



Vorticity and spin polarization

A theoretical perspective

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What is vorticity?



Vortex in a coffee cup

What is vorticity?



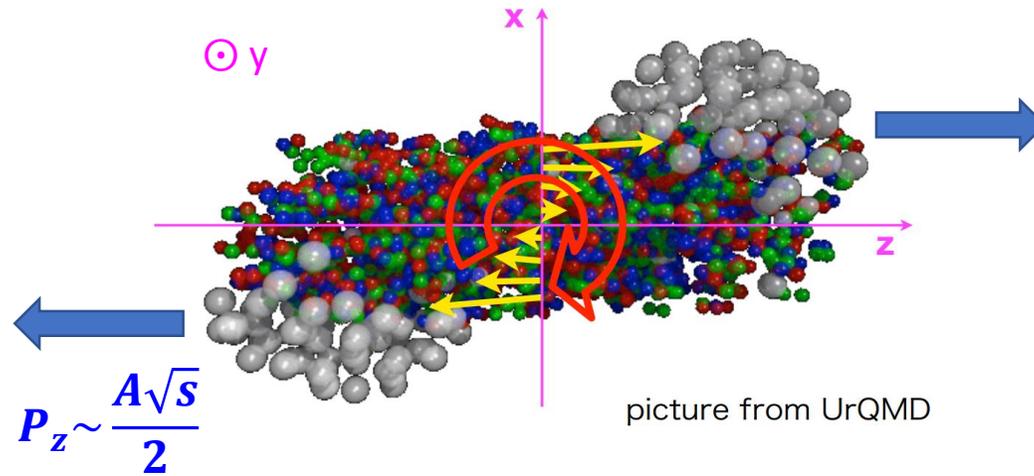
Vortex in a coffee cup

$$\boldsymbol{\omega} = \frac{1}{2} \nabla \times \boldsymbol{v}$$

Fluid vorticity

(Local angular velocity)

Vorticity by global angular momentum

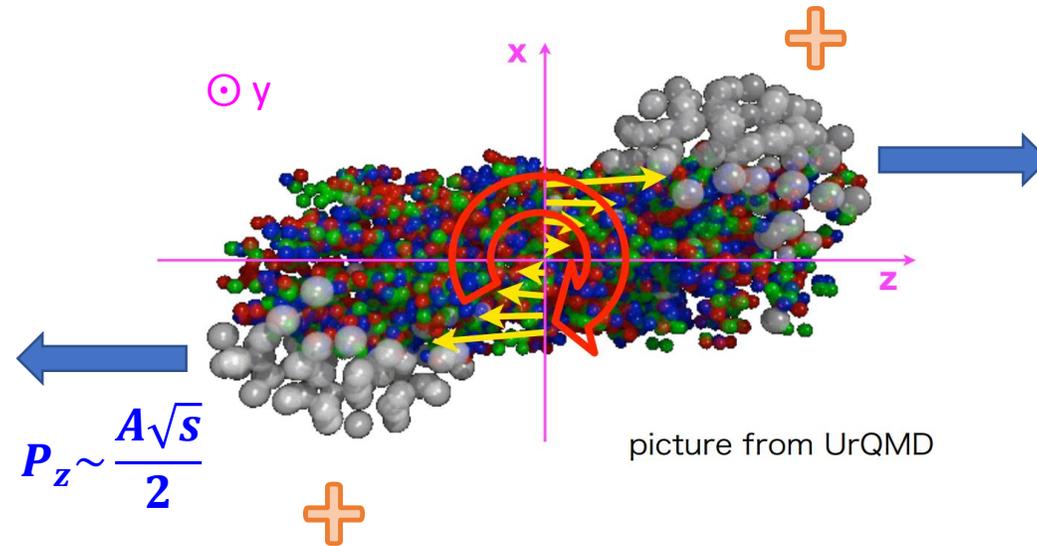


$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

Global angular momentum

(RHIC Au+Au 200 GeV, $b=10$ fm)

Vorticity by global angular momentum



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Global angular momentum

(RHIC Au+Au 200 GeV, $b=10$ fm)

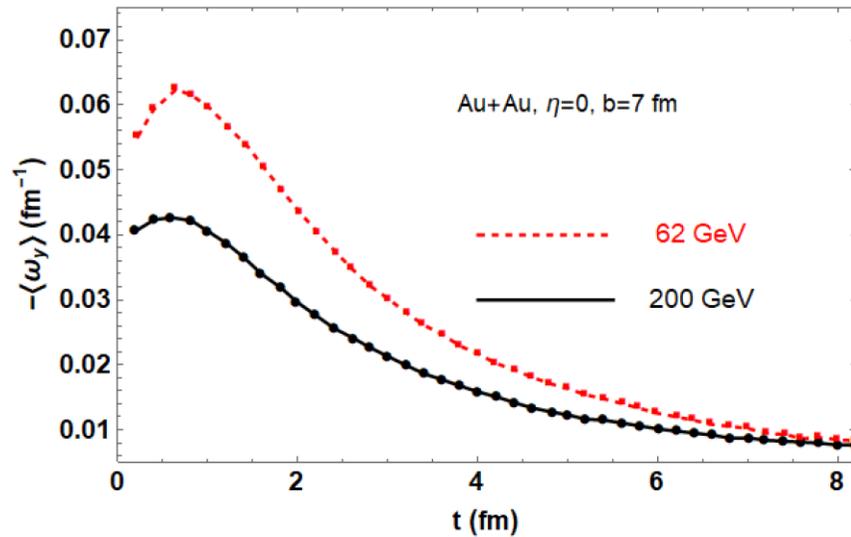
$$eB \sim \gamma \alpha_{EM} \frac{z}{b^2} \sim 10^{18} \text{ G}$$

Strong Magnetic field

(See Liao's plenary talk)

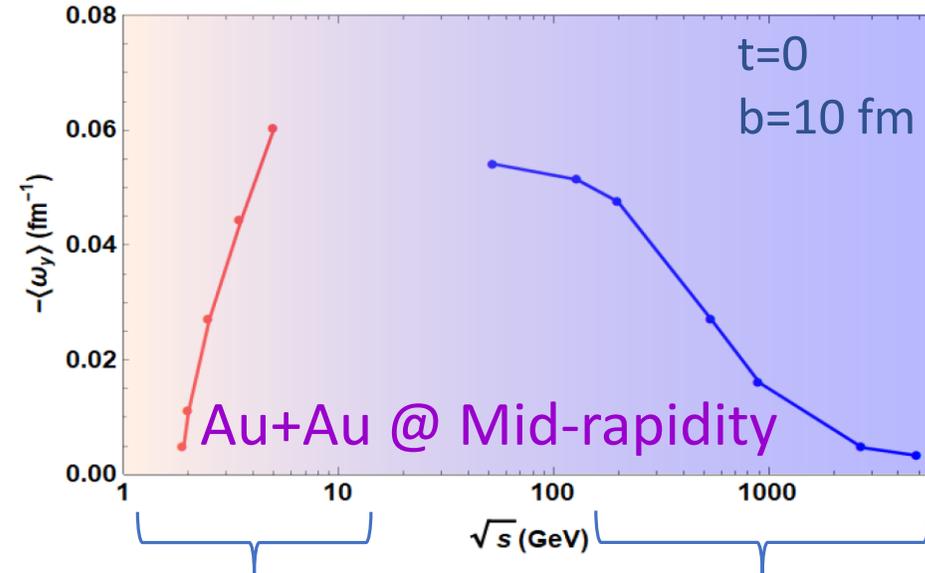
Vorticity by global angular momentum

Time dependence



AMPT (Jiang-Lin-Liao PRC2016)

Energy dependence

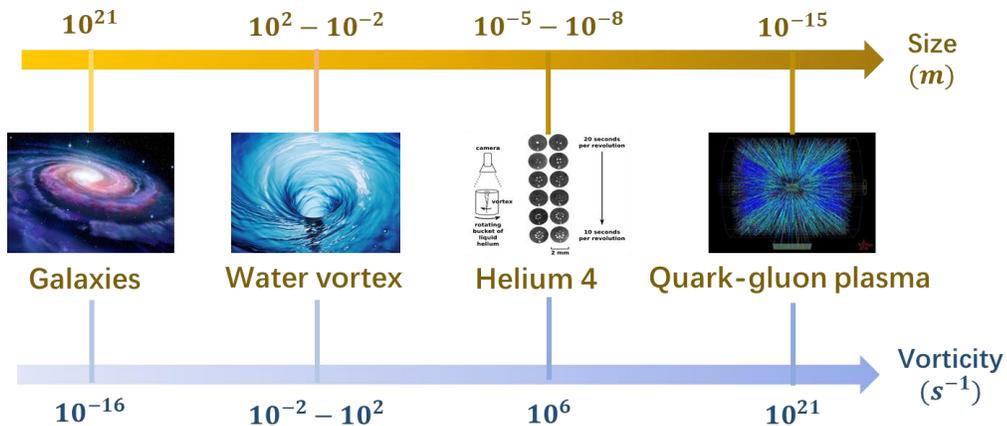


UrQMD/IQMD

(Deng-XGH-Ma-Zhang 2019)

HIJING

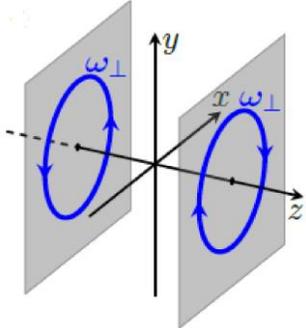
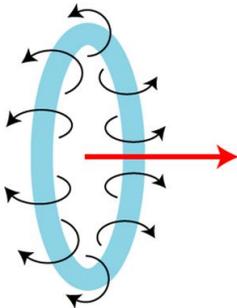
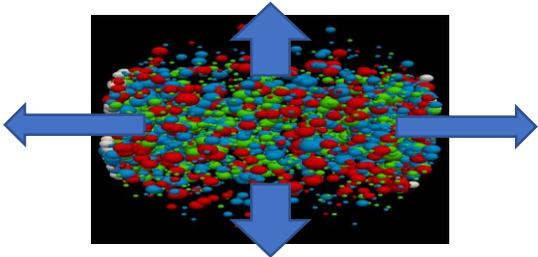
(Deng-XGH PRC2016)



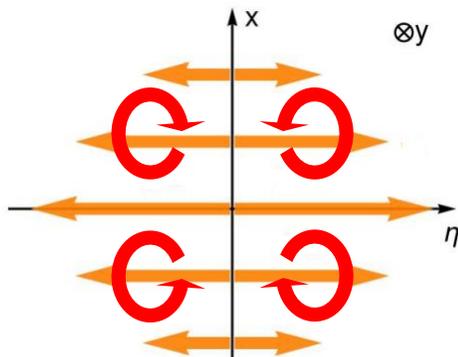
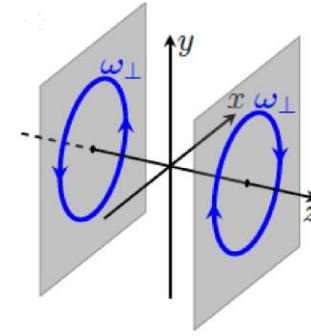
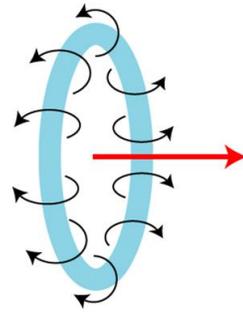
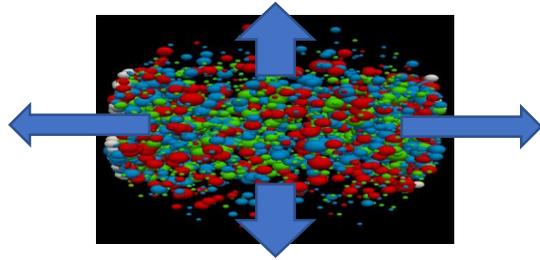
- **Most vortical fluid** $\langle |\omega_y| \rangle \sim 10^{21} s^{-1}$
- Relativistic suppression at high energy

See also: Becattini et al EPJC2015, Csernai et al PRC2013, PRC2014, Ivanov et al PRC2017, PRC2019,

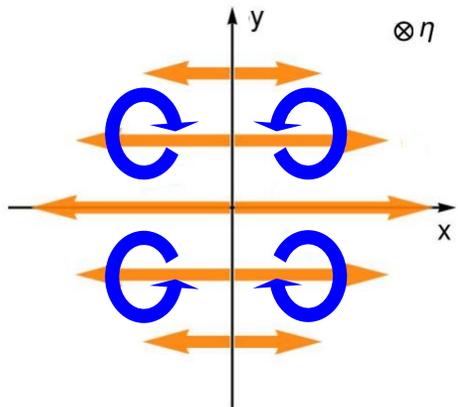
Vorticity by inhomogeneous expansion



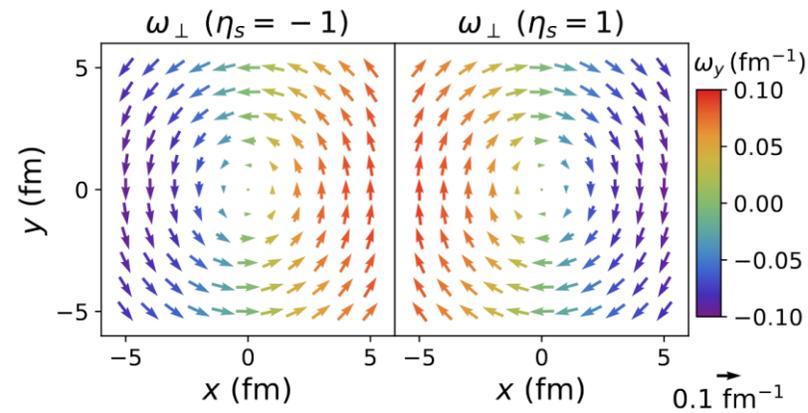
Vorticity by inhomogeneous expansion



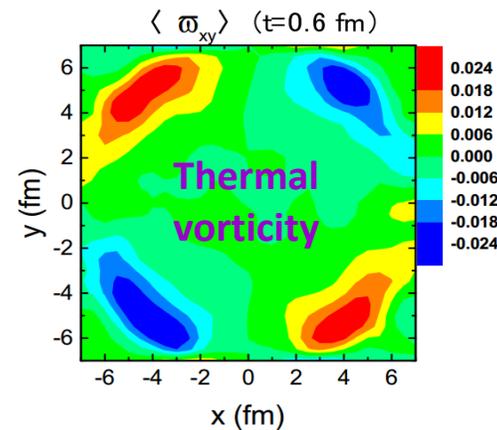
Transverse



Longitudinal



(Xia-Li-Wang PRC2017)



(Wei-Deng-XGH PRC2019)

See also: Karpenko-Becattini EPJC2017, Csernai etal PRC2014, Teryaev-Usubov PRC2015, Ivanov-Soldatov PRC2018,

We have a good understanding of the vorticity formation based on transport models and hydrodynamic models.

How to detect it experimentally?

- Spin polarization
- Chiral vortical effects
-

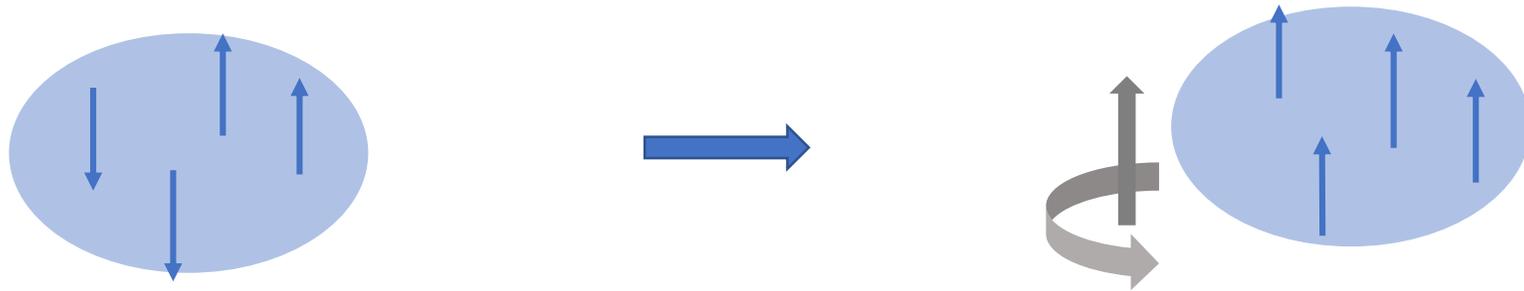
(Experimental perspectives covered in Lisa's plenary talk)

How vorticity polarizes spin?

Early idea: Liang-Wang PRL2005, Voloshin 2004

Vorticity interpretation (at thermal equilibrium)

$$H = H_0 - \boldsymbol{\omega} \cdot \mathbf{J} \quad \longrightarrow \quad \frac{dN}{dp} \sim e^{-(H_0 - \boldsymbol{\omega} \cdot \mathbf{J})/T}$$



$$\longrightarrow \quad P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} \sim \frac{\omega}{T}$$

How vorticity polarizes spin?

More rigorous derivation (Becattini et al AP2013, Fang et al PRC2016)

$$P^\mu(p) = \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\nu \frac{\int d\Sigma_\lambda p^\lambda f'(x, p) \varpi_{\rho\sigma}(x)}{\int d\Sigma_\lambda p^\lambda f(x, p)} + O(\varpi^2)$$

- Valid at global equilibrium. $f(x, p)$ is the distribution function (Fermi-Dirac)
- Thermal vorticity $\varpi_{\rho\sigma} = \left(\frac{1}{2}\right) (\partial_\sigma \beta_\rho - \partial_\rho \beta_\sigma)$
- Spin polarization is enslaved to thermal vorticity, not dynamical
- Friendly for numerical simulation

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At local equilibrium, does the polarization depend on the pseudo-gauge?

In any other pseudo-gauge, corrections are expected of the order

$$\Delta S^\mu(p) \propto \epsilon^{\mu\nu\rho\sigma} p_\sigma \int_\Sigma d\Sigma_\tau \dots (\Omega_{\nu\rho} - \partial_\nu \beta_\rho)$$

OAM

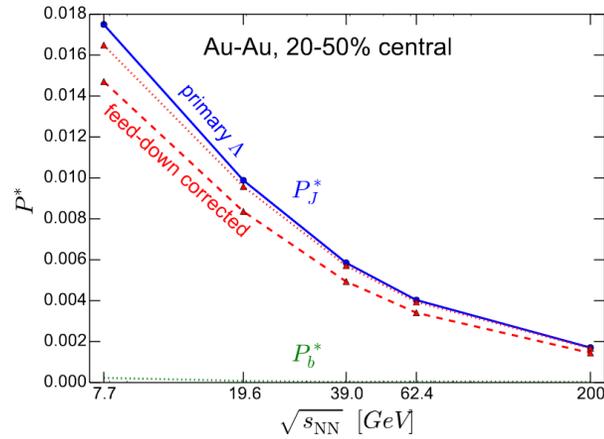
Spin

See Becattini's talk Nov. 6

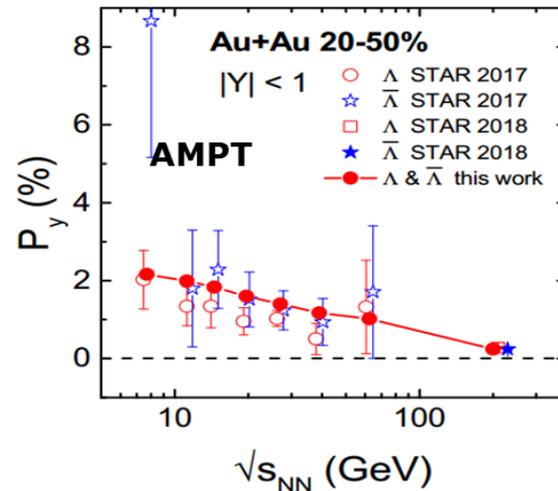
Global Λ spin polarization

The global polarization: Experiment = Theory (use formula on previous page)

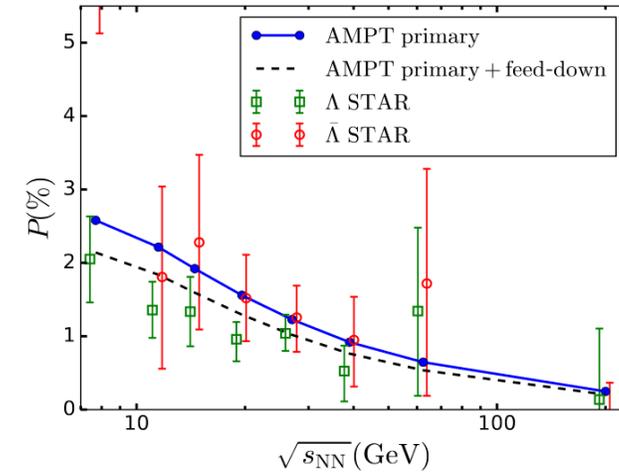
(Karpenko-Becattini EPJC2016)



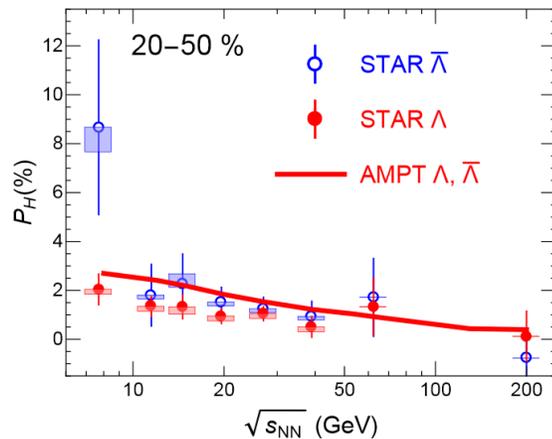
(Wei-Deng-XGH PRC2019)



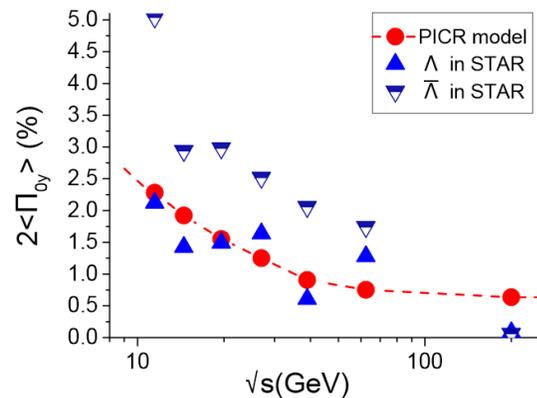
(Li-Pang-Wang-Xia PRC2017)



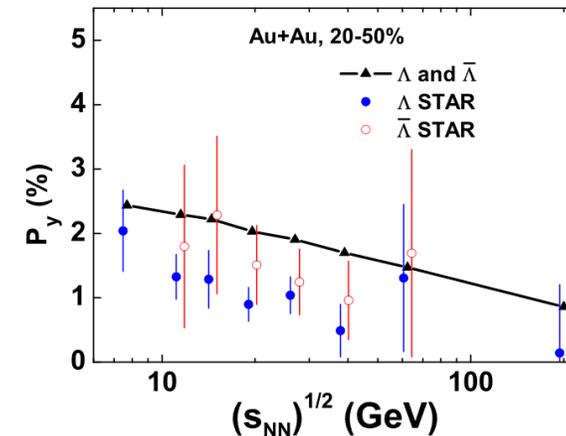
(Shi-Li-Liao PLB2018)



(Xie-Wang-Csernai PRC2017)



(Sun-Ko PRC2017)



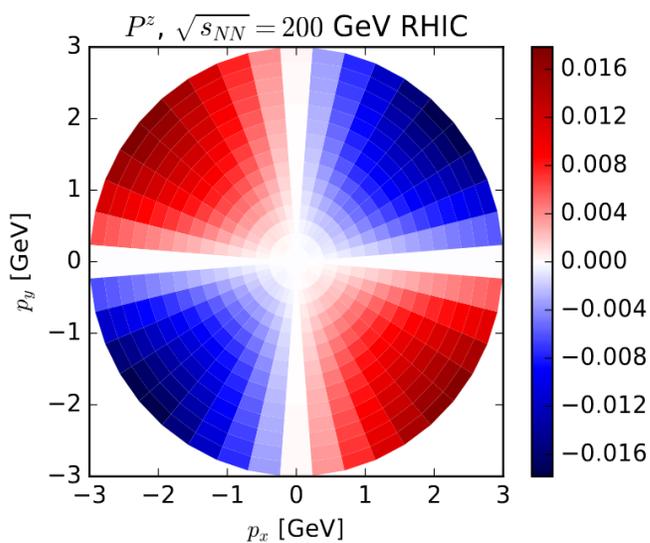
Differential Λ spin polarization

The global Λ polarization reflects the total amount of angular momentum retained in the mid-rapidity region. **How is it distributed in different ϕ ?**

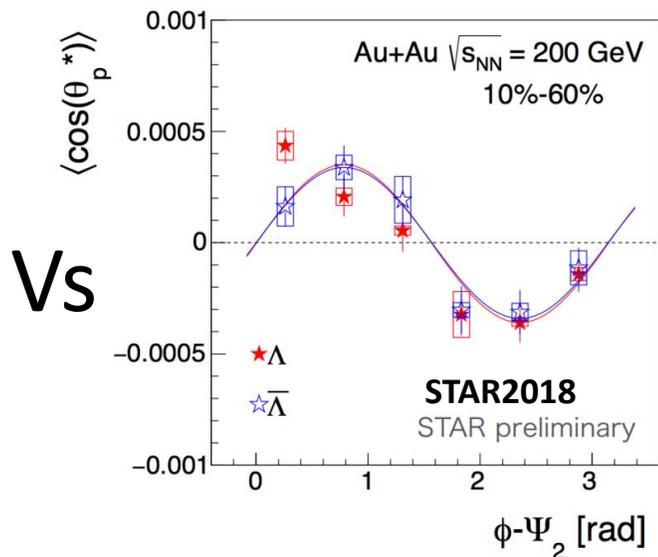
- Spin harmonic flow:
$$\frac{1}{2\pi} \frac{dP_{y,z}}{d\phi} = P_{y,z} + 2f_{2y,z} \sin(2\phi) + 2g_{2y,z} \cos(2\phi) + \dots$$

1) longitudinal polarization vs ϕ

(Becattini-Karpenko PRL2018)



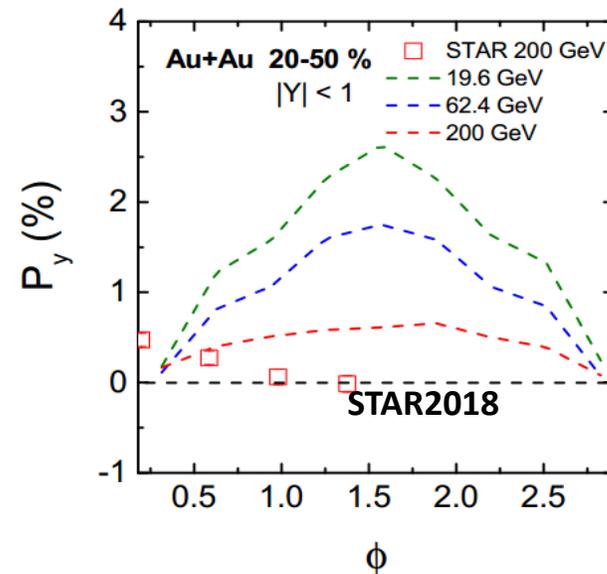
$$f_{2z}^{\text{ther}} < 0$$



$$f_{2z}^{\text{exp}} > 0$$

2) Transverse polarization vs ϕ

(Wei-Deng-XGH PRC2019)



$$g_{2y}^{\text{ther}} < 0, g_{2y}^{\text{exp}} > 0$$

We have a spin “sign problem”!

Differential Λ spin polarization

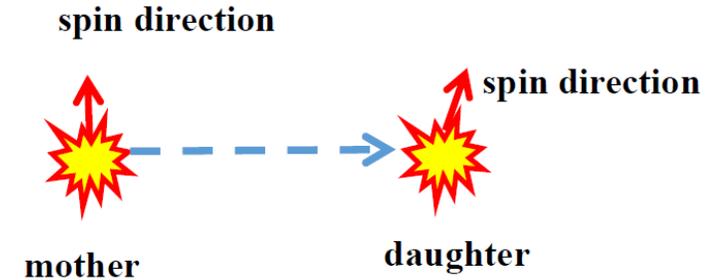
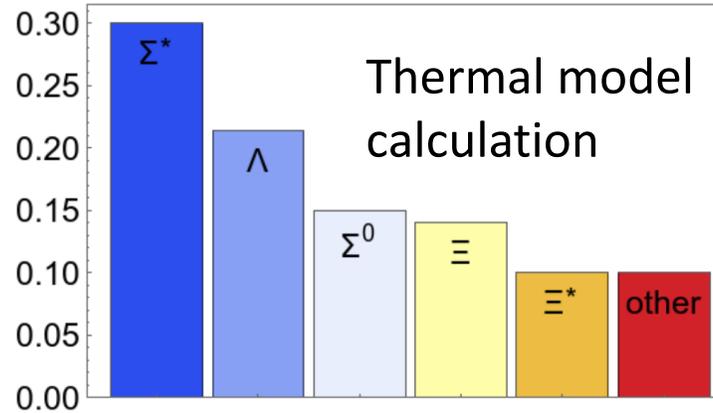
Attack the spin sign problem from theory side:

- Understand the vorticity (☺?)
- **Effect of feed-down decays**
(Measured Λ may from decays of heavier particles)
- Go beyond equilibrium treatment (spin as a dynamic d.o.f)
spin hydrodynamics
spin kinetic theory
- Initial condition
(Initial polarization, initial flow,)
- Other possibilities
(**chiral vortical effect** (Liu-Sun-Ko 2019), **mesonic mean-field**(Csernai-Kapusta-Welle PRC2019),
other spin chemical potential (Wu-Pang-XGH-Wang PRR2019, Florkowski etal2019),
contribution from gluons,)

See Wang's talk Nov. 6

The feed-down effects

About 80% of final Λ 's are from decays of higher-lying particles



Spin polarization transfer (Xia-Li-XGH-Huang PRC2019, Becattini-Cao-Speranza EPJC2019)

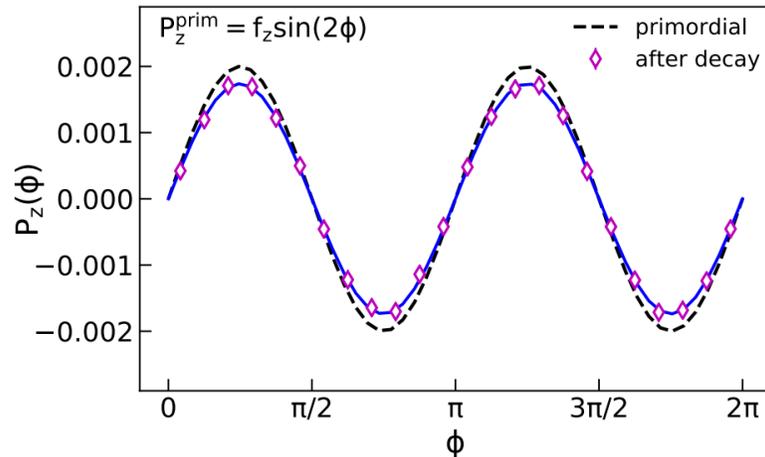
	spin and parity	$(1/N)dN/d\Omega^*$	\mathbf{P}_D	$\langle \mathbf{P}_D \rangle / \mathbf{P}_P$
strong decay	$1/2^+ \rightarrow 1/2^+ 0^-$	$1/(4\pi)$	$2(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*) \hat{\mathbf{p}}^* - \mathbf{P}_P$	-1/3
strong decay	$1/2^- \rightarrow 1/2^+ 0^-$	$1/(4\pi)$	\mathbf{P}_P	1
strong decay	$3/2^+ \rightarrow 1/2^+ 0^-$	$3 \left[1 - 2\Delta/3 - (1 - 2\Delta) \cos^2 \theta^* \right] / (8\pi)$	Too long to be	1
strong decay	$3/2^- \rightarrow 1/2^+ 0^-$	$3 \left[1 - 2\Delta/3 - (1 - 2\Delta) \cos^2 \theta^* \right] / (8\pi)$	shown; see ref.	-3/5
weak decay	$1/2 \rightarrow 1/2 \ 0$	$(1 + \alpha P_P \cos \theta^*) / (4\pi)$		$(2\gamma + 1)/3$
EM decay	$1/2^+ \rightarrow 1/2^+ 1^-$	$1/(4\pi)$	$-(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*) \hat{\mathbf{p}}^*$	-1/3

Some decay channels can lead to spin-polarization flip!

The feed-down effects

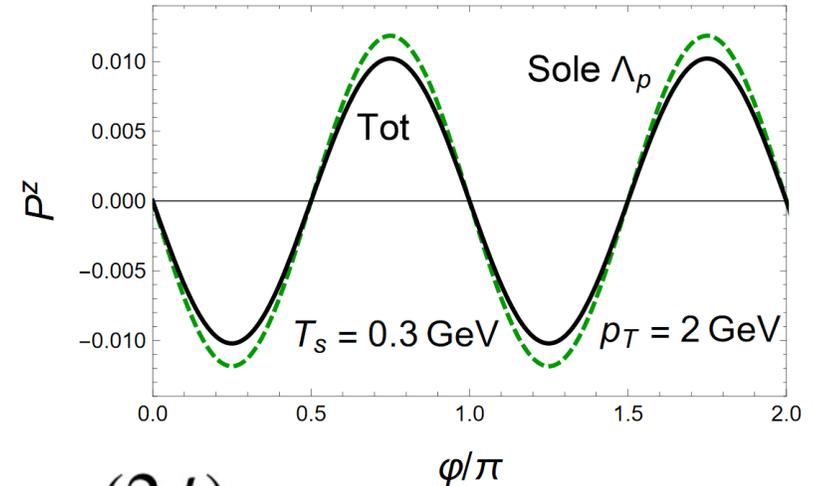
- Longitudinal polarization

(Xia-Li-XGH-Huang PRC2019)



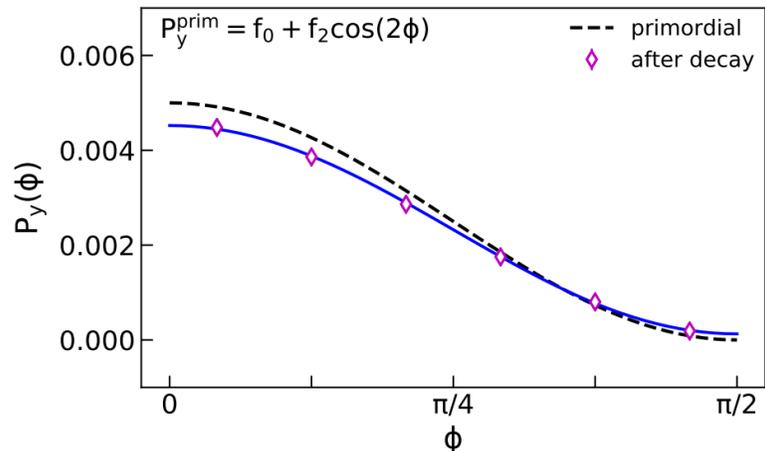
$$P_z = f_z \sin(2\phi)$$

(Becattini-Cao-Speranza EPJC2019)



- Transverse polarization

$$P_y = f_0 + f_2 \cos(2\phi)$$



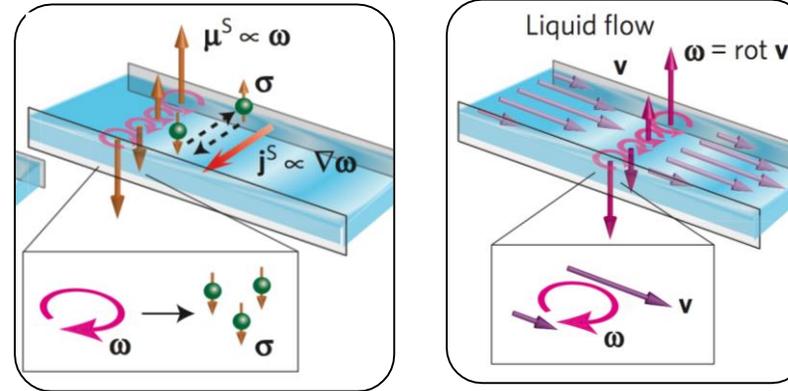
Conclusion:

- Feed-down effects suppress $\sim 10\%$ Λ primordial spin polarization
- Do not solve the spin sign problem

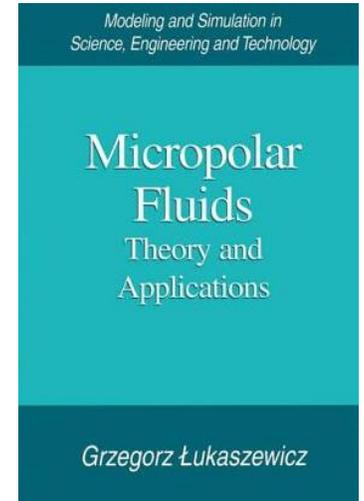
Spin hydrodynamics

Framework for collective spin dynamics. Spin as a (quasi-)hydrodynamic variable

- Widely used in non-relativistic **spintronics**, **micropolar fluid**,



(Takahashi et al Nat.Phys.2016)



- Relativistic **ideal** spin hydrodynamics initiated by Florkowski et al

$$\partial_{\mu} T^{\mu\nu} = 0$$

$$\partial_{\lambda} S^{\lambda,\mu\nu} = 0$$

$$T^{\mu\nu} = (\varepsilon + P)u^{\mu}u^{\nu} - Pg^{\mu\nu}$$

$$S^{\lambda,\mu\nu} = \frac{wu^{\lambda}}{4\zeta}\omega^{\mu\nu}$$

(Florkowski et al PRC2018)

Spin hydrodynamics

Relativistic **dissipative** spin hydrodynamics (Hattori-Hongo-XGH-Matsuo-Taya PLB2019)

- Identify (quasi-)hydrodynamic variables: \mathbf{T} and \mathbf{u}^μ (4 for translation), $\omega^{\mu\nu} = -\omega^{\nu\mu}$ (spin chemical potential, 3 for rotation, 3 for boost).
- Derivative expansion. Apply 2nd law of thermodynamics.
- Constitutive relations up to $\mathcal{O}(\partial)$

$$T_{(0)}^{\mu\nu} = e u^\mu u^\nu + p (g^{\mu\nu} + u^\mu u^\nu)$$

heat current

shear viscosity

bulk viscosity

$$T_{(1)}^{\mu\nu} = -2\kappa \left(D u^{(\mu} + \beta \partial_\perp^{(\mu} \beta^{-1} \right) u^{\nu)} - 2\eta \partial_\perp^{<\mu} u^{\nu>} - \zeta (\partial_\mu u^\mu) \Delta^{\mu\nu}$$

$$-2\lambda \left(-D u^{[\mu} + \beta \partial_\perp^{[\mu} \beta^{-1} + 4u_\rho \omega^{\rho[\mu} \right) u^{\nu]} - 2\gamma \left(\partial_\perp^{[\mu} u^{\nu]} - 2\Delta_\rho^\mu \Delta_\lambda^\nu \omega^{\rho\lambda} \right)$$

boost heat current

rotational viscosity

- Hydrodynamic equations

$$\partial_\mu \left(T_{(0)}^{\mu\nu} + T_{(1)}^{\mu\nu} + \mathcal{O}(\partial^2) \right) = 0$$

Energy-momentum conservation

$$\partial_\mu (u^\mu s^{\alpha\beta}) = T_{(1)}^{\beta\alpha} - T_{(1)}^{\alpha\beta} + \mathcal{O}(\partial^2)$$

Angular momentum conservation

$$p = p(e, s^{\alpha\beta})$$

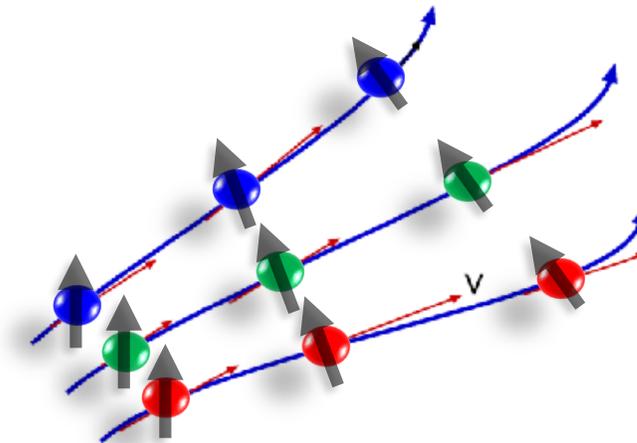
Equation of state

Spin hydrodynamics

Future:

- Causal and stable (Israel-Stewart) 2nd order spin hydrodynamics
- Flow frame choice and pseudo-gauge choice, especially for Belinfante gauge
- Calculation of rotational viscosity and boost heat conductivity (insight to QCD)
- Formulate spin hydrodynamics for large vorticity at $O(1)$ and with magnetic field
- Derive spin hydrodynamics from kinetic theory (see Shi's talk Nov. 5)
- Spin Cooper-Frye formula
- Application: numerical spin hydrodynamics for Λ polarization

See Florkowski's talk Nov. 6



Spin kinetic theory

Semi-classical transport theory for spin of massive particles

- One can start with Kadanoff-Baym equation for the two-point functions or its Wigner transformed version.
- The leading spin effect appears at $\mathcal{O}(\hbar)$. Truncate at $\mathcal{O}(\hbar)$. 3 eqs. for spin 1 for particle #

$$\bullet \delta(p^2 - m^2 \mp \hbar \Sigma_S^{\alpha\beta} F_{\alpha\beta}) \left\{ \left[p^\mu \Delta_\mu \pm \frac{\hbar}{2} \Sigma_S^{\mu\nu} \Xi_{\mu\nu} \right] (f \pm f_A) + \hbar f \Xi_{\mu\nu} \Sigma_S^{\mu\nu} \right\} = \text{col.}$$

$$\bullet \left\{ f_A p \cdot \Delta \theta^\mu - F^{\mu\nu} f_A \theta_\nu + \theta^\mu (p \cdot \Delta f_A) - \frac{\hbar}{2m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \Delta_\nu \Delta_\rho f \right\} \delta(p^2 - m^2) = \text{col}$$

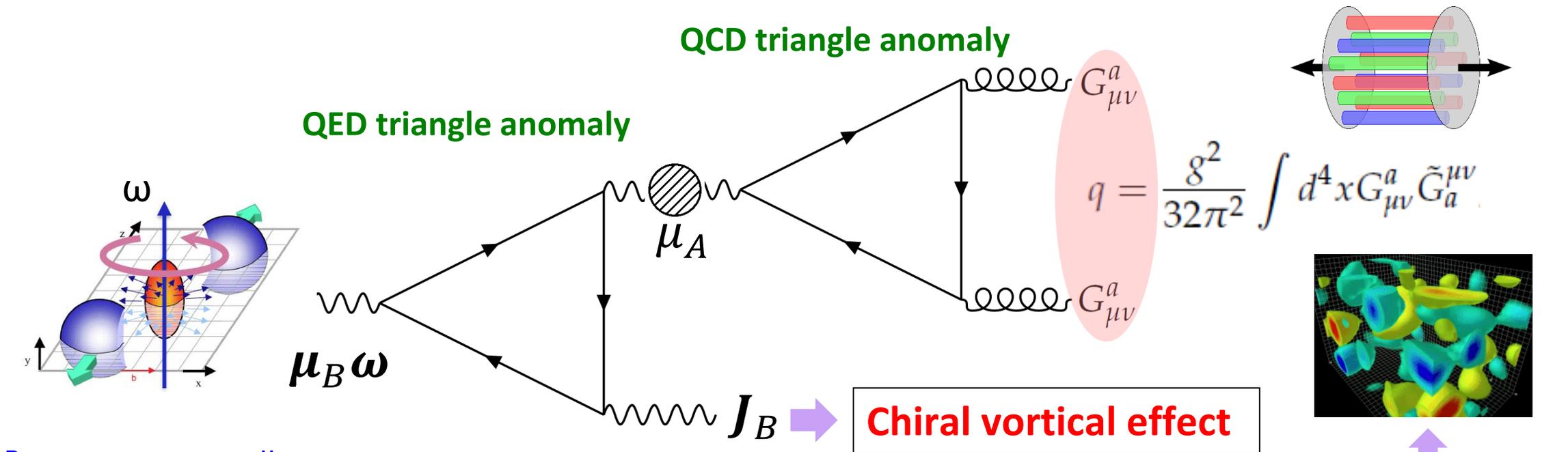
$$\bullet \Sigma_S^{\mu\nu} = \frac{1}{2m} \epsilon^{\mu\nu\rho\sigma} \theta_\rho p_\sigma \quad \bullet \theta^\mu \theta_\mu = -1 \quad \bullet p^\mu \theta_\mu = 0$$

- Collision effect is unknown in general. For heavy quark: (Li-Yee PRD2019)
- **Future:** derive collision effect, connect to spin hydrodynamics, apply to numerical simulation, ...

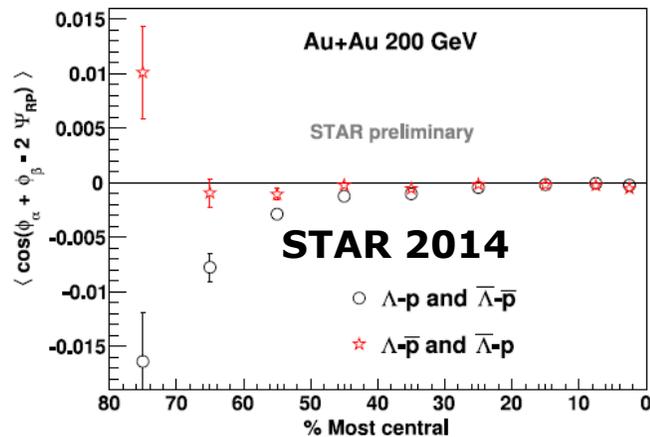
See Gao' talk Nov. 5, Li's, Wang's,
Weickgenannt's talks Nov. 6

(Gao-Liang PRD2019, Weickgenannt etal PRD2019, Hattori-Hidaka-Yang 2019,
Wang-Guo-Shi-Zhuang PRD2019, Liu-Mameda-XGH 2019)

Vorticity can probe QCD topological fluctuation

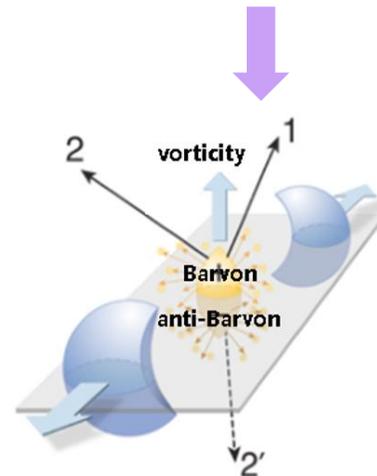


Beam energy scan II



???

v_2 induced background



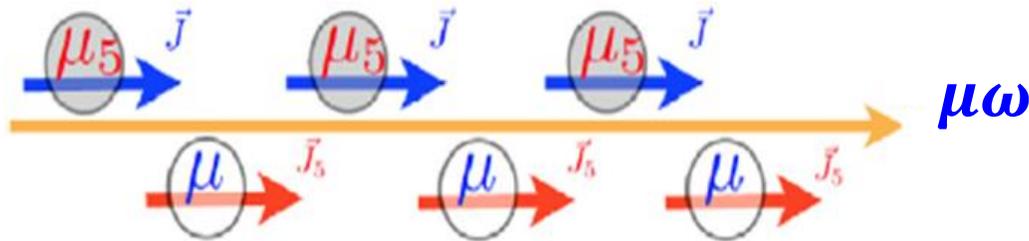
(Torabian-Yee JHEP2009,
Erdmenger etal JHEP2009,
Son-Surowka PRL2009,
Banerjee etal JHEP2011)

Vorticity can probe QCD topological fluctuation

- Collective mode from chiral vortical effects: **chiral vortical wave**

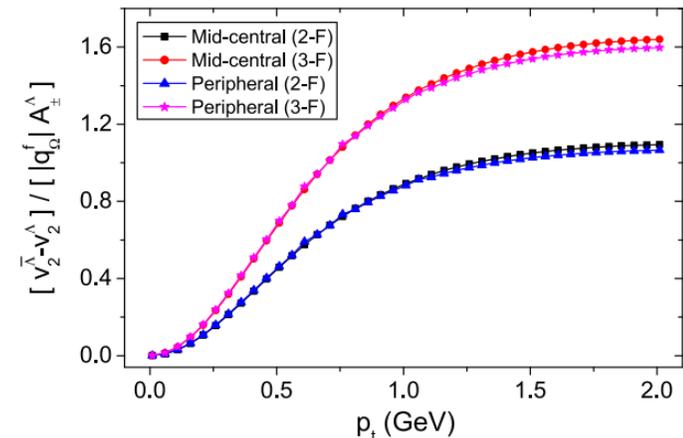
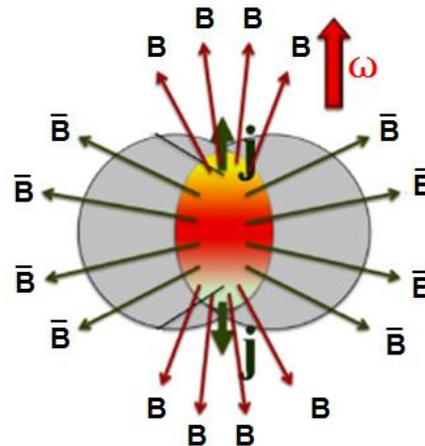
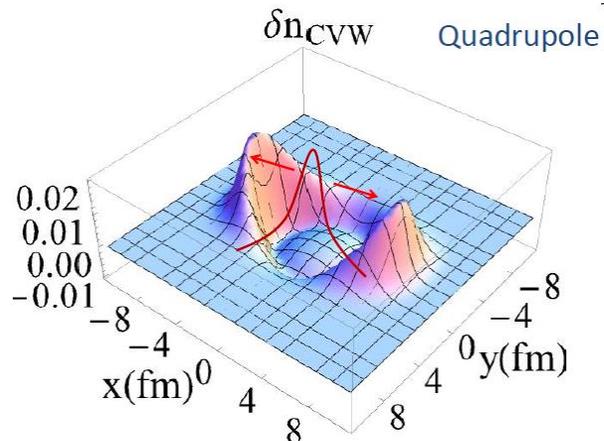
$$J_A = \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2} \omega, \quad J_V = \frac{1}{\pi^2} \mu_V \mu_A \omega$$

(Jiang-XGH-Liao PRD2015)



$$\partial_t n_{L,R} + \nabla \cdot \vec{J}_{L,R} = 0$$

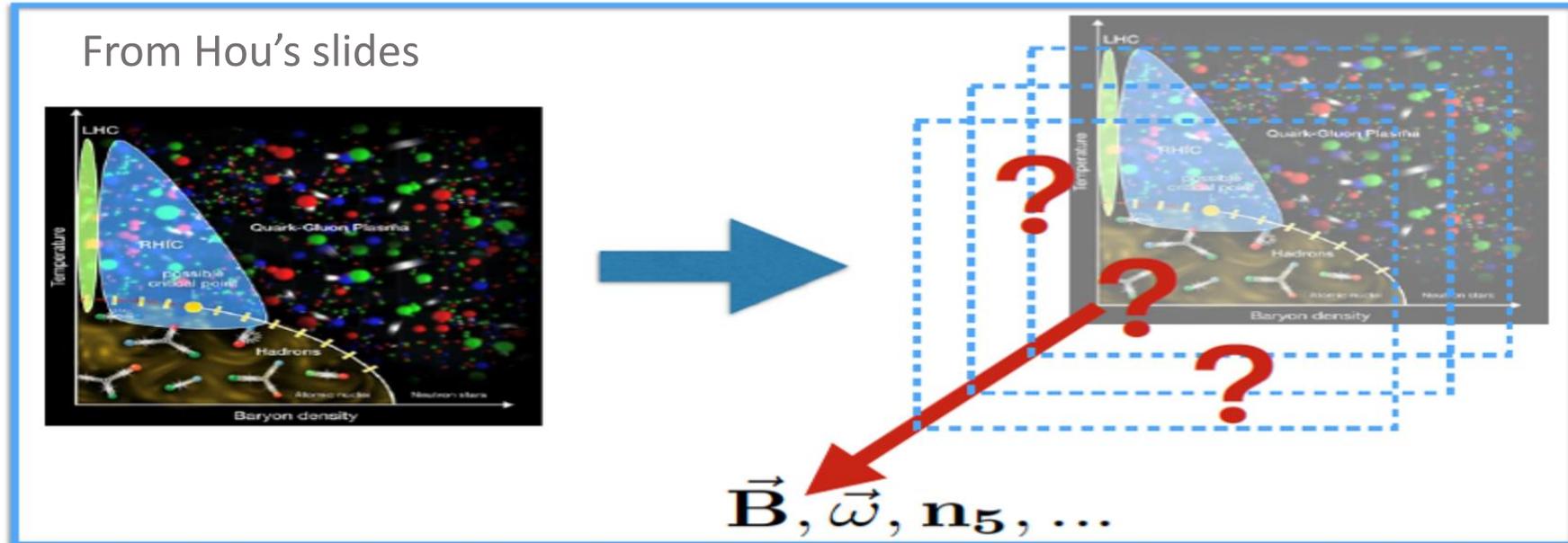
- Can we observe it? **Beam energy scan II**



Stronger v_2 of $\bar{\Lambda}$ than Λ

Rotation induced phase transitions

Vorticity = local rotation. How about a global rotation?



Opening up new dimensions: Toward Hyper-Phase-Diagram!

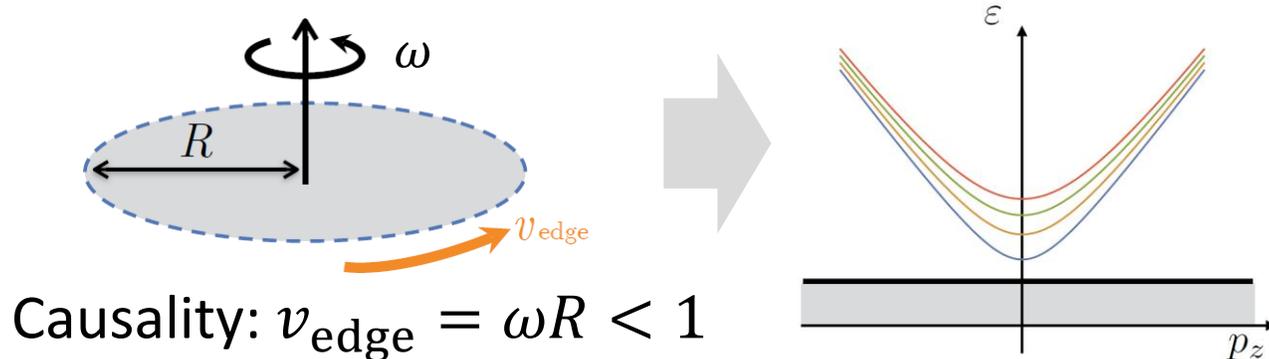
See Hou's talk Nov.05

Rotation induced phase transitions

Analogy and difference between rotation and density

$$H_{\text{rot}} = H - \omega J_z \qquad H_{\mu} = H - \mu N$$

- This indicates ωJ_z plays similar role as chemical potential term μN . However
- Uniformly rotating system must be finite!



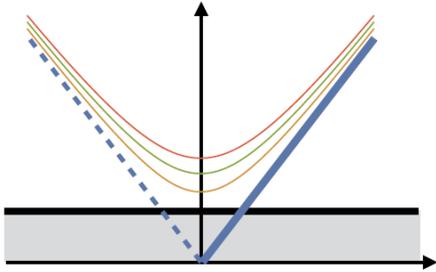
- Excitation gap due to finite size: J_z/R
- Effective chemical potential: $\omega J_z < J_z/R$
- **Pure uniform rotation does not excite any modes**

(Ebihara-Fukushima-Mameda PLB2017)

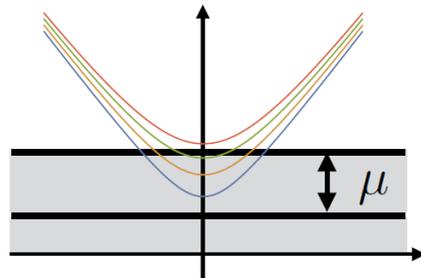
Rotation induced phase transitions

To see uniform rotation effect, we need T, μ, B, \dots

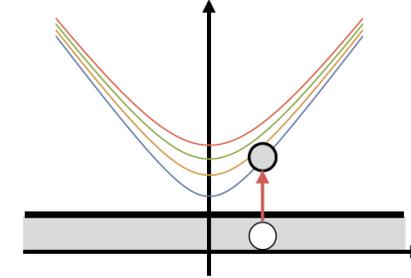
Figures drawn by Mameda



B : Chen etal 2015, Liu-Zahed 2017, Chen-Mameda-XGH 2019, Cao-He 2019, ...

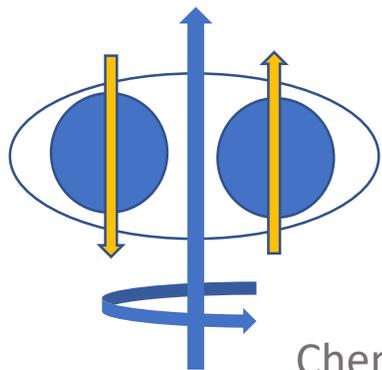


μ : XGH-Nishimura-Yamamoto 2017, Zhang-Hou-Liao 2018, ...

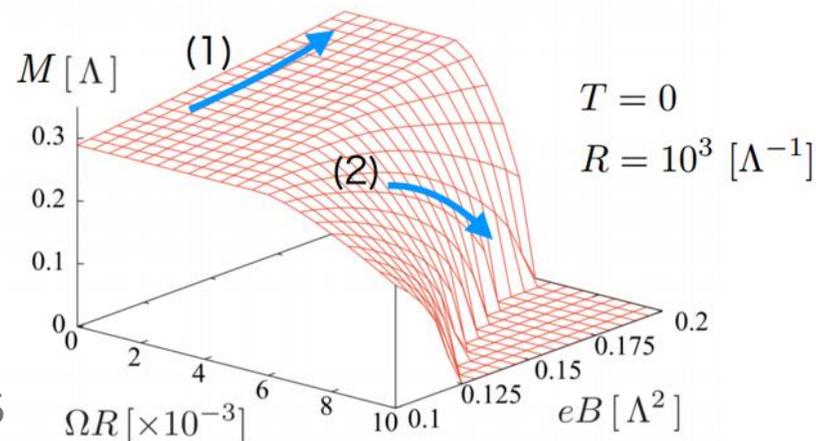


T : Jiang-Liao 2016, Chernodub-Gongyo 2017, Wang etal 2019, ...

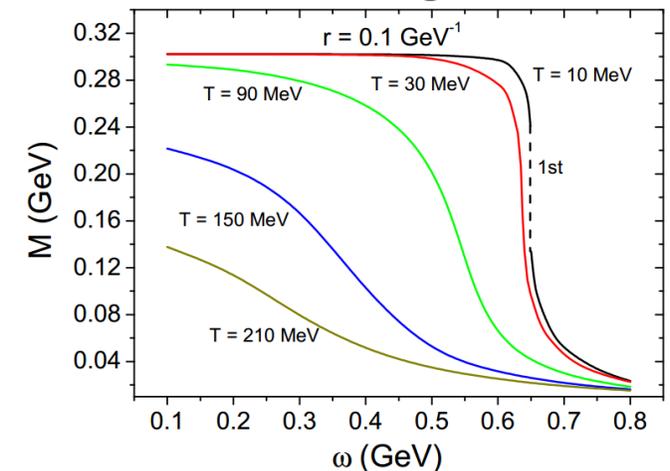
Rotation disfavors spin-0 condensates, e.g., chiral condensate



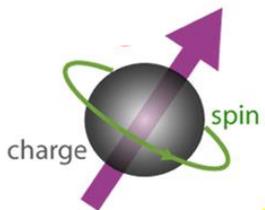
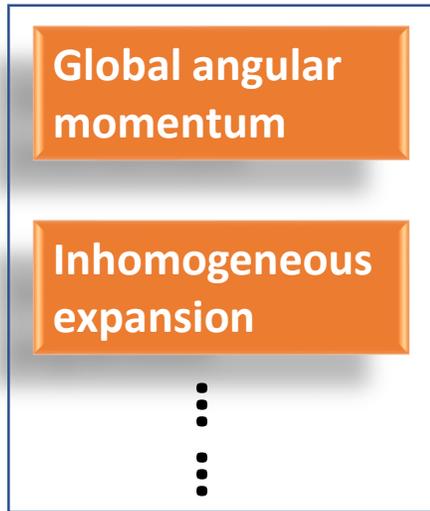
Chen etal 2015



Jiang-Liao 2016



Summary



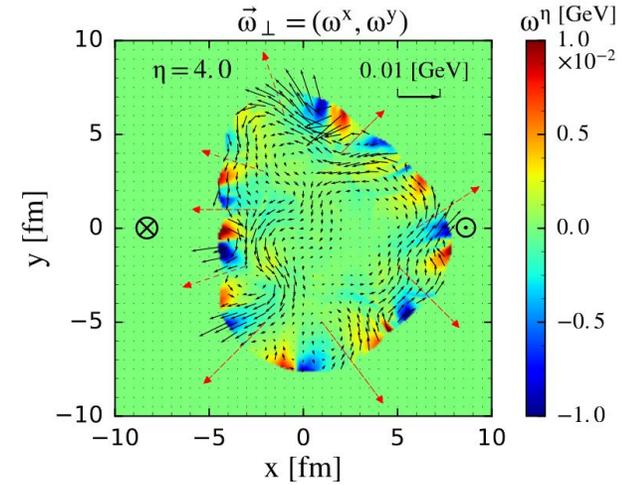
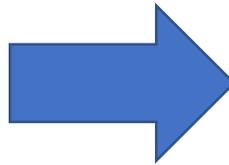
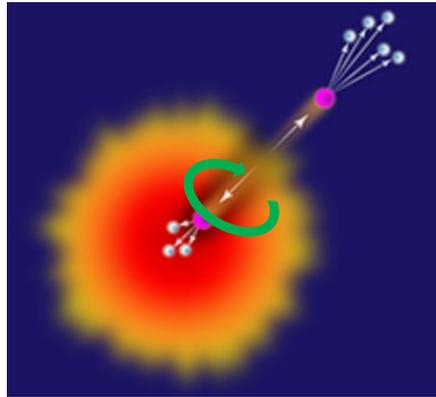
Heavy-ion physics: electronics era to spintronics era
Puzzles, challenges, but opportunities

Thank you

Back up

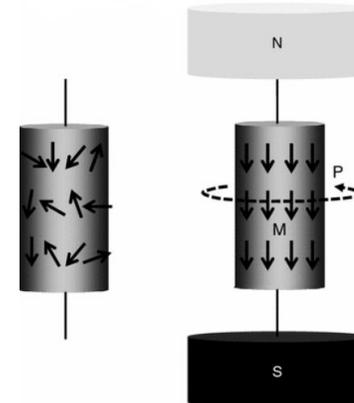
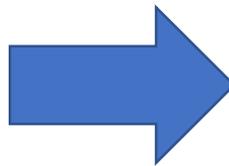
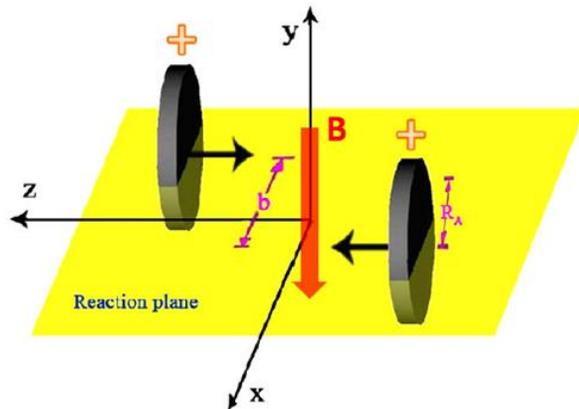
Other sources of vorticity

1) Jet



(Pang-Peterson-Wang-Wang PRL2016)

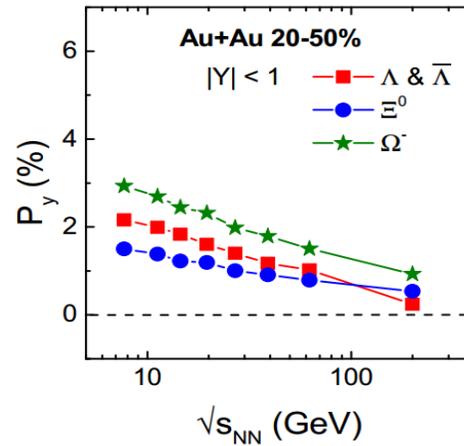
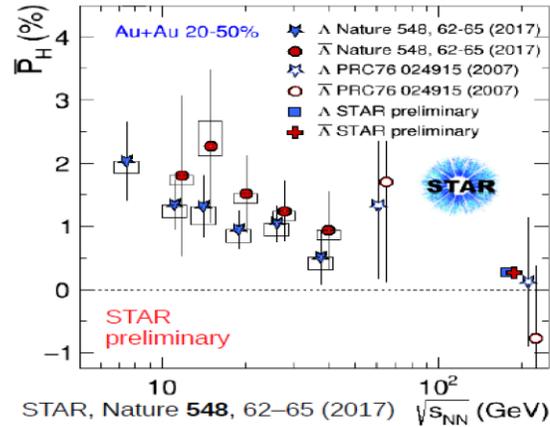
2) Magnetic field



Einstein-de-Haas effect

Can we measure ...

- Is Λ and $\bar{\Lambda}$ polarization difference due to magnetic field?



Magnetic moments $\mu_\Omega : \mu_\Xi : \mu_\Lambda = 3 : 2 : 1$.
Test magnetic contribution.

(Wei-Deng-XGH PRC2019)

- Can we measure the smoke-ring type vortices?

