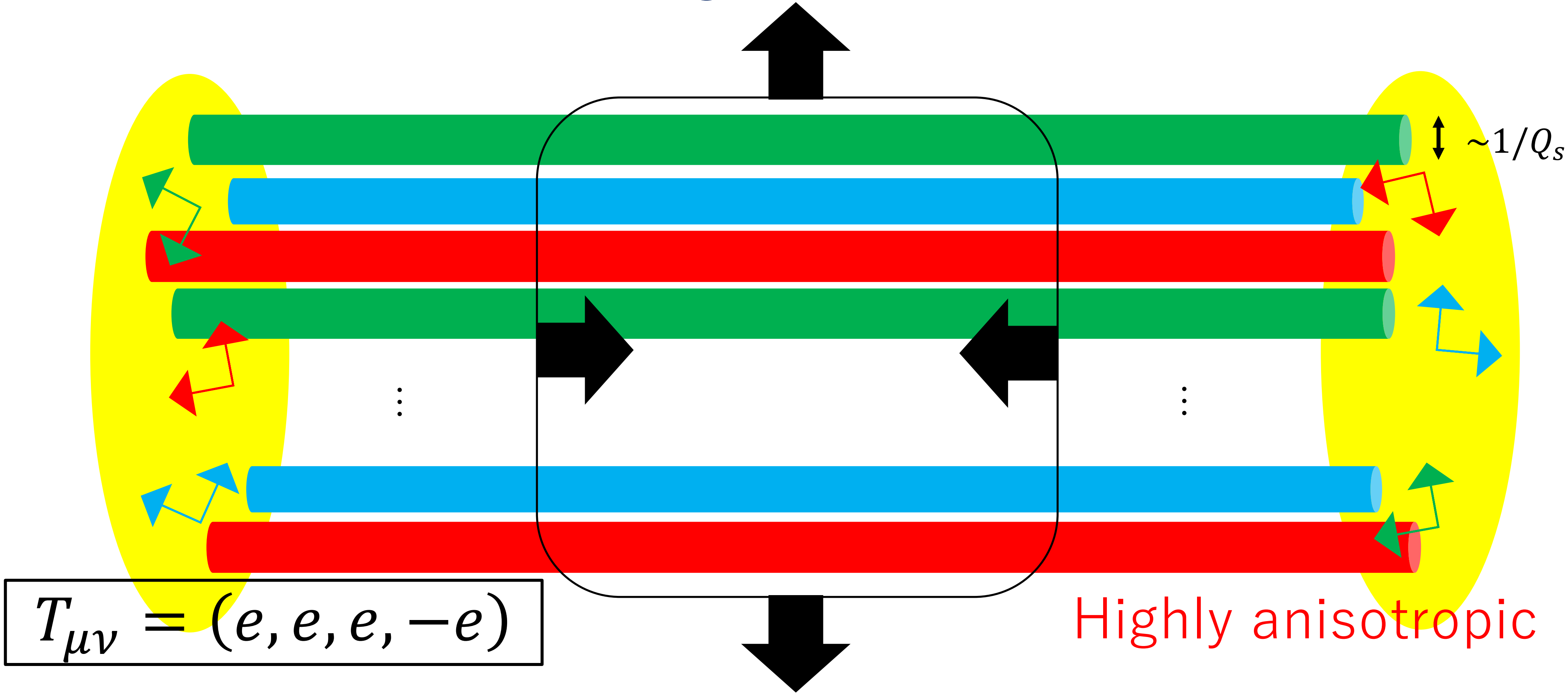
Approach to thermalization and hydrodynamics

Yukinao Akamatsu (Osaka University)

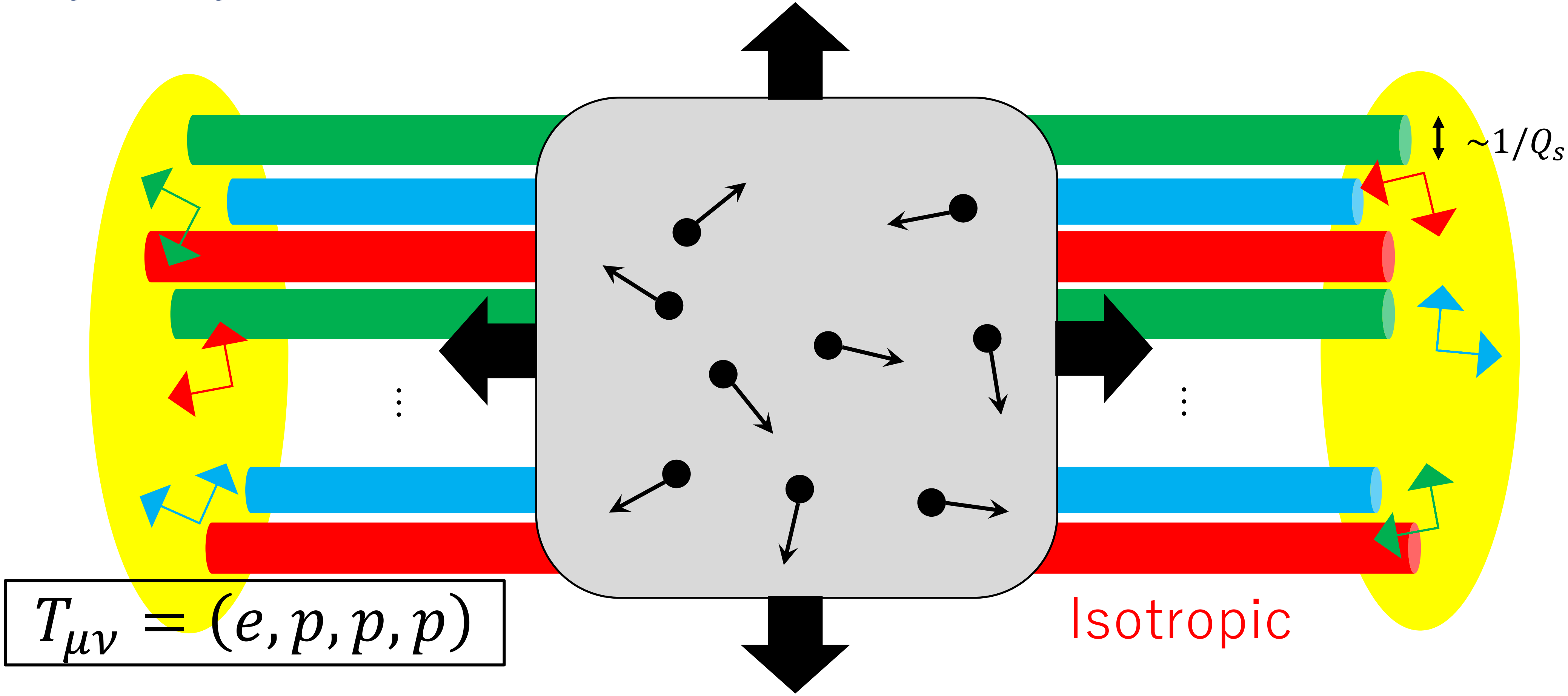
What is the problem?

- CGC initial condition: Longitudinal color E & B fields   Lappi-McLerran 06



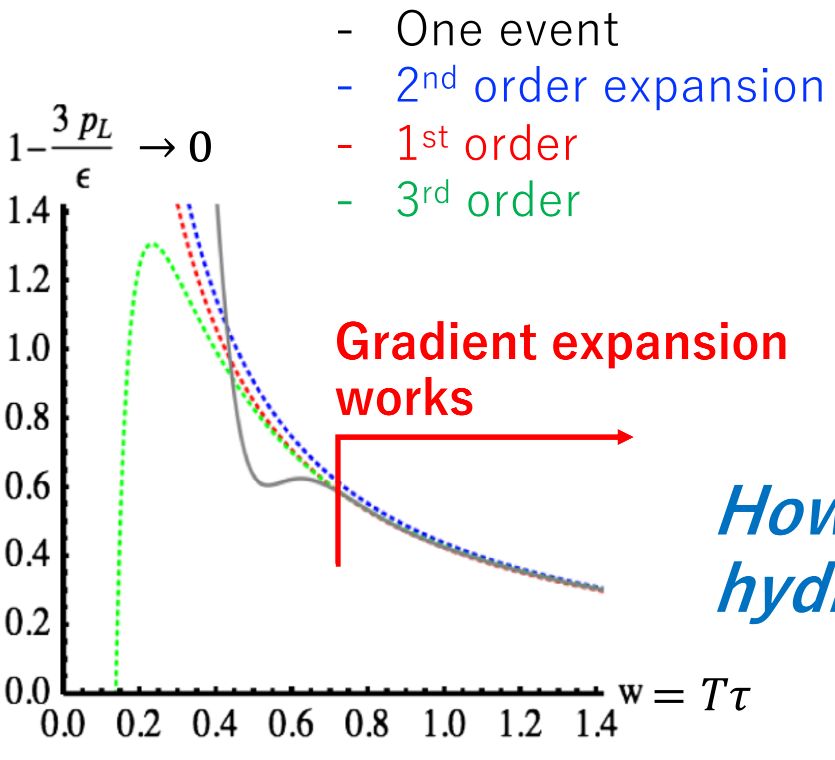
What is the problem?

- Hydrodynamic initial condition at ~1fm/c



Isotropization in Bjorken expansion

- Holographic approach



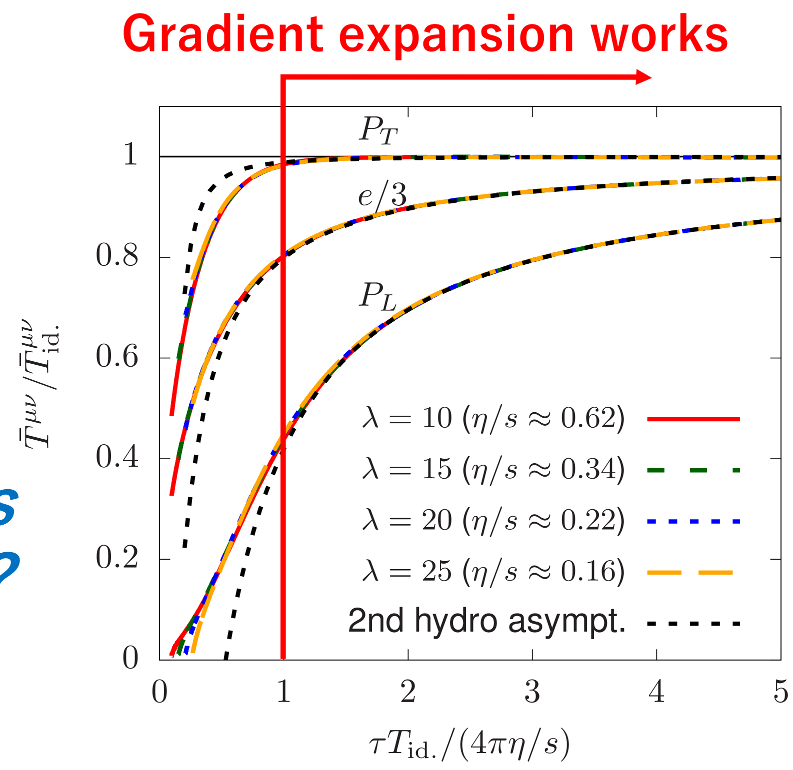
- QCD effective kinetic theory

$$\frac{p_L}{p_T} \approx 0.4$$

$$\frac{\eta}{sT\tau} \approx 0.1$$

very anisotropic!

How far from equilibrium is hydrodynamics applicable?



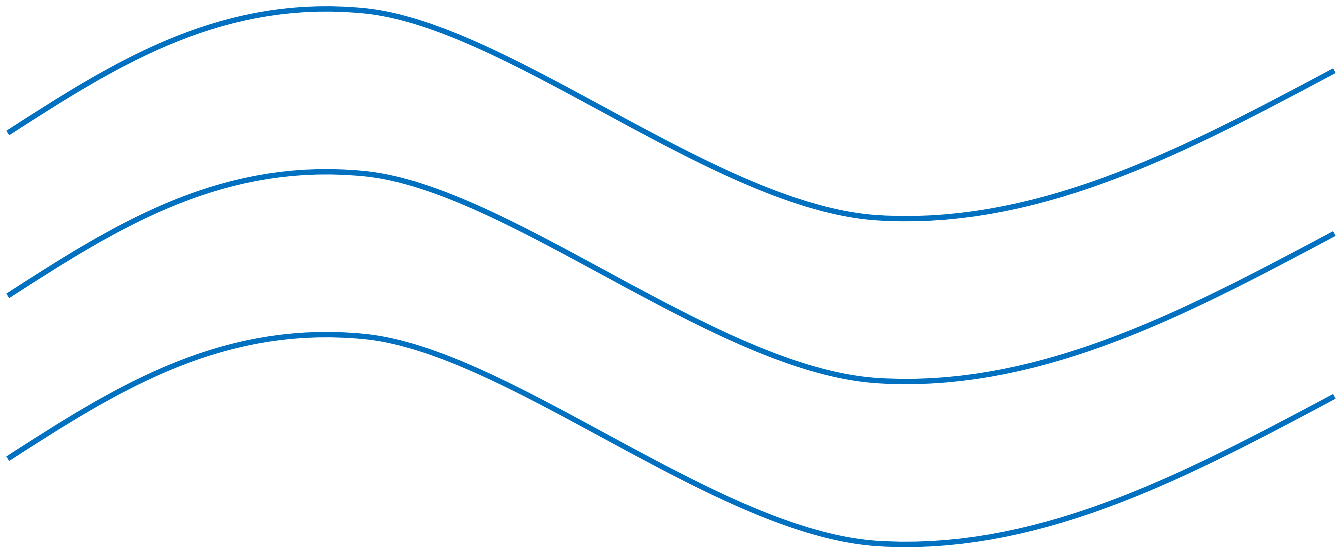
Heller-Janik-Witaszczyk 12

Kurkela-Mazeliauskas-Paquet-Schlichting-Teaney 19

Modern view of hydro applicability

- Hydrodynamics is a low energy effective theory near equilibrium

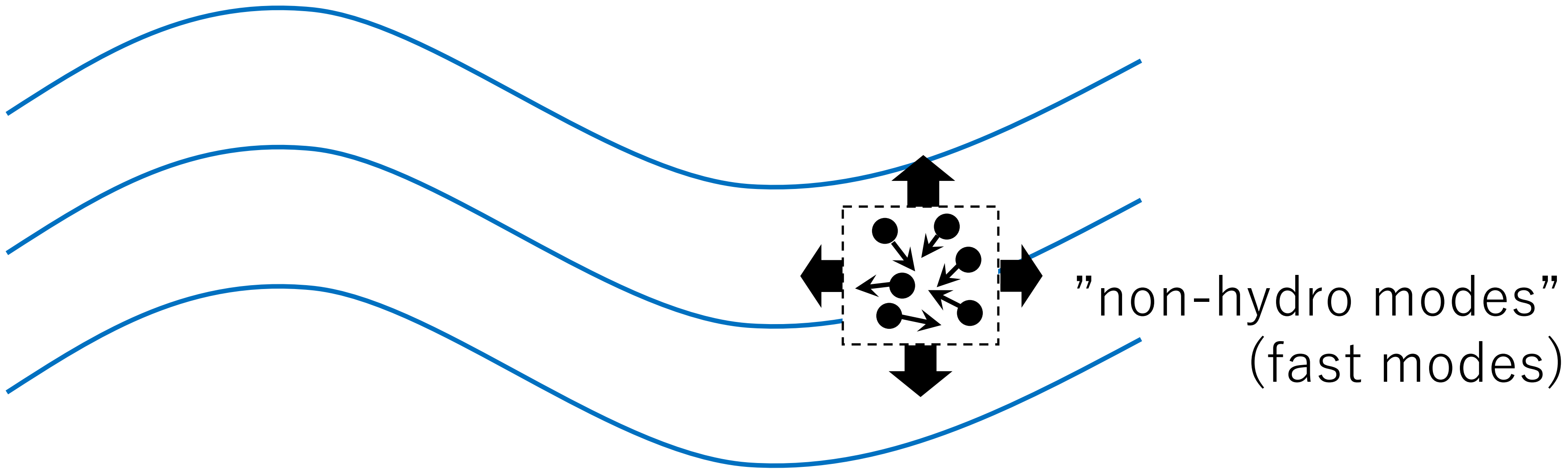
Hydro modes (conserved densities, NG modes, gauge fields)



Based on gradient expansion $\frac{L_{micro}}{L_{macro}} \ll 1$ and symmetry

Modern view of hydro applicability

- Why gradient expansion?

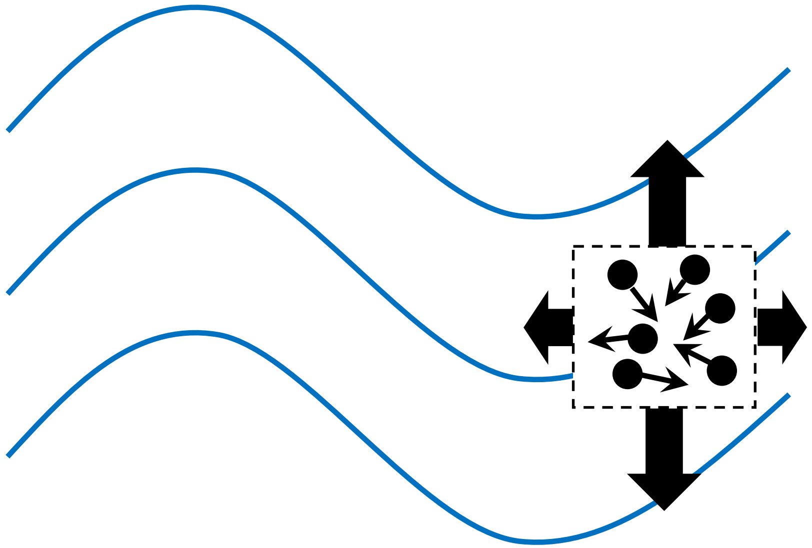


"Non-hydro modes" are **quickly** adjusted to the surrounding macroscopic condition if its variation is **small** → $\delta T_{ij} \sim \eta \partial_i u_j + \cdots$

Modern view of hydro applicability

- What if the gradient is large?

Hydro modes (conserved densities, NG modes, gauge fields)



”non-hydro modes”
(fast modes)

Once the ”non-hydro modes” are adjusted to the **large** gradient, they are **not dynamical anymore** → $\delta T_{ij} \sim \delta T_{ij}(\partial u)$

Modern view of hydro applicability

- Hydro-like description can be extended further when
  1. Non-hydro modes are ineffective
  2. Their non-perturbative response to large gradient is known

Romatschke 18

I will talk about

1. Hydrodynamic Attractor – non-equilibrium frontier
2. Hydrodynamic Fluctuations – small size frontier

What I will not talk about:

- Initial color fields
- Decoherence of initial color fields to particles
- Plasma instabilities
- Transverse expansions

1. Hydrodynamic Attractor

Examples of hydrodynamic attractor

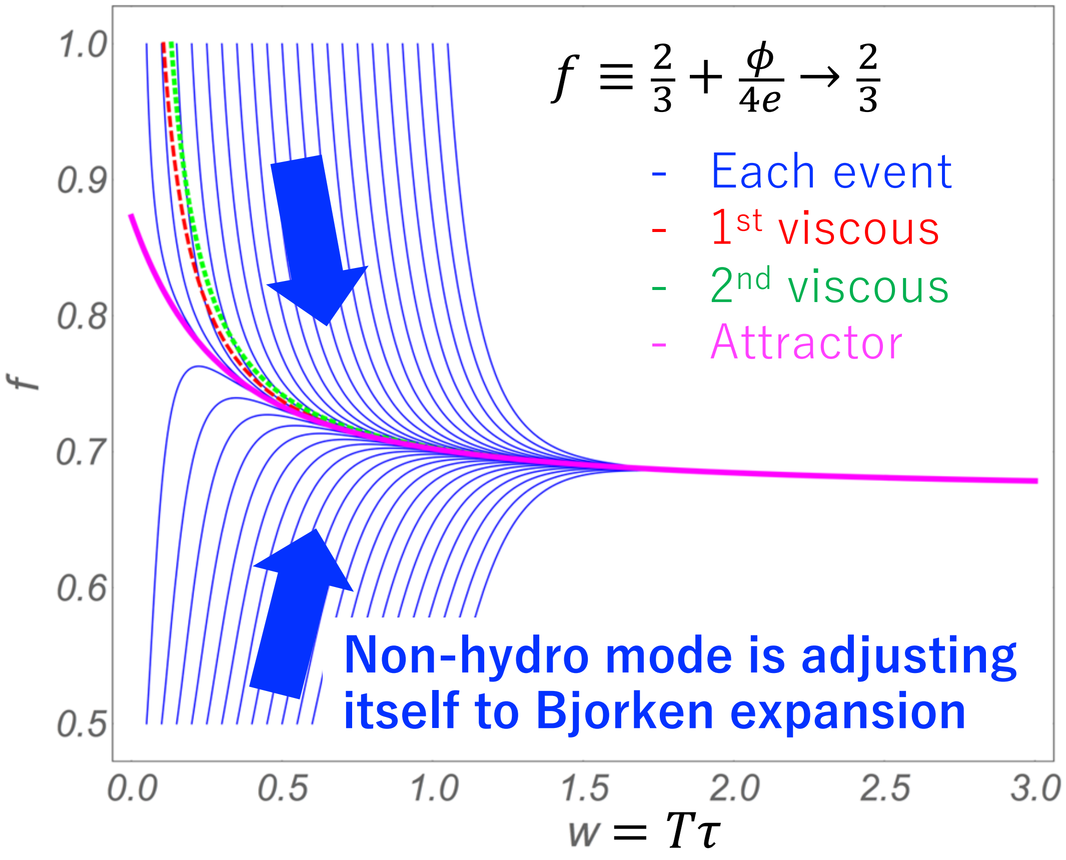
- Conformal causal hydrodynamics in Bjorken expansion
  1. Hydro mode = energy density

$$-\frac{de}{d\tau} = \frac{e+p(e)-\phi}{\tau}$$

  2. Non-hydro mode = shear mode ϕ
     - **Relaxation** vs. **expansion** + nonlinear

$$-\tau_\pi \frac{d\phi}{d\tau} = \phi - \frac{4\eta}{3\tau} + \frac{4\tau_\pi \phi}{3\tau} + \frac{\lambda_1 \phi^2}{2\eta^2}$$

Heller-Spalinski 15



Attractor characterizes the solutions even beyond gradient expansion

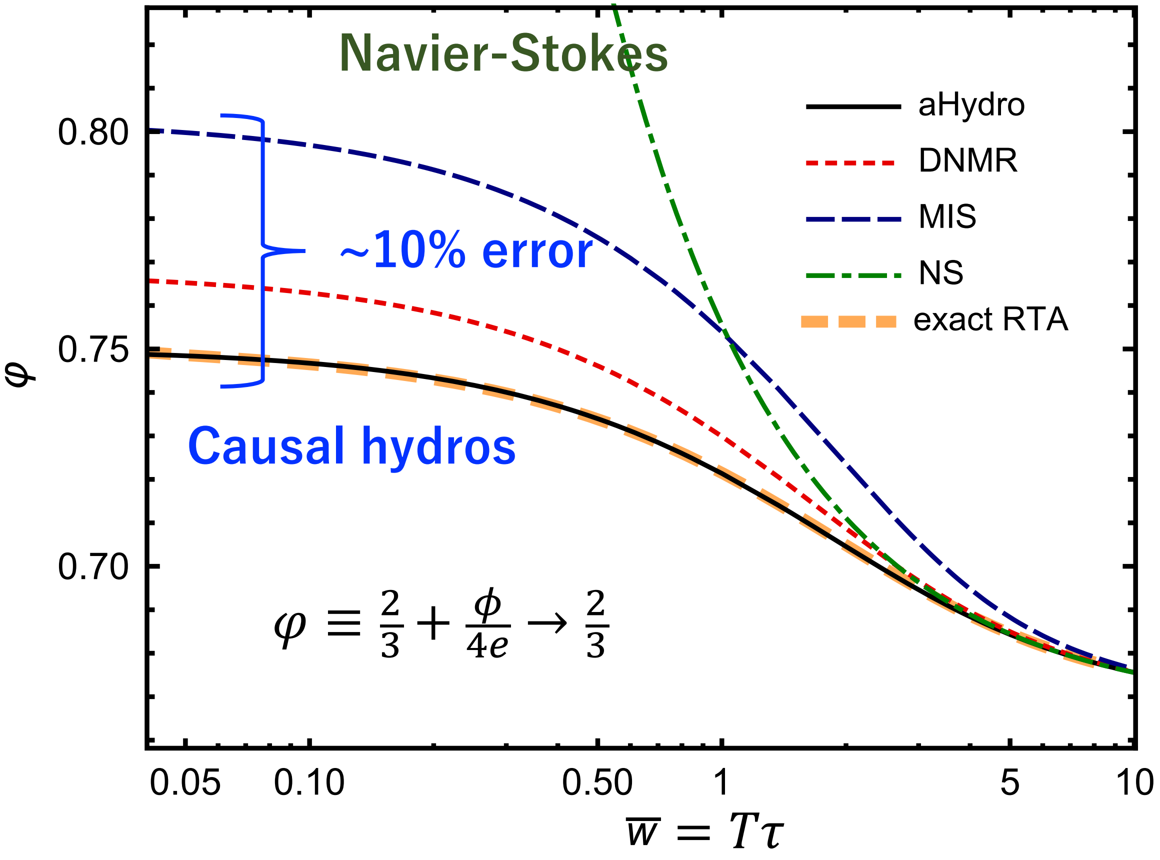
Examples of hydrodynamic attractor

- RTA kinetic theory and its approximations
  (RTA = relaxation time approximation)

  1. Non-hydro modes
     - Kinetic theory contains ∞ of them
     - Causal hydros have only one

  2. Causal hydros work ~10% errors

Strickland-Noronha-Denicol 18



Causal hydro = a far-from-eq. hydro!

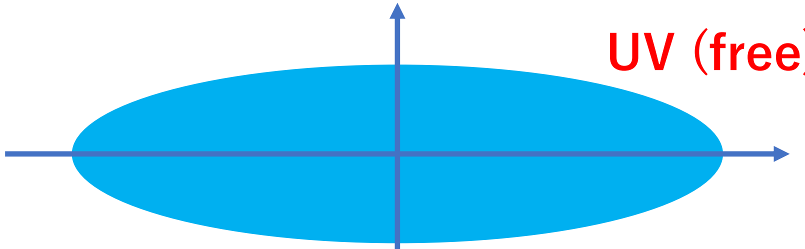
Why does causal hydro work well already from early times?

Why causal hydro works from early times

- Asymptotic behaviors of RTA kinetic theory

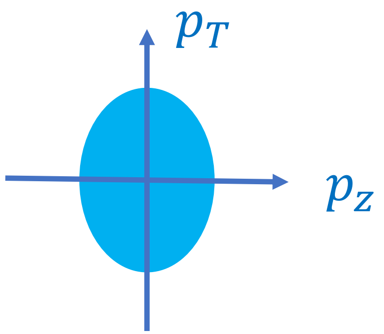
Blaizot-Yan 18, 19
See also Kurkela-Wiedemann-Wu 19

1. Free streaming asymptotics

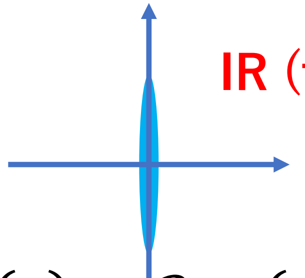


$$e(\tau) = p_L(\tau) \propto \tau^{-2}$$
$$\because e + p_L = 2e$$

$0^+ \leftarrow \tau$

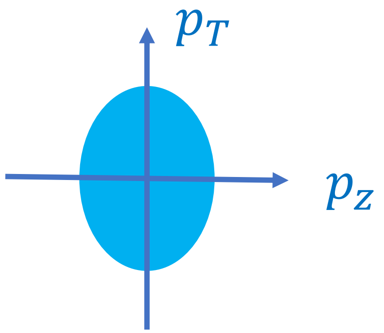


$\tau \to \infty$

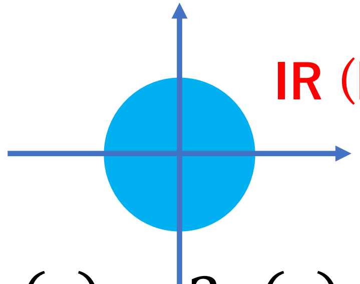


$$e(\tau) = 2p_T(\tau) \propto \tau^{-1}$$
$$\because e + p_L = e$$

2. Hydrodynamic asymptotics



$\tau \to \infty$



$$e(\tau) = 3p(\tau) \propto \tau^{-4/3}$$
$$\because e + p_L = 4e/3$$

Exponents of physical quantities characterize the asymptotics

Why causal hydro works from early times

- Fixed point analysis

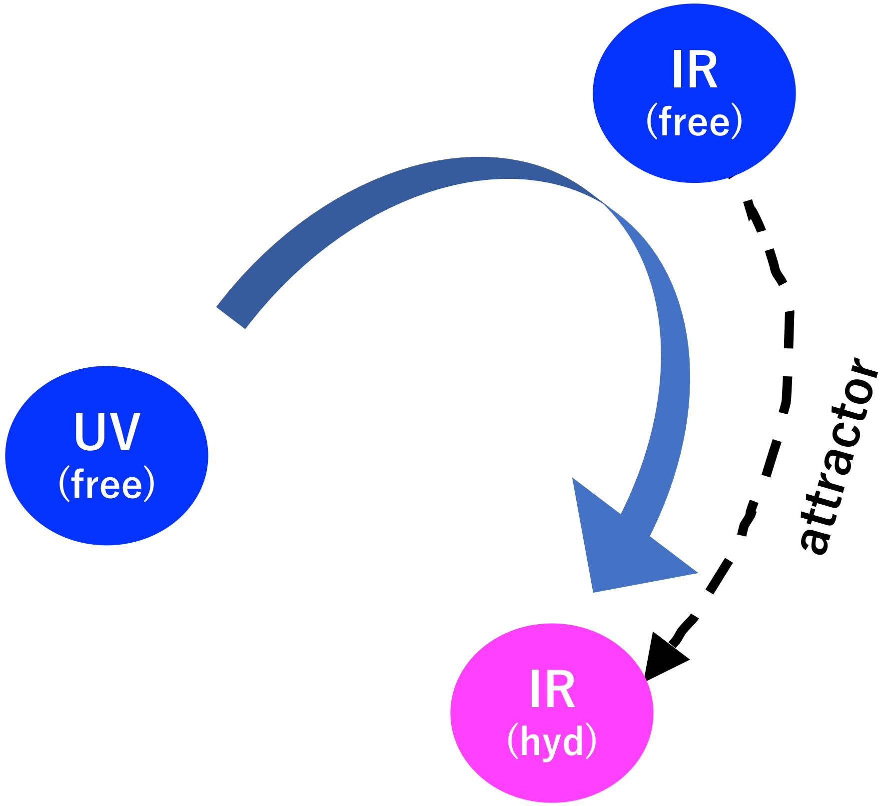
Blaizot-Yan 18, 19

1. Logarithmic growth rates of energy density and anisotropy

$$g_0(\tau) \equiv \frac{d \log e}{d \log \tau}, \qquad g_1(\tau) \equiv \frac{d \log(p_L - p_T)}{d \log \tau}$$

(g_0, g_1) at fixed points	RTA kinetic theory	Causal hydro (DNMR) (2-moment truncation)
UV (free)	(-2,-2)	**(-2.21, -2.21)**
IR (free)	(-1,-1)	**(-0.93, -0.93)**
IR (hyd)	(-4/3, -2)	(-4/3, -2)

Close! ⇔



Causal hydro captures global features of the RTA kinetic theory solutions, even at early times

Why causal hydro works from early times

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Blaizot-Yan 18, 19

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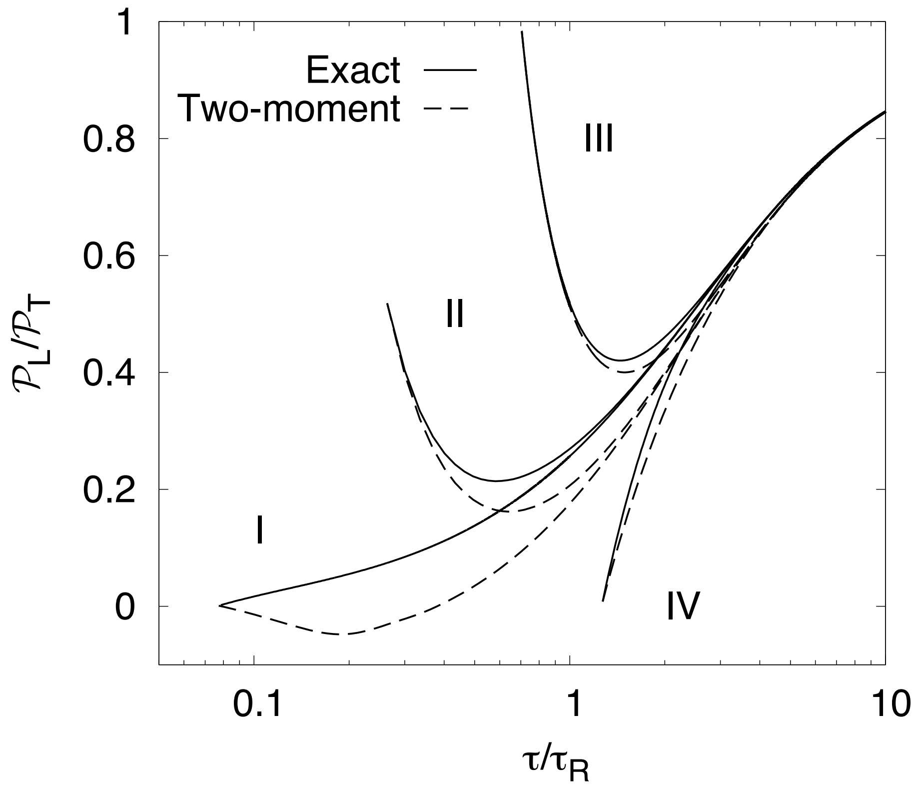


Causal hydro captures global features of the RTA kinetic theory solutions, even at early times

Lessons so far

1. Out-of-equilibrium behavior is characterized by hydrodynamic attractor even beyond the gradient expansion

2. Hydrodynamic attractor of (RTA) kinetic theory is approximated well by causal hydro, which only has single non-hydro mode

3. This unexpected success of causal hydro is because it shares the same fixed points with the (RTA) kinetic theory

Recent works on far-from-equilibrium hydro

- Attractors and far-from-equilibrium hydro
  - Lublinsky-Shuryak 07!
  - Heller-Janik-Witaszczyk 12, 13, Heller-Spalinski 15, Romatschke 17, 18, Behtash-CruzCamacho-Martinez 18, Heller-Kurkela-Spalinski-Svensson 18, Strickland-Noronha-Denicol 18, Strickland 18, Behtash-CruzCamacho-Kamata-Martinez 19, Behtash-Kamata-Martinez-Shi 19, **Denicol**-**Noronha** 19 (Wed 8:40-, 15:00-), **Jaiswal**-Chattopadhyay-Jaiswal-Pal-Heinz 19 (poster NT7), **Du** (poster NT4)

- New ideas and applications of attractors
  - Fixed points: Blaizot-Yan 18, 19, Kurkela-Wiedemann-Wu 19
  - Pre-scaling: **Mazeliauskas**-Berges 19 (Wed 14:40-)
  - Adiabatic hydrodynamics: **Brewer**-Yan-Yin 19 (Wed 14:20-)
  - Phenomenology: Giacalone-Mazeliauskas-Schlichting 19, Kurkela-Mazeliasukas 19

New ideas on far-from-equilibrium hydro

- Pre-scaling in overpopulated anisotropic plasma

Mazeliauskas-Berges 19
(Wed 14:40-)

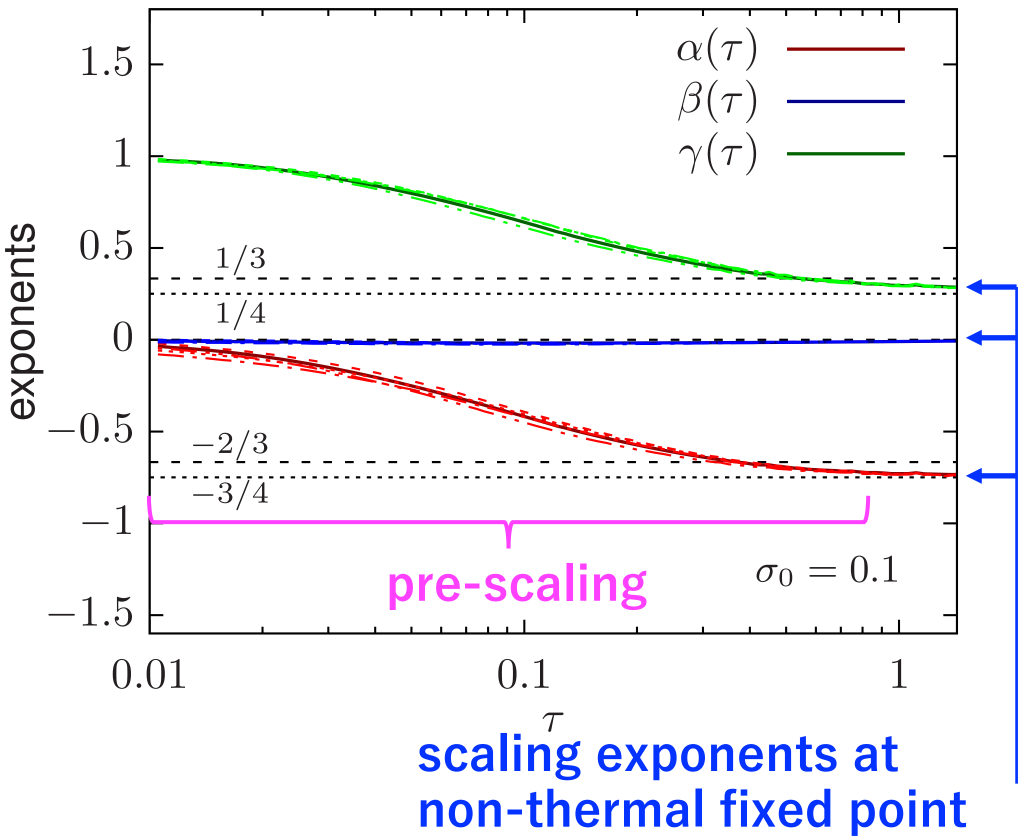
1. QCD effective kinetic theory

$$C^{1\leftrightarrow 2}[f] + C^{2\leftrightarrow 2}[f]$$

2. Non-thermal fixed point
   - The 1st stage of bottom up thermalization

3. Scaling behavior established earlier

$$f(p_T, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_T, \tau^{\gamma(\tau)} p_z)$$



Pre-scaling away from non-thermal fixed point suggests an attractor behavior

New ideas on far-from-equilibrium hydro

- Adiabatic hydrodynamics
  1. Trace the slowest configuration
     ~Quantum mechanics with energy gap
  2. RTA kinetic theory
     - **Free streaming** vs. **relaxation**

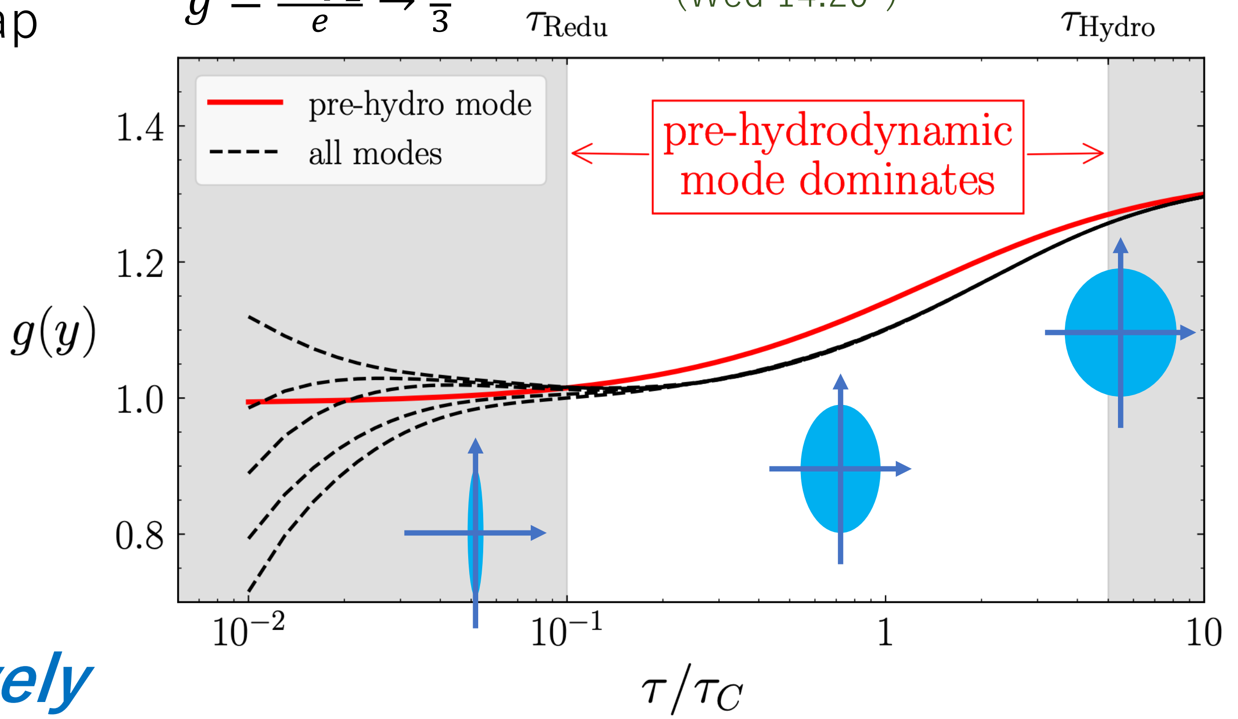
$$\frac{d}{dt}f = -[\mathcal{H}_F + \lambda(t)\mathcal{H}_R]f$$

$f(t) \approx f_{0;\lambda(t)}$ slowest configuration

Instantaneous ground state effectively selects the attractor solution

Brewer-Yan-Yin 19
(Wed 14:20-)

$g = \frac{e+p_L}{e} \to \frac{4}{3}$

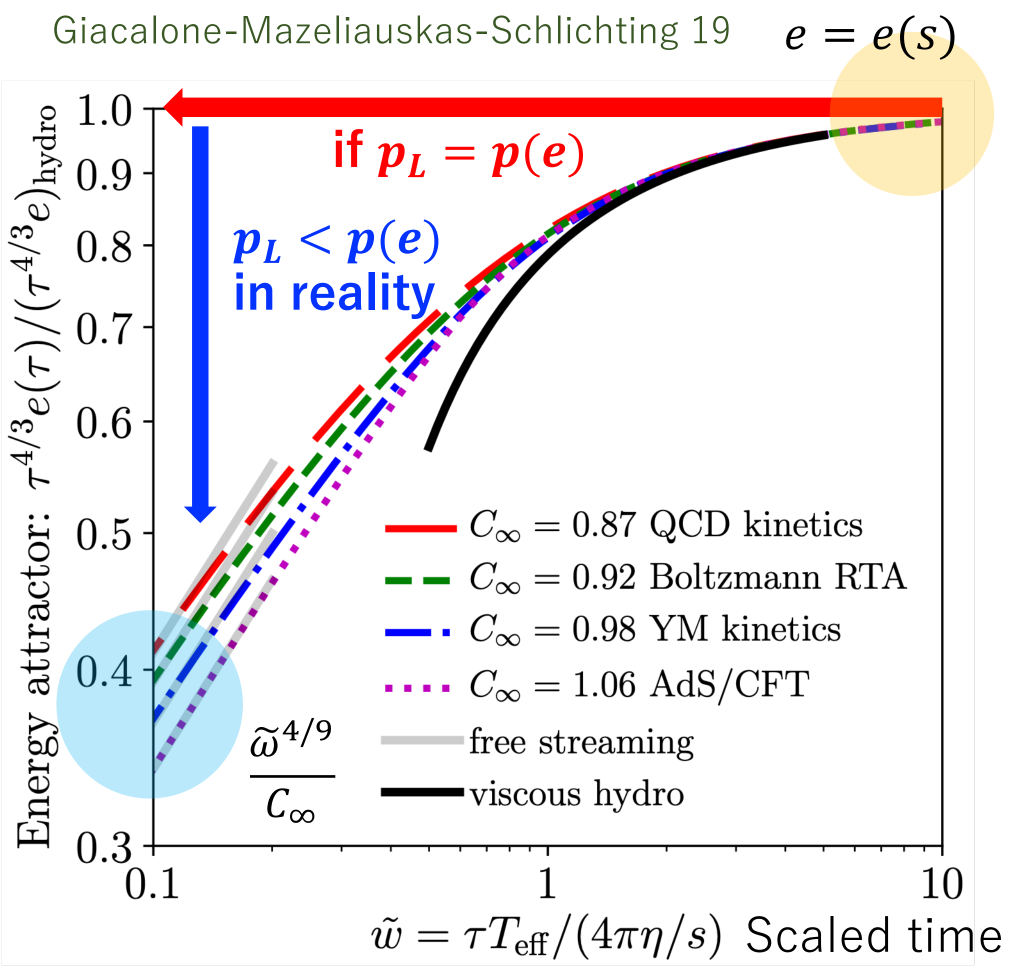


Application of hydrodynamic attractor

- Energy attractor
  1. Insensitive to models by scaling with equilibrium relaxation time
     $$\tilde{\omega} = \frac{\tau}{(4\pi\eta/sT_{\rm eff})} = \frac{\tau}{\tau_R(\tau)}$$
  2. Relates initial energy density to late-time energy & entropy densities
     $$(e\tau^{4/3})_{\rm ini} \sim (e\tau^{4/3})_{\rm hydro}\tilde{\omega}^{4/9}$$
     $$(s\tau)_{\rm hydro} \sim \left(\frac{4\pi\eta}{s}\right)^{1/3} \cdot \nu_{\rm eff}^{1/3} \cdot (e\tau)_{\rm ini}^{2/3}$$

Dominant entropy production is estimated by the attractor

Giacalone-Mazeliauskas-Schlichting 19



$T_{\rm eff} \propto e(\tau)^{1/4}$

Application of hydrodynamic attractor

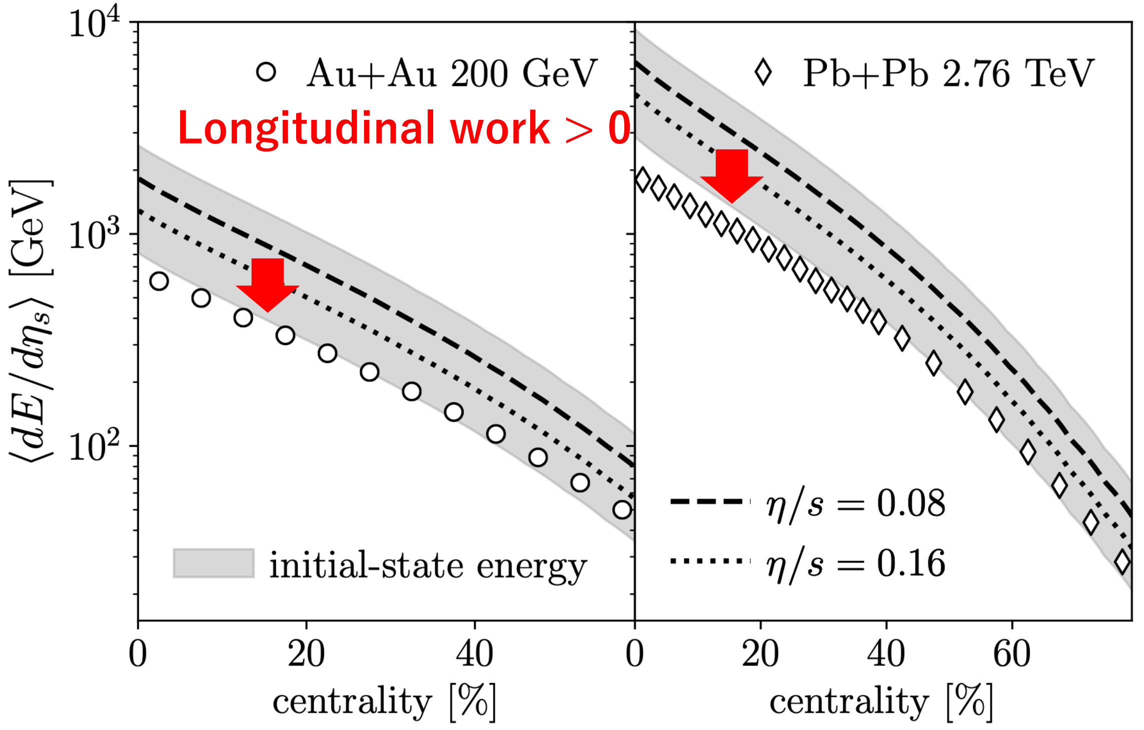
- How much has (pre-)QGP worked?
  1. Using entropy-multiplicity relation

$$(e\tau)_{\mathrm{ini}} \sim 30 \cdot \left(\frac{4\pi\eta}{s}\right)^{-1/2} \cdot \nu_{\mathrm{eff}}^{-1/2} \cdot \left(\frac{dN_{\mathrm{ch}}}{A_T d\eta}\right)^{3/2}$$

  2. Longitudinal work estimated
     - Observed energy per rapidity
     - Multiplicity → initial energy density

Viscosity can be constrained by using independent data

Giacalone-Mazeliauskas-Schlichting 19



Application of hydrodynamic attractor

- More formulae
  1. Hydrodynamization time $\tilde{\omega}_{\rm hydro} = \frac{\tau_{\rm hydro}}{\tau_R(\tau_{\rm hydro})} = 1$ Schlichting-Teaney 19

$$\frac{\tau_{\rm hydro}}{R} \sim \left(\frac{dN_{\rm ch}/dy}{63}\right)^{-1/2} \cdot \underbrace{\left(\frac{\eta/s}{2/4\pi}\right)^{3/2}}_{\sim 1} \cdot \underbrace{\left(\frac{\nu_{\rm eff}}{40}\right)^{1/2}}_{\sim 1}$$

< 1: System thermalizes for $dN_{\rm ch}/dy > 63$

  2. Chemical equilibration time $\tilde{\omega}_{\rm chem} = \frac{\tau_{\rm chem}}{\tau_R(\tau_{\rm chem})} = 1.2$ Kurkela-Mazeliauskas 19

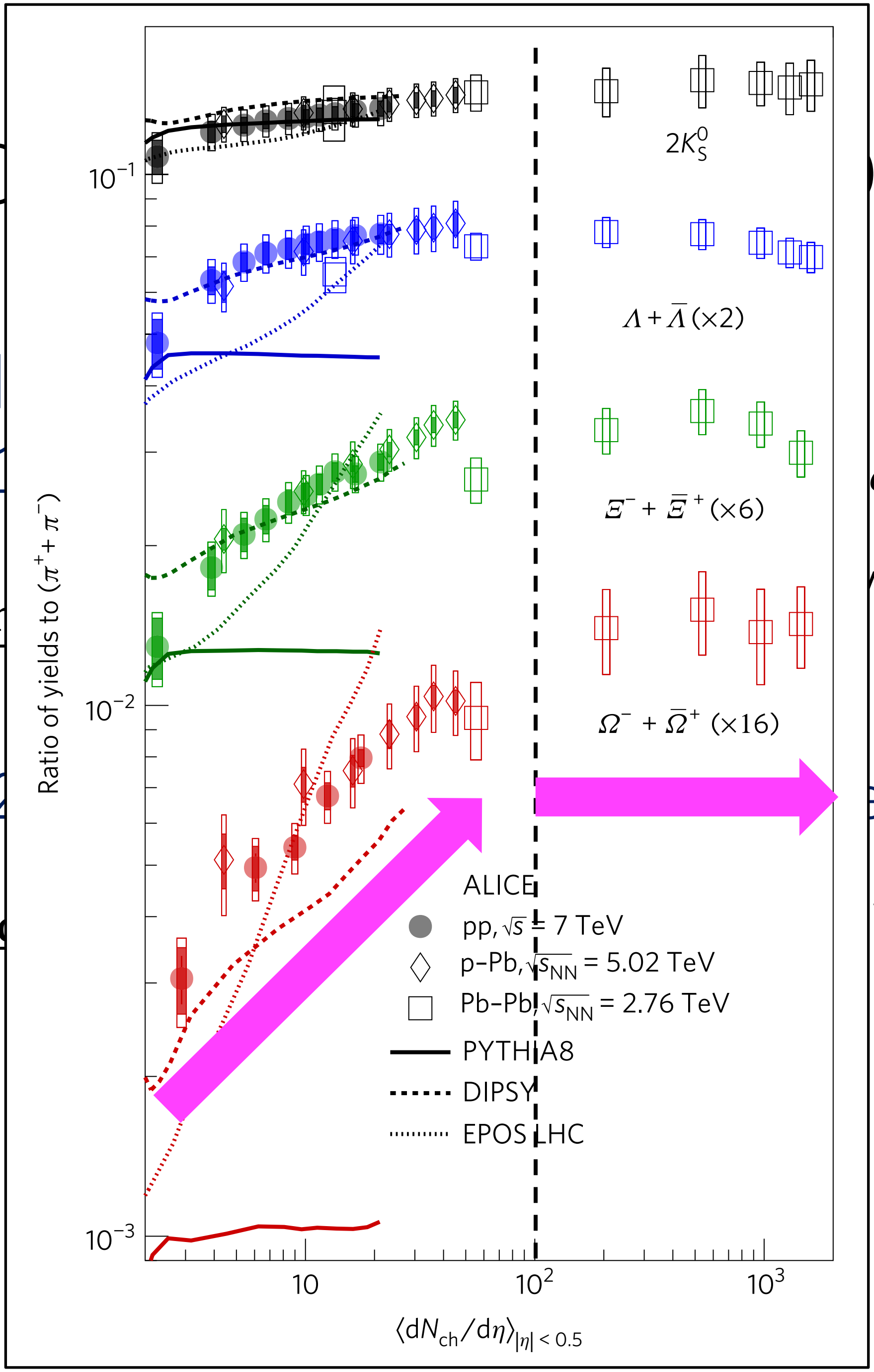
$$\frac{\tau_{\rm chem}}{R} \sim \left(\frac{dN_{\rm ch}/dy}{110}\right)^{-1/2} \cdot \underbrace{\left(\frac{\eta/s}{2/4\pi}\right)^{3/2}}_{\sim 1} \cdot \underbrace{\left(\frac{\nu_{\rm eff}}{40}\right)^{1/2}}_{\sim 1}$$

<1: Chemically equilibrated for $dN_{\rm ch}/dy > 110$

*Strangeness saturates at $dN_{\rm ch}/dy \sim 100$ (ALICE)

Minimum size for formation of equilibrated system under expansion

Ap… dynamic attractor



- M…

1…

$\tilde{\omega}_{\text{hydro}} = \dfrac{\tau_{\text{hydro}}}{\tau_R(\tau_{\text{hydro}})} = 1$ Schlichting-Teaney 19

τ_h … $^{2} \cdot \left(\dfrac{\nu_{\text{eff}}}{40}\right)^{1/2}$ **< 1: System thermalizes for $dN_{ch}/dy > 63$**

~ 1

2… e $\tilde{\omega}_{\text{chem}} = \dfrac{\tau_{\text{chem}}}{\tau_R(\tau_{\text{chem}})} = 1.2$ Kurkela-Mazeliauskas 19

τ_c … $^{2} \cdot \left(\dfrac{\nu_{\text{eff}}}{40}\right)^{1/2}$ **<1: Chemically equilibrated for $dN_{ch}/dy > 110$**

~ 1

*Strangeness saturates at $dN_{ch}/dy \sim 100$ (ALICE)

Minimum size for formation of equilibrated system under expansion

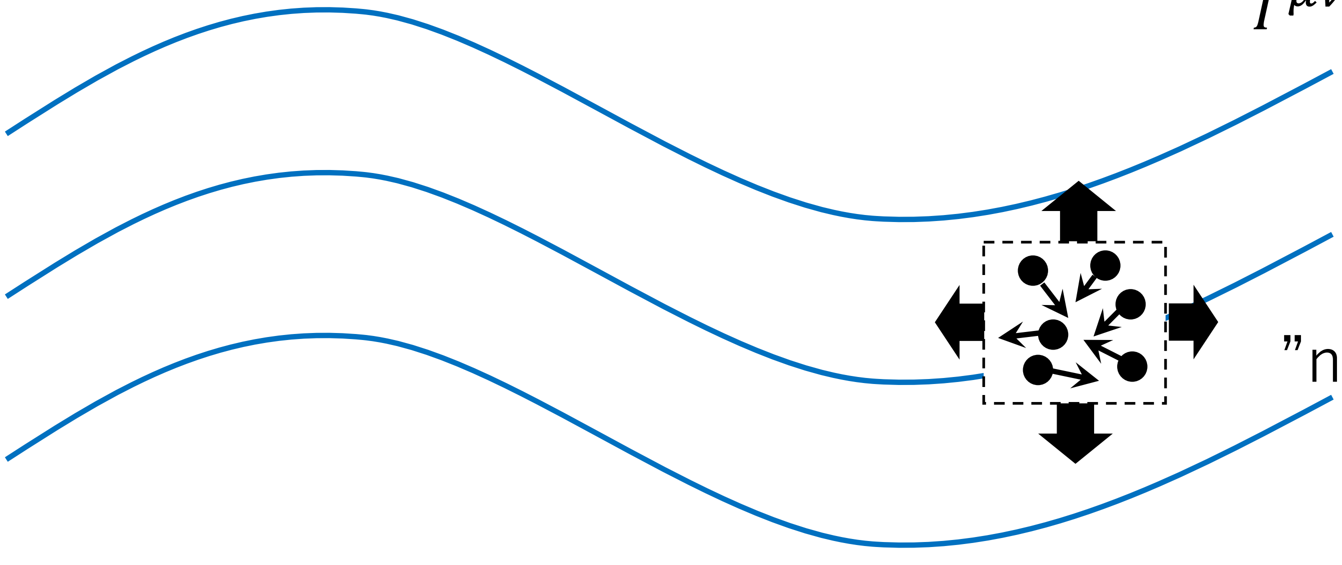
2. Hydrodynamic Fluctuations

Fate of non-hydro modes

- After enough time has passed

Landau-Lifshitz
Kapusta-Mueller-Stephanov 12

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{id}} + T^{\mu\nu}_{\mathrm{vis}} + S^{\mu\nu}_{\mathrm{noise}}$$



”non-hydro modes”
(fast modes)

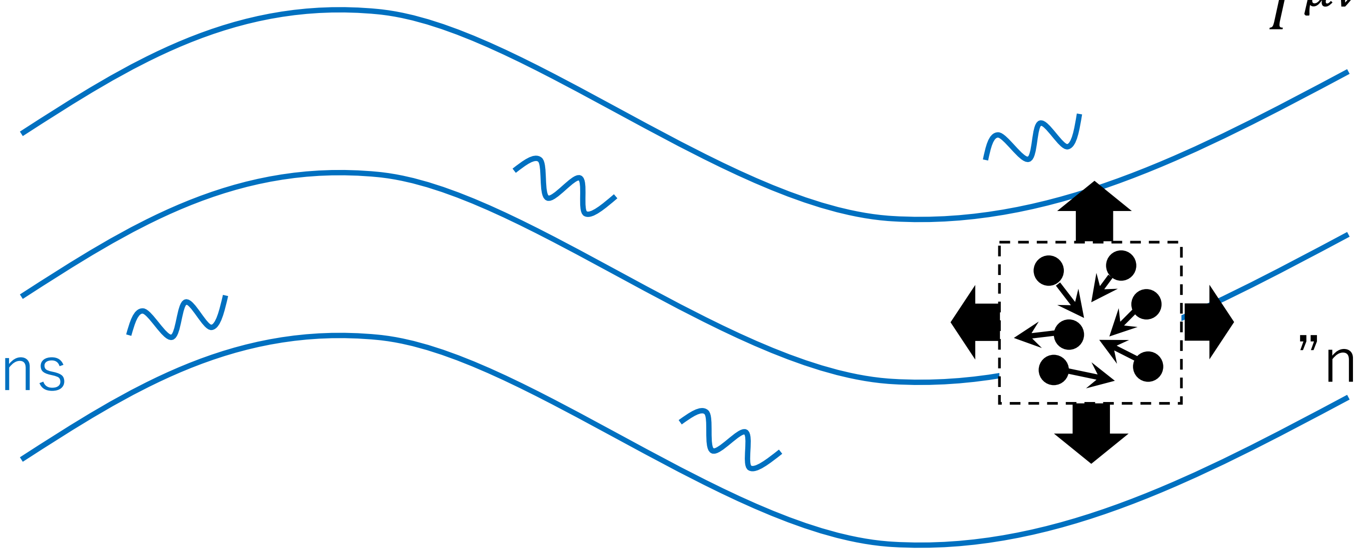
“Non-hydro modes” are almost equilibrated
= **linear response + noise** (fluctuation-dissipation theorem)

Fate of non-hydro modes

- Hydrodynamic fluctuations are excited

Landau-Lifshitz
Kapusta-Mueller-Stephanov 12

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{id}} + T^{\mu\nu}_{\mathrm{vis}} + S^{\mu\nu}_{\mathrm{noise}}$$



hydro fluctuations
(particle-like)

"non-hydro modes"
(fast modes)

Kinetic regime k_* = relaxation and expansion balance
Hydro-kinetic theory = dynamics of **particle-like** modes at k_*

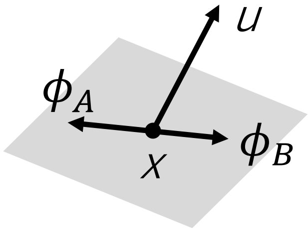
Akamatsu-Mazeliauskas-Teaney 17, 18
See also Martinez-Schafer 18, 19

New development in hydro-kinetic theory

- Hydro fluctuations in general background

An-Basar-Stephanov-Yee 19
(Wed 9:20-, Tue 14:00-)

1. Careful consideration of equal-time and rest frames
    - Confluent correlator / derivatives



2. Phonons as particles

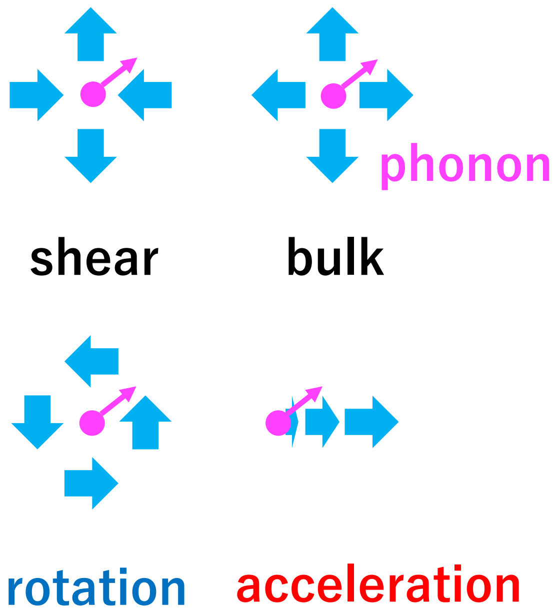
$$\frac{d}{dt}p_a = -E(2v_b\omega_{ba} + a_a) - (\partial_a u_b)p_b - \nabla_a E$$

Coriolis **inertial** **"Hubble"** **potential**

3. No particle interpretation for diffusive modes

$$\omega = \cancel{c_s k} - i\gamma_\eta k^2$$

local flow configurations

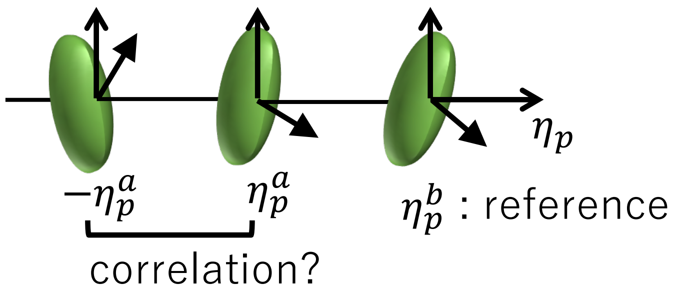


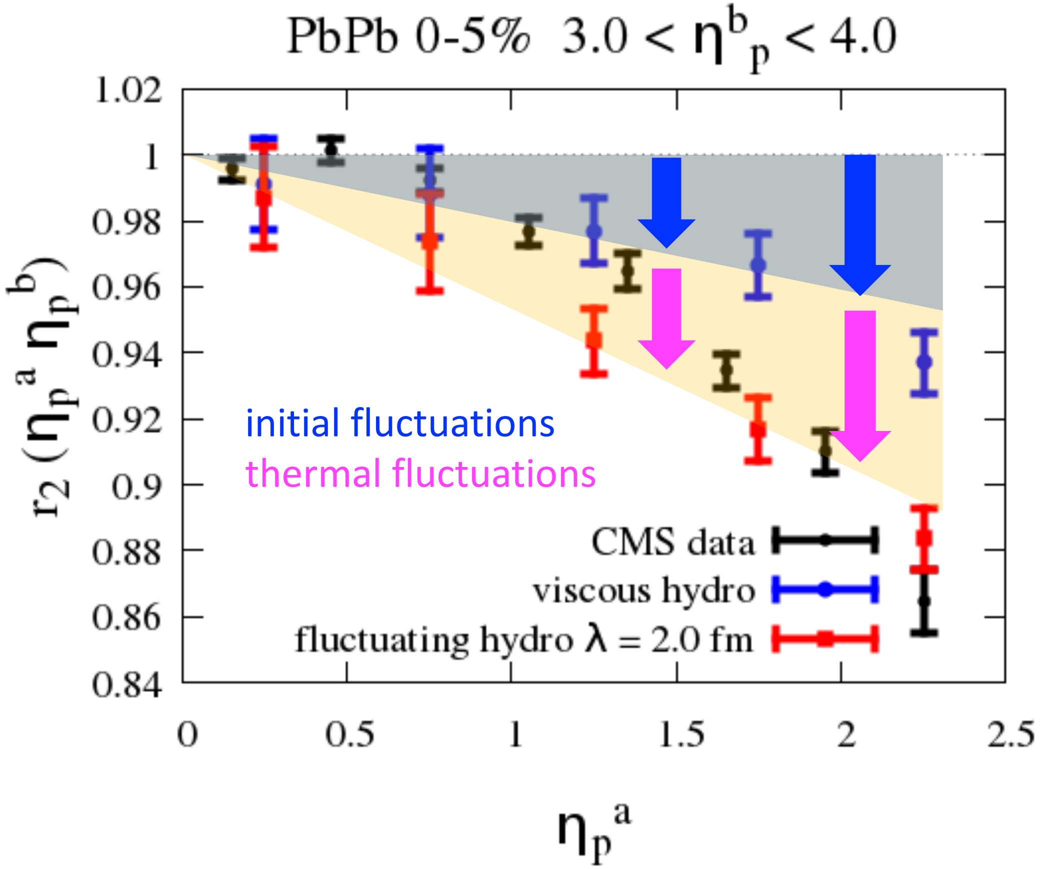
Fluctuating hydro = hydro + phonon gas + ⋯
A new simulation method?

Simulating fluctuating hydrodynamics

Sakai-Murase-Hirano
(Tue 9:40-)

- Rapidity decorrelation
  1. Initial longitudinal fluctuations
     - Decay of hadronic strings using PYTHIA
  2. Thermal fluctuations
     - Hydrodynamics + noise (smeared by λ)
     - **No free parameter** (except for λ)





Thermal fluctuations essential to study initial longitudinal fluctuations

Modified fluctuation-dissipation relation

Murase 19 (Poster CD24)
See also Hirano-Kurita-Murase 19

- Fluctuations in causal hydro
  1. Noise in constitutive relation

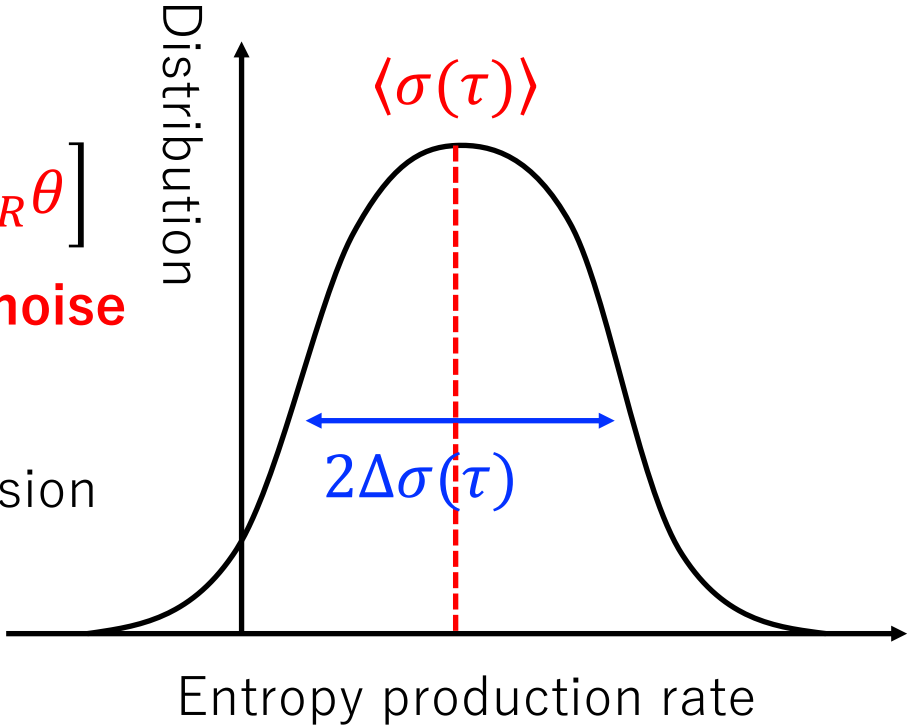
$$(1+\tau_R D)\pi = \pi_{NS} + \xi$$

$$\langle \xi(x)\xi(x') \rangle = T\kappa\delta(x-x')\left[2 + \tau_R D \ln\frac{T\kappa}{\tau_R} - \tau_R\theta\right]$$

"non-instantaneous" noise

  2. Distribution of produced entropy
     - Fluctuation Theorem applied to Bjorken expansion

$$R(\tau) = \frac{2\langle\sigma(\tau)\rangle}{\left(\Delta\sigma(\tau)\right)^2(\tau-\tau_0)} \to 1 \quad (\tau \gg \tau_0)$$

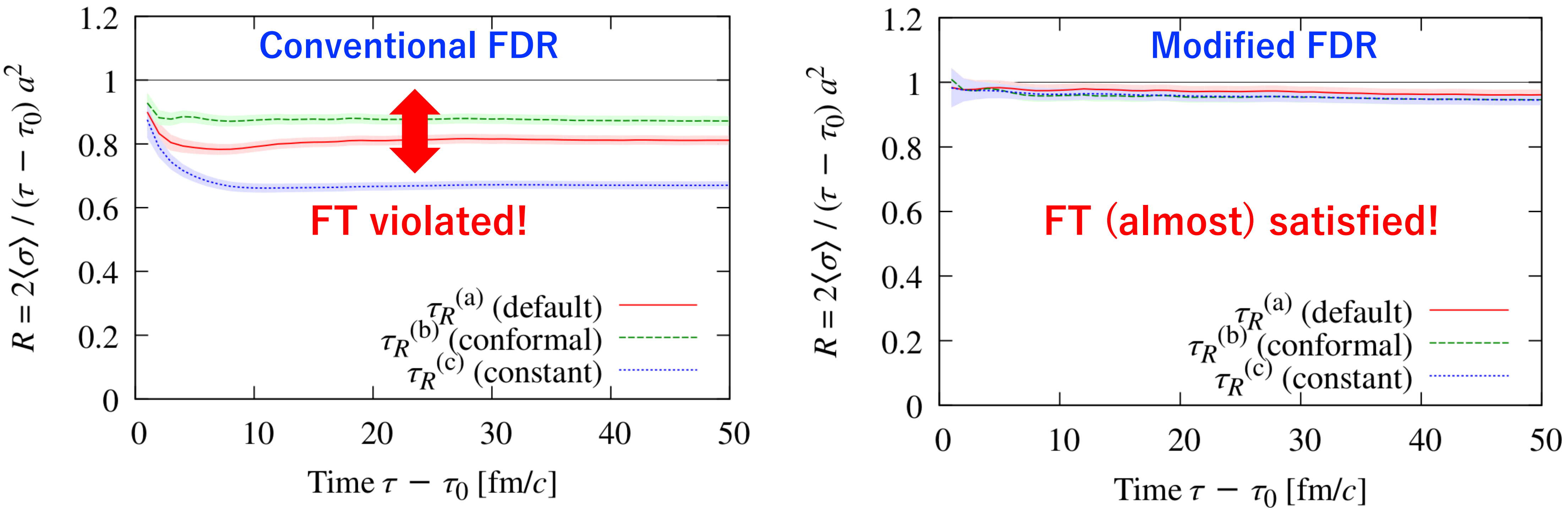


$$\sigma(\tau) = \frac{S(\tau) - S(\tau_0)}{\tau - \tau_0}$$

Modified fluctuation-dissipation relation

Murase 19 (Poster CD24)
See also Hirano-Kurita-Murase 19

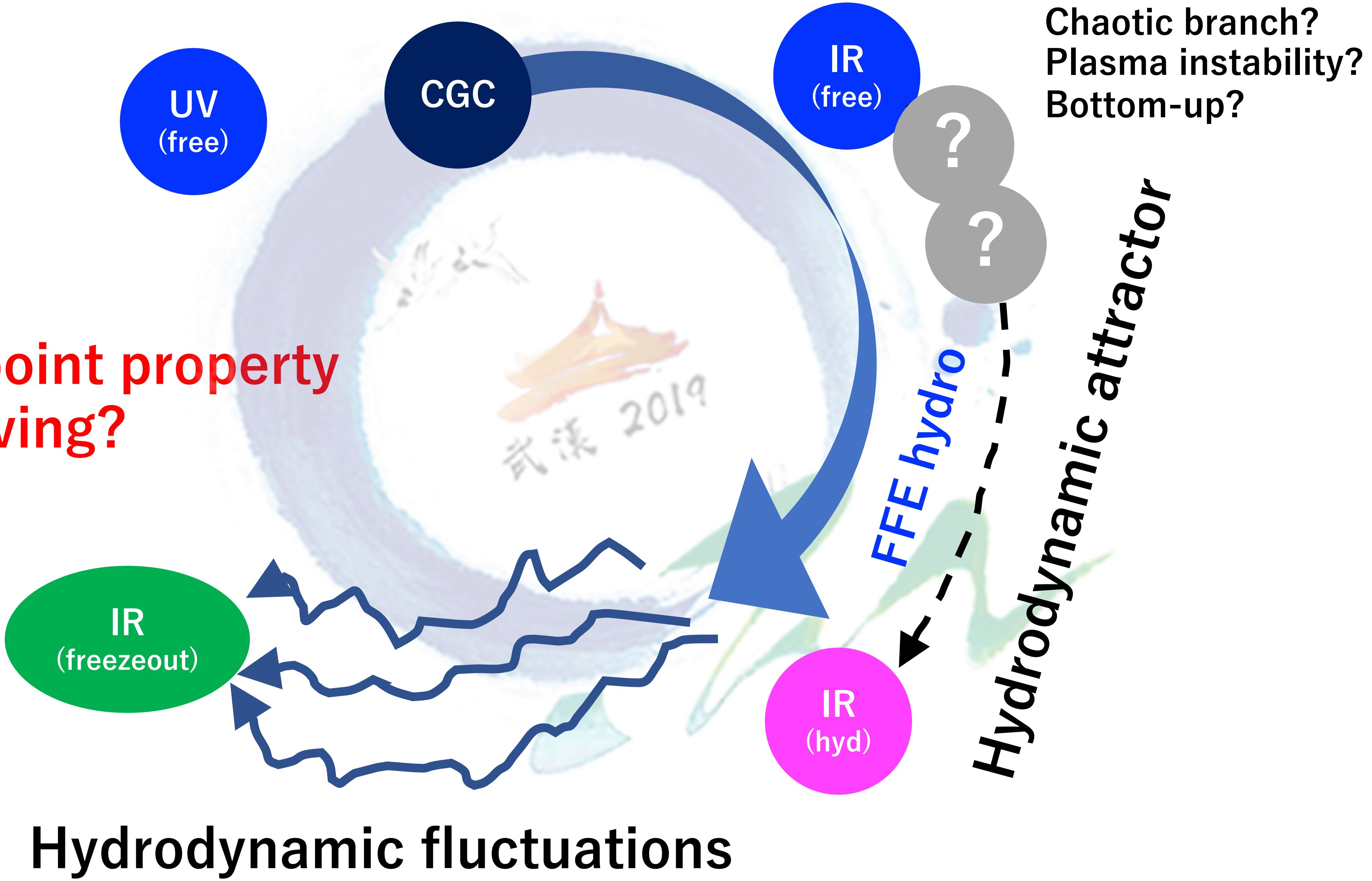
- Fluctuations in causal hydro
  3. Test of Fluctuation Theorem $\boldsymbol{R(\tau) \to 1}$



Modified FDR necessary for correct distribution of entropy production

Summary

Overall picture I have as of 19/11/7



Which fixed point property are we observing?

Acknowledgements

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Michal Heller
Tetsufumi Hirano
Masakiyo Kitazawa
Aleksi Kurkela

Aleksas Mazeliauskas
Koichi Murase
Azumi Sakai
Soeren Schlichting
Chun Shen
Derek Teaney
Li Yan
Yi Yin

+ students in Nuclear Theory group at Osaka University
+ all the authors of the interesting papers in this field

2019 Yagi Award

https://ithems.riken.jp/en/about/yagi-award

"Kohsuke Yagi Quark Matter Award" (Yagi Award) is based on the donation to iTHEMS from bereaved family of late Professor Kohsuke Yagi who was a renowned Japanese nuclear physicist. Responding to the family request, the award aims to support early career scientists with Japanese nationality, to promote and expand country's nuclear physics research field. It will be awarded to junior Japanese physicists under age of 40 who give plenary talk at the "Quark Matter: International Conference on Ultra-relativistic Nucleus-Nucleus Collisions" held in every 1.5 years.

Prof. Kohsuke Yagi (1934-2014)
Quark Matter 1997, Chair

謝謝

Thank You

BACK UP

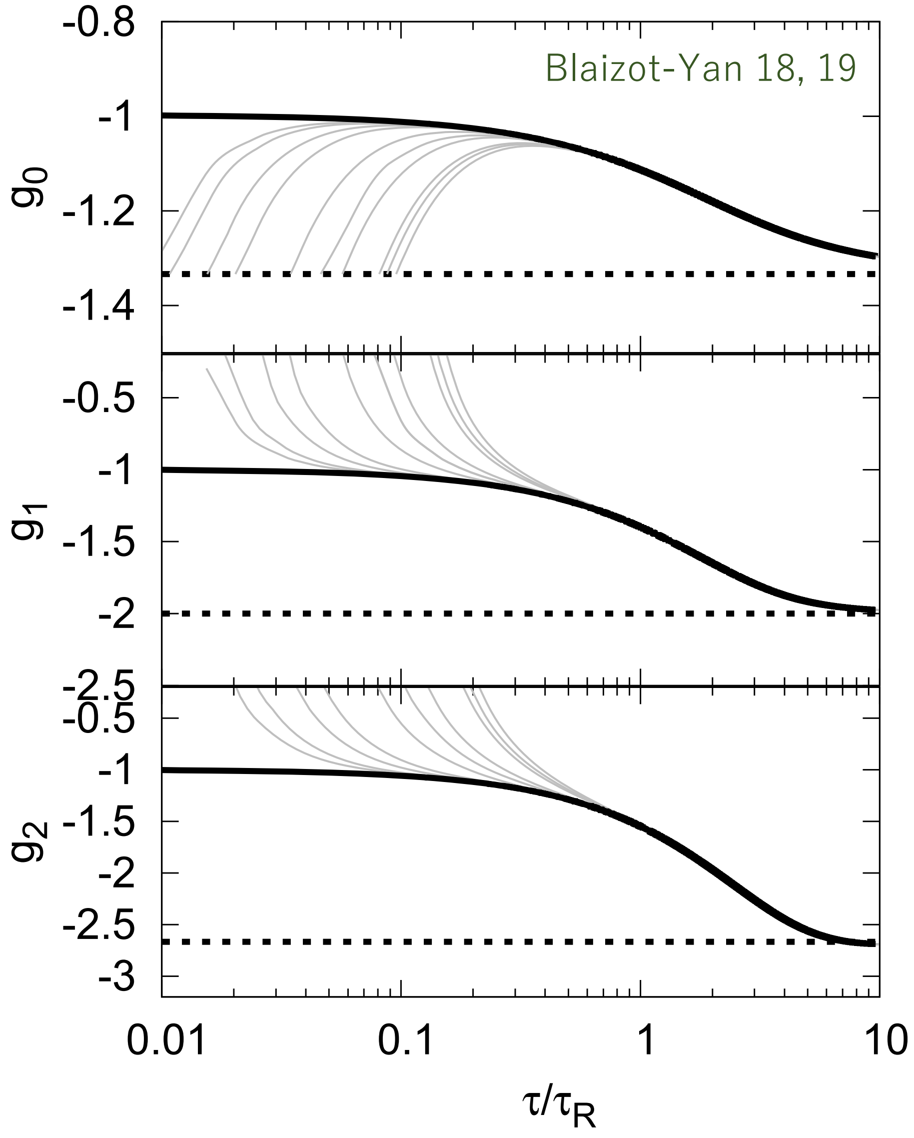
Why causal hydro works from early times

- Fixed point analysis
    1. Logarithmic growth rates of energy density and anisotropy

$$g_0(\tau) \equiv \frac{d\log e}{d\log\tau}$$

$$g_1(\tau) \equiv \frac{d\log(p_L - p_T)}{d\log\tau}$$

Flow and fixed points in the space of g_n



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Blaizot-Yan 18, 19

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2. Free-streaming asymptotics of causal hydro

$$\tau\frac{d}{d\tau}\begin{pmatrix} e \\ p_L - p_T \end{pmatrix} = \begin{bmatrix} -4/3 & -2/3 \\ -8/15 & -38/21 \end{bmatrix}\begin{pmatrix} e \\ p_L - p_T \end{pmatrix}$$

eigenvalues → exponents = g_0 and g_1

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IR (free)	(-1,-1)	**(-0.93, -0.93)**
IR (hyd)	(-4/3, -2)	(-4/3, -2)

Enough to just solve 2 x 2 matrix problem

Effective viscosity

- To improve causal hydro
  1. Use attractor solution for the truncated order

Blaizot-Yan 18, 19

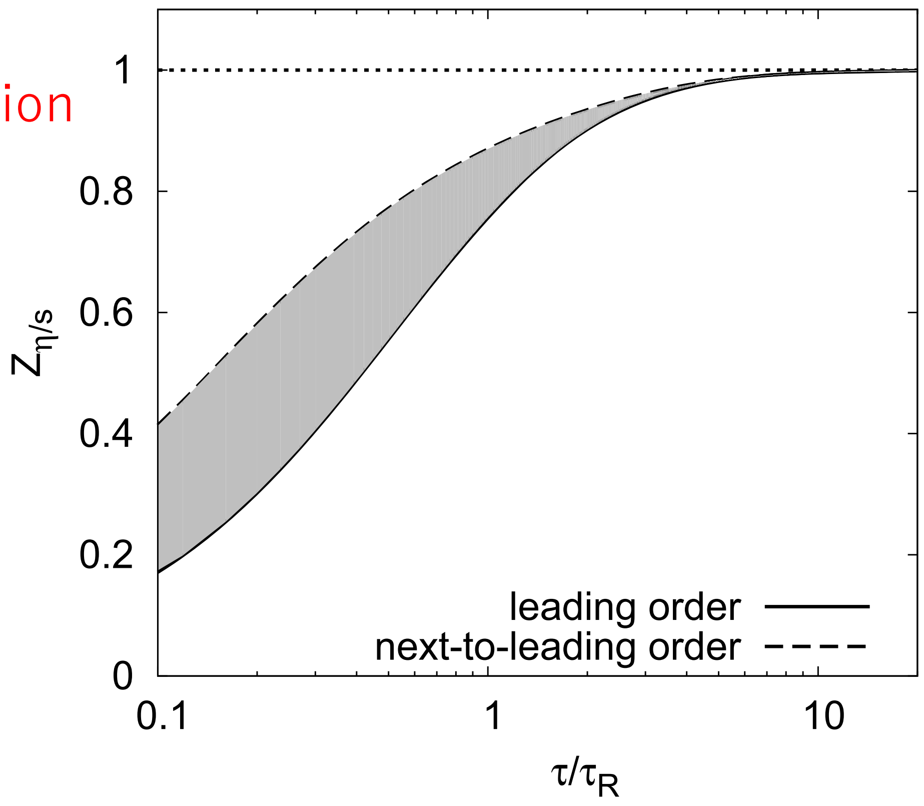
$$\frac{\partial}{\partial\tau}\mathcal{L}_0 = -\frac{1}{\tau}(a_0\mathcal{L}_0 + c_0\,\mathcal{L}_1)$$

Use attractor solution

$$\frac{\partial}{\partial\tau}\mathcal{L}_1 = -\frac{1}{\tau}(a_1\mathcal{L}_1 + b_1\mathcal{L}_0) - \left[1 + \boxed{\frac{c_1\tau_R}{\tau}\frac{\mathcal{L}_2}{\mathcal{L}_1}}\right]\frac{\mathcal{L}_1}{\tau_R}$$

  2. Effective relaxation time and viscosity

$$\tau_R^{\mathrm{eff}} \equiv Z_{\eta/s}\tau_R \rightarrow \left(\frac{\eta}{s}\right)_{\mathrm{eff}} = Z_{\eta/s}\left(\frac{\eta}{s}\right)$$



Far-from-equilibrium effective viscosity in Bjorken expansion < equilibrium viscosity

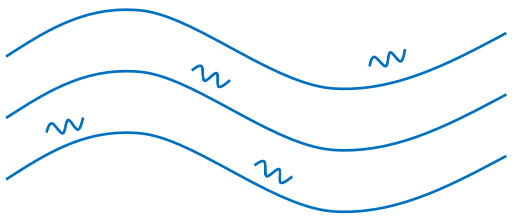
Hydro-kinetic theory

- Hydro fluctuations in Bjorken expansion
  1. Kinetic regime: **relaxation** vs. **expansion**

$$\gamma_\eta k_*^2 = \frac{1}{\tau}$$

$$k_* = \frac{1}{\sqrt{\gamma_\eta \tau}} \gg \frac{1}{\tau}$$

particle-like mode



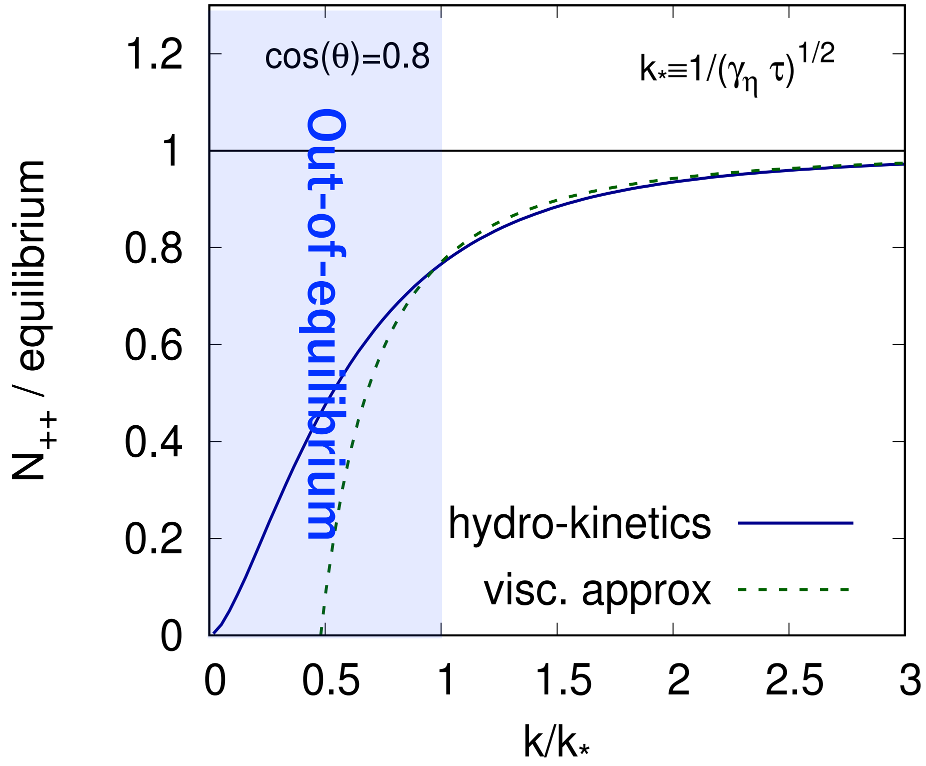
  2. Hydro-kinetic equation

$$N_k(t) = \langle |\delta v_k(t)|^2 \rangle$$

$$\frac{d}{d\tau} N_k = -\gamma_\eta k^2 (N_k - N_{\mathrm{eq}}) - \frac{1}{\tau} N_k$$

Akamatsu-Mazeliauskas-Teaney 17, 18
See also Martinez-Schafer 18, 19

Sound Modes



Hydro-kinetic theory

- Hydro fluctuations in Bjorken expansion
  3. Pressure from kinetic regime

Akamatsu-Mazeliauskas-Teaney 17, 18
See also Martinez-Schafer 18, 19

$$T_{zz} \sim \underbrace{[p_0(\Lambda) + T\Lambda^3]}_{} + \frac{1}{\tau}\underbrace{\left[\eta_0(\Lambda) + T\Lambda\frac{e_0+p_0}{\eta_0}\right]}_{} + \Delta T_{zz} + \frac{\#}{\tau^2}$$

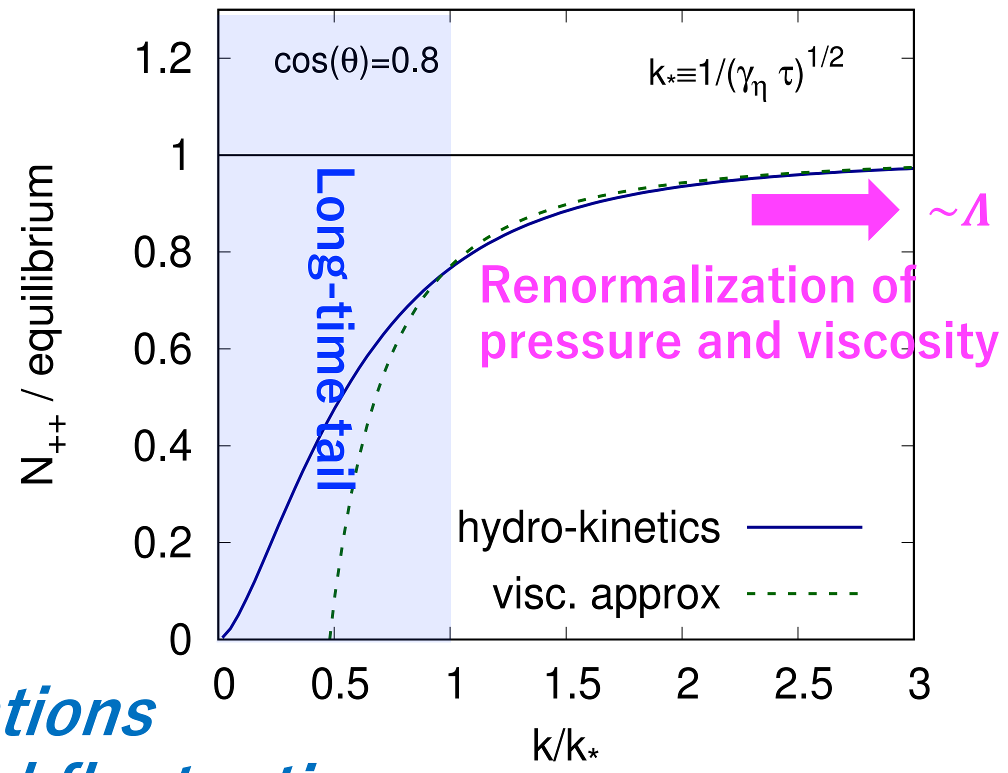
physical pressure and viscosity

$$\Delta T_{zz} \sim T k_*^3 \sim \frac{T}{(4\pi\gamma_\eta\tau)^{3/2}}$$

long time tail

Decays due to shrinking phase space volume of k*

Sound Modes



Long-time tail from out-of-equilibrium fluctuations and renormalization from (almost) equilibrated fluctuations