Initial conditions for heavy-ion collisions

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From the violence of a nuclear collision...to the calm of a quark-gluon fluid

How is thermalization achieved in QCD?
Approaches to thermalization

Two “clean” theoretical limits:

- Holographic thermalization (based on duality of strongly coupled \( g^2 N_c \rightarrow \infty, \ N_c \rightarrow \infty \))
  
  N=4 SUSY YM to classical gravity in AdS\(_5 \times S_5 \))

The AdS/CFT correspondence provides valuable insight into universal features of non-equilibrium dynamics in QCD. Examples: transport coefficients and hydrodynamization

- Highly occupied (occupancy \( f \gg 1 \)) QCD at weak coupling \( g^2 \rightarrow 0, \ g^2 f \sim 1 \)

Our focus: **non-equilibrium strongly correlated QCD at weak coupling**
The nuclear wavefunction at high energies
What happens when you boost a proton or nucleus to high energies?

What the proton or nucleus "looks like" in QCD depends on boost and resolution scale.

As the proton is boosted, "parton" fluctuations live longer and longer, manifesting themselves as bremsstrahlung in scattering.
Generating strong fields by multi-particle production

Bremsstrahlung is **ubiquitous** in QCD because phase space logs compensate for the suppression in coupling: \( \alpha_S \ln \left(1/x\right) \sim 1 \) and/or \( \alpha_S \ln \left(Q^2/\Lambda^2_{QCD}\right) \sim 1 \)

Appropriate limit for multi-particle production:
Regge limit of QCD
\( s \to \infty, Q^2 = \text{fixed} \gg \Lambda^2_{QCD}, x \to 0 \)

A fascinating equilibrium of splitting and recombination should eventually result. It is a considerable theoretical challenge to calculate this equilibrium in detail...

Gluons at maximal phase space occupancy \( n \sim 1/\alpha_s \), resist close packing by recombining and screening their color charges -- **gluon saturation**

Emergent dynamical saturation scale \( Q_S(x) \gg \Lambda_{QCD} \)

Asymptotic freedom! \( \alpha_s(Q_S) \ll 1 \) provides weak coupling window into infrared
Gluon saturation

Boost

log(1/x) or log(s)

Resolution

log(Q^2)

Saturation

Unitarization boundary f α_s ~ 1

BFKL

DGLAP
Saturation: dipole model formulation in DIS

Equivalent formulation:
Strong screening of color charge of a quark-antiquark dipole

\[ \sigma_{\gamma^*P} = \int d^2r_\perp \int dz |\psi_{T,L}(r_\perp, z, Q^2)|^2 \sigma_{q\bar{q},P}(r_\perp, x) \]

Golec-Biernat Wusthoff model

\[ \sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0 \left[ 1 - \exp \left( -r_\perp^2 Q_s^2(x) \right) \right] \]

Color transparency for \( r_\perp^2 Q_s^2 \ll 1 \) \( (\sigma \propto A) \)

Color opacity ("black disk") for \( r_\perp^2 Q_s^2 \gg 1 \) \( (\sigma \propto A^{2/3}) \)

QCD picture of "shadowing"...

**QCD picture of "shadowing"...**

\[
Q_s^2(x) = Q_0^2 \left( \frac{x_0}{x} \right) ^\lambda
\]

Parameters:
\[ Q_0 = 1 \text{ GeV}; \; \lambda = 0.3; \; x_0 = 3 \times 10^{-4}; \; \sigma_0 = 23 \text{ mb} \]
Nuclear “oomph” of the saturation scale

Dipole couples coherently with color charges in different nucleons in path of its scattering: \( Q_s^2 \sim A^{1/3} \)

\[ x \leq 0.01 \]

EIC: Electron-Ion Collider

Aschenauer et al., arXiv:1708.01527
Classicalization in the Regge limit: the Color Glass Condensate EFT

**Born-Oppenheimer** separation between fast and slow modes

**CGC**: Effective Field Theory of classical static quark/gluon sources and dynamical gluon fields

Remarkably, physics of extreme quantum fluctuations becomes classical because of high gluon occupancy...

McLerran, RV (1994)
Classicalization in the Regge limit: the Color Glass Condensate EFT

EFT allows one to compute many-body correlations just as in condensed matter physics

Wilsonian RG:
2+1-D B-JIMWLK hierarchy of equations for multi-point ”Wilson line” dipole, quadrupole, etc. correlators -- right degrees of freedom

*Universal infrared classical dynamics of QCD in the infrared?*

Balitsky (1996)
Kovchegov (1999)
Classicalization in the Regge limit: the Color Glass Condensate EFT

A closed form non-linear (Balitsky-Kovchegov) equation describes how $q\bar{q}$ “dipole” probe evolves with energy – provides clean demonstration of unitarization in strong fields

Its dynamics can be mapped* to that of the Fischer-Kolmogorov (FKPP) eqn. describing the evolution of non-linear wave fronts. Rich synergy with stat. mech.


* small caveat
Photons and di-jets to NLO+NLLx precision in the CGC EFT

Compton amplitude for $eA \rightarrow \gamma + \text{dijets} + X$

Virtual photon fluctuates into quark-antiquark dipole and a photon state that scatters off a \textit{gluon shockwave} fluctuation of nuclear target

Differential DIS computations in the CGC EFT now available to $O(\alpha_s^3 \ln(1/x))$ accuracy

Can be tested to $\sim 10\%$ \textit{accuracy} at an Electron-Ion Collider
Boiling the QCD vacuum in heavy-ion collisions

Nuclei as heavy as bulls
Through collisions
Generate new states of matter

TD Lee
Nobel Laureate (1957)
A “Standard model” model of a heavy-ion collision

Glasma: Out of equilibrium QGP formed from decay of colliding CGCs
Problem: Compute particle production in QCD with *strong time dependent* sources “Schwinger-Keldysh” framework

Gelis, Lappi, RV
arXiv:0708.0047, 0804.2630; 0807.1306 [hep-ph]
Jeon, arXiv:1308.0263
Big Bang

Stars and galaxies that can be observed today were born as a result of the evolution of the universe.

Present time (13.7 billion years since the Big Bang)

WMAP data (3x10^5 years)

Hot Era

Inflation

Phase transition completed

The universe began in an endless state

Created from "nothing"

Little Bang

QGP

CGC/Glasma

Plot by T. Hatsuda
Big Bang vs. Little Bang

Decaying Inflaton with occupation # $1/g^2$ ↔ Decaying Glasma with occupation # $1/g^2$

Explosive amplification of low momentum small fluctuations (preheating) ↔ Explosive amplification of low momentum small fluctuations (Weibel instabilities)


Other common features: turbulence, topological defects,...
The Glasma at leading order

Collisions of lumpy gluon "shock" waves

Leading order solution: Solution of QCD Yang-Mills eqns in presence of light-cone (valence) sources

\[ D_\mu F^{\mu\nu,\alpha} = \delta^{\nu+} \rho^a_A(x_\perp)\delta(x^-) + \delta^{\nu-} \rho^a_B(x_\perp)\delta(x^+) \]

\[ \langle \rho^a_{A(B)}(x_\perp)\rho^a_{A(B)}(y_\perp) \rangle = Q^2_{S,A(B)}\delta^{(2)}(x_\perp - y_\perp) \]

The saturation scale \( Q_s(x,b_T) \) is the only scale in the problem
The Glasma: colliding gluon shock waves

Glasma color fields

Glasma color fields matched to viscous hydrodynamics

Schenke, Tribedy, Venugopalan, PRL108 (2012)

Note: 1 fm/c = $3 \times 10^{-24}$ seconds!
At NLO: Decoherence from quantum fluctuations

Dusling, Epelbaum, Gelis, RV (2011)

“Toy” example: scalar $\Phi^4$ theory

\[ \phi(\tau, \eta, x_\perp) = \phi_{\text{cl}}(\tau, x_\perp) + \frac{1}{2} \int \frac{d\nu}{2\pi} d\mu_k \ c_{\nu k} e^{i\nu\eta} \chi_k(x_\perp) H_{\nu\nu}(\lambda_k \tau) + c.c \]

Satisfies the “small fluctuation” equation

\[ [-\partial_\perp^2 + V''(\phi_0)] \chi_k = \lambda_k^2 \chi_k \]

These quantum modes satisfy an “eigenstate thermalization” criteria conjectured by Berry (and developed by Srednicki and others) as essential for thermalization of a quantum fluid

Gaussian random variable

\[ \langle c_{\nu k} c_{\mu l} \rangle = 0 \]

\[ \langle c_{\nu k} c_{\mu l}^* \rangle = 2\pi \delta(\nu - \mu) \delta_{kl} \]
Decoherence from quantum fluctuations

Conformal scalar 1+1-D $\Phi^4$ theory:

Energy density and pressure without averaging over fluctuations

Energy density and pressure after averaging over fluctuations

Dusling, Epelbaum, Gelis, RV
arXiv:1009.4363

Such “classical-statistical” quantum averaging decoheres the classical fields – *scrambling information* – resulting in a “pre-thermal” micro-canonical distribution. Strongly correlated dynamics subsequently described in terms of single particle dists.
From Glasma to Quark Gluon Plasma

Longitudinally expanding Glasma fields are unstable to quantum fluctuations... leading to an explosive “Weibel” instability.

This instability leads to rapid decoherence and overpopulation of all momentum modes.

Classical-statistical lattice simulations of 3+1-D gluon fields exploding into the vacuum

Berges, Schenke, Schlichting, RV, NPA 931 (2014) 348
There is a natural \textit{competition} between \textit{interactions} and the \textit{longitudinal expansion} which renders the system \textit{anisotropic} on large time scales.

\textbf{Longitudinal Expansion:}
\begin{itemize}
  \item Red-shift of longitudinal momenta $p_z$
    \hspace{1cm} \rightarrow \text{increase of anisotropy}
  \item Dilution of the system
\end{itemize}

\textbf{Interactions:}
\begin{itemize}
  \item Isotropize the system
\end{itemize}
Pressure becomes increasingly anisotropic

Initial condition for gauge field amplitude varying occupancy $n_0$ and prolateness $\xi_0$

$$f(p_\perp, p_z, t_0) = \frac{n_0}{\alpha_s} \Theta \left( Q - \sqrt{p_\perp^2 + (\xi_0 p_z)^2} \right)$$

$P_L/P_T$ approaches universal $\tau^{-2/3}$ behavior
Result: universal non-thermal fixed point

Conjecture: \[ f(p_\perp, p_z, t) = t^\alpha f_S(t^\beta p_T, t^\gamma p_z) \]

Moments of longitudinal momentum distribution extracted over range of time slices lie on universal curves

Distribution as function of \( p_T \) displays 2-D thermal behavior
Overoccupied expanding Glasma: particles or fields?

For $1 < f < 1/\alpha_S$ a dual description is feasible either in terms of kinetic theory or classical-statistical dynamics ...

Mueller, Son (2002)
Jeon (2005)
"Big whorls have little whorls, which feed on their velocity, and little whorls have lesser whorls, and so on to viscosity."
The Glasma and over-occupied quantum gases

Simulations of self-interacting scalar fields with identical initial conditions demonstrates remarkable *universality of longitudinally expanding world’s hottest and coolest fluids*

In a wide inertial range, scalar & gauge fields have identical scaling exponents & functions

\[
\alpha = -\frac{2}{3}, \beta = 0, \gamma = 1/3
\]

\[
f(p_T, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_T, \tau^\gamma p_z)
\]

\[
\tau = \sqrt{t^2 - z^2}
\]
The Glasma and over-occupied quantum gases

Similar non-thermal fixed points discovered in cold atom experiments - albeit only for static geometry so far

$^87$Rb BEC in a quasi 1D optical trap

$$f_\theta(k, t) = t^\alpha f_S(t^\beta k)$$

$\alpha = 0.33 \pm 0.08 \quad \beta = 0.54 \pm 0.06$

Oberthaler BEC Labs
Kinetic theory of the Glasma

Different scenarios when occupancy $f \leq 1$:

- Elastic multiple scattering dominates in the Glasma
  
  BMSS: Baier, Mueller, Schiff, Son

- Rescattering influenced by plasma (Weibel) instabilities
  
  DB: Bodeker
  KM: Kurkela, Moore

- Transient Bose condensation + multiple scattering
  
  BGLMV: Blaizot, Gelis, Liao, McLerran, Venugopalan

*Gell-Mann’s totalitarian principle*: Anything that is not forbidden is allowed
Non-thermal fixed point in overpopulated QGP

Increasing anisotropy

And the winner is... bottom-up thermalization (caveat, caveat, caveat,...)

Decreasing occupancy with expansion

BMSS: Baier, Mueller, Schiff, Son
BD: Bodeker
KM: Kurkela, Moore
BGLMV: Blaizot, Gelis, Liao, McLerran, Venugopalan

Berges, Boguslavski, Schlichting, Venugopalan. PRD89 (2014) 114007
From nuts to soup: bottom-up thermalization

Thermalized soft bath of gluons for

\[ \tau > \frac{1}{\alpha_S^{5/2} Q_S} \]

Thermalization temperature of

\[ T_i = \alpha_S^{2/5} Q_S \]

Glasma / bottom-up prediction:

*In the Regge limit of QCD, matter thermalizes almost instantaneously.*

\[ \tau_{therm} \to 0 \text{ as } Q_S \to \infty \]
From nuts to soup: bottom-up thermalization

Mazeliauskas, QM 2018
arXiv:1807.05586
VISCOUS FLOW AT LHC


Flow harmonics $v_n$

ATLAS 20-30% central
viscosity: $\eta/s = 0.2$

CGC to QGP: from large to small systems

Bottom-up results plotted as function of scaled “hydrodynamization” variable match smoothly to viscous hydro even when system is quite anisotropic

Kurkela, Mazeliauskas, Schlichting, Paquet, Teaney, arXiv:1805.00961

From bottom-up analysis, regime of validity of hydro is limited for small systems:

Kurkela, Wiedemann, Wu, arXiv:1905.05139

Hydrodynamization:

Heller, Kurkela, Spalinski, Svensson, arXiv:1609.04803
Bazow, Heinz, Martinez, arXiv:1507.06595
Romatschke, arXiv:1704.08699
Strickland, Noronha, Denicol, arXiv:1709.06644
Early time probes: photons from the Glasma

Potentially significant contributions to photon production from the different stages (classical/quantum) of bottom-up thermalization of the Glasma

However there are significant uncertainties in the computations of both the thermal and the glasma rates

Topology in heavy-ion collisions: The Chiral Magnetic Effect

External (QED) magnetic fields - $10^{18}$ Gauss, of Magnetar strength!

Over barrier topological (sphaleron) transitions ... analogous to proposed mechanism for electroweak baryogenesis

massless quarks in hot medium

Topological transition $Q_W = n_L - n_R$

CME current generated

Kharzeev, McLerran, Warringa (2007)
Kharzeev, Fukushima, Warringa (2008)
Topology in ion-ion collisions: Chiral Magnetic Effect

External B field dies rapidly. Lifetime of hot matter ~ 10 Fermi: effect most significant, for transitions at early times

Consistent (caveat emptor!) with heavy-ion results from RHIC & LHC

CME studies a major part of RHIC’s upcoming beam energy scan (BES II) - possibly definitive results from comparative study of isobar collisions


BNL CME task force report: V. Skokov et al., arXiv:1608.00982
CME in condensed matter systems?

Dirac semi-metal: Zirconium Penta-Telluride

Axial charge separation in external B field

\[ \mathbf{J}_{\text{CME}} = \frac{e^2}{2\pi^2} \mu_5 \mathbf{B} \]

Effect of chiral anomaly

\[ \mu_5 \propto \mathbf{E} \cdot \mathbf{B} \]

\[ J_{\text{i,cme}}^{i} = \sigma_{\text{cme}}^{ik} E^{k} \]

\[ \sigma_{\text{cme}}^{ik} \propto B^{i} B^{k} \Rightarrow \sigma_{\text{cme}}^{zz} \propto B^{2} \]

Uncovering the topology of the QCD vacuum: Sphaleron transitions

Sphaleron: spatially localized, unstable finite energy classical solutions
(σφαλερος - "ready to fall")

EW theory: Klinkhamer, Manton, PRD30 (1984) 2212

Distinct energy degenerate QCD vacua characterized by topological Chern-Simons number $N_{CS}$

Sphaleron transition rate:

$$\Gamma^e = \lim_{\delta t \to \infty} \left\langle \frac{(N_{CS}(t + \delta t) - N_{CS}(t))^2}{V \delta t} \right\rangle_{eq}$$

Moore, Tassler, arXiv:1011.1167
Overoccupied gauge fields in a box

Thermalization extensively studied in this context employing classical-statistical simulations

Berges, Schlichting, Sexty, PRD86 (2012) 074006
Schlichting PRD86 (2012) 065008
York, Kurkela, Lu, Moore, PRD89 (2014) 074036
Overoccupied gauge fields in a box

Clean separation of scales develop *a la* thermal field theory:

- Temperature ($T$)
- Electric (Debye) screening ($gT$)
- Magnetic screening ($g^2 T$) scales

Berges, Scheffler, Sexty, PRD77 (2008) 034504
Mace, Schlichting, Venugopalan, PRD93 (2016), 074036
Berges, Mace, Schlichting, PRL118 (2017)
Topological transitions in the Glasma

Distribution of Chern-Simons charge localizes around integer values as UV modes are removed

“Cooled” Glue configurations in the Glasma are topological!
Topological transitions in the Glasma

Sphaleron transition rate scales with string tension squared

Very suggestive of non-trivial infrared structure of QCD far out of equilibrium
Exploding sphalerons


Sphaleron transition rate very large in the Glasma
- much larger than equilibrium rate

Couple sphaleron background with fermions & external EM fields to simulate ab initio the Chiral Magnetic Effect!
Classical-statistical simulations

Emergence of chiral magnetic wave

\[
\left( \frac{\vec{j}_V}{\vec{j}_A} \right) = \frac{N_c e B}{2\pi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix}
\]
From classical-statistical simulations to chiral kinetic theory and anomalous hydro

Results from RHIC isobar run imminent: exciting time for CME search!
Summary

- This talk covered only a small fraction of the developments in our understanding of the Initial Stages of ion-ion collisions. (Indeed, there is now a dedicated conference series by this name - next edition, Weizmann Institute, Israel, January 2021)

- I hope it provides some context to understanding the exciting developments over the week and wish you a successful and enjoyable conference!