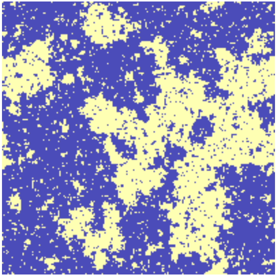
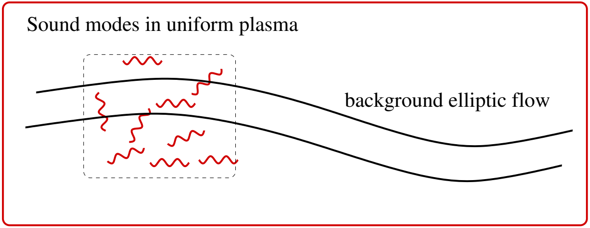
Dynamics of Critical Fluctuations in Nucleus-Nucleus Collisions

Derek Teaney
Stony Brook University



- Yukinao Akamatsu, Aleksas Mazeliauskas, Fanglida Yan, Yi Yin, Misha Stephanov, Gokce Basar, Maneesha Pradeep, Xin An, Thomas Schäfer, Mauricio Martinez, Mayank Singh, Volker Koch, Marcus Bluhm, Marlene Nahrgang, J Stachel, PBM

Hydrodynamic and thermal fluctuations:



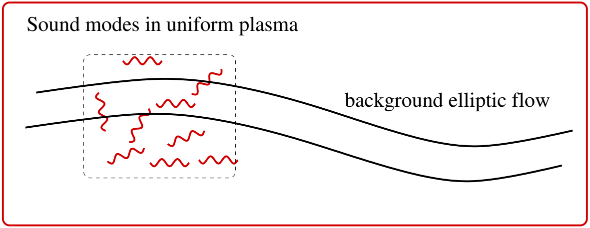
These short wavelength modes are part of the bath:

$$W^{\rm eq}_{ee}(\boldsymbol{k},t) \equiv \underbrace{\langle e^*(\boldsymbol{k},t)e(\boldsymbol{k},t)\rangle}_{\text{energy-density flucts}} = T^2 c_v$$

$$W^{\rm eq}_{nn}(\boldsymbol{k},t) \equiv \underbrace{\langle n^*(\boldsymbol{k},t)n(\boldsymbol{k},t)\rangle}_{\text{baryon flucts}} = T\chi\delta^{ij}$$

Want to measure these thermal fluctuations . . .
But, long wavelength modes will always be out of equilibrium,
and reflect the initial state not the CP.

Hydrodynamic and thermal fluctuations:



These short wavelength hydro modes are part of the bath:

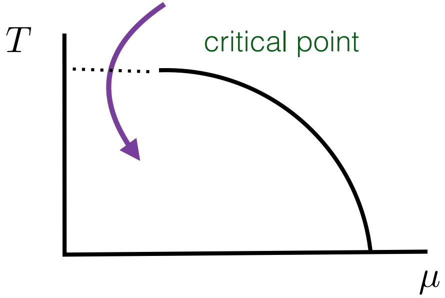
- For CP search we study flucts in baryon/entropy ratio, $\hat{n} \equiv s\delta(n/s)$

$$W^{\rm eq}_{\hat{n}\hat{n}}(\boldsymbol{k},t) \equiv \underbrace{\langle \hat{n}^*(\boldsymbol{k},t)\hat{n}(\boldsymbol{k},t)\rangle}_{\text{baryon/entropy flucts}} = \left(\frac{n}{s}\right)^2 c_p$$

Want to measure these thermal fluctuations . . .
But, long wavelength modes will always be out of equilibrium,
and reflect the initial state not the CP.

Transits of the critical point: a parametric analysis

see also, Berdnikov & Rajagopal; Mukherjee, Venugopalan, Yin; Y. Akamatsu, DT,F. Yan, Y. Yin, 1811.05081



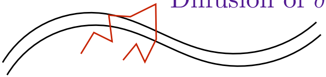
Finite relaxation rate?
What if we miss?

- How does the finite expansion rate limit the critical flucts?

$$\epsilon \equiv \underbrace{\tau_o}_{\text{micro time}} \times \underbrace{\partial_\mu u^\mu}_{\text{expansion rate } 1/\tau_Q} = \frac{\tau_o}{\tau_Q}$$

The critical fluctuations will reach a maximum of order, $1/\epsilon^{\rm some-power}$.

Estimate of the longest wavelength of thermalized fluctuations



- The longest wavelength that can be equilibrated by diffusion is

$$\underbrace{\ell_*^2}_{\text{the longest wavelength}} = \underbrace{\ell_o^2}_{\text{microlength}} \times \underbrace{\tau_Q/\tau_o}_{\text{(total time)/(microtime)}}$$

- Find, with $D_0 \equiv \ell_0^2/\tau_0$ the diffusion coefficient away from the CP:

$$\ell_* \equiv \sqrt{D_0\tau_Q} \sim \ell_o\epsilon^{-1/2}$$

The wavelength of thermal fluctuations are short, but still hydrodynamic:

$$\underbrace{\ell_o}_{\text{microlength}} \ll \underbrace{\ell_*}_{\text{hydro-kinetic} \sim \ell_0/\epsilon^{1/2}} \ll \underbrace{L}_{\text{system-size} \sim \ell_0/\epsilon}$$

Estimate of the longest wavelength of thermal critical correlations:

- The coefficient D decreases near the CP, and ℓ_* is an over-estimate
- We will see that typical critical wavelength is ℓ_{kz}

$$\underbrace{\ell_o}_{\text{micro-length}} \quad \lll \quad \underbrace{\ell_o \epsilon^{-0.19}}_{\text{kibble-zurek } \ell_{kz}} \quad \lll \quad \underbrace{\ell_o \epsilon^{-0.5}}_{\text{cutoff } \ell_{\mathrm{max}}}$$

Numerically these evaluate to with $\epsilon = 1/5$ and $\ell_0 = 1.2\,\mathrm{fm}$

$$1.2\,\mathrm{fm} \lll 1.6\,\mathrm{fm} \lll 2.7\,\mathrm{fm}$$

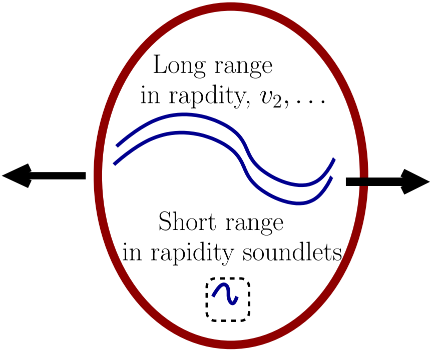
- To translate to rapidity divide by τ_Q, and use $\epsilon = \ell_0/\tau_Q$

$$\underbrace{\epsilon^{0.82}}_{\ell_{\mathrm{kz}} \text{ range in } \eta} \quad \lll \quad \underbrace{\epsilon^{0.5}}_{\ell_{\mathrm{max}} \text{ range in } \eta} \quad \lll 1$$

The critical rapidity correlation length is parametrically smaller than unity

Picture and hydro-kinetics:

Akamatsu, Mazeliauskas, DT, 1606.07742



- Need a kinetic equation for the phase-space density of these sound modes:

$$\underbrace{W_{++}(t, \boldsymbol{x}, \boldsymbol{k})}_{\text{distribution of sound modes}}$$

$$\underbrace{\ell_o}_{\text{microscopic}} \ll \underbrace{\ell_*}_{\text{soundlets}} \ll \underbrace{L \sim \ell_0/\epsilon}_{\text{macroscopic flow } v_2, v_3 \ldots}$$

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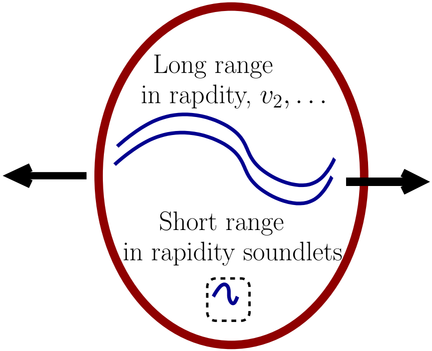


- The phase-space density of $\hat{n}$ reflects the critical flucts:

$$\underbrace{W_{\hat{n}\hat{n}}(t, \boldsymbol{x}, \boldsymbol{k})}_{\text{distribution of } n/s}$$

Will definitely need hydro-kinetic equation for this!

$$\underbrace{\ell_o}_{\text{microscopic}} \quad \ll \quad \underbrace{\ell_*}_{\text{soundlets}} \quad \ll \quad \underbrace{L \sim \ell_0/\epsilon}_{\text{macroscopic flow } v_2, v_3 \ldots}$$

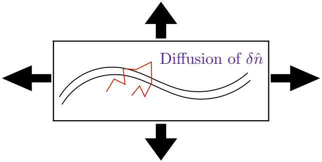
Hydrokinetic equation for $W^{\hat{n}\hat{n}}(k,t)$

see also hydro+ by Stephanov,Yin 1712.10305

- Start from dissipative hydro with noise

$$\partial_\mu(T^{\mu\nu}_{\text{Ideal}} + T^{\mu\nu}_{\text{diss}} + \xi^{\mu\nu}) = 0$$

$$\partial_\mu(j^{\mu}_{\text{Ideal}} + j^{\mu}_{\text{diss}} + \xi^{\mu}) = 0$$



- Can derive time evolution equations for the correlators

$$W^{ee} = \langle \delta e^*(k,t)\delta e(-k,t)\rangle \quad W^{nn} = \langle \delta n^*(k,t)\delta n(-k,t)\rangle \quad \text{, etc}$$

- From W^{nn}, W^{ee} derive an equation for $W^{\hat{n}\hat{n}}$:

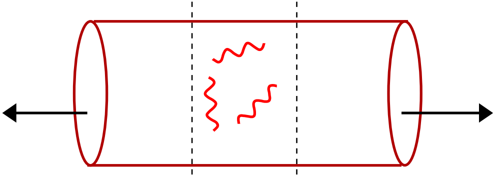
$\hat{n} \equiv s\delta(n/s)$

$$\partial_t W^{\hat{n}\hat{n}} = -\underbrace{\frac{\lambda k^2}{c_p}}_{\lambda\text{ is therm. conduct.}}(W^{\hat{n}\hat{n}} - c_p) + \text{grad corrections}$$

From stochastic hydro, find that $W^{\hat{n}\hat{n}}$ obeys a relaxation equation + gradient corrections

Hydrokinetics: Bjorken expansion

Akamatsu et al; Martinez and Shäfer



$$\frac{dT^{\tau\tau}}{d\tau} = -\frac{T^{\tau\tau} + T^{zz}}{\tau}$$

- The kinetic equation for the phase space distribution read:

$$\underbrace{\frac{\partial}{\partial\tau} W_{++}}_{\text{phase-space-dist of sound}} = -\underbrace{\frac{4\eta}{3sT}\left(k_\perp^2 + k_z^2\right)\left[W_{++} - \frac{T^2 c_v}{\tau}\right]}_{\text{relaxation to equilibrium}} - \frac{1}{\tau}\Big[\underbrace{2 + c_s^2 + \frac{k_z^2}{k_\perp^2 + k_z^2}}_{\text{expansion}}\Big] W_{++}$$

- And then the fluctuations modify the stress tensor: $\ell_* = \sqrt{4\pi D_\eta \tau}$

$$\langle T^{zz}\rangle = p - \underbrace{\frac{\frac{4}{3}\eta + \zeta}{\tau}}_{\text{1st order visc.}} + \underbrace{(e+p)\frac{1.083}{s\,\ell_*^3}}_{\text{flucts}} + \dots$$

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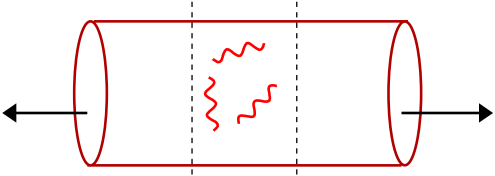

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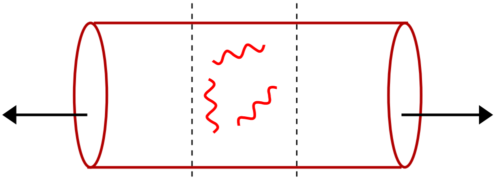

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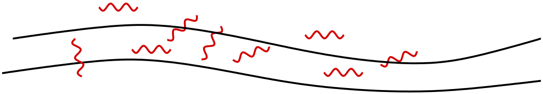
Hydrokinetics: general background flow

An, Basar, Stephanov, Yee 1902.09517



relaxation to equilibrium

$$u \cdot \bar{\nabla} W_{\pm} = \mp c_s \hat{q} \cdot \bar{\nabla} W_{\pm} - \gamma_L q^2 (W_{\pm} - Tw) + \left(\pm \left(c_s - \frac{\dot{c}_s}{c_s} \right) |q| a_\mu + (\partial_{\perp\mu} u_\nu) q^\nu + 2c_s^2 q^\lambda \omega_{\lambda\mu} \right) \frac{\partial W_{\pm}}{\partial q_\mu}$$
$$- \left((1 + c_s^2 + \dot{c}_s)\theta + \theta_{\mu\nu} \hat{q}^\mu \hat{q}^\nu \pm \frac{1 + 2c_s^2}{c_s} \hat{q} \cdot a \right) W_{\pm} \,, \qquad (4.$$

Describes how the shear strain and vorticity drive the
fluctuations from equilibrium

1. These equations can be formulated as a particle scheme, which are propagated on top of existing hydro codes
2. Still need to work out how to turn hydro-particles to real particles:

D. Oliinychenko, V. Koch, arXiv:1902.09775

Stochastic hydro for higher cumulants:

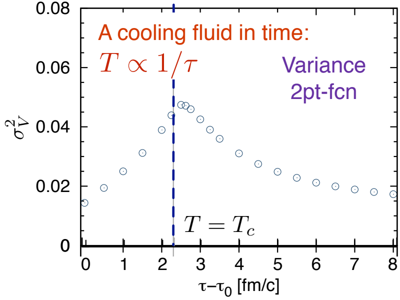
Nahrgang, Blum, Schafer, Bass 1804.05728

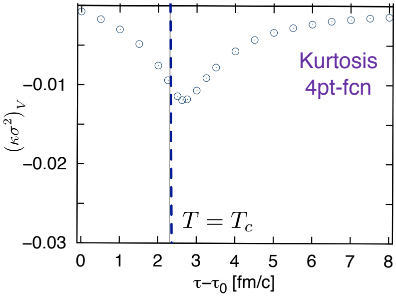
The stochastic diffusion equation of Model B near the critical point is

$$\partial_t n = \lambda \nabla^2 \left(\frac{\delta \mathcal{F}}{\delta n} \right) + \underbrace{\nabla \cdot \vec{\xi}}_{\text{noise}}$$

with $\mathcal{F}$ given by the Landau-Ginzburg functional with $c_2 \propto T - T_c$

$$\mathcal{F}[\delta n] = T \int \mathrm{d}^3x \; \frac{K}{2} (\nabla n)^2 + c_2 \delta n^2 + c_3 \delta n^3 + c_4 \delta n^4$$





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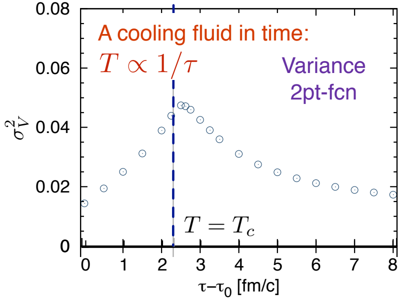

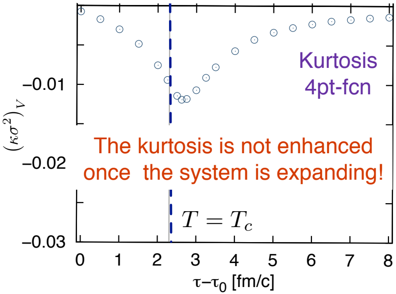

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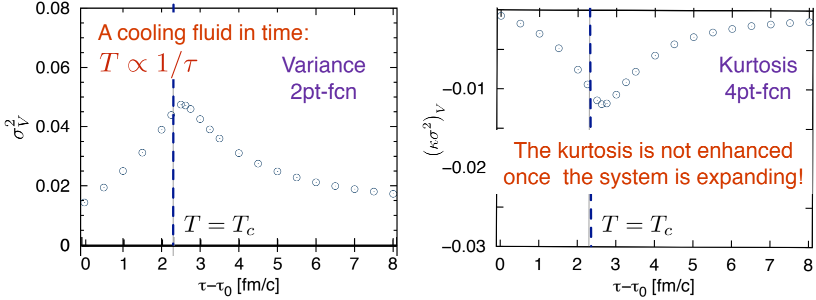
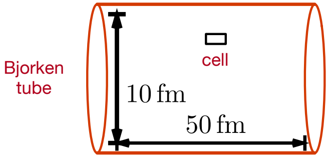
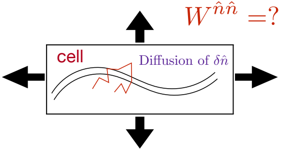


see also Plumberg, Pratt 1712.09298 ; M. Singh, *et al* 1807.05451; K. Murase, Kurita, T. Hirano 1809.04773

Evolution of two point functions $W^{\hat{n}\hat{n}}$ near the critical point





- Only need to simulate $W^{\hat{n}\hat{n}}$ in a single fluid cell!
- Define $t = 0$ as the time in the LRF the cell goes through the CP

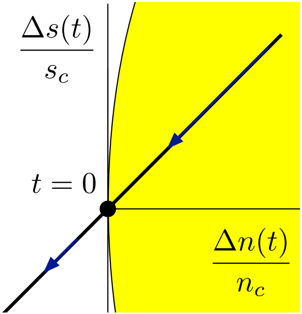
Entropy conservation: $$\frac{\Delta s(t)}{s_c} = \frac{t}{\tau_Q}$$

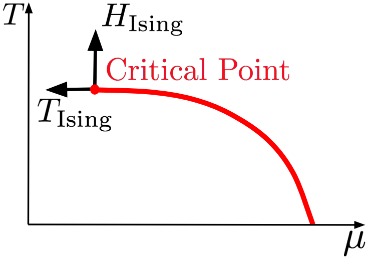
Baryon conservation: $$\frac{\Delta n(t)}{n_c} = \frac{t}{\tau_Q}$$

with $\Delta n \equiv n - n_c$ and $\Delta s \equiv s - s_c$

How does $W^{\hat{n}\hat{n}}$ evolve on these trajectories, and with a finite expansion rate, $\partial_\mu u^\mu \equiv 1/\tau_Q$?

Trajectories in n, s plane and mapping QCD to the Ising model





- The EOS and correlation length $\xi(t)$ vs. time are known, after specifying the QCD to Ising map:

$$\Delta s \longleftrightarrow \Delta M_{\mathsf{Ising}} \qquad \Delta T_{\mathsf{QCD}} \longleftrightarrow \Delta H_{\mathsf{Ising}}$$

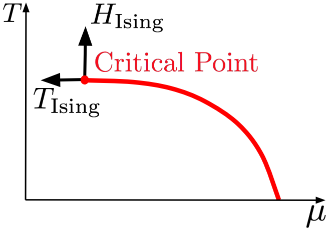
$$\Delta n \longleftrightarrow \Delta e_{\mathsf{Ising}} \qquad -\Delta\mu_{\mathsf{QCD}} \longleftrightarrow \Delta T_{\mathsf{Ising}}$$

Most modeling has used this simple map, and not a linear combination

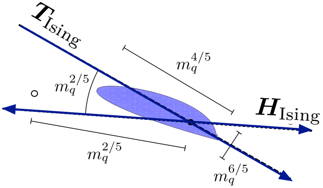
 M. Pradeep, Stephanov 1905.13247

- Close to the chiral limit $m_q = 0$, the CP is close to a tri-critical point.
- This leads to the following expectations:

Usual Modeling



New theoretical expectation



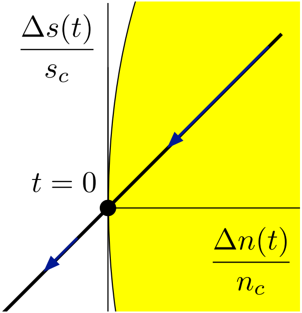
- Changes (non-universal) estimates of bulk viscosity near CP:

Martinez, Schäfer, Skokov 1906.11306

$$\frac{\zeta}{s} \simeq \underbrace{(0.00042 \leftrightarrow 0.8)}_{\text{usual} \leftrightarrow \text{new}} \left(4\pi\frac{\eta_0}{s}\right)\left(\frac{\xi}{\xi_0}\right)^{2.8}$$

How do the two point functions evolve while transiting the CP?

We transit the CP:



$$\Delta n \longleftrightarrow \Delta e_{\mathsf{Ising}}$$

$$\Delta s \longleftrightarrow \Delta M_{\mathsf{Ising}}$$

And solve the hydro-kinetic equations for

$$\partial_t W^{\hat{n}\hat{n}} = - \underbrace{\frac{\lambda k^2}{c_p(t)}}_{\text{relaxation rate } \Gamma(k,t)} \left(W^{\hat{n}\hat{n}} - c_p(t)\right)$$

Critical slowing down:

Son,Stephanov;Berdnikov,Rajagopal

$$\partial_t W^{\hat{n}\hat{n}} = - \underbrace{\frac{\lambda k^2}{c_p}}_{\text{relaxation rate } \Gamma(k)} (W^{\hat{n}\hat{n}} - c_p)$$

- Substituting $c_p = \chi_o \left(\xi/\ell_o\right)^{2-\eta}$ and into the EOM with $k\xi \sim 1$

$$\Gamma(\xi^{-1}) \equiv \underbrace{\frac{\lambda}{\chi_o {\ell_o}^2}}_{1/\tau_o} \times \underbrace{\frac{1}{(\xi(t)/\ell_o)^{4-\eta}}}_{\text{goes to 0 at CP}}$$

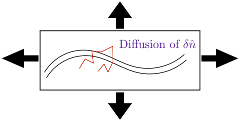
- The correlation length diverges, $\xi(t) = \ell_o(\frac{\tau_Q}{t})^{a\nu}$ $\quad a \equiv 1/(1-\alpha), a\nu \simeq 0.71$

$$\Gamma = \frac{1}{\tau_0}\left(\frac{t}{\tau_Q}\right)^{a\nu z} \quad \text{where} \quad z = 4 - \eta.$$

At CP, the hydro fluctuations relax infinitely slowly,
while the rate of change in equilibrium is fast $\partial_t \xi/\xi \sim 1/t$

Mukherjee, Venugopalan, Yin 1506.00645

- When is this rate in change in equilibrium comparable to the relaxation rate Γ ?



$$\underbrace{\left|\frac{\partial_t \xi}{\xi} \sim \frac{1}{t}\right|}_{\text{rate-of change of } \xi(t)} = \underbrace{\frac{(|t|/\tau_Q)^{a\nu z}}{\tau_o} = \Gamma}_{\text{relaxation rate}}$$

- This determines a characteristic Kibble-Zurek time t_{kz}

$$t_{kz} = \epsilon^{1/(a\nu z+1)}\tau_Q = \epsilon^{0.26}\tau_Q$$

- Kibble-Zurek length is the correlation length at this time

$$\ell_{kz} = \xi(t_{kz}) = \ell_o \epsilon^{-a\nu/(a\nu z+1)} = \ell_o \epsilon^{-0.19}$$

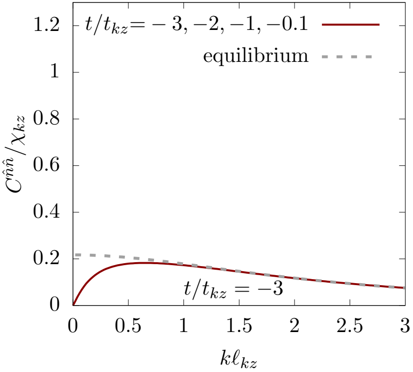
$$\underbrace{\ell_o}_{\text{micro-length}} \ll \underbrace{\ell_o\epsilon^{-0.19}}_{\text{kibble-zurek } \ell_{kz}} \ll \underbrace{\ell_o\epsilon^{-0.5}}_{\text{cutoff } \ell_{\max}}$$

Solution for $W^{\hat{n}\hat{n}}$

see also Shanjin Wu, Zeming Wu, Huichao Song 1711.09518; Rajagopal et al

- All lengths and times are in units of the Kibble-Zurek length and time
- The size of the fluctuations grow to $\chi_{kz} = \chi_0 \ell_{\rm kz}^{2-\eta}$

$$W^{\hat{n}\hat{n}} \propto C^{\hat{n}\hat{n}}$$



Before critical point

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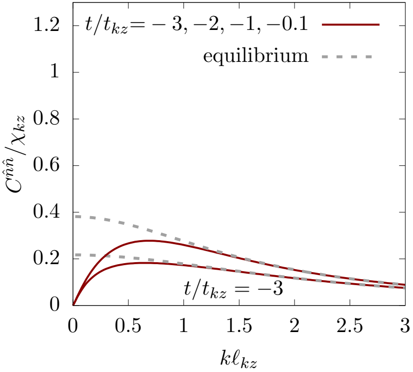

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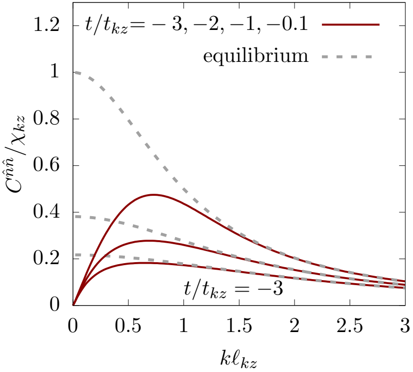

Before critical point

Solution for $W^{\hat{n}\hat{n}}$

see also Shanjin Wu, Zeming Wu, Huichao Song 1711.09518; Rajagopal et al

- All lengths and times are in units of the Kibble-Zurek length and time
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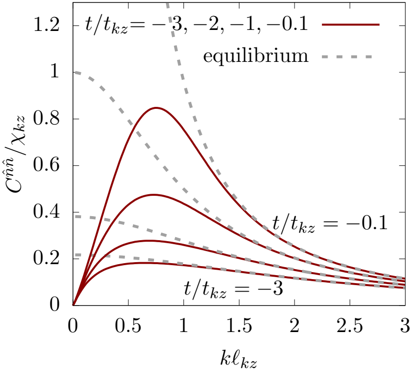

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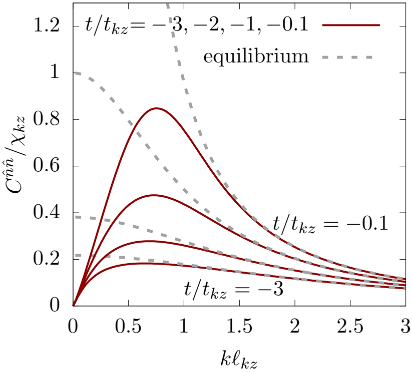
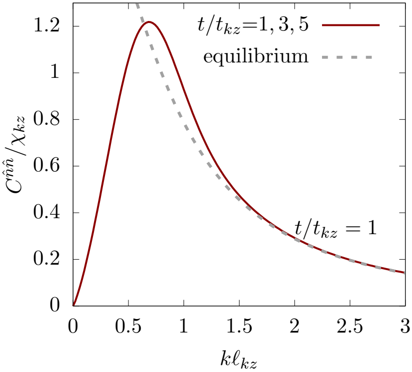


Before critical point



After critical point

Solution for $W^{\hat{n}\hat{n}}$

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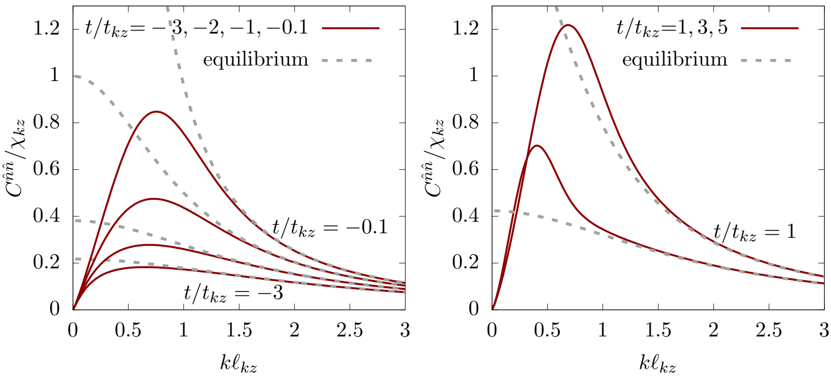

Before critical point

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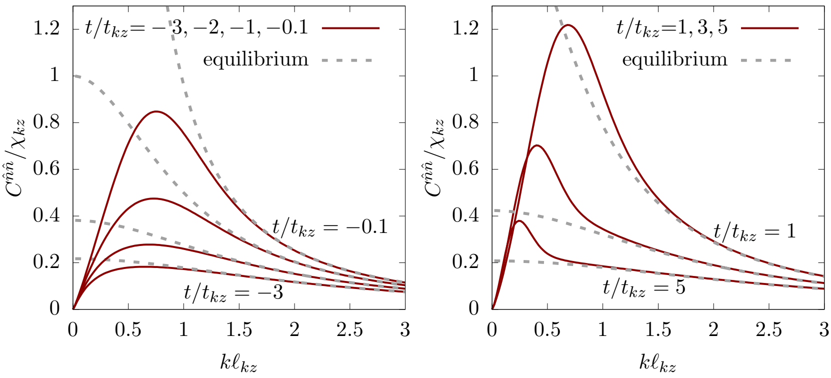


Before critical point

After critical point

Summary: a parametric picture of the CP transit

1. The small parameter is the microscopic length to macroscopic length:

$$\epsilon = \frac{\tau_0}{\tau_Q} = \frac{\text{micro scale}}{\text{macro scale}} \simeq \frac{1}{7}$$

2. Hierarchy of scales:

$$\underbrace{\ell_0}_{\text{microlength}} \ll \underbrace{\ell_{kz}}_{\text{longest critical-fluct}} \ll \underbrace{\ell_*}_{\text{hydro-kinetics}} \ll \underbrace{\ell_{\rm hydro}}_{v_2 \text{ etc}}$$

which are of relative order

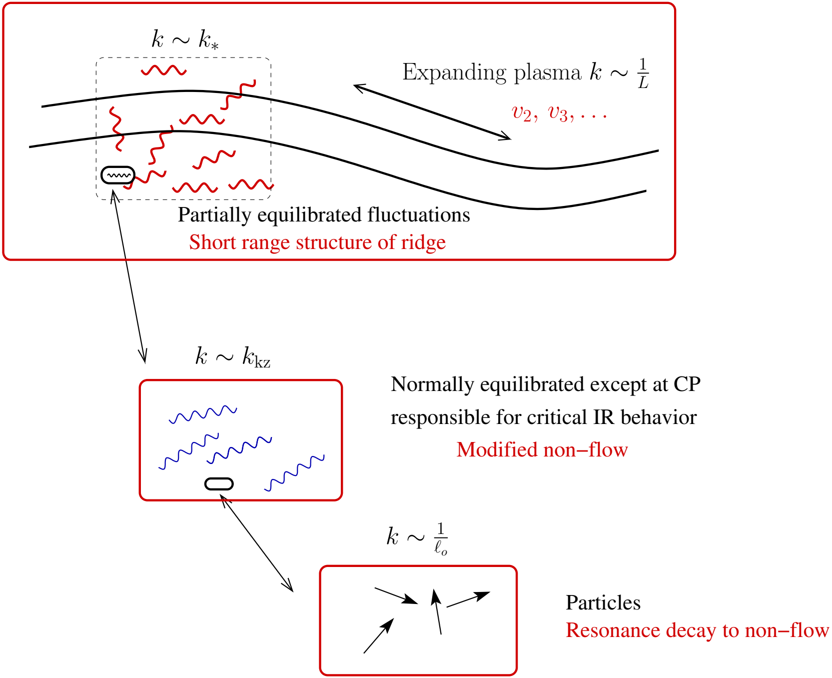
$$\epsilon \ll \epsilon^{0.82} \ll \sqrt{\epsilon} \ll 1 \qquad \text{or} \qquad 0.14 \ll 0.21 \ll 0.38 \ll 1$$

3. The fluctuations are larger than the baseline susceptibility χ_0:

$$\frac{C^{\hat{n}\hat{n}}}{\chi_0} \sim \left(\frac{\ell_{\rm kz}}{\ell_0}\right)^{2-\eta} = \epsilon^{-0.37} \sim 2.0$$

but live only a (parametrically) short time

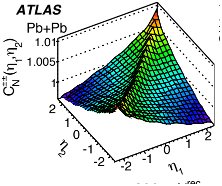
$$t_{\rm kz} \sim \epsilon^{0.26}\,\tau_Q \quad \text{or} \quad t_{\rm kz} \sim 0.6\,\tau_Q$$


$k \sim k_*$
Expanding plasma $k \sim \frac{1}{L}$
$v_2,\ v_3, \ldots$
Partially equilibrated fluctuations
Short range structure of ridge
$k \sim k_{\rm kz}$
Normally equilibrated except at CP
responsible for critical IR behavior
Modified non−flow
$k \sim \frac{1}{\ell_o}$
Particles
Resonance decay to non−flow

How to measure such short range correlations?

$$C(\eta_1, \eta_2) = \frac{\left\langle \frac{dN}{d\eta_1} \frac{dN}{d\eta_2} \right\rangle}{\left\langle \frac{dN}{d\eta_1} \right\rangle \left\langle \frac{dN}{d\eta_2} \right\rangle}$$

Correlation function



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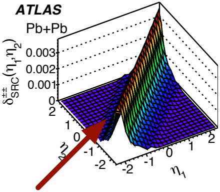
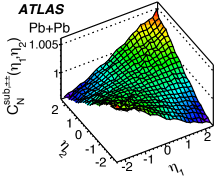
Short Range = "Non-flow"



+

Long range rapidity flucts



Find the CP in here
at lower energy

(perhaps with machine learning
Steinheimer et al 1906.06562)

What's in the "non-flow" correlations? How is it correlated with flow?
Try to probe $\ell_{\rm kz}$. Study pairs with $\Delta p \sim 100\,\mathrm{MeV} \sim k_{\rm kz} \sim \frac{1}{2\,\mathrm{fm}}$

Thank you!