FLUCTUATIONS AND CORRELATIONS
(EXPERIMENT)

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$$\chi^2_B = \frac{\langle \Delta N_B^2 \rangle - \langle \Delta N_B \rangle^2}{VT^3}$$
Outline

Why fluctuations

- Fluctuations of conserved quantities
- Baselines from LQCD

From data to Susceptibilities on the lattice

- Centrality selection and participant fluctuations
- Acceptance selection and conservation laws

Experimental results

- Artefact of conservation laws on fluctuation measurement
- Cumulants vs. correlation functions

Summary
Why Fluctuations?

To probe the structure of strongly interacting matter
Locate phase boundaries
Search for critical phenomena
...

**E-by-E fluctuations are predicted within Grand Canonical Ensemble**

\[
\frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle^2} = \frac{T \chi_T}{V} \quad \chi_T = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T
\]

direct link to the EoS

\[
\langle N^2 \rangle - \langle N \rangle^2 = \kappa_2(N) = T^2 \frac{\partial^2 \ln Z}{\partial \mu^2}
\]

probing the response of the system to external perturbations


A. Rustamov, Quark Matter 2019, Wuhan, China 4-9 November
Why Fluctuations?

fingerprints of criticality for $m_{u,d} = 0$ survive at crossover with $m_{u,d} \neq 0$


$T_c^{LQCD} = 156.5 \pm 1.5$ MeV

$\langle \bar{\psi} \psi \rangle_l = \frac{T}{V} \frac{\partial \ln Z}{\partial m}$

$\chi_l = \frac{\partial}{\partial m} \langle \bar{\psi} \psi \rangle_l$


A. Rustamov, Quark Matter 2019, Wuhan, China 4-9 November
Understanding the QCD phase transition

Freeze-out at the phase boundary

\[ T_{f_0}^{ALICE} = 156.5 \pm 1.5 \text{MeV} \pm 3 \text{MeV}(sys) \]
\[ T_{c}^{LQCD} = 156.5 \pm 1.5 \text{MeV} \]

Experimental plan:

- measuring fluctuations of net-baryons along the QCD phase boundary

Open questions:

- the order of the phase transition
- existence of the critical endpoint
- ...

Fluctuations of conserved charges

\[ \langle N_B \rangle = 400, \quad \langle N_{\bar{B}} \rangle = 100 \]

fluctuations of net-baryons appear only inside finite acceptance

event generator used from:
P. Braun-Munzinger, A. Rustamov, J. Stachel,
QM18, NPA 982 (2019) 307-310
Basic definitions

\[ X = N_B - N_{\bar{B}} \]

\[ \mu_r \equiv \langle (X - \langle X \rangle)^r \rangle = \sum_X (X - \langle X \rangle)^r P(X) \]

first four cumulants

\[ \kappa_1 = \langle X \rangle, \quad \kappa_2 = \mu_2, \quad \kappa_3 = \mu_3, \quad \kappa_4 = \mu_4 - 3\mu_2^2 \]

Uncorrelated Poisson limit: \( \langle N_B N_{\bar{B}} \rangle = \langle N_B \rangle \langle N_{\bar{B}} \rangle \)

Net-Baryons \( \rightarrow \) Skellam

\[ \kappa_n = \langle N_B \rangle + (-1)^n \langle N_{\bar{B}} \rangle \]

\[ \frac{\kappa_{2n+1}}{\kappa_{2k}} = \tanh \left( \frac{\mu}{T} \right) = \frac{\langle N_B \rangle - \langle N_{\bar{B}} \rangle}{\langle N_B \rangle + \langle N_{\bar{B}} \rangle} \]
Baselines from LQCD

for a thermal system in a fixed volume $V$ within the Grand Canonical Ensemble

$$\hat{\lambda}_2^B = \frac{\langle \Delta N_B^2 \rangle - \langle \Delta N_B \rangle^2}{VT^3} \equiv \frac{\kappa_2(\Delta N_B)}{VT^3}$$

$$\hat{\lambda}_n^{N=B,S,Q} = \frac{1}{VT^3} \frac{\partial^n \ln Z(V, T, \mu_B, Q, S)}{\partial (\mu_N/T)^n}$$

Assumptions:

- Volume is fixed in each event
- Conservations are imposed on the averages

Reality:

- Volume fluctuates from $E$-to-$E$
- Conservations depend on the acceptance

$$\frac{\kappa_4^{exp}(\Delta N_B)}{\kappa_2^{exp}(\Delta N_B)} \neq \frac{\hat{\lambda}_4^B}{\hat{\lambda}_2^B}$$

$$\frac{\kappa_3^{exp}(\Delta N_B)}{\kappa_2^{exp}(\Delta N_B)} \neq \frac{\hat{\lambda}_3^B}{\hat{\lambda}_2^B}$$


A. Rustamov, Quark Matter 2019, Wuhan, China 4-9 November

smaller than in HRG for $T > 150$ MeV

F. Karsch; QM17, arXiv:1706.01620
O. Kaczmarek; QM17, arXiv:1705.10682
Cross-correlators

\[ \chi^X_{1} = \frac{1}{VT^3} \frac{\partial \ln Z(V,T,\mu_{B,Q,S})}{\partial (\mu_X/T)} \langle N_X - N_{\bar{X}} \rangle \]

\[ \chi^X_{2} = \frac{1}{VT^3} \frac{\partial^2 \ln Z(V,T,\mu_{B,Q,S})}{\partial (\mu_X/T)^2} \langle (N_X - N_{\bar{X}})^2 \rangle \]

\[ \chi^{X\bar{Y}}_{11} = \frac{1}{VT^3} \frac{\partial^2 \ln Z(V,T,\mu_{B,Q,S})}{\partial (\mu_X/T) \partial (\mu_Y/T)} \langle (N_X - N_{\bar{X}})(N_Y - N_{\bar{Y}}) \rangle \]

significant deviations from HRG already for the second mixed cumulants


R. Bellwied et al., arXiv:1910.14592
bridging the gap between measurements and theory predictions
Measurement details

Non-dynamical contributions

- e-by-e fluctuations of wounded nucleons (volume fluctuations)
  - depends on centrality selection methods
  - has to be estimated for each experiment


Efficiency correction methods of cumulants

- depend on the response functions of detectors
- has to be studied for each experiment


keep efficiency as high as possible!

what should be the optimum acceptance to confront with theory calculations?
Acceptance selection

To achieve requirements of GCE, cuts in $p_T$, $\Delta y$ or $\Delta \eta$ are imposed

$\Delta \eta > \Delta \eta_{thr}$
- conservations dominate

$\Delta \eta < \Delta \eta_{thr}$
- dynamical fluctuations may disappear (Poisson limit)

$\kappa_2 \left( n_B - n_\bar{B} \right) \over \left< n_B + n_\bar{B} \right>$

1

conservation laws or genuine physics

$\Delta \eta_{thr}$

$\Delta \eta$
experimental results
Measurement techniques

Cut based approach

use additional detector information or reject a given phase space bin

( challenge: decrease in efficiency)

Identity method (ALICE, NA49, NA61/SHINE)

\[
\begin{align*}
\omega_\pi (x_i) &= \frac{\rho_\pi (x_i)}{\rho_\pi (x_i) + \rho_K (x_i)} \quad & W_\pi = \sum_{i=1}^{5} \omega_\pi (x_i) \\
\omega_K (x_i) &= \frac{\rho_K (x_i)}{\rho_\pi (x_i) + \rho_K (x_i)} \quad & W_K = \sum_{i=1}^{5} \omega_K (x_i)
\end{align*}
\]

\( W_\pi, W_K \) - proxies for particle multiplicities

M. Gazdzicki et al., PRC 83, 054907 (2011)
M. I. Gorenstein, PRC 84, 024902 (2011)
M. Arslandok, A. Rustamov, NIM A946, 162622 (2019)
Results from ALICE (Identity method)

$R_1 = \frac{\kappa_2(n_p - n_{\bar{p}})}{\langle n_p + n_{\bar{p}} \rangle} \quad R_2 = \frac{\kappa_2(n_p)}{\langle n_p \rangle}$

Note: $\langle n_p + n_{\bar{p}} \rangle = \kappa_2 (Skellam)$

At LHC $R_1$ is protected against volume fluct. $R_2$ can be described by volume fluctuations

$R_1 = \frac{\kappa_2(n_p) + \kappa_2(n_{\bar{p}})}{\langle n_p + n_{\bar{p}} \rangle} - 2 \frac{cov(n_p, n_{\bar{p}})}{\langle n_p + n_{\bar{p}} \rangle}$

unity in ideal HRG

$cov(n_p, n_{\bar{p}}) = \langle n_p n_{\bar{p}} \rangle - \langle n_p \rangle \langle n_{\bar{p}} \rangle$
Conservations laws

\[ \kappa_2(n_B - n_{\bar{B}}) = \kappa_2(n_B) + \kappa_2(n_{\bar{B}}) - 2\text{cov}(n_Bn_{\bar{B}}) \]

\[ \text{cov}(n_Bn_{\bar{B}}) = \langle n_Bn_{\bar{B}} \rangle - \langle n_B \rangle \langle n_{\bar{B}} \rangle \]

★ Global baryon number conservation:
★ \( y_b \) and \( y_{\bar{B}} \) are produced independently

\[ R_1 = 1 - \alpha \]

A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901

★ Local baryon number conservation:

\[ |y_B - y_{\bar{B}}| < \Delta y_{\text{corr}}/2 \]


conservation laws introduce subtle correlations between baryons and antibaryons
Results from ALICE (Identity method)

The data are best described by global baryon number conservation: $R_1 = 1 - \alpha$

However, the results are also consistent with $\Delta y_{corr} = 5$

HIJING corresponds to $\Delta y_{corr} = 2$, not consistent with the data

the ALICE data suggest long range correlations
vanishing 3\textsuperscript{rd} cumulants at LHC energies.
consistent with expectations

\textbf{essential crosscheck before proceeding to higher moments ($\kappa_4, \kappa_6$)}

Mesut Arslanbek, Tue, Search for the CP I
Results from STAR

Note different notation: $k\sigma^2 \equiv \kappa_4/\kappa_2$

- Non-monotonic energy dependence
- $\kappa_4/\kappa_2$ measurement at 54.4 GeV follows the trend established by the BES-I results

Ashish Pandav, Tue, Search for the CP I
Baryon number conservation

Assumption:

- $\kappa_3/\kappa_2$ is entirely driven by baryon number conservation

Prediction:

- Excitation function for $\kappa_4/\kappa_2$

above 11.5 GeV CE suppression describes the $\kappa_4/\kappa_2$ data

Cumulants vs. correlation functions

\[
\rho(x_1, x_2) = \rho(x_1)\rho(x_2) + C_2(x_1, x_2)
\]
\[
\rho(x_1, x_2, x_3) = \rho(x_1)\rho(x_2)\rho(x_3) + \rho(x_1)C_2(x_2, x_3) + \cdots + C_3(x_1, x_2, x_3)
\]

\(\rho\) - distribution functions

\(C_n\) - integrated correlation functions

\[
\kappa_2 = \kappa_1 + C_2 \\
\kappa_3 = \kappa_1 + 3C_2 + C_3 \\
\kappa_4 = \kappa_1 + 7C_2 + 6C_3 + C_4
\]


driven by 4-proton correlation function, \(C_4\)

A. Bzdak, V. Koch, V. Skokov and N. Strodthoff, NPA 967, 465 (2017)

dominated by 2-proton correlation function, \(C_2\)

(consistent with baryon number conservation)

A. Rustamov, Quark Matter 2019, Wuhan, China 4-9 November
Results from STAR

\[
\frac{C_6}{C_2} \text{ for most central collisions}
\]

- negative for \( \sqrt{s_{NN}} = 200 \text{ GeV} \)
- positive for \( \sqrt{s_{NN}} = 54.4 \text{ GeV} \)

Negative value of six-order cumulant due to O(4) chiral criticality

Ashish Pandav, Tue, Search for the CP I

A. Bazavov et al., Phys.Rev. D95 (2017) 054504
S. Borsanyi et al., arXiv:1805.04445
Results from STAR and LQCD

- STAR data are not consistent with LQCD
- would be interesting to see also $\kappa_5$ from STAR

Heng-Tong Ding, Mon, Plenary
Dennis Bollweg, Wed, QCD at finite temperature III
Ashish Pandav, Tue, Search for the CP I

negative value of six-order cumulant due to O(4) chiral criticality

A. Bazavov et al., Phys.Rev. D95 (2017) 054504
S. Borsanyi et al., arXiv:1805.04445
G.A. Almasi, B. Friman and K. Redlich,
PRD 96, no. 1, 014027 (2017)
Results from NA61/SHINE

$\sqrt{s_{NN}} \approx 17\text{GeV}$

$p + p \approx \text{Be} + \text{Be}$ both in $\kappa_3/\kappa_2$ and $\kappa_4/\kappa_2$

EPOS predictions agree with the measured data

No critical behavior in Be+Be collisions

Maja Mackowiak, Tue, Search for the CP I

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Results from HADES

GSI/FAIR covers important energy regime to understand the excitation function of cumulants

For the smaller acceptance ($y_0 \pm 0.2$)
- $\kappa_3/\kappa_2$ - is below unity
- $\kappa_4/\kappa_2$ - is negative

For the larger acceptance ($y_0 \pm 0.4$)
- $\kappa_3/\kappa_2$ - becomes negative
- $\kappa_4/\kappa_2$ - approaches unity, still consistent with negative

the results are obtained with NLO volume corrections, based on:


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Near Future Experiments

**ALICE upgrade**
- New ITS: better vertexing
- Faster TPC: MWPC → GEMs
- Record minimum-bias Pb-Pb data at 50 kHz (currently < 1 kHz)
- Order of magnitude more events
- Measuring $\kappa_6$, may be beyond

**STAR upgrade, BES - II**
- iTPC: $|\eta| < 1.5$
- Better dE/dx resolution
- Lower momentum acceptance
- EPD: $2.1 < |\eta| < 5.1$
- Centrality determination
- ~ factor 20 more statistics

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**Michael Weber: ALICE plenary**

**Zhangbu Xu: STAR plenary**

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Near Future Experiments

CBM at FAIR (fixed target)  MPD at NICA (collider)

both experiments plan to explore fluctuations and correlations of conserved charges

Subhasis Samanta, Tue, Search for the CP I
MPD: Adam Kisiel, Tue, Future facilities
CBM: Viktor Klochkov, Tue, Future facilities
Event-by-event fluctuation signals are promising tools to explore the phase structure of a matter under the study.

To confront experiment with theory, a number of non-dynamical contributions are to be accounted for:
- Fluctuations of participant nucleons
- Conservation of baryon and/or strangeness number

The measured second cumulants of net-protons at ALICE are, after accounting for global baryon number conservation, in agreement with the corresponding second cumulants of the Skellam distribution.

LQCD predicts a Skellam behavior for the second cumulants of net-baryons at $T_{pc} = 156 \text{ MeV}$.

Contributions due to local baryon number conservation at LHC energies are small.

The ALICE data suggest long range correlations.

Measured third cumulants are consistent with zero.

*Essential crosscheck before proceeding to higher moments* ($\kappa_4$, $\kappa_6$)
Summary

- The STAR data show non-monotonic energy dependence in $\kappa_3/\kappa_2$ and $\kappa_4/\kappa_2$.
- The new results at 54.4 GeV on $\kappa_4/\kappa_2$ are consistent with the BES-I results.
- For the most central collisions at 200 GeV, the value of $\kappa_6$ is negative while at 54 GeV it stays positive.
  - Not consistent with the LQCD predictions for crossover.
  - Would be interesting to see $\kappa_5$ results.

- NA61/SHINE results on net-charge fluctuations are similar for p-p and Be-Be collisions.
  - Evidences for critical behavior are not observed so far.

- HADES results strongly depend on rapidity interval, but are always below the corresponding cumulant ratios as measured by STAR at 7.7 GeV.
Thank you for your attention
Bonus slides
Acceptance selection

\[ \alpha = \frac{\langle n_B \rangle^{acc}}{\langle N_B \rangle^{4\pi}} , \quad \kappa_4 \frac{(n_B - n_B)}{\kappa_2 (n_B - n_B)} = 1 - 6\alpha (1 - \alpha) \left[ 1 - \frac{2}{\langle N_B + N_B \rangle_{CE}} \langle N_B \rangle_{GCE} \langle N_B \rangle_{GCE} - \langle N_B \rangle_{CE} \langle N_B \rangle_{CE} \right] \]

deviations from unity are driven by different mechanisms

\[ 5 \times 10^8 \text{ events} \]

A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901
Modeling net-baryon fluctuations

Phenomenological approach
due to isospin randomization at $\sqrt{s_{NN}} > 10 GeV$

$$\phi(n_p, n_{\bar{p}}; n_B, n_{\bar{B}}) = \phi(n_B, n_{\bar{B}})B(n_p; n_B, r)B(n_{\bar{p}}; n_{\bar{B}}, \bar{r})$$

$$r = \langle n_p \rangle / \langle n_B \rangle \quad \bar{r} = \langle n_{\bar{p}} \rangle / \langle n_{\bar{B}} \rangle$$

in this case net-baryon fluctuations can be easily obtained from corresponding net-proton measurements


Experimental approach
measurement of fluctuations of other baryons
to improve understanding of net-baryon baseline
to study correlated baryon-strangeness fluctuations

event generator used from:
Net-pions and Net-kaons

**net-pions**

**ALICE Preliminary, Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV**

0.6 < $p$ < 1.5 GeV/c, centrality 0-5%

- ratio, stat. uncert.
- syst. uncert.
- HIJING

**net-kaons**

**ALICE Preliminary, Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV**

0.6 < $p$ < 1.5 GeV/c, centrality 0-5%

- ratio, stat. uncert.
- syst. uncert.
- HIJING

**Warning:** Skellam is not a proper baseline for net-pions and net-kaons

Resonance pion and kaon production is likely to explain the measured trend.

**perfect agreement with HIJING**

**reasonable agreement with HIJING**
in experiments we use net-protons as a proxy for net-baryons

moreover, protons contain feed-down contributions from weak decays
The ALICE apparatus and data sets

Inner Tracking System (ITS): tracking, vertexing
Time Projection Chamber (TPC): tracking, PID
V0 detector (V0A, V0C): centrality selection

minimum bias Pb-Pb data sets

<table>
<thead>
<tr>
<th>$\sqrt{s_{NN}}$ [TeV]</th>
<th>events</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.76</td>
<td>$13 \times 10^6$</td>
</tr>
<tr>
<td>5.02</td>
<td>$59 \times 10^6$</td>
</tr>
</tbody>
</table>
Participant (volume) fluctuations

$2.8 < \eta < 5.1$  
$-3.7 < \eta < -1.7$

search for the critical behavior in 6\textsuperscript{th} and higher cumulants for vanishing net baryon densities is achievable with the upcoming ALICE data

Increased statistics x100 in RUN3 and RUN4

A. Bazavov et al., Phys.Rev. D95 (2017) 054504
S. Borsanyi et al., arXiv:1805.04445

Results from ALICE (Identity method)

Comparison between two energies

\[ \kappa_{\langle p-p, p \rangle} \]

Different momentum ranges

\[ \kappa_{\langle p-p, p \rangle} \]

\( \Delta \eta \)

\( \Delta \eta \)

- Deviation from unity due to baryon number conservation
- Both data sets are consistent with \( \Delta y_{\text{corr}} = 5 \)
- More deviation from unity for larger momentum range
- Consistent with baryon number conservation

More details in: M. Arslanbek

A. Rustamov, Quark Matter 2019, Wuhan, China 4-9 November
Participant (volume) fluctuations

\[ \hat{\chi}_2^B = \frac{\langle \Delta N_B^2 \rangle - \langle \Delta N_B \rangle^2}{V T^3} \equiv \frac{\kappa_2(\Delta N_B)}{V T^3} \]

\[ \kappa_2^{\text{exp}}(\Delta N_B) = \kappa_2(\Delta N_B) + \frac{\langle N_B - \langle N_B \rangle \rangle}{\langle N_W \rangle} \kappa_2(N_W) \]

\[ \kappa_n^{\text{exp}}(\Delta N_B) = \kappa_n(\Delta N_B) + F(\kappa_1, \kappa_2, \ldots, \kappa_n(N_W)) \]

0-5% contributions from volume fluctuations have to be subtracted for each experiment.

A. Bialas, and M. Bleszynski, W. Czyz, NPB 111 (1976) 461.

A. Rustamov, Quark Matter 2019, Wuhan, China 4-9 November
Net-Lambda cumulants (Identity Method)

$$\kappa_2(n - \bar{n}) \equiv C_2(n - \bar{n})$$

ALICE Preliminary

Pb–Pb, \( \sqrt{s_{NN}} = 5.02 \) TeV

1 < \( p_{T,\Lambda} < 4 \) GeV/c, \( |\eta_\Lambda| < 0.5 \)

A. Ohlson, QM18, NPA982 (2019) 299-302

Similar trend as for net-protons

A. Rustamov, Quark Matter 2019, Wuhan, China 4-9 November
Higher cumulants (cut-based)

Both ALICE and STAR attempting to improve $p_T$ acceptance measured with the cut-based approach in a rather small $p_T$ acceptance

N. Behera, QM18, NP A982 (2019) 851-854

measured with the cut-based approach in a rather small $p_T$ acceptance

Both ALICE and STAR attempting to improve $p_T$ acceptance