New Developments in Lattice QCD

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Search for Criticality

HTD, F. Karsch, S. Mukherjee, arXiv:1504.05274

Sign Problem at $\mu_B=\pm 0$

Taylor Expansion

Imaginary $\mu_B$

STAR data: X.F. Luo, 1503.02558, X.F. Luo and N. Xu, 1701.02105

Toshihiro Nonaka, Poster

4th to 2nd order cumulant ratio

6th to 2nd order cumulant ratio

Toshihiro Nonaka, ATHIC 2018, EMMI 2019

Ashish Pandav, 11:20 Tue

STAR Preliminary
QCD criticality at $\mu_B=0$: relevance to CEP

Not relevant: 1st order chiral phase transition region as it becomes small and is away from the physical point

Relevant: 2nd order chiral phase transition belonging to O(4) universality class as axial $U(1)$ symmetry is not effectively restored at the critical temperature ($\chi_{l,\text{disc}} \neq 0$)

QCD phase diagram in 3D: quark mass, $\mu_B$, $T$

- $T_{pc}^{tri}$: transition $T$ at the tri-critical point
- $T_{CEP}^C$: transition $T$ at the critical end point
- $T_{c}^0$: chiral phase transition $T$ at $m_q=0$ and $\mu_B=0$
- $T_{pc}$: $156.5(1.5)$MeV, chiral crossover $T$ at $\mu_B=0$

Random Matrix Model & NJL suggests:

$$T_{c}^{tri} - T_{c}^{CEP}(m_q) \propto m_q^{2/5}$$

$T_{c}^0(\mu_B)$ decreases as $\mu_B$ up to NLO from LQCD


M. A. Halasz et al., PRD 58 (1998) 096007

M. Buballa, S. Carignano, PLB791(2019)361

O. Kaczmarek et al., PRD83 (2011) 014504

P. Hegde & HTD, PoS LATTICE2015 (2016) 141
QCD phase diagram in 3D: quark mass, $\mu_B$, $T$

$T_{pc}$ : 156.5(1.5)MeV, chiral crossover $T$ at $\mu_B=0$

$T_{CEP}$ : transition $T$ at the critical end point

$T_{c}$ : chiral phase transition $T$ at $m_q=0$ and $\mu_B=0$

$T_{tri}$ : transition $T$ at the tri-critical point

\[
T_{tri} - T_{CEP}^C (m_q) \propto m_q^{2/5}
\]

Random Matrix Model & NJL suggests:

$T_{C}^{tri}$ decreases as $\mu_B$ up to NLO from LQCD

Indication

$T_{C}^{0}(\mu_B)$ decreases as $\mu_B$ up to NLO from LQCD

$T_{C}^{0} > T_{C}^{tri} > T_{C}^{CEP}$
O(4) scaling analyses: Nf=2+1, Nt=6,8 & 12 lattices, $M_\pi \gtrsim 55$ MeV thermodynamic, continuum & chiral extrapolated

Chiral crossover $T_{pc}^{phys} = 156.5(1.5)$ MeV

$T_{pc}(H) = T^0_c \left(1 + \frac{z_p}{z_0} H^{1/\beta_8} \right)$

Chiral phase transition $T$

$T^0_c = 132^{+3}_{-6}$ MeV


$\chi(M) \propto \frac{\partial^2 \ln Z}{\partial m_q^2}$

Continuum extrapolation

Infinite volume & chiral extrapolation

$T_c^0$ is ~ 25 MeV smaller than the chiral crossover $T$!

See QCD-inspired model calculations:


High order fluctuations & critical behavior

\[ \chi_{lmn}^{BQS} \equiv \chi_{lmn}^{BQ}(T) = \frac{\partial^{l+m+n} P(T, \hat{\mu})/T^4}{\partial \hat{\mu}_B \partial \hat{\mu}_Q \partial \hat{\mu}_S} \bigg|_{\hat{\mu} = 0} \]

Many 8th order fluctuations turn to be negative at \( T \geq 135-140 \) MeV

Suggests zeros in the complex plane: no phase transition in the above T window

Supports for \( T_{c\text{CEP}}^{S} < 135 - 140 \) MeV \( \sim T_{c}^{0} \)
Singularity in the complex plane: Radius of convergence in $\mu_B$

Idea testing on Nt=4 coarse lattices using 2-stout improved staggered fermions

Try to have a direct determination of the leading singularity of the pressure, i.e. the closest Lee-Yang zero, via reweighting to a complex chemical potential

Attila Pasztor
12:20 Wed

Singularity in the complex plane: Radius of convergence in $\mu_B$ based on O(4) universality and LQCD results on chiral phase transition

Universal parameters: O(4) critical exponents, scaling functions
Scaling functions: singularity in the complex plane—Lee-Yang edge singularity

The singularity limits the convergence of the Taylor series in $\mu_B$

Non-universal parameters:
LQCD-determined chiral phase transition $T_{c^0}$, curvature of transition line

Vladimir Skokov
16:40 Wed
Lattice data suggests

\[ T_{CEP}^c < 135 \text{ – } 140 \text{MeV} \]

\[ \mu_B^{CEP} > 300 \text{MeV} \]
Chiral crossover line: \( T_{pc}(\mu_B) = T_{pc}(0) \left( 1 - \kappa_2 \left( \frac{\mu_B}{T} \right)^2 - \kappa_4 \left( \frac{\mu_B}{T} \right)^4 \right) \)


Imaginary \( \mu_B \): Wuppertal-Budapest preliminary: Jana Guenther, 11:40 Tue

courtesy from Jana Guenther


Imaginary \( \mu_B \): Wuppertal-Budapest preliminary: Jana Guenther, 11:40 Tue

\[ n_S = 0, \quad \frac{n_Q}{n_B} = 0.4 \]
Chiral crossover line: \( T_{pc}(\mu_B) = T_{pc}(0) \left( 1 - \kappa_2 \left( \frac{\mu_B}{T} \right)^2 - \kappa_4 \left( \frac{\mu_B}{T} \right)^4 \right) \)

ALICE data point: \( T_f = 156.5(1.5) \text{ MeV} \)

STAR data points:

Similar chiral crossover line: see from Jana Guenther, 11:40 Tue
Explore the QCD phase diagram through fluctuations of conserved charges $x=B,Q,S$

$$\frac{M_x(\sqrt{s})}{\sigma_x^2(\sqrt{s})} = \frac{\langle N_x \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi^x_1(T,\mu_B)}{\chi^x_2(T,\mu_B)} = R^{x}_{12}(T,\mu_B)$$

$$\frac{S_x(\sqrt{s}) \sigma_x^3(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^3 \rangle}{\langle N_x \rangle} = \frac{\chi^x_3(T,\mu_B)}{\chi^x_1(T,\mu_B)} = R^{x}_{31}(T,\mu_B)$$

$$\frac{\kappa_x(\sqrt{s}) \sigma_x^2(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^4 \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi^x_4(T,\mu_B)}{\chi^x_2(T,\mu_B)} = R^{x}_{42}(T,\mu_B)$$

$$\frac{S^h_x(\sqrt{s}) \sigma_x^5(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^5 \rangle}{\langle N_x \rangle} = \frac{\chi^x_5(T,\mu_B)}{\chi^x_1(T,\mu_B)} = R^{x}_{51}(T,\mu_B)$$

$$\frac{\kappa^h_x(\sqrt{s}) \sigma_x^4(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^6 \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi^x_6(T,\mu_B)}{\chi^x_2(T,\mu_B)} = R^{x}_{62}(T,\mu_B)$$

**LQCD**

generalized susceptibilities

$$\chi^x_n(T,\mu_B) = \frac{1}{VT^3} \frac{\partial^n \ln Z(T,\bar{\mu})}{\partial(\mu_x/T)^n}$$

**HIC**

mean: $M_x$

variance: $\sigma_x^2$

skewness: $S_x$

kurtosis: $\kappa_x$

hyper-skewness: $S^h_x$

hyper-kurtosis: $\kappa^h_x$

Proxies:

proton, charge particles, kaons
Many caveats

Non-equilibrium effects
S. Mukherjee, R. Venugopalan, Yi Yin PRL(2016)...

Proton v.s. Baryon
M. Kitazawa and M. Asakawa, PRC(2012)...

Detector effects: cuts in acceptance & kinematics...
A. Bzdak, V. Koch, PRC (2012)
V. Skokov et al., PRC (2013)...

Final-state interactions in the hadronic phase
J. Steinheimer et al., PRL (2013)...

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Baseline from thermal equilibrated QCD
Comparison to Hadron Resonance Gas Model (HRG)

See various 2nd & 4th order cumulant ratios in Jishnu Goswami’s Poster

At $T \approx T_{pc}$ ideal HRG describes cumulants up to 2nd order at $\mu_B \lesssim 120$ MeV

QM-HRG needed to describe 2nd cumulant ratios where e.g. Not-PDG-listed strange-baryons’ contributions manifest

An ideal HRG cannot describe cumulants for >2nd orders

Cumulant ratios at $\mu_B = 0$: contact to ALICE data interesting

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Anar Rustamov Plenary Wed, Mesut Arslandok 11:00 Tue
Bands in QCD: chiral crossover $T$ region

\[
T_{pc}(\mu_B) = T_{pc}(0) \left( 1 - \kappa_2 \left( \frac{\mu_B}{T} \right)^2 - \kappa_4 \left( \frac{\mu_B}{T} \right)^4 \right)
\]
QCD v.s. Experimental data: skewness ($R_{31}$) & kurtosis ($R_{42}$) ratios

Bands in QCD: chiral crossover T region

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\[ \sqrt{s_{NN}}=54.4 \text{ GeV} \text{ data points fall on the fits to the old data} \]
QCD v.s. Experimental data: skewness ($R_{31}$) & kurtosis ($R_{42}$) ratios

Consistency of $\sqrt{s_{NN}}$=54.4 GeV data with QCD at $T$=155-158 MeV

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QCD v.s. Experimental data: skewness ($R_{31}$) & kurtosis ($R_{42}$) ratios

Consistency of $\sqrt{s_{NN}}=54.4$ GeV data with QCD at $T=155$-158 MeV

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Consistency of $\sqrt{s_{NN}}=54.4$ GeV data with QCD at $T=155-158$ MeV

$\sqrt{s_{NN}}=54.4$ GeV data points fall on the fits to the old data

At fixed $R_{12}$, $R_{31}$ & $R_{42}$ increase with decreasing $T$
QCD v.s. Experimental data: skewness ($R_{31}$) & kurtosis ($R_{42}$) ratios

Consistency of $\sqrt{s_{NN}}=54.4$ GeV data with QCD at $T=155-158$ MeV

$\sqrt{s_{NN}}=54.4$ GeV data points fall on the fits to the old data

At fixed $R_{12}$, $R_{31}$ & $R_{42}$, increase with decreasing $T$

Inconsistency of $T_{f}=165(4)$ MeV from yields and their ratios at $\sqrt{s_{NN}}=200$ GeV

Statistics ? Non-equilibrium?…?…?
QCD v.s. Experimental data: hyper-skewness ($R_{51}^B$) & hyper-kurtosis ($R_{62}^B$) ratios

$$R_{62}^B(\mu_B/T) = \frac{\chi_6^B(\mu_B/T)}{\chi_2^B(\mu_B/T)} = \frac{\chi_6^B(0)}{\chi_2^B(0)} + \frac{1}{2} \left( \frac{\mu_B}{T} \right)^2 \left( \frac{\chi_8^B(0)}{\chi_4^B(0)} - \frac{\chi_6^B(0)}{\chi_2^B(0)} \frac{\chi_4^B(0)}{\chi_2^B(0)} \right) + \mathcal{O} \left( \frac{\mu_B}{T} \right)^4$$

R$_{51}^B$ and R$_{62}^B$: Statistics-hungry quantities

LQCD: Both hyper-skewness and hyper-kurtosis are negative down to $\sqrt{s_{NN}}=39$ GeV

Nice consistency seen in skewness & kurtosis at $\sqrt{s_{NN}}=54.4$ GeV with QCD while deviations seen at both $\sqrt{s_{NN}}=54.4$ & 200 GeV
Excited bottomonia up to 3S and 2P

NRQCD, 48^3x12 lattices, 1st lattice QCD study of up to 3S and 2P bottomonia

NRQCD, 48^3x12 lattices, 1st lattice QCD study of up to 3S and 2P bottomonia

Thermal width extracted from temporal correlators:

$$\Gamma_{Y(1S)}(T) < \Gamma_{\chi_{b0}(1P)}(T) < \Gamma_{Y(2S)}(T) < \Gamma_{\chi_{b0}(2P)}(T) < \Gamma_{Y(3S)}(T)$$

Compatible to Sequential dissociation picture

Rasmus Larsen et al., 1910.07374, 1908.08437

Rasmus Larsen 17:00 Wed
Fate of $J/\Psi$ and $\Upsilon$

Quenched QCD, 1st continuum extrapolated results

- e.g. $T=1.1\, T_c$ continuum extrapolation based on $192^3\times 64$, $144^3\times 48$, $120^3\times 40$, $96^3\times 32$ lattices

1. At $T\geq 1.1\, T_c$: No resonance peaks of $J/\Psi$ is needed
2. At $T\geq 1.5\, T_c$: No resonance peaks of $\Upsilon$ is needed

Olaf Kaczmarek
14:40 Tue
Ratios of vector correlators contributed from transport peaks between charm and bottom quarks

\[ \frac{G_{\text{trans}}^{\text{charm}}(\tau T = 0.5)/\chi_q^{\text{charm}}}{G_{\text{trans}}^{\text{bottom}}(\tau T = 0.5)/\chi_q^{\text{bottom}}} \]

Quenched QCD, continuum extrapolated results

\[ G_{\text{trans}}: \text{correlators after resonance and continuum contribution fitted using pNRQCD+cont are subtracted} \]

Using Lorentzian ansatz for the transport peak:

\[ \frac{G_{\text{trans}}^{\text{charm}}/\chi_q^{\text{charm}}}{G_{\text{trans}}^{\text{bottom}}/\chi_q^{\text{bottom}}} \approx \frac{M_{\text{bottom}}}{M_{\text{charm}}} \cdot \frac{\tan^{-1}(T/\eta_{\text{charm}})}{\tan^{-1}(T/\eta_{\text{bottom}})} \]

\[ M_{\text{bottom}}/M_{\text{charm}} \approx 3 \]

\[ \frac{\tan^{-1}(T/\eta^{\text{charm}})}{\tan^{-1}(T/\eta^{\text{bottom}})} < 1 \]

\[ \eta^{\text{charm}} > \eta^{\text{bottom}} \]

Flavor hierarchy of drag coefficients $\eta$

Olaf Kaczmarek
14:40 Tue
SU_L(2)xSU_R(2) and U_A(1) Symmetries

Continuum extrapolated results in 2+1 flavor QCD with m_π=140 MeV

Meson screening masses

U_A(1) susceptibility

SU_L(2)xSU_R(2) chiral symmetry: Degeneracy of Vector and Axial Vector at T≃T_pc
Axial U(1) symmetry: Degeneracy of Pseudo scalar and Scalar at T-200 MeV

[HotQCD] arXiv:1908.09552 [hep-lat]
Summary

Negative 6th and 8th order cumulants as well as \( T_{c}^{0}=132 \) MeV suggests a possible critical end point can located only at

\[
T_{c}^{CEP} < 135 - 140 \text{MeV} \quad \mu_{B}^{CEP} > 300 \text{MeV}
\]

hyper-skewness and hyper-kurtosis ratios are obtained in NLO in \( \mu_{B} \)

\[
\sqrt{s_{NN}}=200 \text{ GeV}: \quad R_{51}^{B} = -0.5(3), \quad R_{62}^{B} = -0.7(3)
\]

\[
\sqrt{s_{NN}}=54.4 \text{ GeV}: \quad R_{51}^{B} = -0.7(4), \quad R_{62}^{B} = -2(1)
\]

Great progress achieved in understanding the fate of quarkonia states as well as heavy quark transports