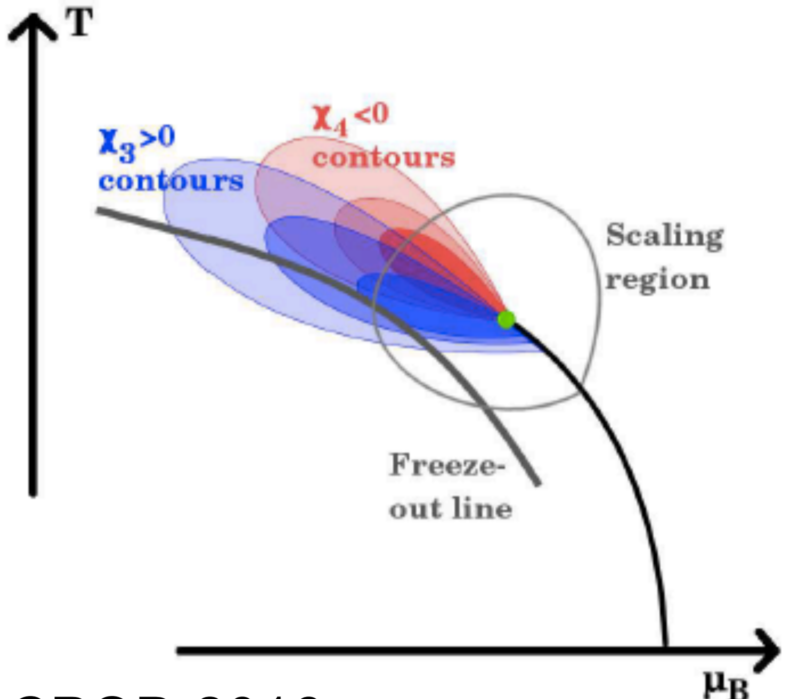



# Expectation from Calculations



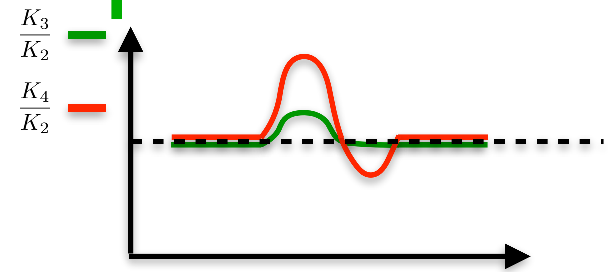




Characteristic "Oscillating pattern" is expected for the QCD critical point but *the exact shape depends on the location of freeze-out with respect to the location of CP*

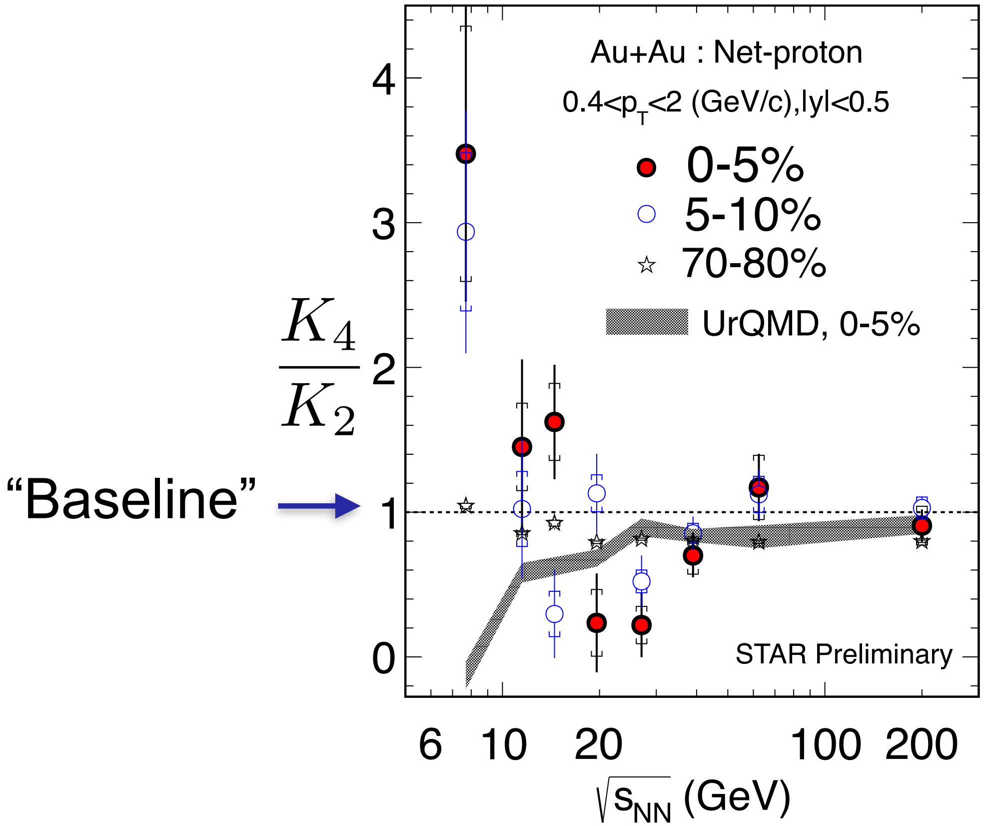
- M. Stephanov, ***PRL107***, 052301(2011)
- V. Skokov, Quark Matter 2012
- J.W. Chen, J. Deng, H. Kohyyama, arXiv: 1603.05198, Phys. Rev. **D93** (2016) 034037

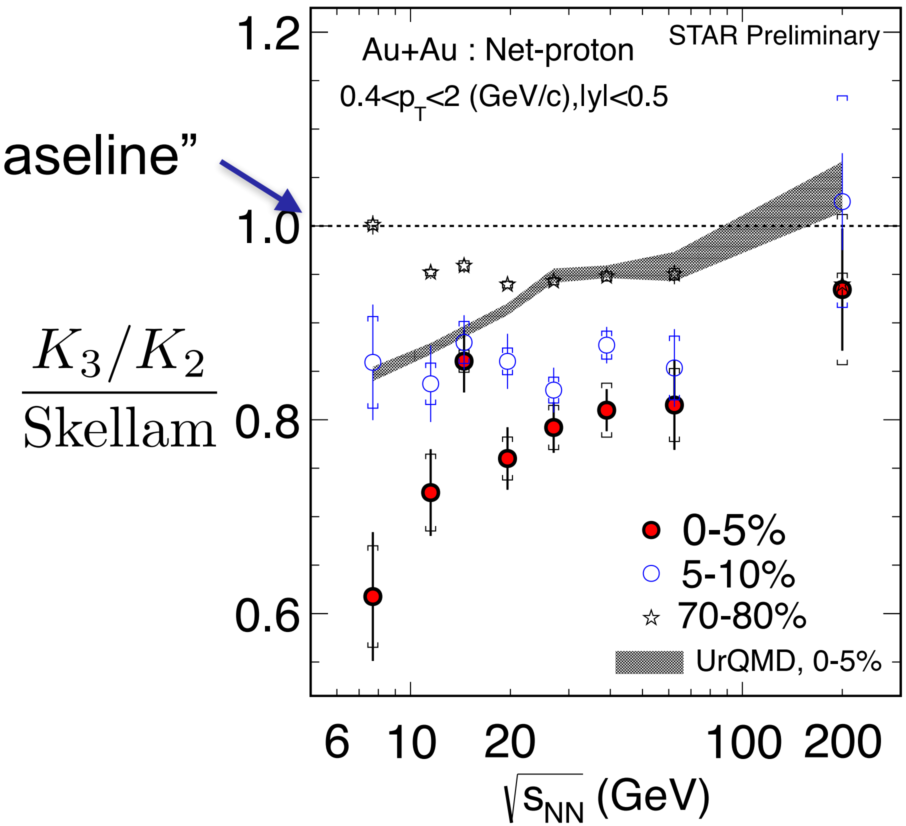
N. Xu, CPOD 2016

# Latest STAR result on net-proton cumulants



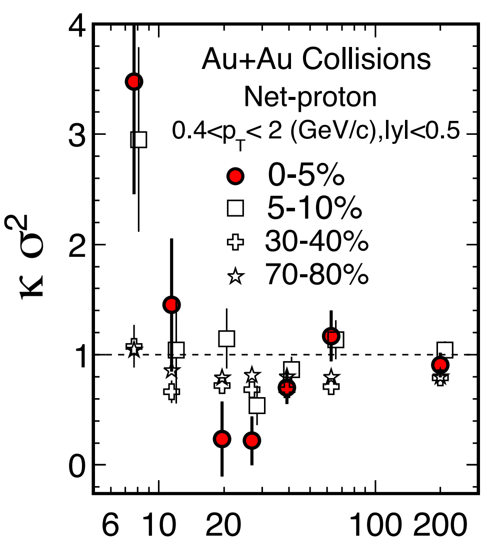

X. Luo, NPA 956 (2016) 75

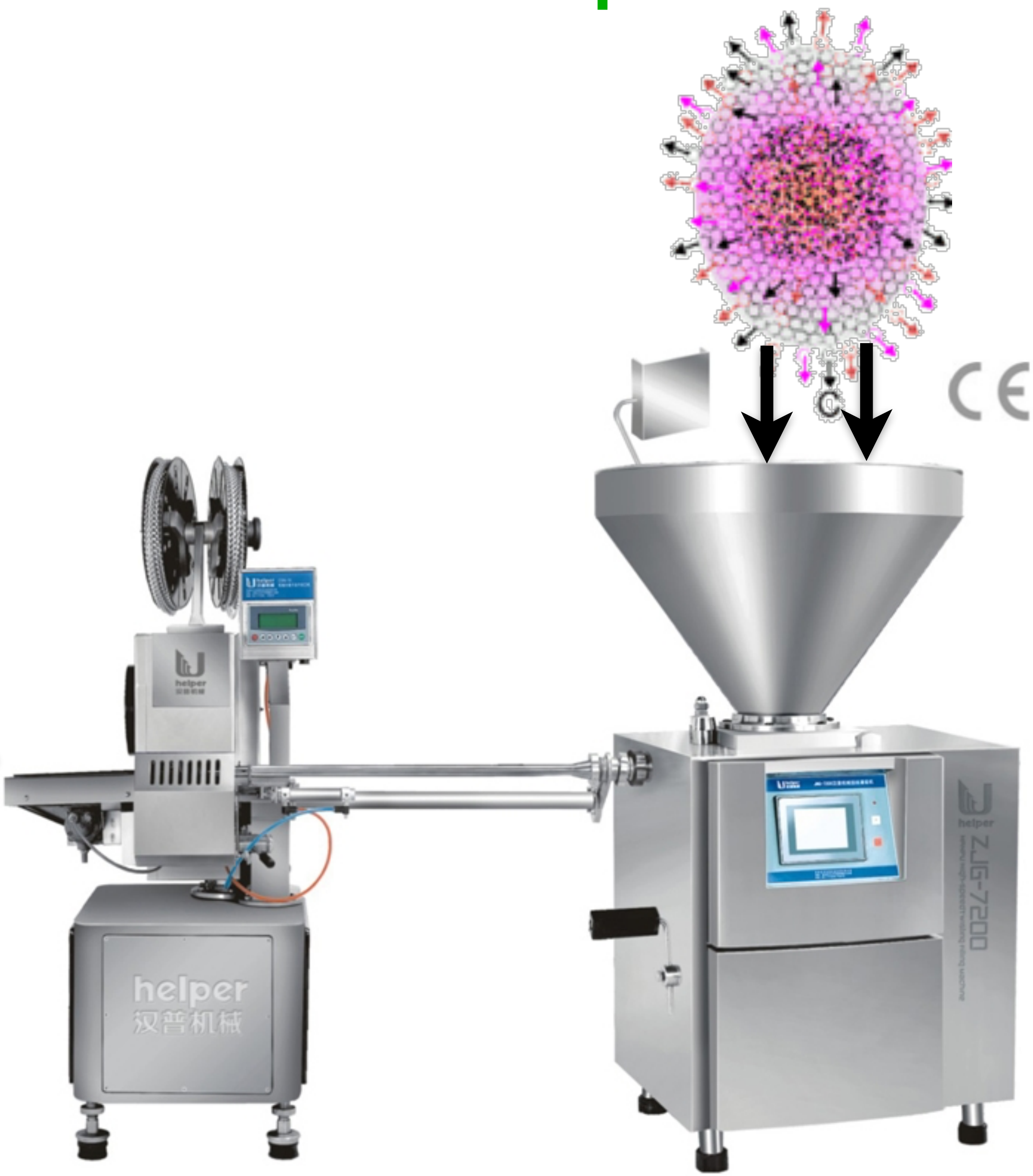


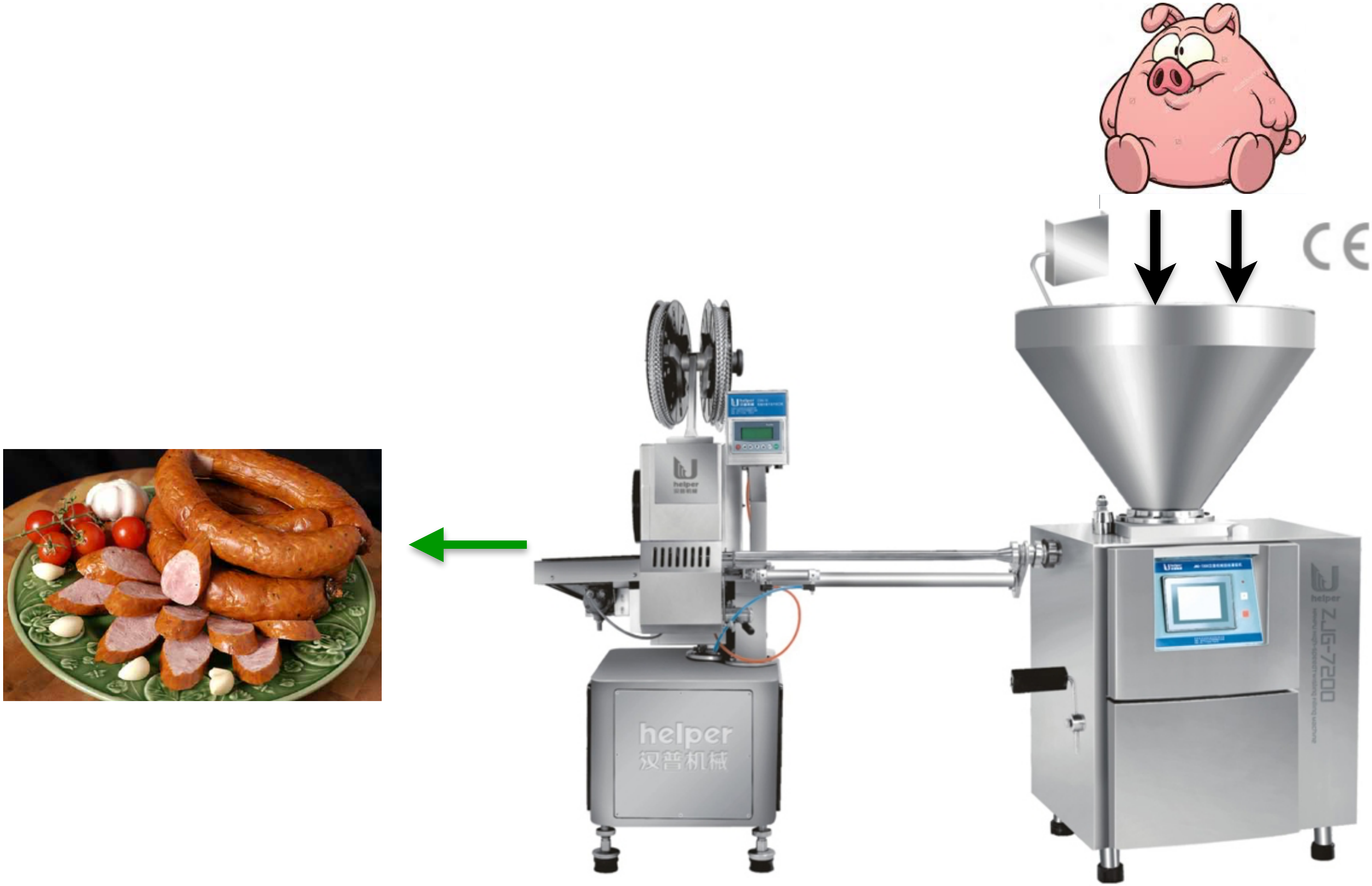

$K_4/K_2$ follows expectation, $K_3/K_2$ no so much…..
URQMD totally fails to get trend for $K_4/K_2$ !

# The measurement process

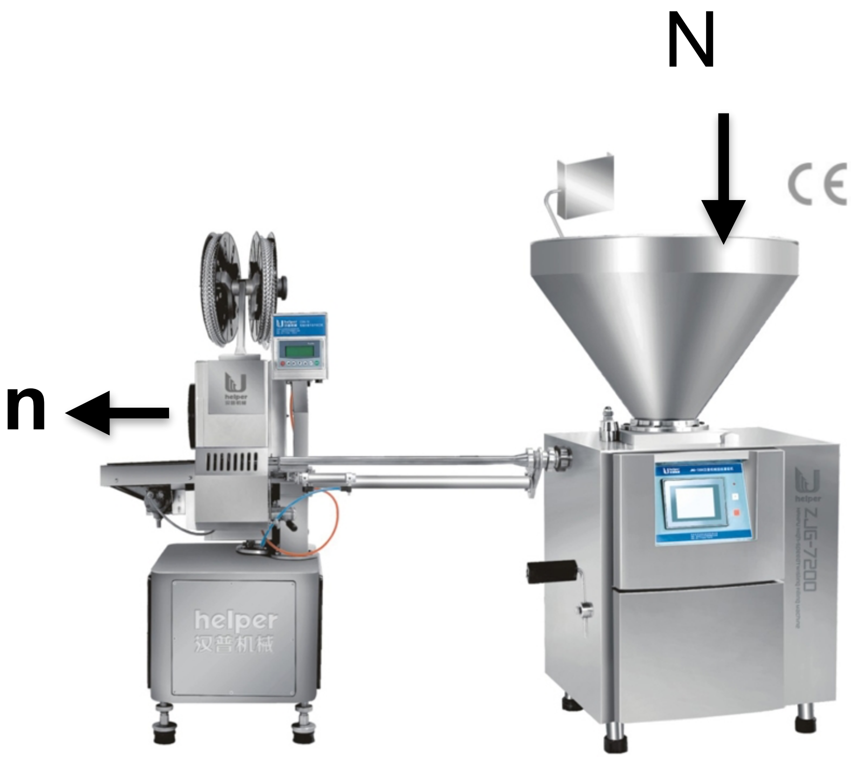


# Or in the real world…..



# Modeling the detector (multiplicities only)




$$B(n, N, \epsilon, \ldots)$$

Detector maps TRUE number of particles onto OBSERVED number of particles

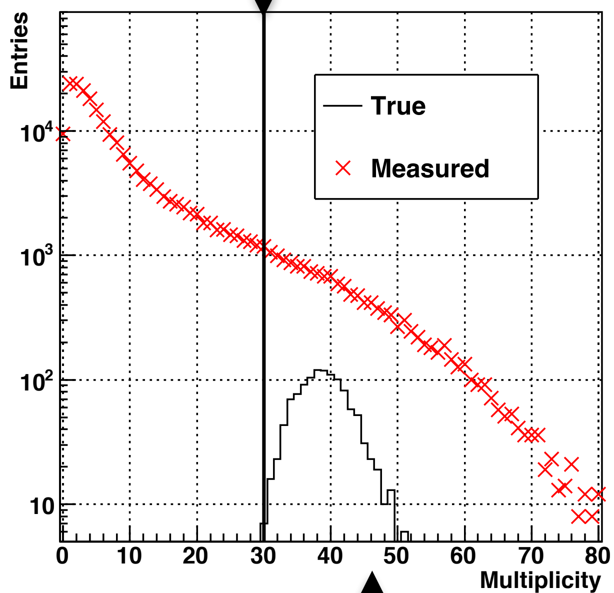
$$\boxed{\text{Observed}}\quad p(n) = \sum_N B(n, N, \epsilon, \ldots) P(N)\quad\boxed{\text{True}}$$

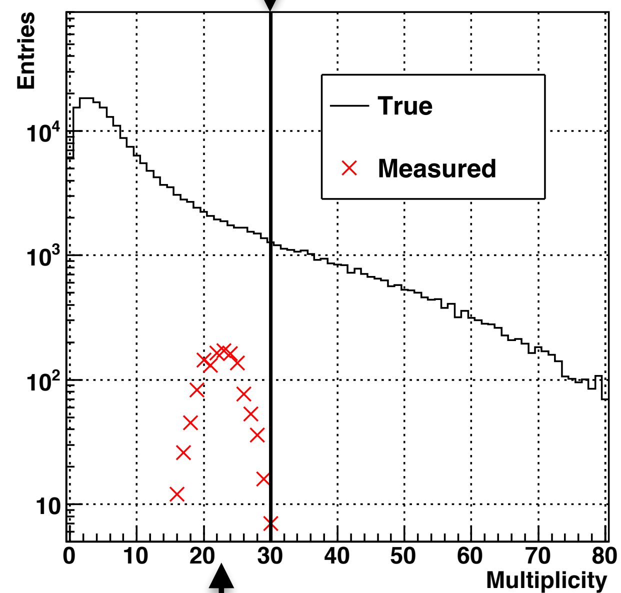
B is matrix which controls the mapping

$$p_n = B_{n,N} P_N$$

measure in this bin

true particles in this bin




True particles contributing

measured particles contributing

J-F. Grosse-Oetringhaus,
K. Reygers arXiv:0912.0023v2

# Unfolding

$$p(n) = \sum_N B(n, N, \epsilon, \ldots) P(N)$$

Observed

True

To get TRUE P(N) we need to invert matrix B so that

$$P(N) = \sum_n B^{-1}(N, n, \epsilon, \ldots) p(n)$$

True

Observed

This is called UNFOLDING

In practice simple inverting does not work!

# Or in the real world…

# Example: Binomial

$p_n$ = $B_{n,N}$ $P_N$

$$\begin{pmatrix} p(0) \\ p(1) \\ p(2) \\ p(3) \\ p(4) \end{pmatrix} = \begin{pmatrix} 1 & 1-\epsilon & (1-\epsilon)^2 & (1-\epsilon)^3 & (1-\epsilon)^4 \\ 0 & \epsilon & 2\epsilon(1-\epsilon) & 3\epsilon(1-\epsilon)^2 & 4\epsilon(1-\epsilon)^3 \\ 0 & 0 & \epsilon^2 & 3\epsilon^2(1-\epsilon) & 6\epsilon^2(1-\epsilon)^2 \\ 0 & 0 & 0 & \epsilon^3 & 4\epsilon^3(1-\epsilon) \\ 0 & 0 & 0 & 0 & \epsilon^4 \end{pmatrix} \begin{pmatrix} P(0) \\ P(1) \\ P(2) \\ P(3) \\ P(4) \end{pmatrix}$$

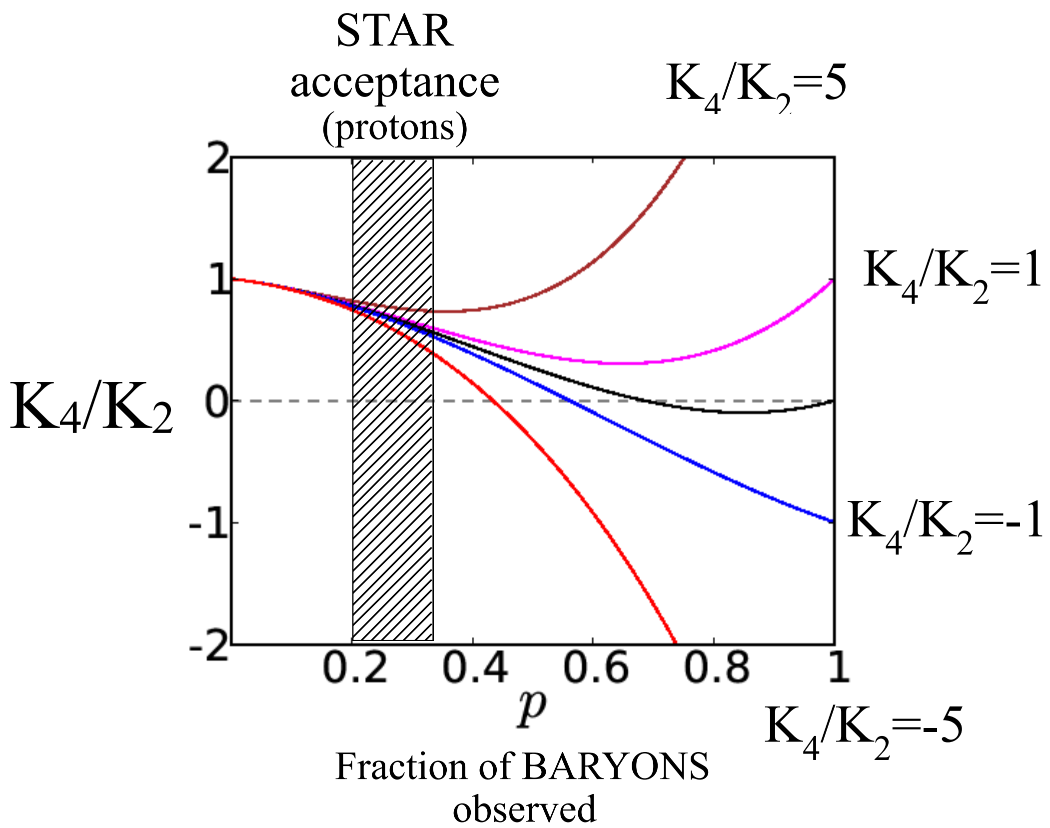
Binomial probability ε<1 is often called "efficiency"

Theoretically: $n_{Obs.} \leq N_{True} \Rightarrow$ B is triangular

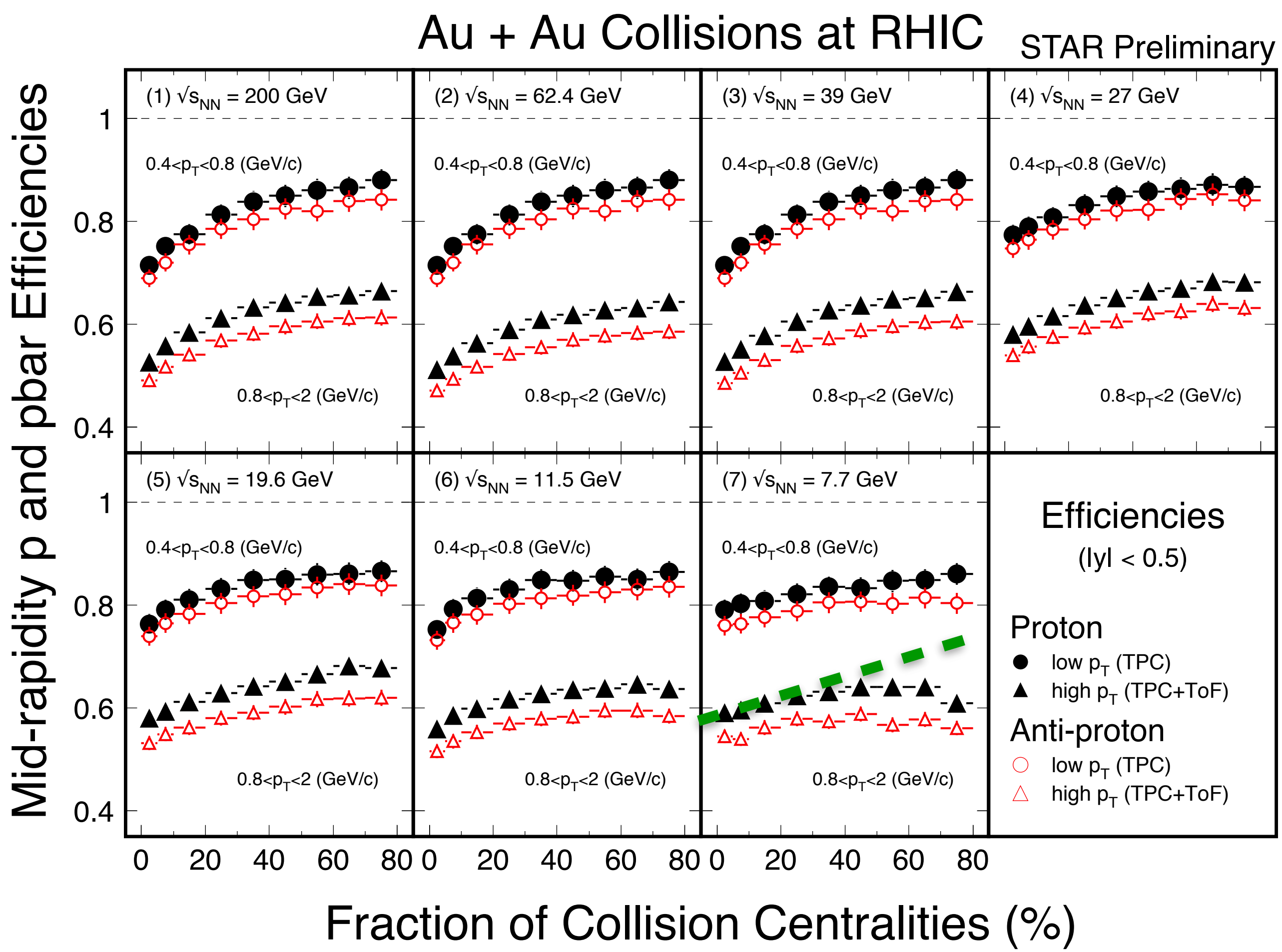
$B_{n,N}$ almost singular ! STAR: 0.6<ε<0.8

In Practice: Who knows… is the detector even "binomial"

# Binomial allows to invert (at least for cumulants)



# Is B(n,N) binomial ?

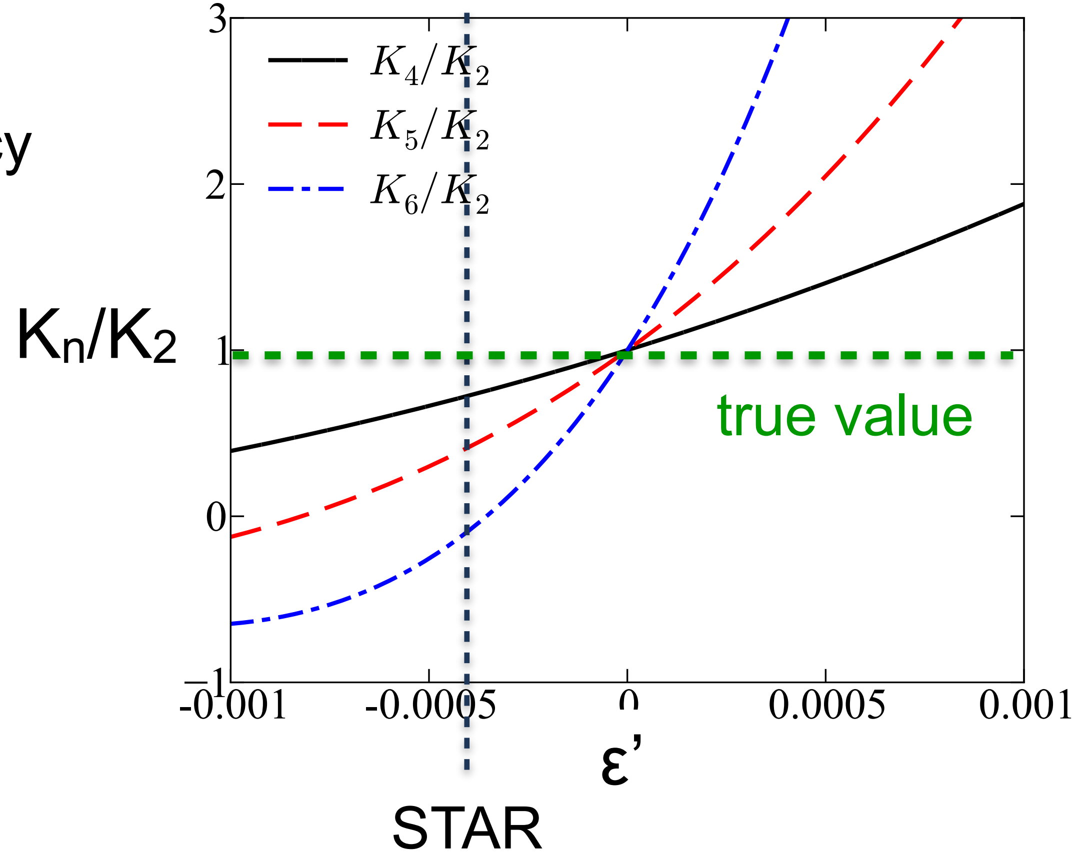



Efficiency depends on multiplicity!

# Binomial distributions and real detectors

The most obvious correction:
Multiplicity dependence of efficiency

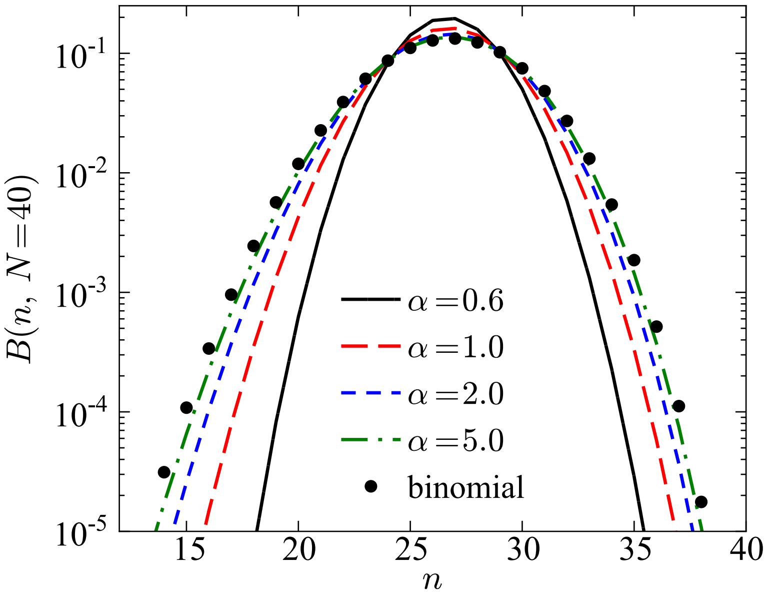
$$\epsilon(N) = \epsilon_0 + \epsilon'(N - \langle N \rangle)$$




More details:
A. Bzdak, R. Holzmann et al.
arXiv:1603.09057

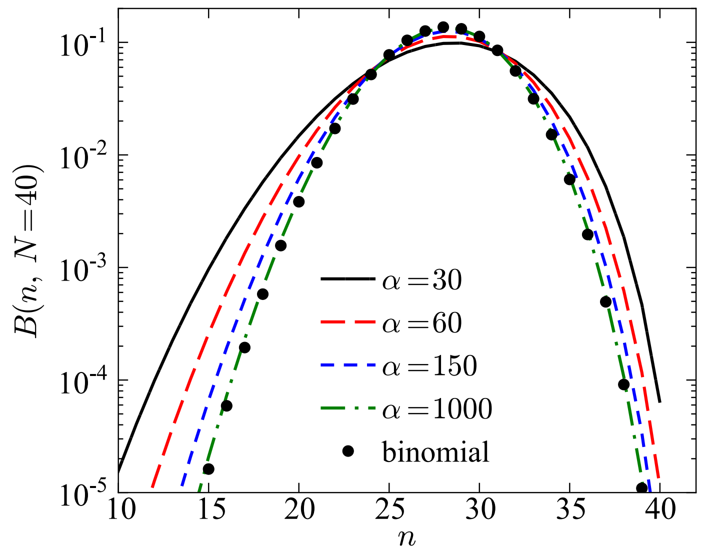
# Other models for B(n,N)

## Hypergeometric




| Hypergeometric | $\alpha = 0.6$ | $\alpha = 1.0$ | $\alpha = 2.0$ | $\alpha = 5.0$ |
|---|---|---|---|---|
| $K_3/K_2$ | 1.16 | 1.12 | 1.07 | 1.03 |
| $K_4/K_2$ | 0.66 | 0.88 | 0.98 | 1.00 |
| $K_5/K_2$ | 2.19 | 1.68 | 1.23 | 1.05 |
| $K_6/K_2$ | -3.99 | -1.38 | 0.31 | 0.89 |

## Beta Binomial




| Beta-binomial | $\alpha = 30$ | $\alpha = 60$ | $\alpha = 150$ | $\alpha = 1000$ |
|---|---|---|---|---|
| $K_3/K_2$ | 1.28 | 1.24 | 1.13 | 1.02 |
| $K_4/K_2$ | 0.82 | 1.45 | 1.35 | 1.07 |
| $K_5/K_2$ | -1.11 | 1.15 | 1.63 | 1.16 |
| $K_6/K_2$ | 5.71 | -0.44 | 1.80 | 1.32 |

# Insights from Theory

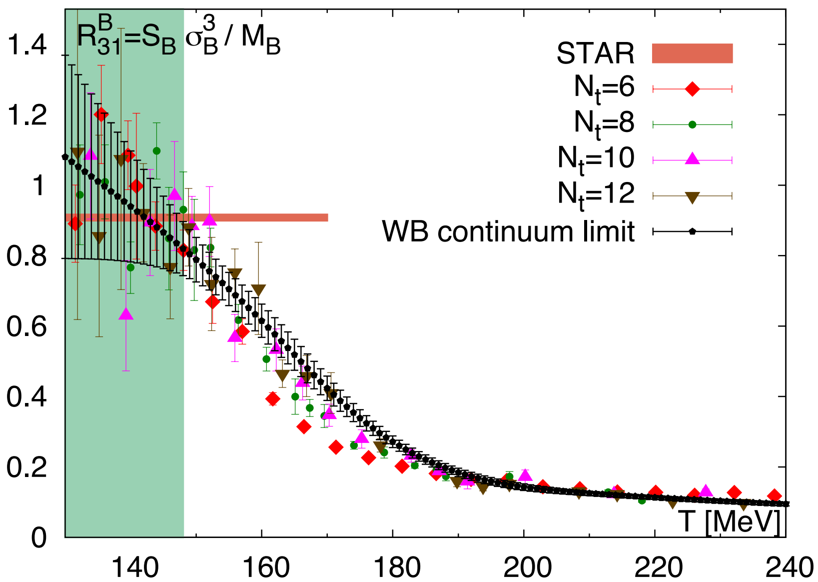
# Compare Data with Lattice QCD and other field theoretical models

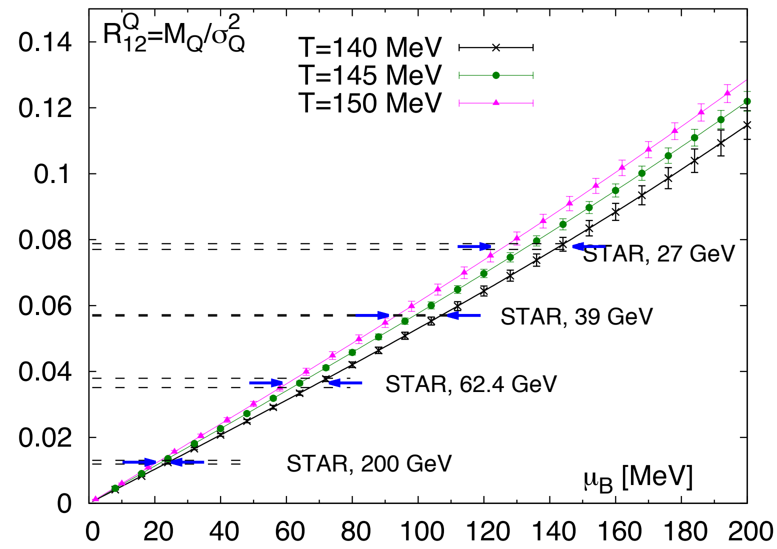
# Compare Data with Lattice QCD and other field theoretical models

- Lattice cannot calculate hadron abundances
- Cumulants are well defined quantities
- Compare cumulants !?
  - Baryon number conservation
  - Experiment measures protons not all baryons
  - Volume is not fixed in experiment
  - Experiment has finite momentum space coverage (usually)

# Compare Data with Lattice QCD

For example: Wuppertal-Budapest (arXiv:1305.5161)
(similar from Hot QCD)



# Baryon number conservation

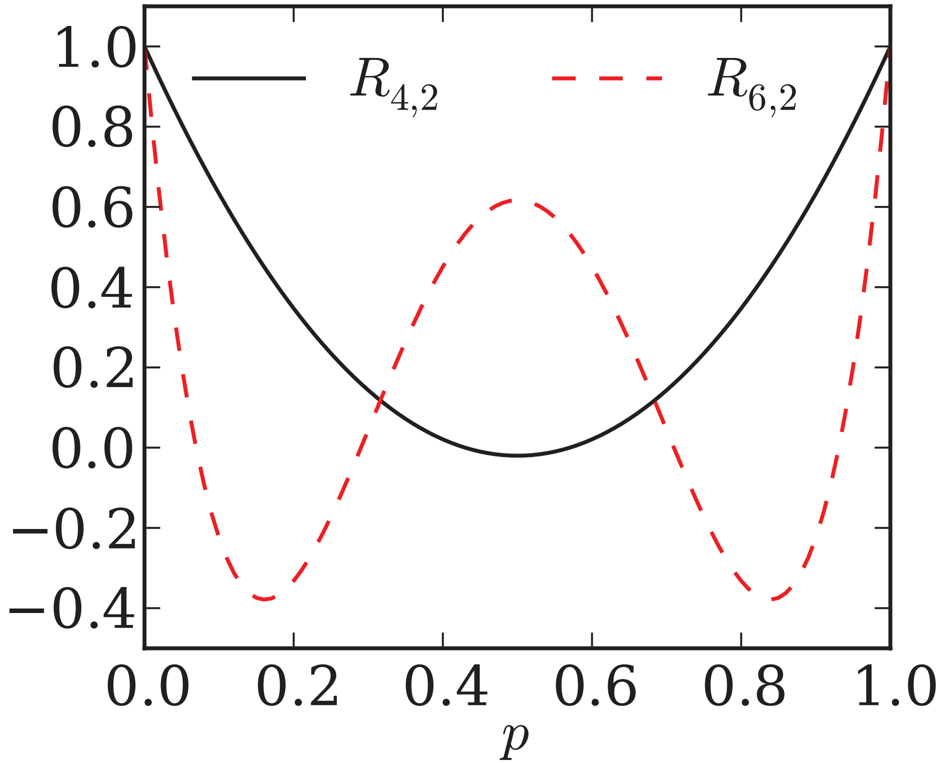
Lattice works in grand-canonical ensemble:

Baryon number conserved only on average

Experiment: Baryon number is conserved event-by-event

Bzdak et al. arXiv:1203.4529

No physics other than baryon number conservation




Fraction of total baryons in detector

$$R_{4,2} = \frac{K_4}{K_2}$$

$$R_{6,2} = \frac{K_6}{K_2}$$

# Protons vs Baryons

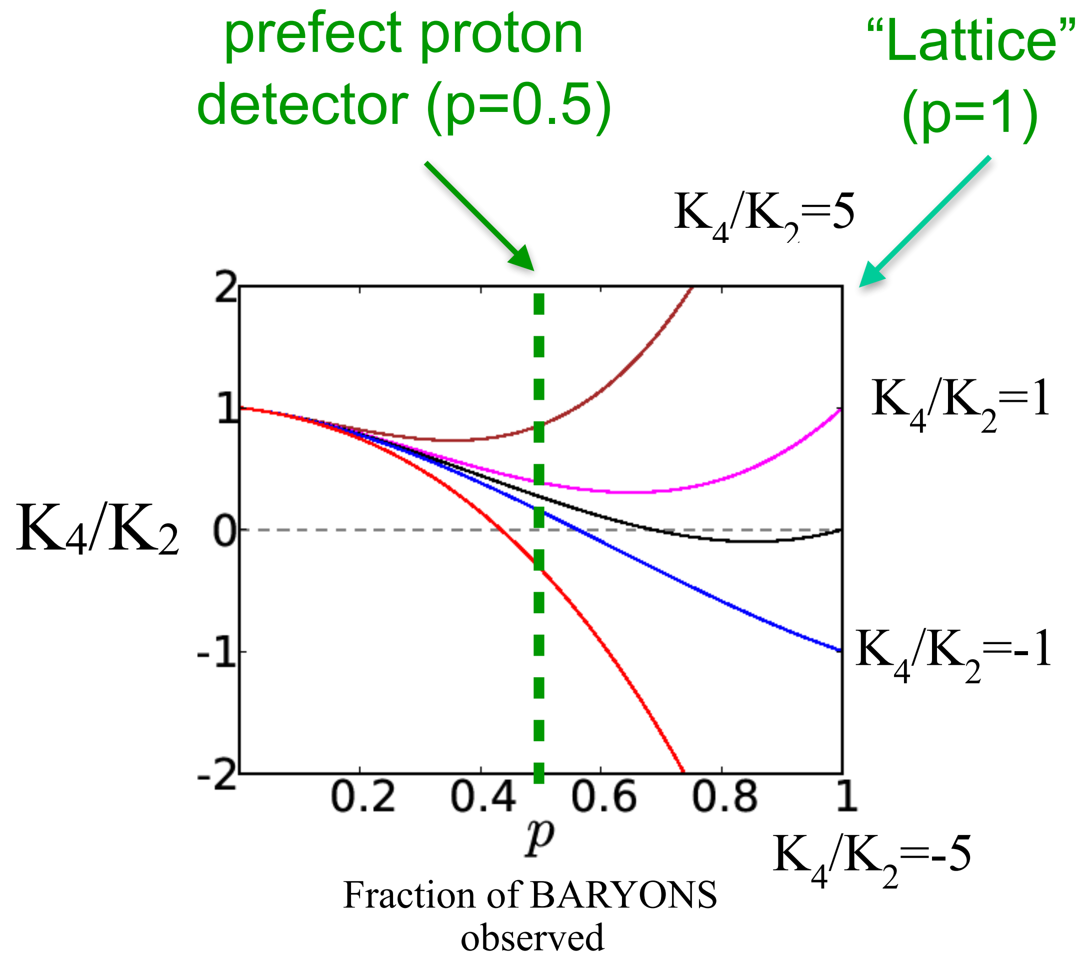
Fast isospin exchange
a.k.a lots of pions:

protons and neutrons follow binomial distribution

$$P(N_p) = \frac{B!}{N_p!(B-N_p)!} p^{N_p}(1-p)^{B-N_p}$$

with p~ 0.5

(Kitazawa, Asakawa arXiv:1107.2755)



# Finite acceptance

Example: “Charge” susceptibility

$$\chi_Q = \int d^3x < \rho(x)\rho(0) >= \int d^3p < \tilde{\rho}(p)\tilde{\rho}(0) >$$

Equivalence of *Integrated* coordinate space and momentum space correlation function

Experiment almost never integrates ALL of momentum space!

Lattice (hopefully) does integrate over all coordinate space

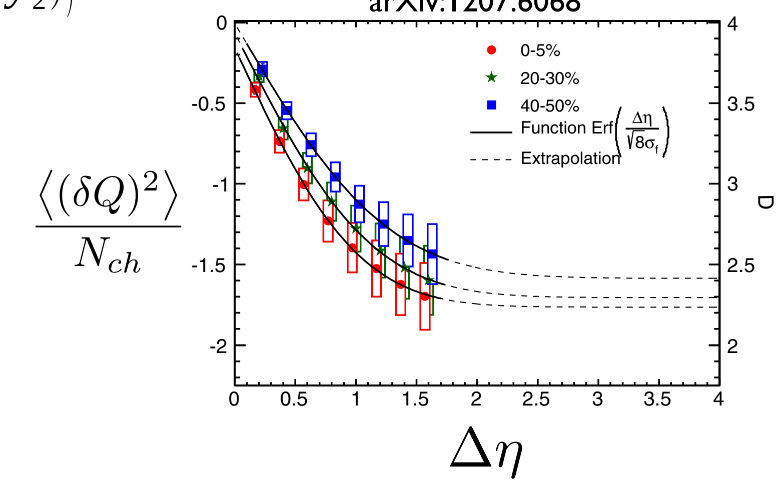
# Correlations: Lattice vs Data

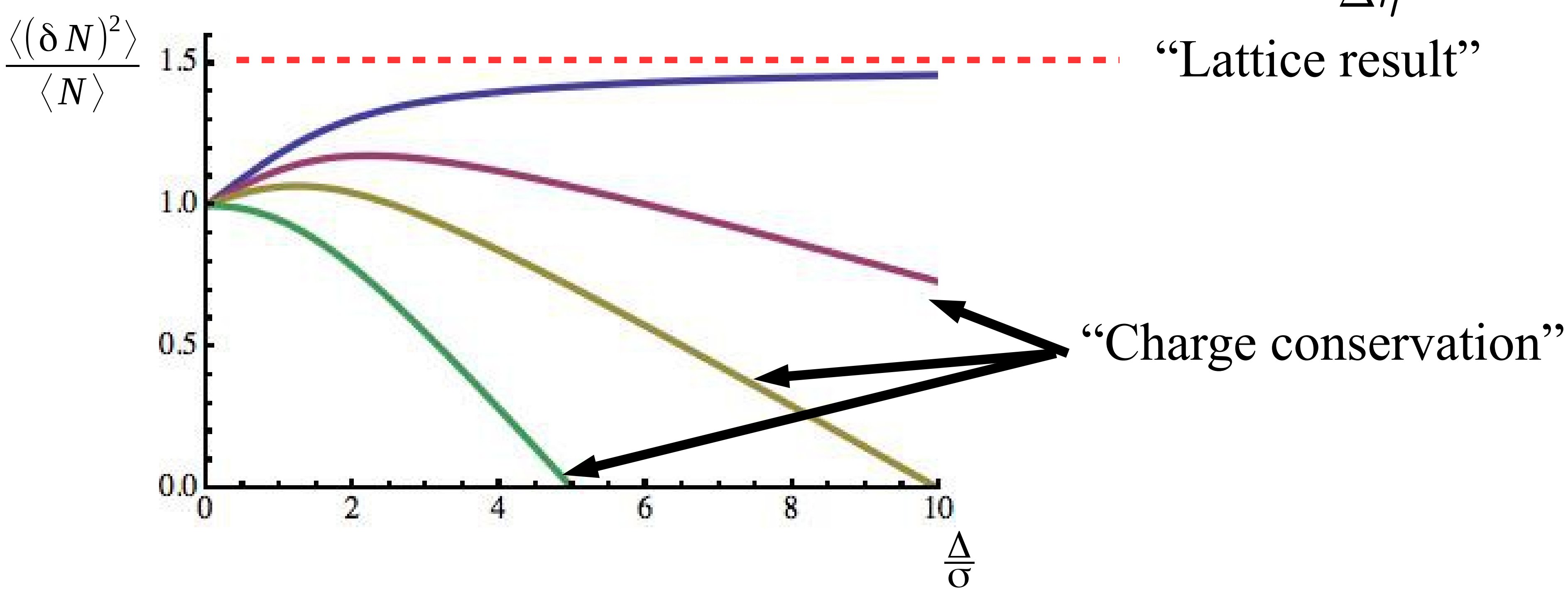
$$\langle n(y_1)(\, n(y_2)-\delta(y_1-y_2)\,)\rangle=\langle n(y_1)\rangle\langle n(y_2)\rangle\left(1+C(y_1,y_2)\right)$$

$$C(y_{1,}y_2)\sim\exp\left(\frac{-(y_1-y_2)^2}{2\sigma^2}\right)$$

$$\frac{\langle(\delta N)^2\rangle}{\langle N\rangle}=1+\langle N\rangle\int_{\Delta/2}^{\Delta/2}C(y1,y2)dy1\ dy_2$$

Alice Charge Flucts

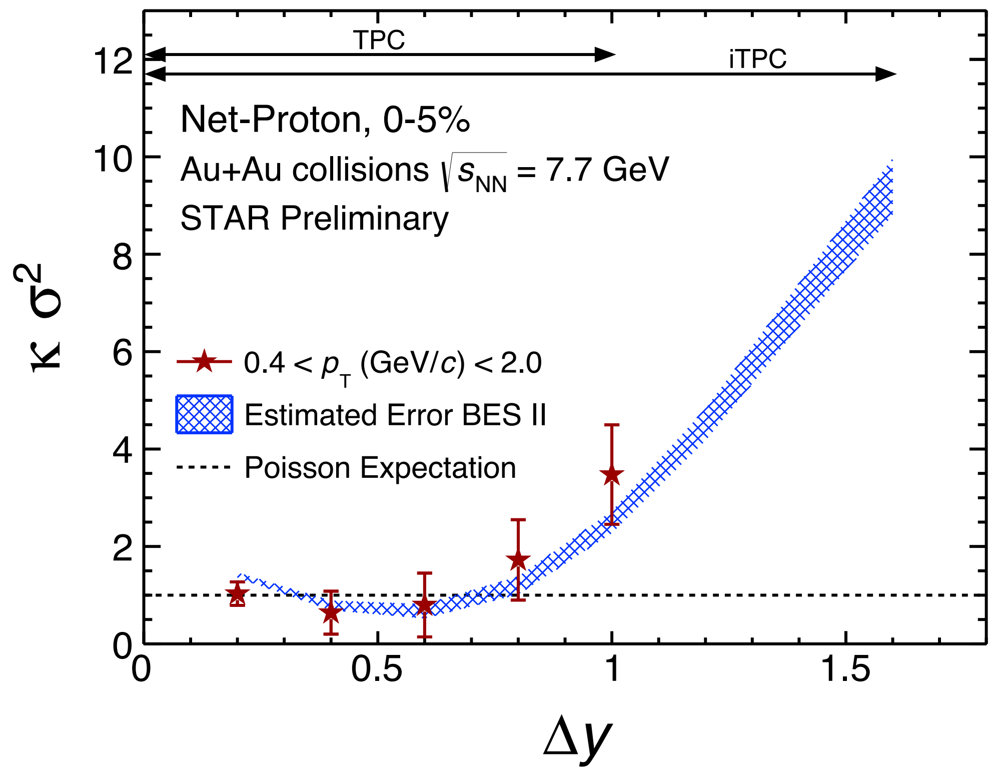


# Dependence on Rapidity window

X. Luo, EMMI Workshop, Nov. 2015

- Kurtosis depends strongly on Rapidity window
- Comparison with Lattice:
  - Lattice catches the full correlation length
  - need to expand rapidity window until signal saturates (after correcting for charge conservation)

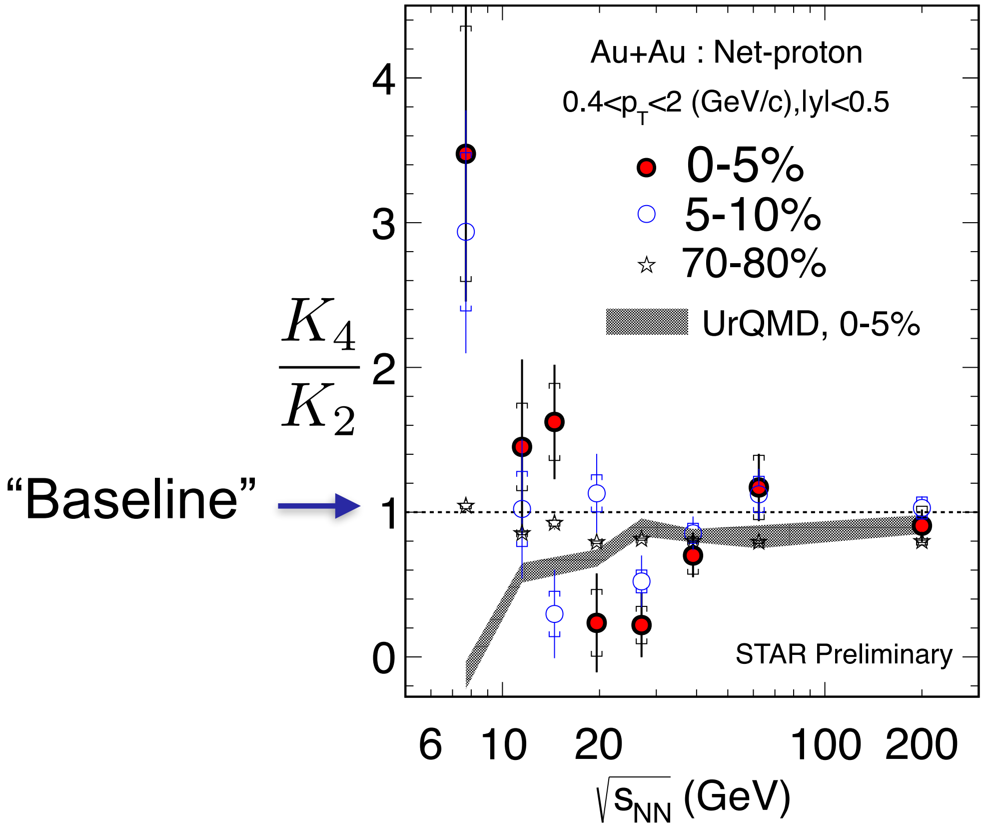


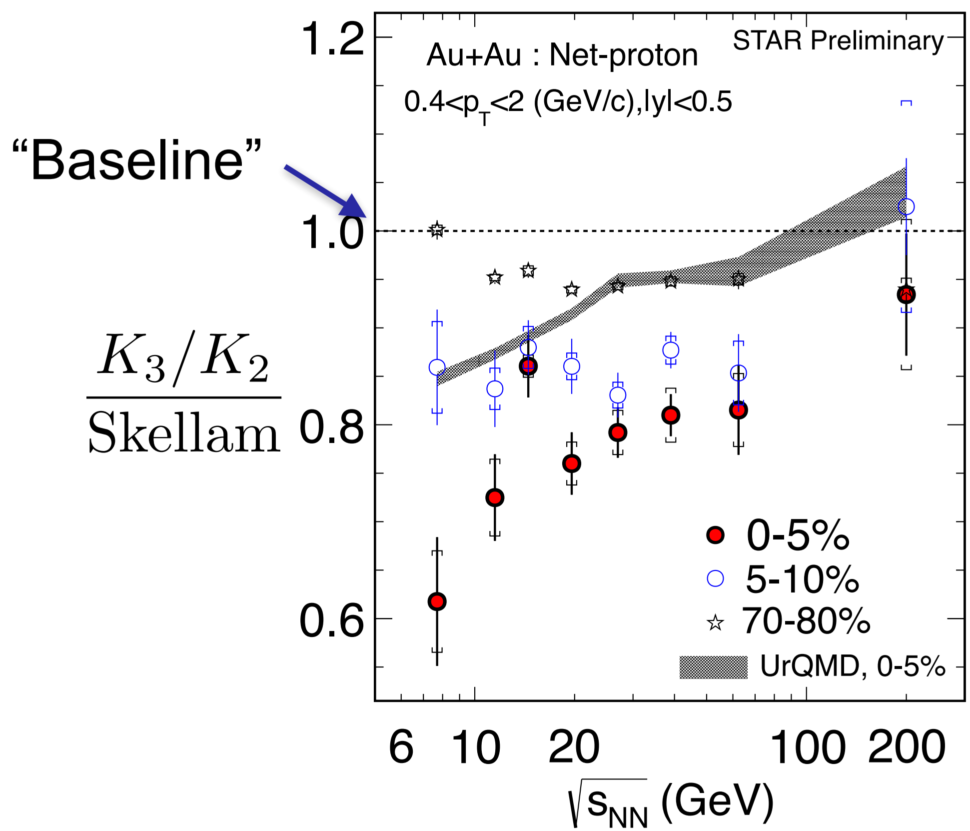

Any comparison of Lattice to Data needs to assure that cumulants reach asymptotic value in experiment.

So far this has NOT ben established for proton cumulants

# Back to data
## assuming that STAR has done their job

X. Luo, NPA 956 (2016) 75




$K_4/K_2$ follows expectation, $K_3/K_2$ no so much…..
URQMD totally fails to get trend for $K_4/K_2$ !

# Further insights: Correlations

Cumulants $$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z$$

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \langle (\delta N)^2 \rangle$$

$$\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2),$$

$C_2$: Correlation Function

$$K_3 = \langle (\delta N)^3 \rangle$$

$$\rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)\underline{C_2(p_2, p_3)} + \rho_1(p_2)\underline{C_2(p_1, p_3)} + \rho_1(p_3)\underline{C_2(p_1, p_2)} + \underline{C_3(p_1, p_2, p_3)}$$

More details: Bzdak et al, arXiv:1607.07375, Lin et al arXiv:1512.09125

# From Cumulants to Correlations (no anti-protons)

Defining integrated correlations function

$$C_n = \int dp_1 \ldots dp_n C_n(p_1, \ldots, p_n)$$

Simple Algebra leads to relation between correlations $C_n$ and $K_n$

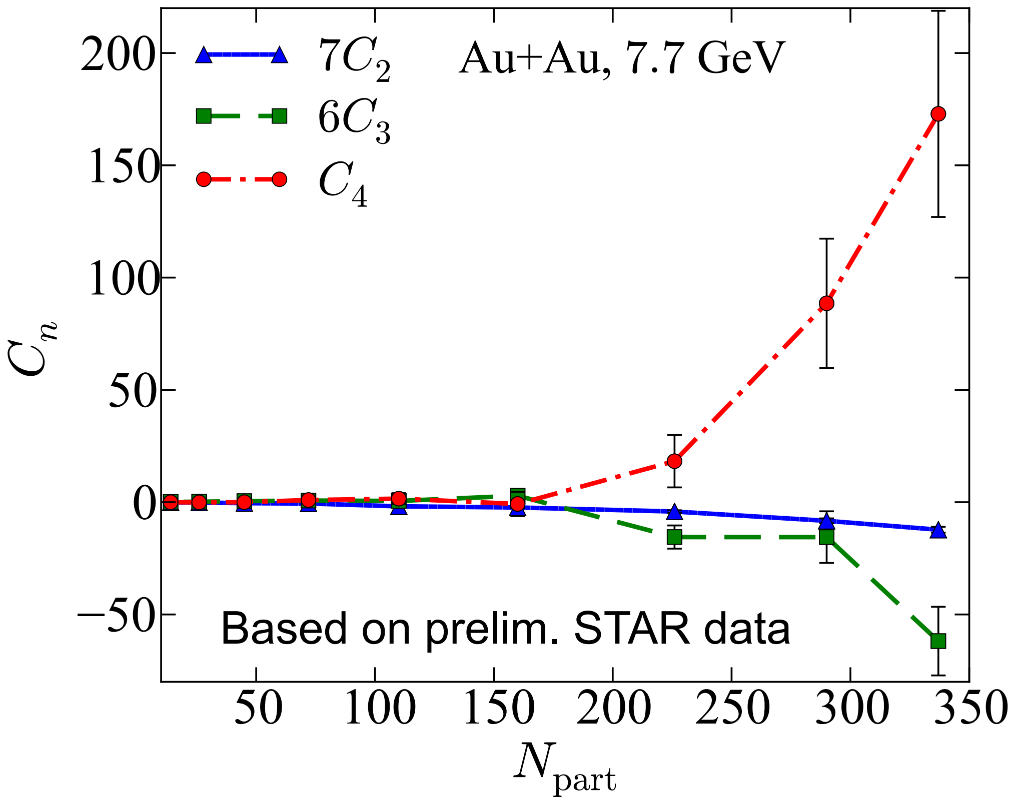
$$C_2 = -K_1 + K_2,$$
$$C_3 = 2K_1 - 3K_2 + K_3,$$
$$C_4 = -6K_1 + 11K_2 - 6K_3 + K_4, .$$

or vice versa

$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

# Preliminary Star Data

(X. Luo, PoS Cpod 2014 (019))




Significant four particle correlations!

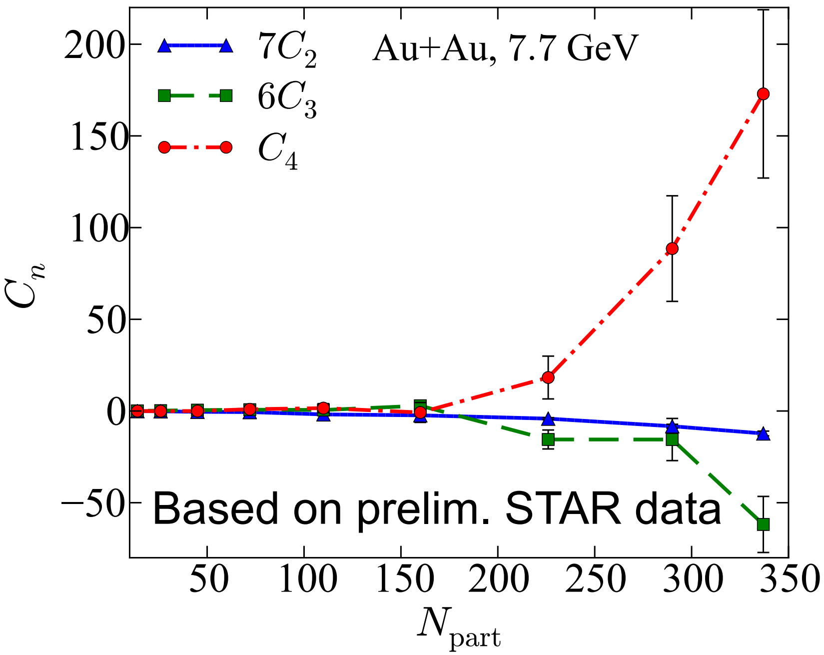
Four particle correlation dominate $K_4$ for central collisions at 7.7 GeV

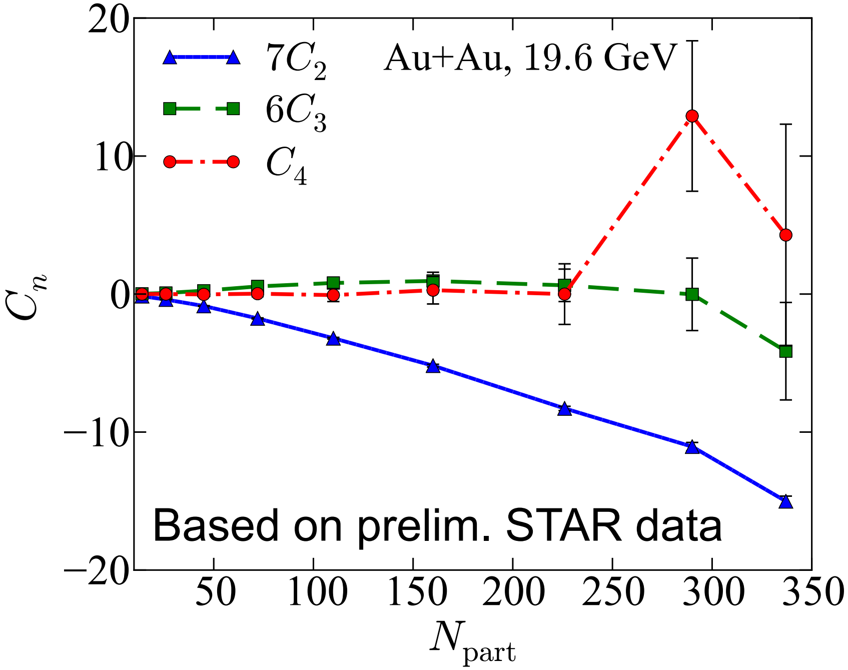
$$K_2 = \langle N \rangle + C_2$$

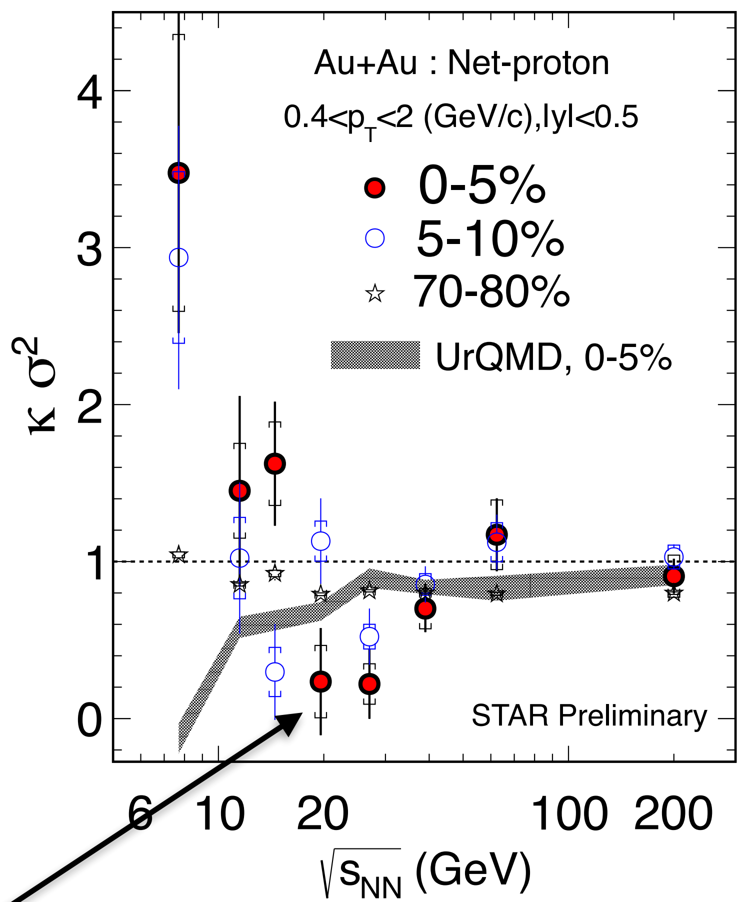
$$K_3 = \langle N \rangle + 3C_2 + C_3$$

$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

# Correlations

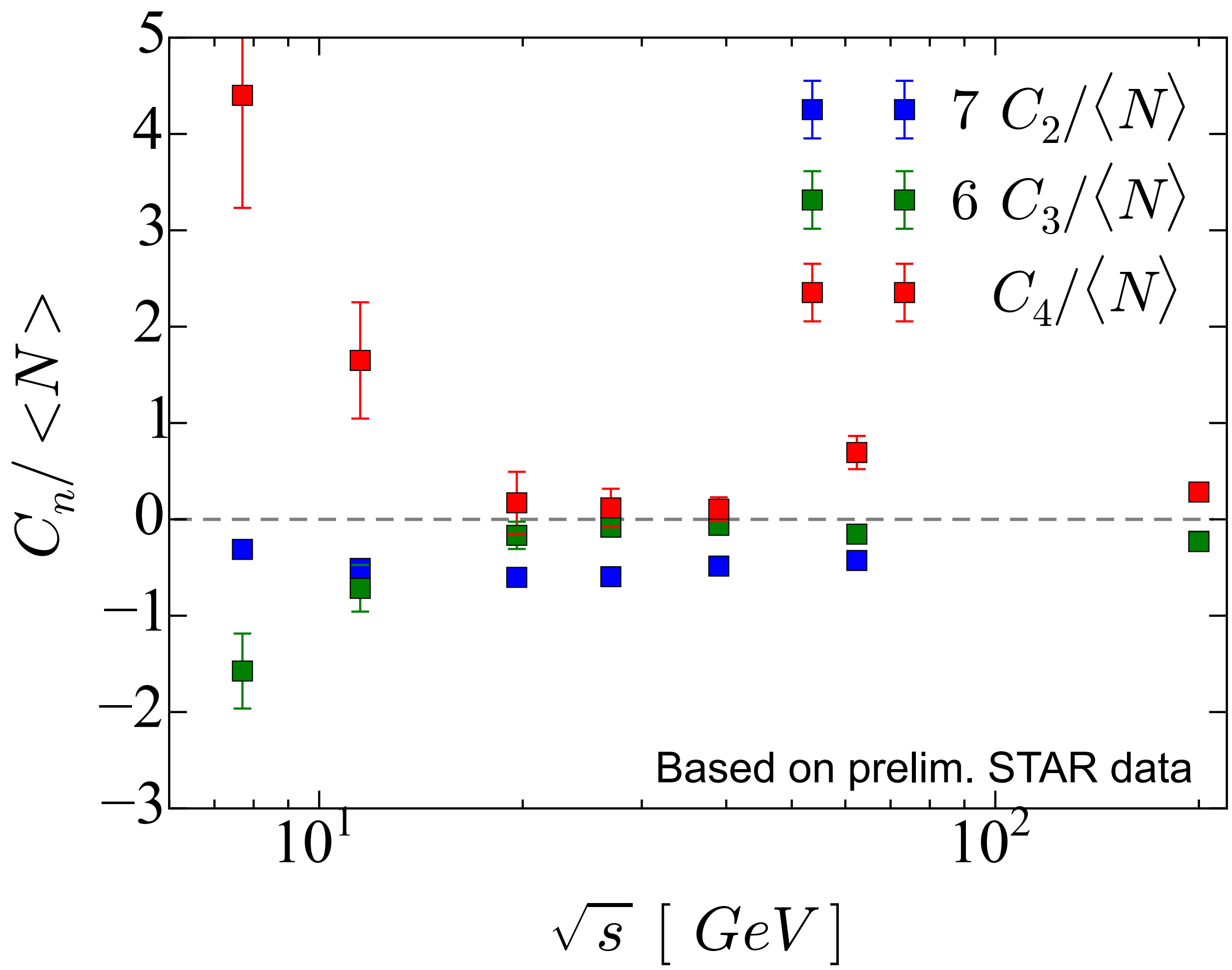



Dip at 19.6 GeV from NEGATIVE $C_2$ !

# Energy dependence




Note: anti-protons are non- negligible above 19.6 GeV

# Rapidity dependence

$$C_k(\Delta Y) = \int_{\Delta Y} dy_1 \dots dy_k \rho_1(y_1) \dots \rho_1(y_k) c_k(y_1, \dots, y_k)$$

Assume: $\rho_1(y) \simeq const.$

short range correlations:

$$c_k(y_1, \dots, y_k) \sim \delta(y_1 - y_2) \dots \delta(y_{k-1} - y_k)$$

$$C_k(\Delta Y) \sim \Delta Y \to K_k \sim \Delta Y$$

Long range correlations:

$$c_k(y_1, \dots, y_k) = const.$$

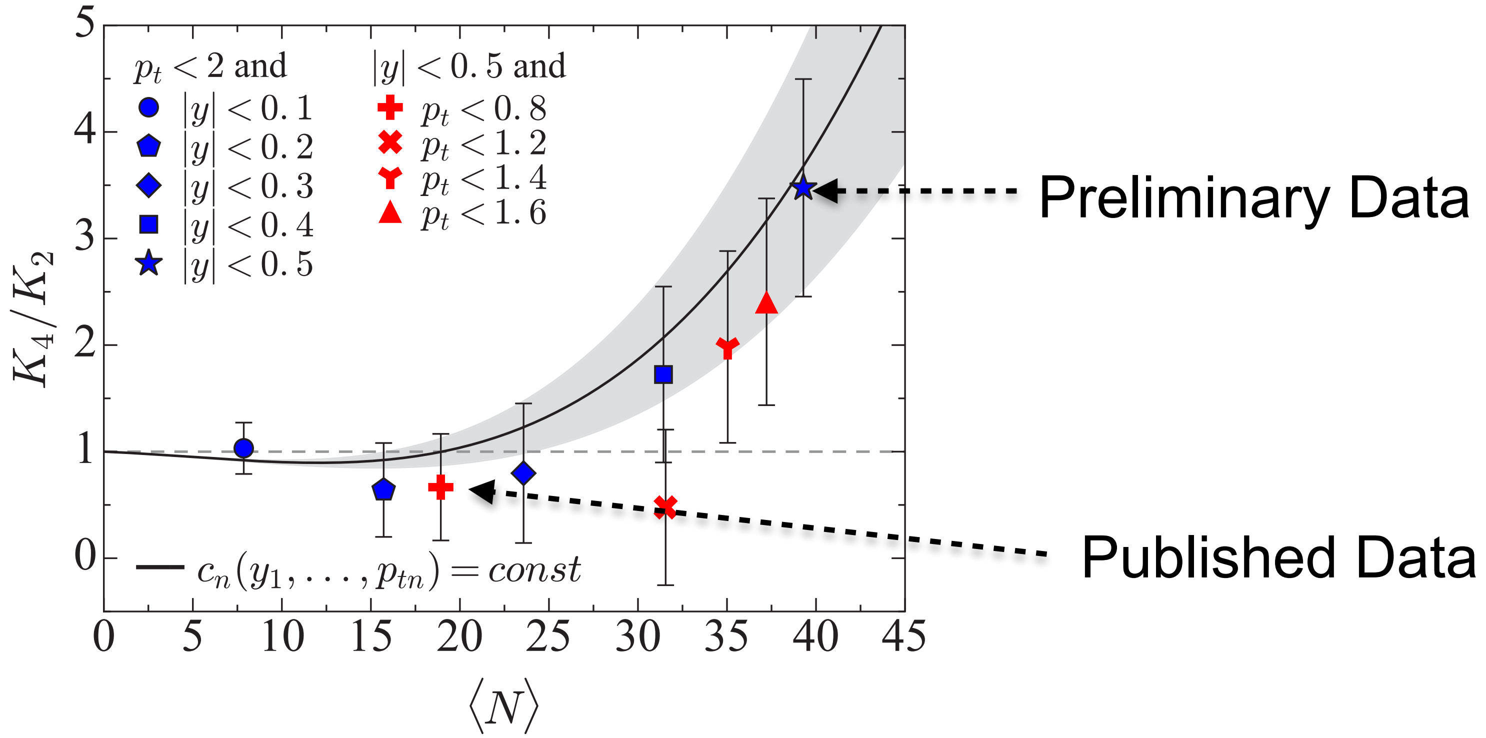
$$C_k(\Delta Y) \sim (\Delta Y)^k \sim \langle N \rangle^k$$

$$\Rightarrow K_n = K_n(\langle N \rangle)$$

# Long range correlations

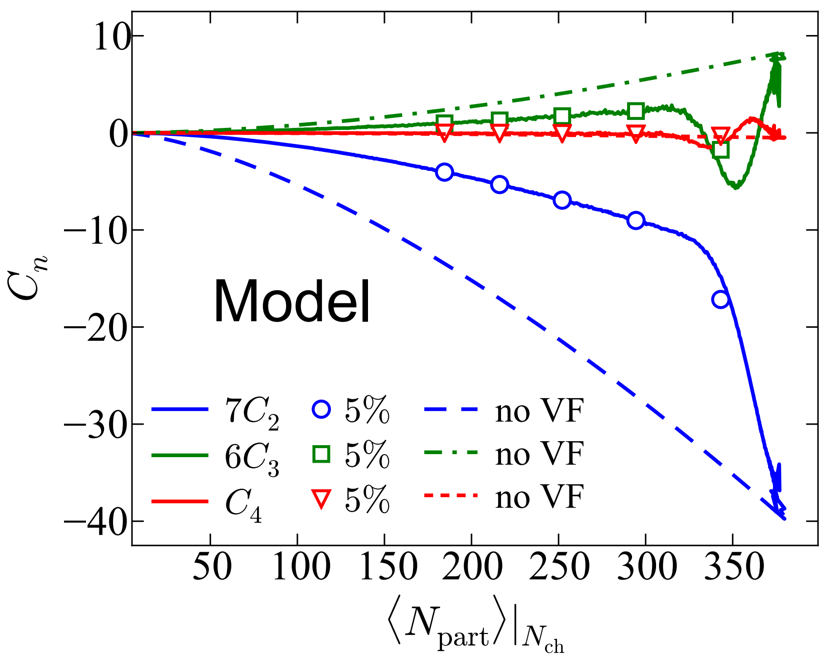
$$C_k = \langle N \rangle^k c_k$$

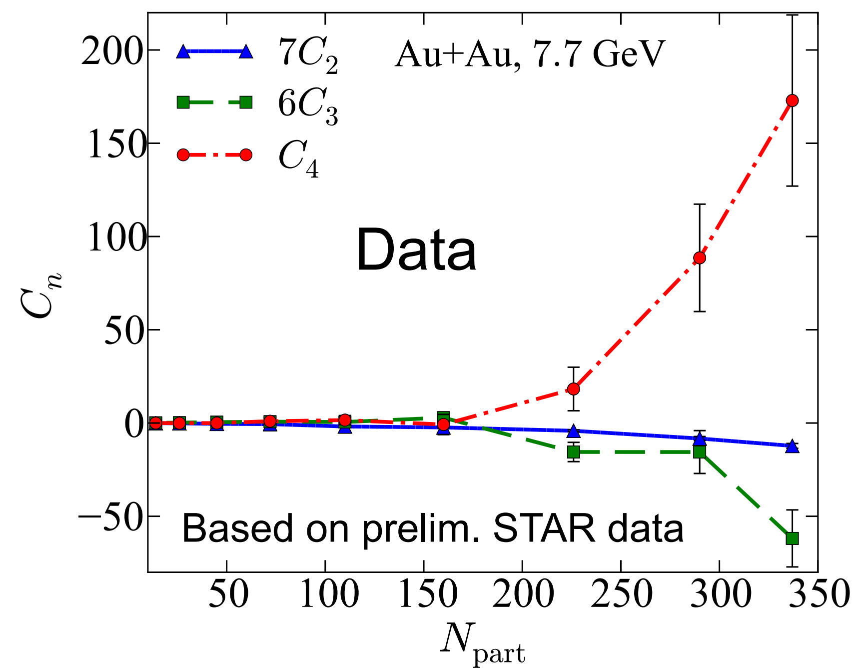
$$c_k = const. \Rightarrow K_n = K_n(\langle N \rangle)$$



# Can we understand these correlations?

- Two particle correlations can be understood by simple Glauber model + Baryon number conservation



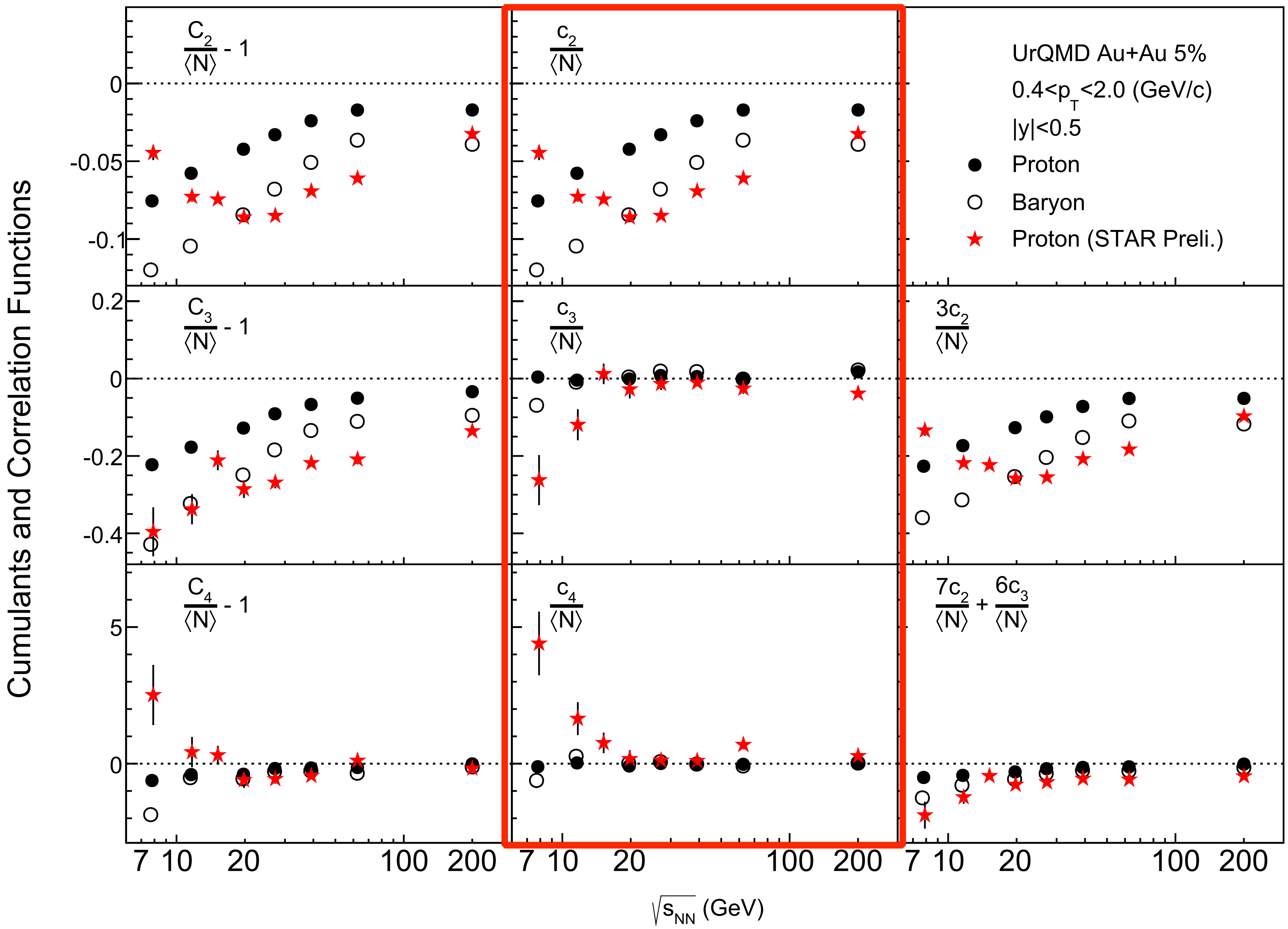




Four particle correlations are orders of magnitudes larger in the data

Also seen in URQMD calculations by He et al. PLB774 (2017) 623

Need to assume the ~40% of protons come from 8-nucleon cluster in order to get magnitude right!

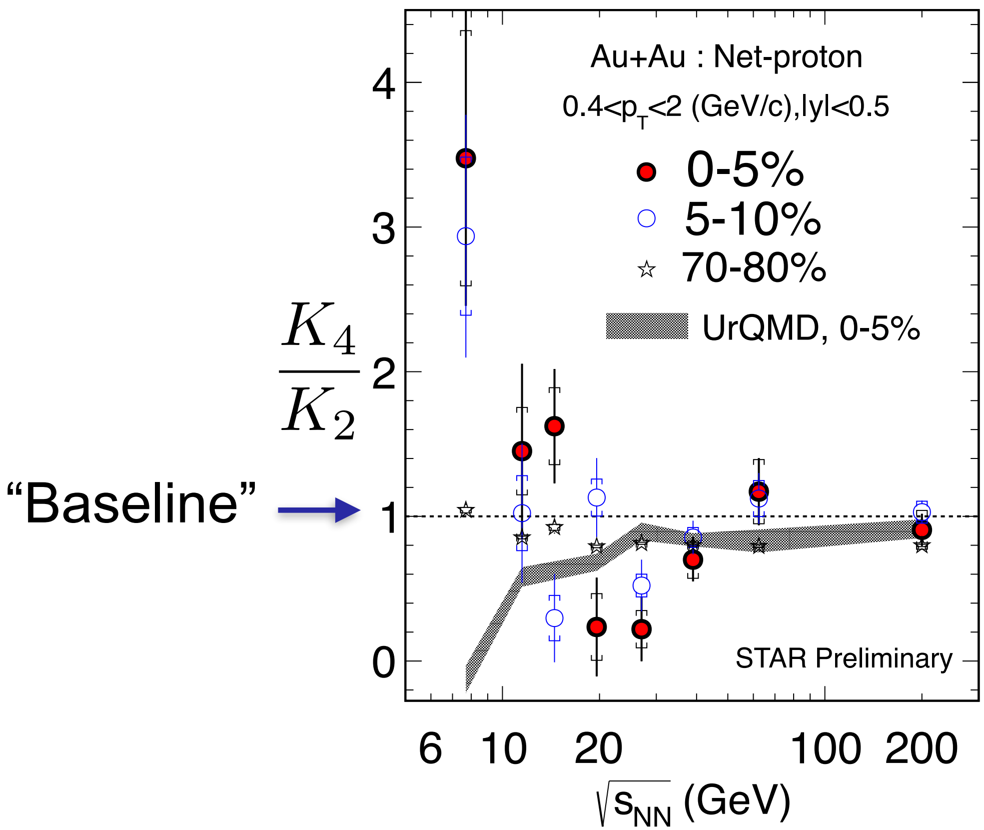
# URQMD

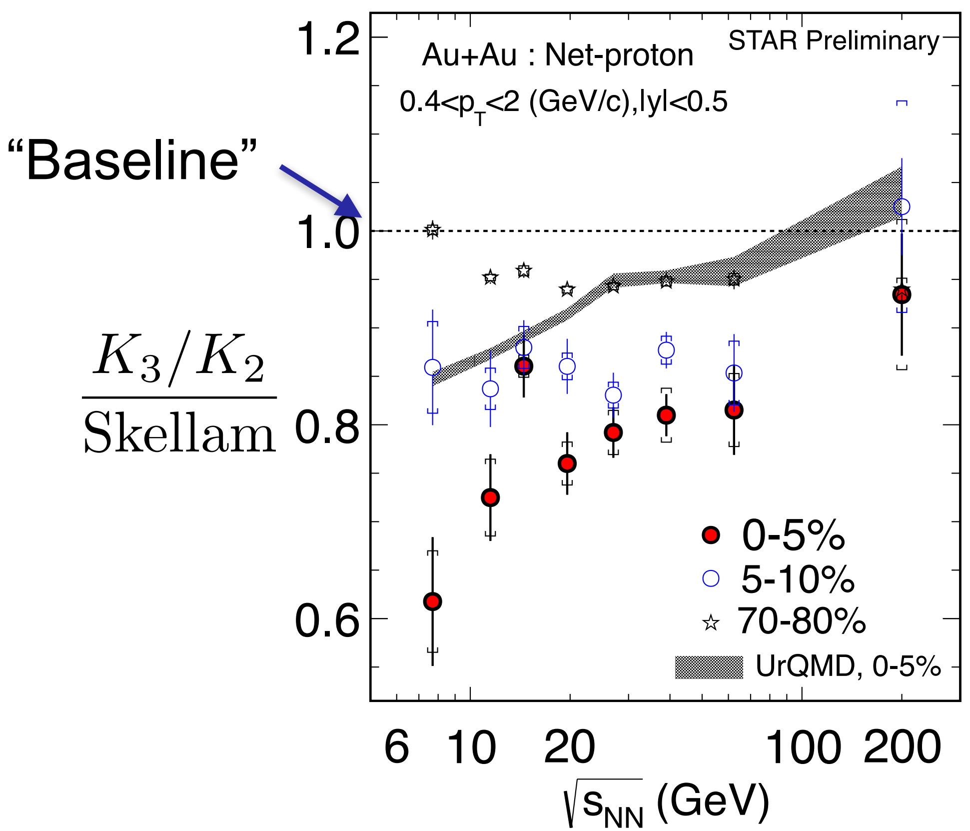



He, Luo PLB774 (2017) 623

# Latest STAR result on net-proton cumulants

X. Luo, NPA 956 (2016) 75



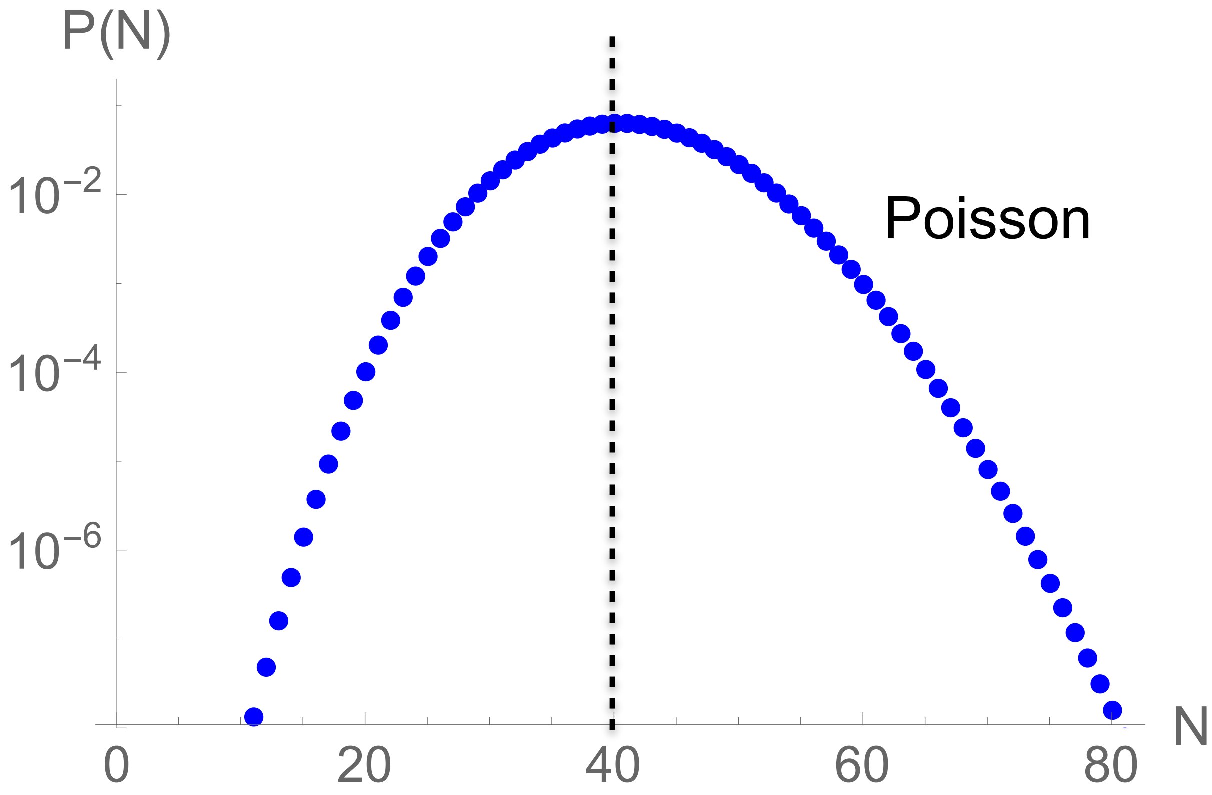




$K_4/K_2$ above baseline $K_3/K_2$ below baseline

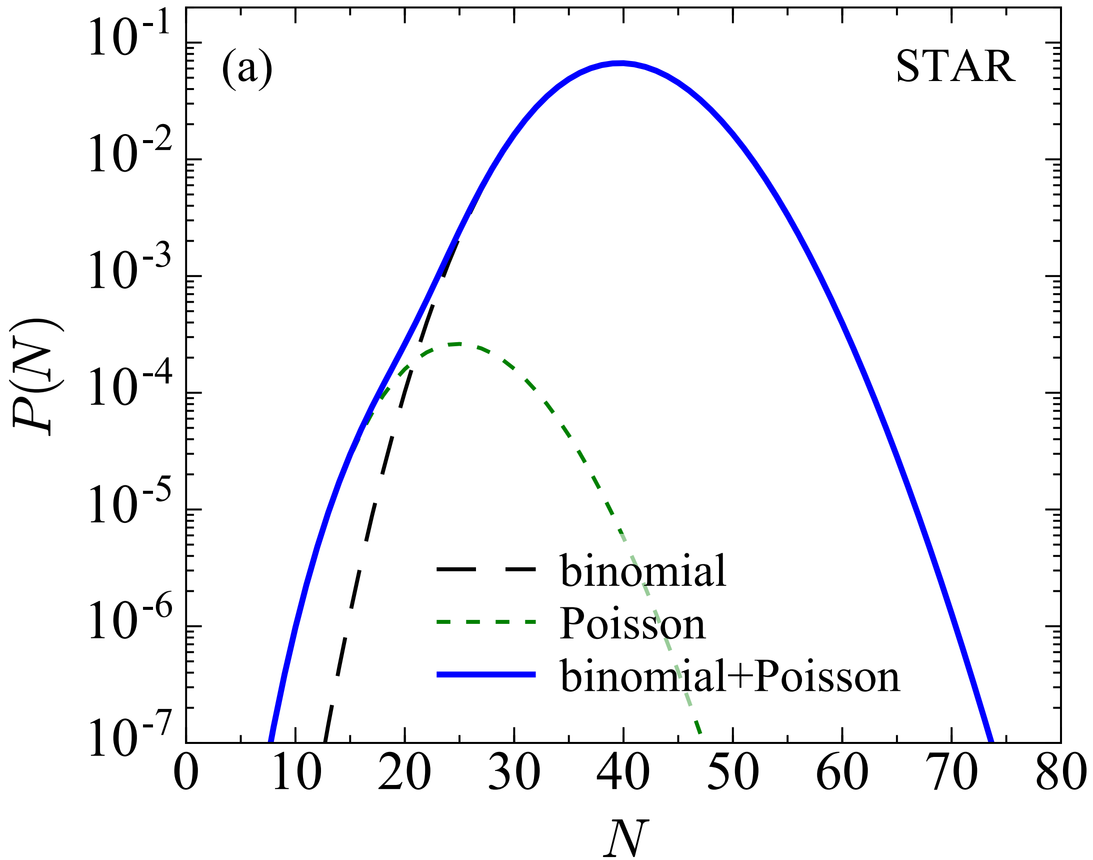
# Shape of probability distribution

$$K_3 < \langle N \rangle \qquad K_3 = \langle N - \langle N \rangle \rangle^3$$

$$K_4 > \langle N \rangle \qquad K_4 = \langle N - \langle N \rangle \rangle^4 - 3 \langle N - \langle N \rangle \rangle^2$$



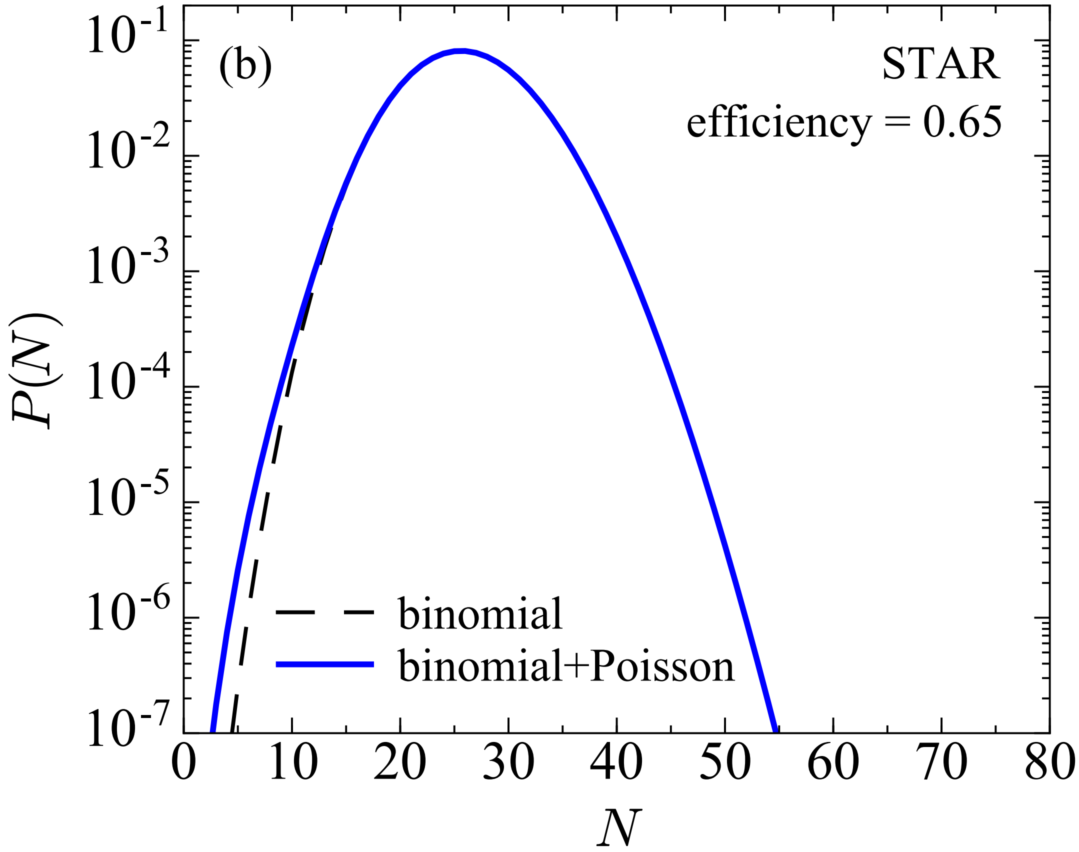
# Simple two component model




Weight of small component: ~0.3%

# Simple two component model

Difficult to see in the real data with efficiency ε=0.65



# Two component model

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

$$\overline{N} = \langle N_{(a)}\rangle - \langle N_{(b)}\rangle$$

$$\begin{aligned}
C_2 =& C_2^{(a)} - \alpha\left\{\overline{C}_2 - (1-\alpha)\overline{N}^2\right\} \\
C_3 =& C_3^{(a)} - \alpha\left\{\overline{C}_3 + (1-\alpha)\left[(1-2\alpha)\overline{N}^3 - 3\overline{N}\,\overline{C}_2\right]\right\} \\
C_4 =& C_4^{(a)} - \alpha\left\{\overline{C}_4 - (1-\alpha)\left[\left(1-6\alpha+6\alpha^2\right)\overline{N}^4 - 6(1-2\alpha)\overline{N}^2\overline{C}_2 + 4\overline{N}\,\overline{C}_3 + 3(\overline{C}_2)^2\right]\right\}
\end{aligned}$$

$$\overline{C}_n = C_n^{(a)} - C_n^{(b)},$$

For Poisson, $C_{(a)}$, $C_{(b)}$=0

Fit to STAR data: $\langle N_{(a)}\rangle \simeq 40,\ \ \langle N_{(b)}\rangle \simeq 25,\ \ \alpha \simeq 0.003$

# Two component model

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

$$\bar{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle > 0$$

For $P_{(a)}$, $P_{(b)}$ Poisson, or (to good approximation) Binomial

$$C_n = (-1)^n K_n^B \bar{N}^n \quad n \geq 2$$

$$K_n^B : \text{ Cumulant of Bernoulli distribution}$$

$$\alpha \ll 1, K_n^B = \alpha \;\Rightarrow\; C_n \simeq \alpha(-1)^n \bar{N}^n$$

$\Rightarrow |C_n| \sim \langle N \rangle^n$ as seen by STAR ( i.e. "infinite" correlation length)

**predict:** $$\boxed{\frac{C_4}{C_3} = \frac{C_5}{C_4} = \frac{C_{n+1}}{C_n} = -\bar{N}}$$ $$\bar{N} \simeq 15$$

Clear and falsifiable prediction: $C_5 \approx -2650 \quad C_6 \approx 41000$

# This model can be tested RIGHT NOW!

Model prediction:

$$C_5 = -2645\,(1 \pm 0.14), \quad C_6 = 40900\,(1 \pm 0.18),$$
$$C_7 = -615135\,(1 \pm 0.26), \quad C_8 = 8520220\,(1 \pm 0.42)$$

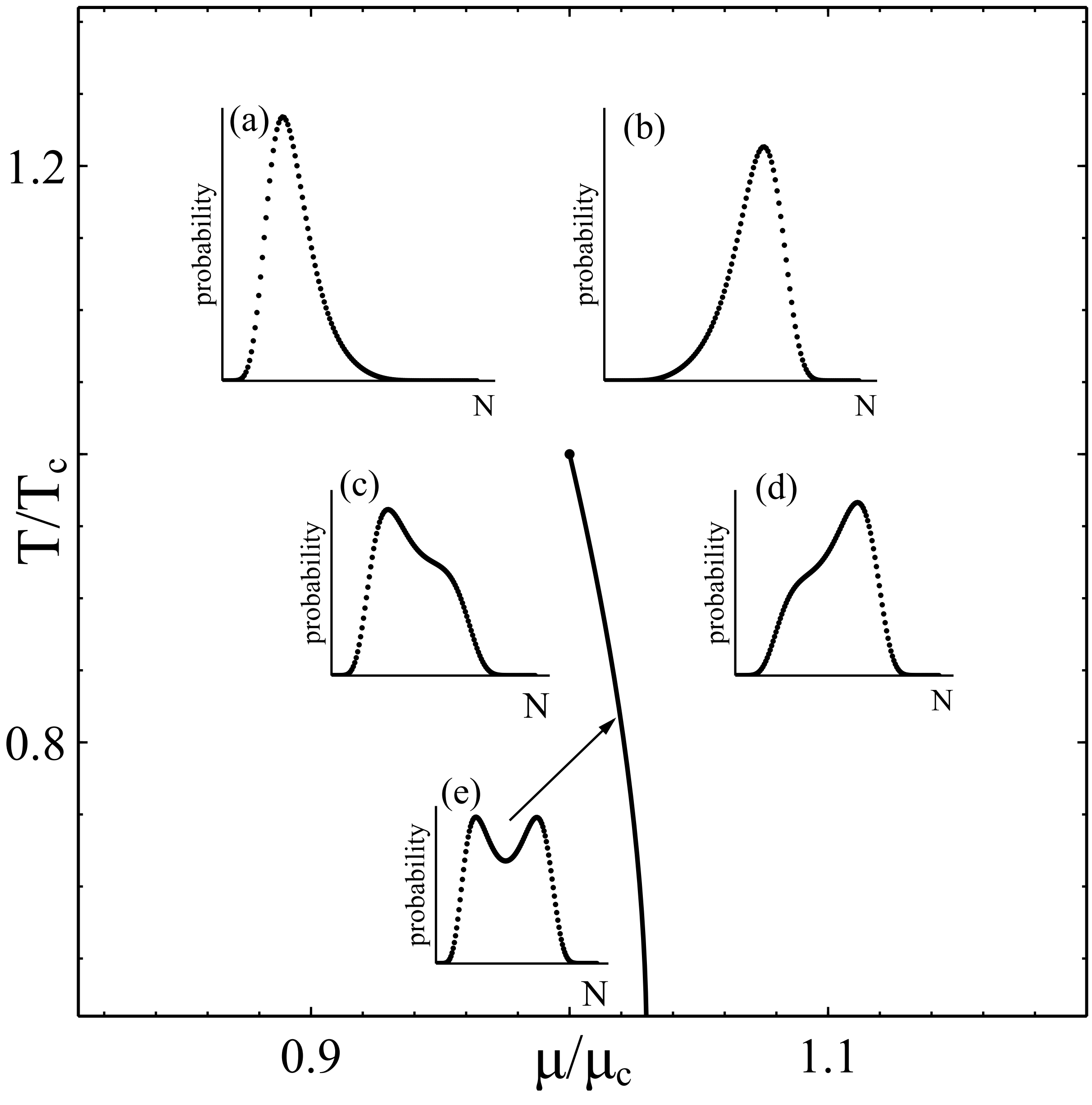
Efficiency corrected

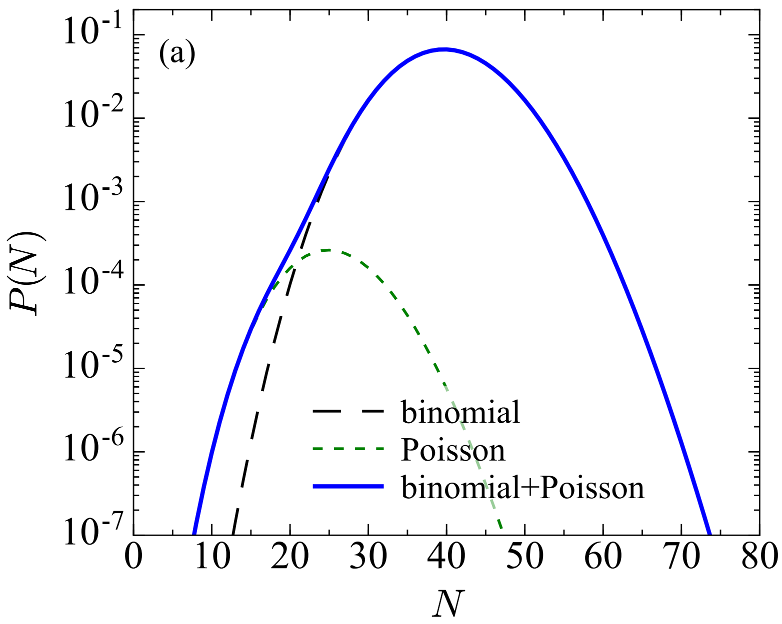
$$C_5 = -307\,(1 \pm 0.31), \quad C_6 = 3085\,(1 \pm 0.41),$$
$$C_7 = -30155\,(1 \pm 0.61), \quad C_8 = 271492\,(1 \pm 1.06),$$

Efficiency UN-corrected

Based on 144393 events (same as STAR 0-5% at 7.7 GEV)

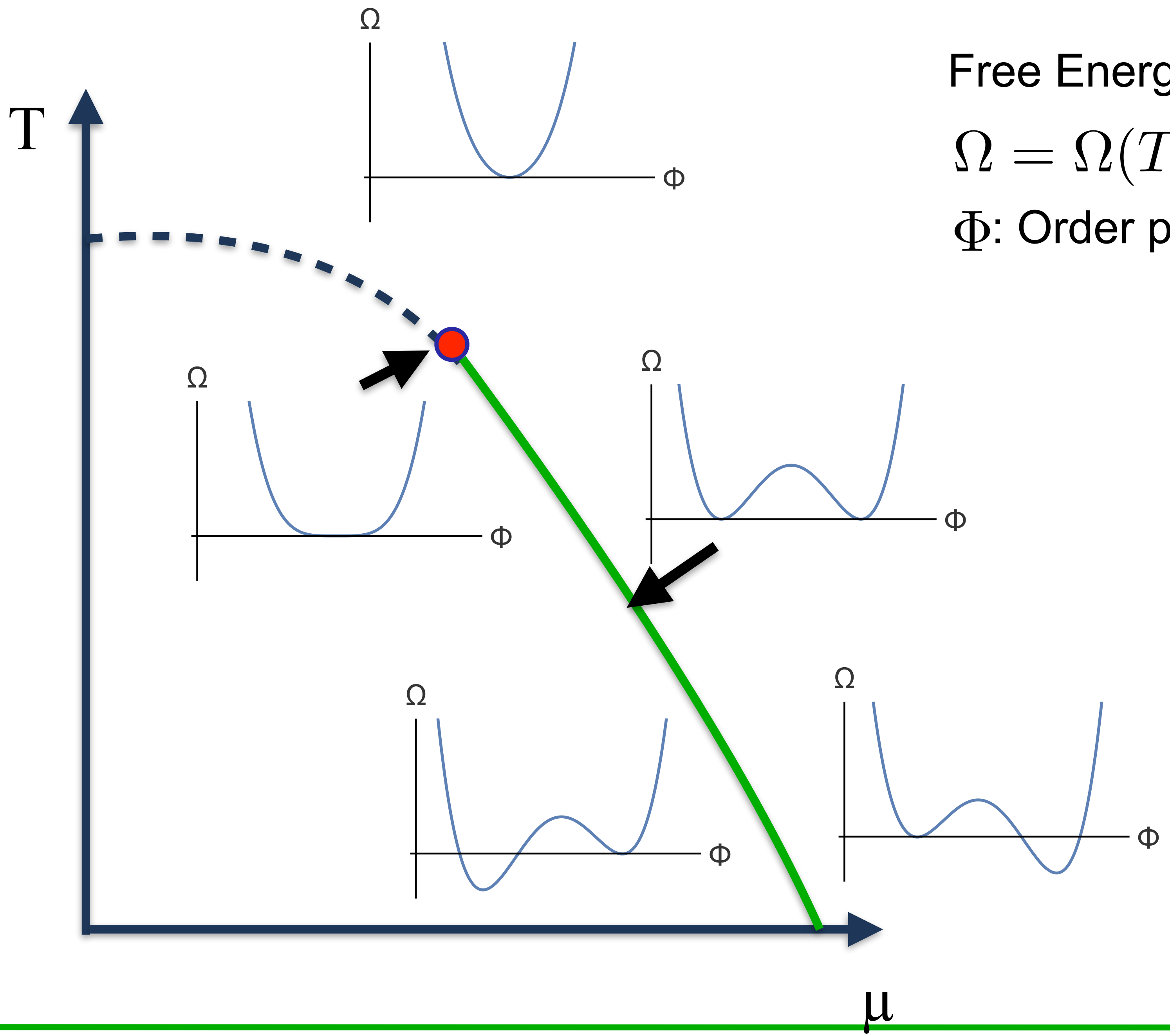
# Speculation




Bzdak et al, arXiv:1804.04463

# Free Energy

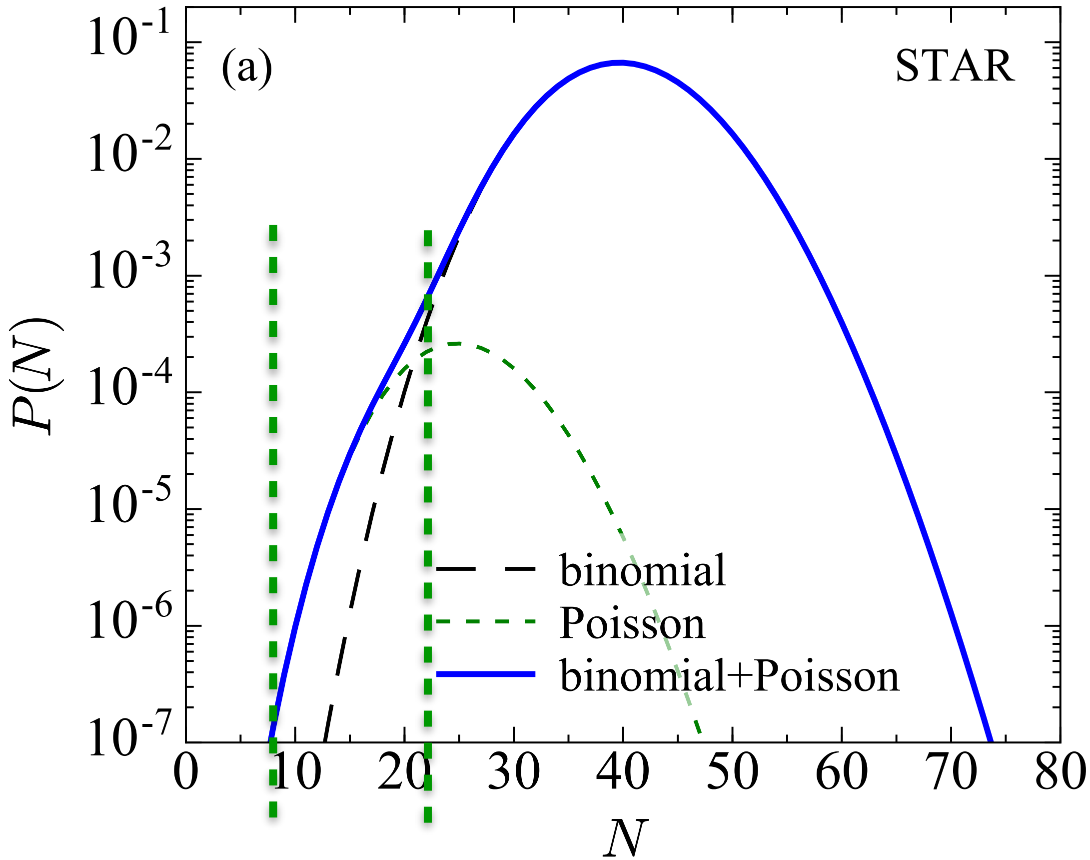



Free Energy:

$$\Omega = \Omega(T, \mu;\ \Phi)$$

$\Phi$: Order parameter

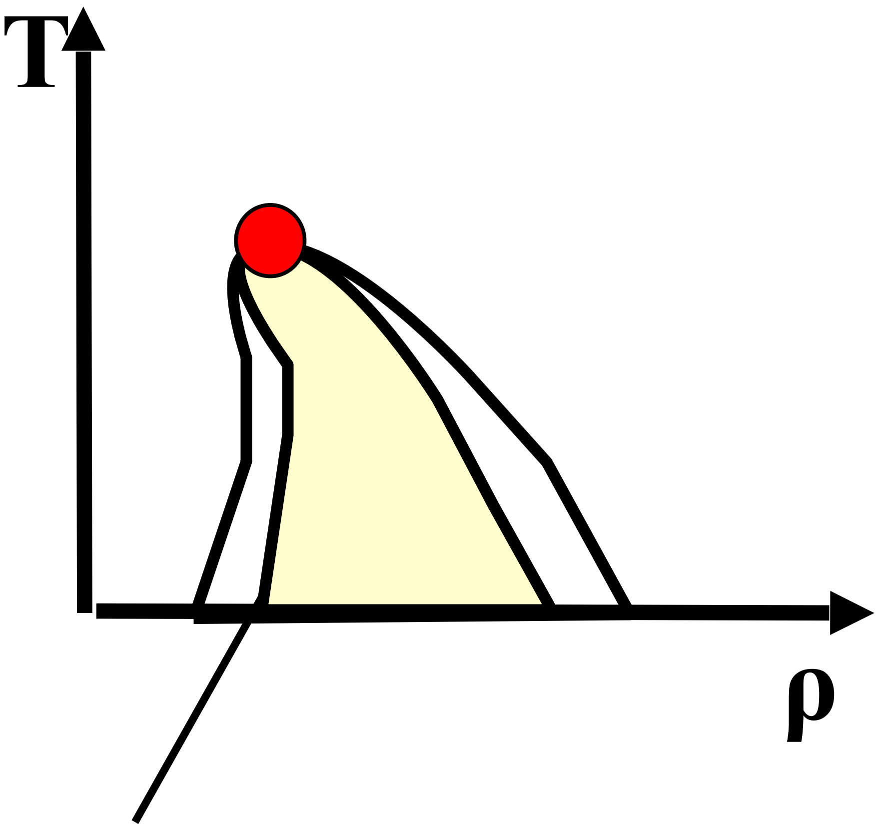
# Simple two component model

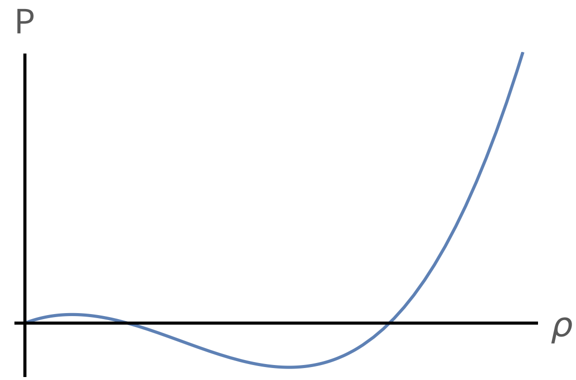



Analyse data for $N_p$ <20

- Is flow etc different?
- “Inspect by eye (<1% of all events)

# Co-existence region




Spinodal instability:
Mechanical instability

$$\frac{\partial P}{\partial \rho} < 0$$

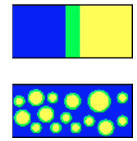
System should spent long time in spinodal region

Exponential growth of clumping

Non-equilibrium phenomenon!

# *Phase-transition dynamics: Density clumping*

Phase transition => 
- Phase coexistence: surface *tension*
- Phase separation: *instabilities*



Introduce a ***gradient*** term:

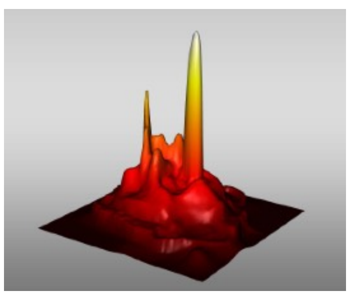
$$p(\boldsymbol{r}) = p_0(\varepsilon(\boldsymbol{r}), \rho(\boldsymbol{r})) - C\rho(\boldsymbol{r})\boldsymbol{\nabla}^2\rho(\boldsymbol{r})$$

*Insert the modified pressure into existing ideal finite-density fluid dynamics code*

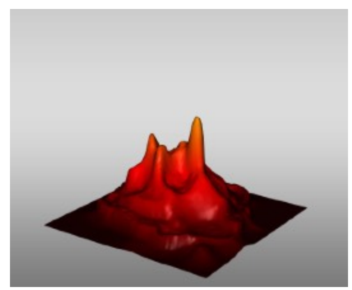
*Use UrQMD for pre-equilibrium stage to obtain fluctuating initial conditions*

Simulate central Pb+Pb collisions at ≈3 GeV/A beam kinetic energy on fixed target, using an Equation of State either ***with*** a phase transition or ***without*** (Maxwell partner):

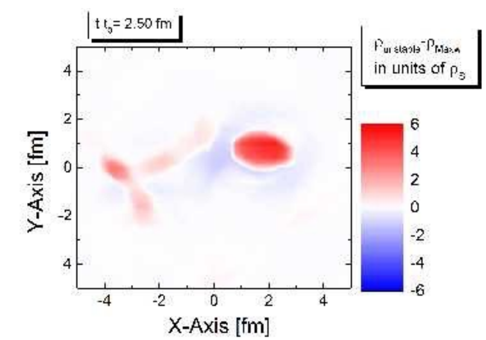
**With** phase transition:

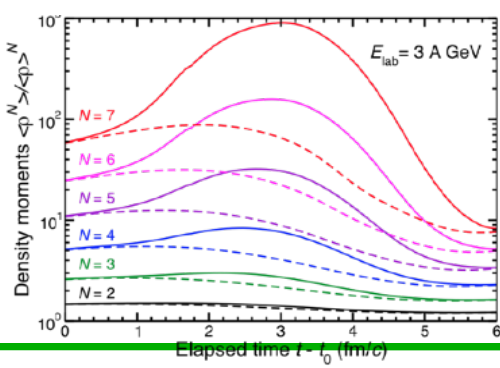


**Without** phase transition:



Density enhancement:

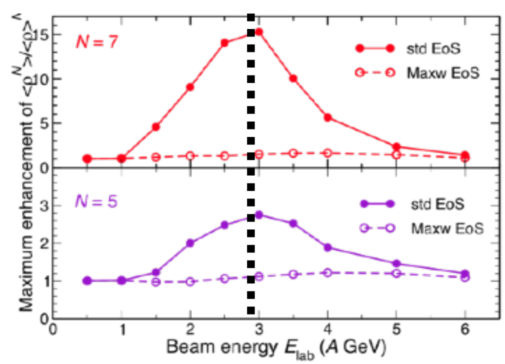



*Evolution of density moments*

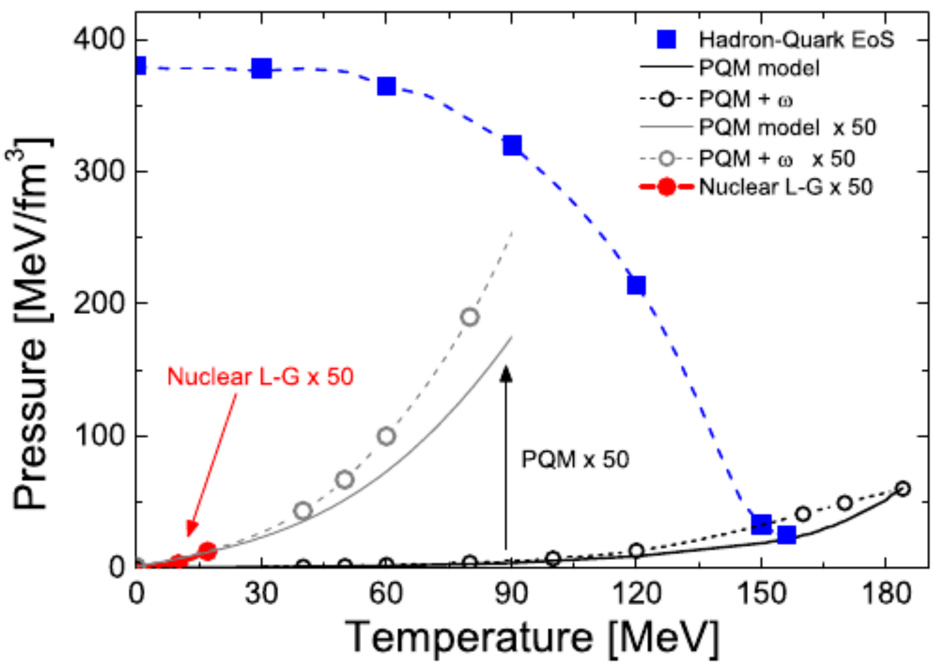
$$\langle\rho^N\rangle \equiv \frac{1}{A}\int \rho(\boldsymbol{r})^N \rho(\boldsymbol{r}) d^3\boldsymbol{r}$$

J. Steinheimer & J. Randrup,
PRL 109, 212301(2012)
PRC 87, 054903 (2013)

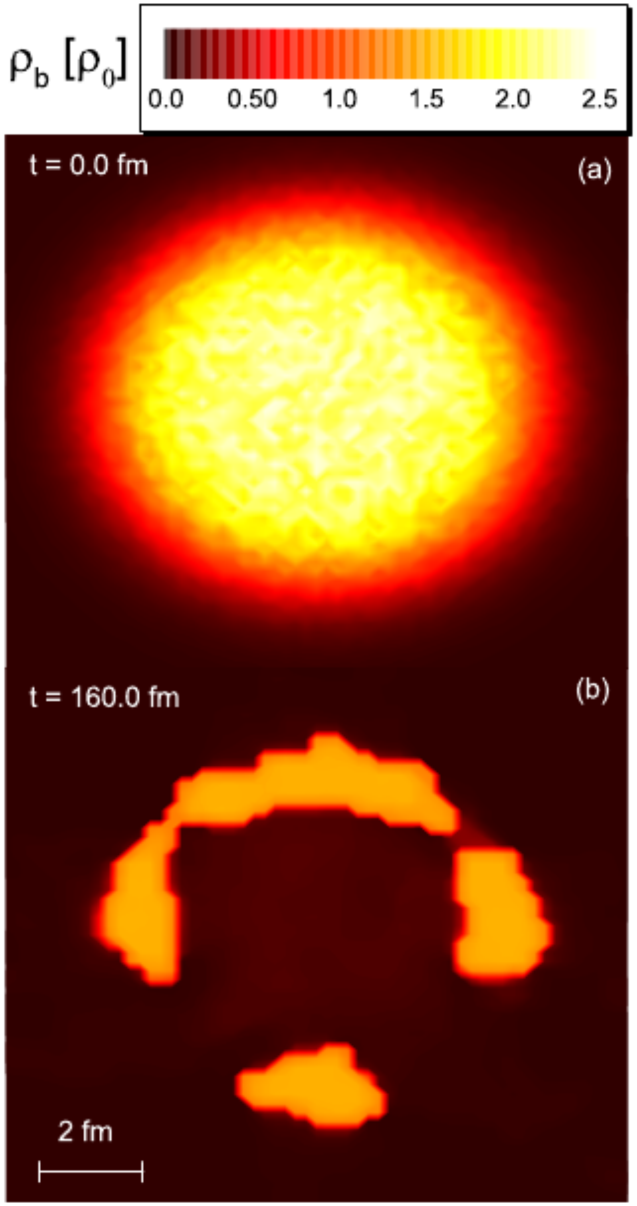



$E_{Lab}$=3 GeV

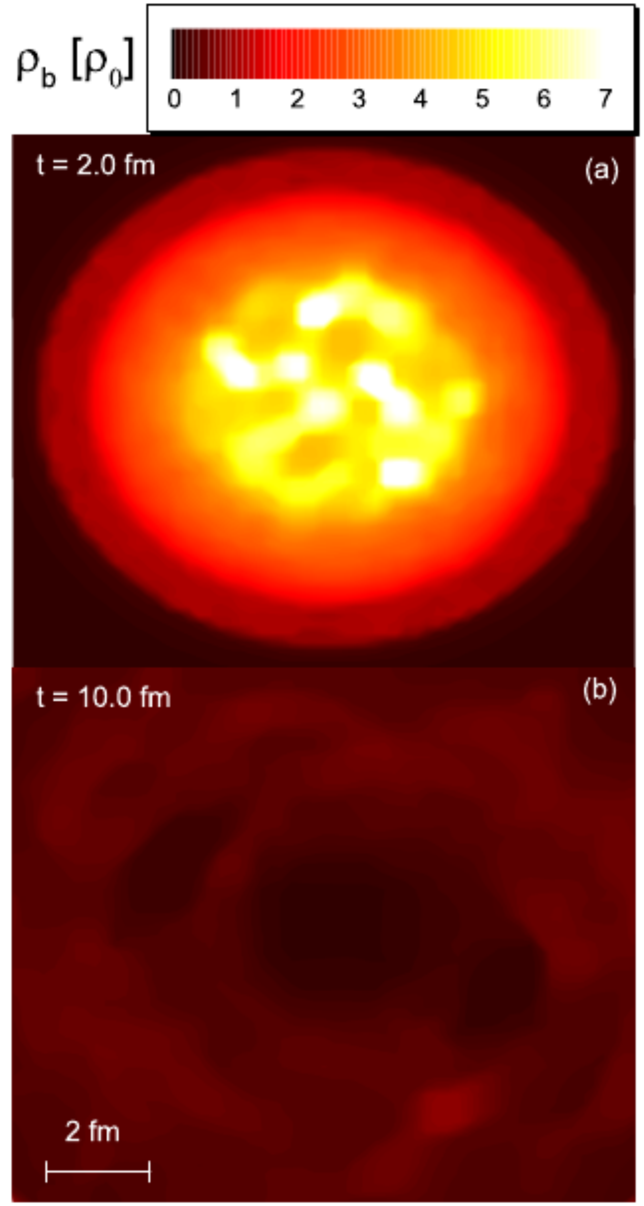
# Consider two Equations of State




Steinheimer et al,
Phys.Rev. C89 (2014) 034901

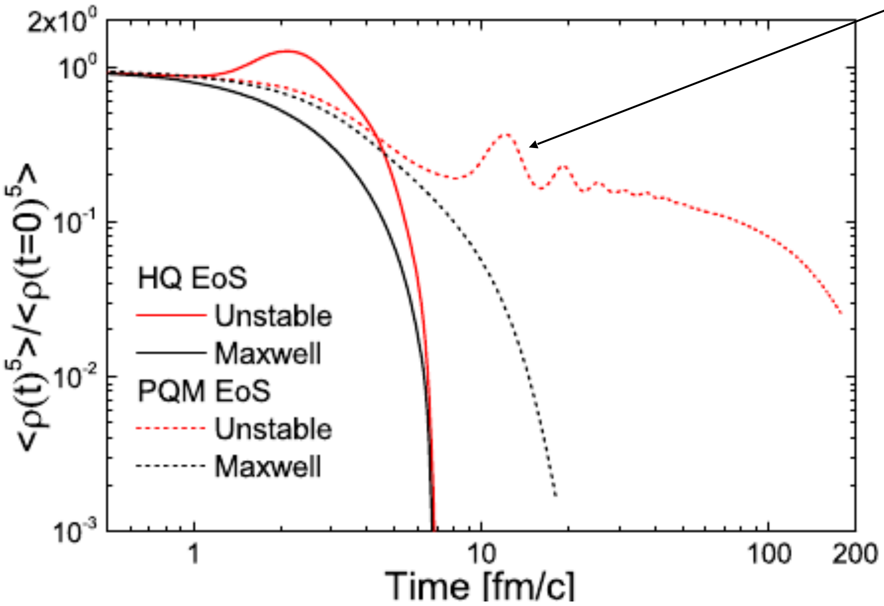



PQM (“liquid-gas”)




“QCD”

# Time evolution

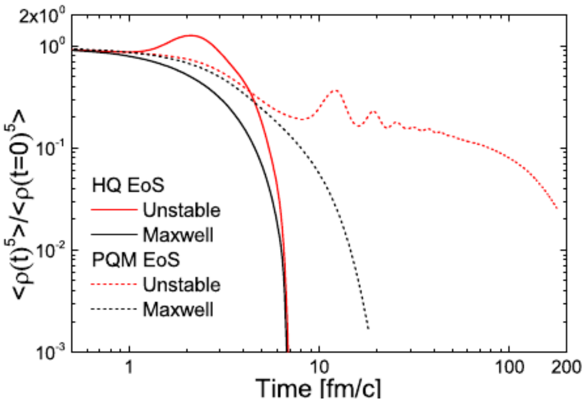



Oscillation of nearly stable droplets for "liquid-gas" EoS

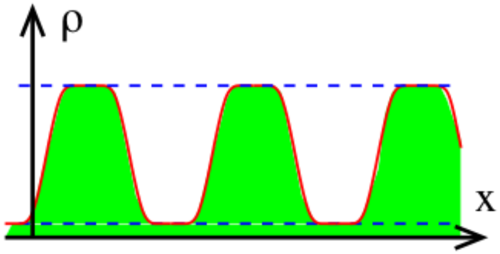
Higher pressure leads to faster evolution of "QCD" EoS.

Steinheimer et al,
Phys.Rev. C89 (2014) 034901

# Cluster a.k.a. nuclei




Even if total baryon number does not fluctuate the baryon **density** does




Therefore measure production of NUCLEI: d, $^3$He, $^4$He, $^7$Li....

$$\langle d\rangle \sim \langle \rho^2\rangle \qquad \langle {}^3He\rangle \sim \langle \rho^3\rangle \qquad \langle {}^7Li\rangle \sim \langle \rho^7\rangle$$

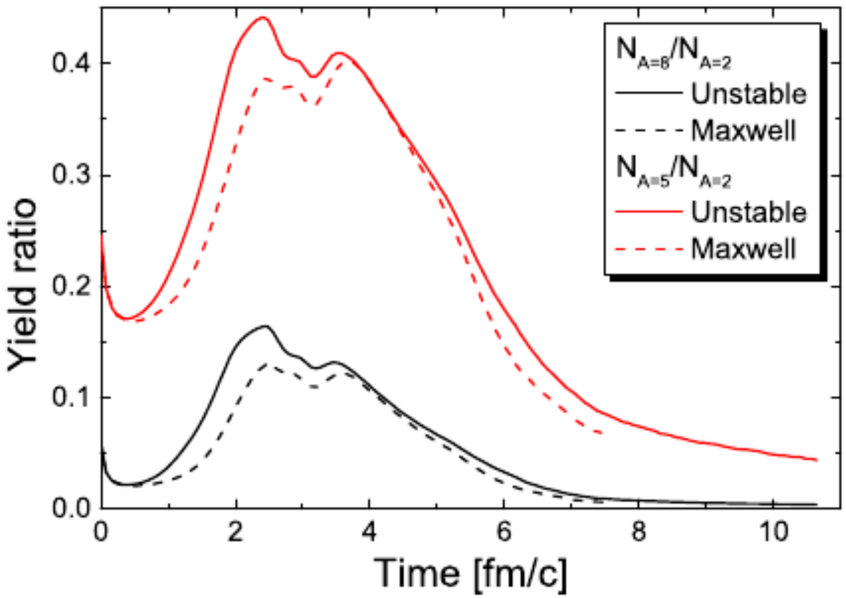
Extracts higher moments of the baryon **density** at freeze out

Nice Idea, but...

# “Cluster” formation

“QCD” EoS

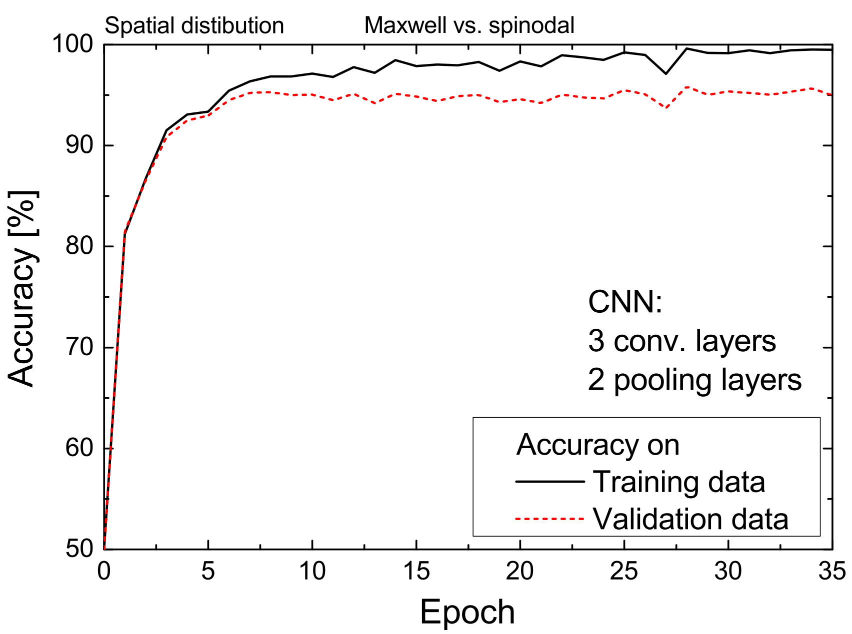
$$\left(\frac{S}{B}\right)_{hadron-gas} < \left(\frac{S}{B}\right)_{QGP-\text{liquid}}$$

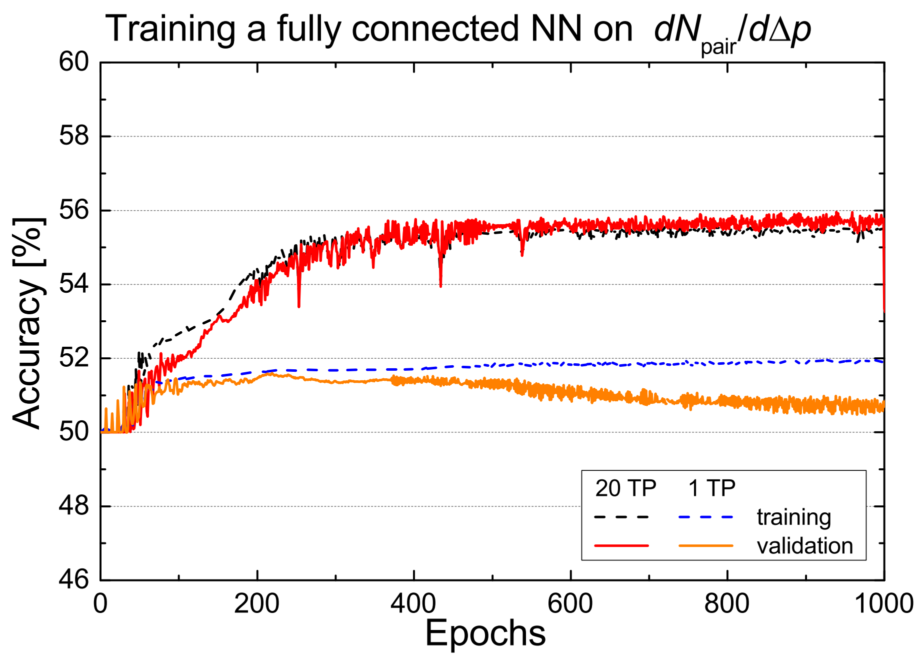



Clumping in coordinate space is compensated by dilution in momentum space $\rightarrow$ tiny effect

Steinheimer et al,
Phys.Rev. C89 (2014) 034901

# “Deep” learning fails as well….




Coordinate space 🙂

Momentum space ☹️

Steinheimer et al. , arXiv:1906.06562

# Summary

- Cumulants measure derivatives of the free energy (equation of state)
  - Sensitive to “wiggles” a.k.a. “remnants” of phase transition
- Experiments are difficult: detector needs to be understood well
- Careful when comparing theory with measured cumulants
- Correlations a.k.a. factorial cumulants provide complementary insights
  - strong four particle correlation at low energies
- Don’t forget the first order phase transition
  - Spinodal instability
- Very active field, both in experiment and theory

VERY INTERESTING TIMES ADHEAD