

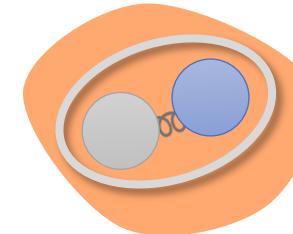
QUARKONIUM PRODUCTION AND SUPPRESSION: THEORY

Alexander Rothkopf

Faculty of Science and Technology
Department of Mathematics and Physics
University of Stavanger



Heavy Quarkonium



a clean
QCD laboratory

Theory advantage: separation of scales
enables powerful effective field theory tools

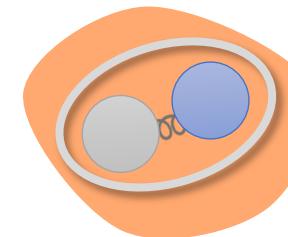
a precision probe
in HIC studies

Experiment advantage: clean signals
via enhanced dilepton decay channels

Heavy Quarkonium

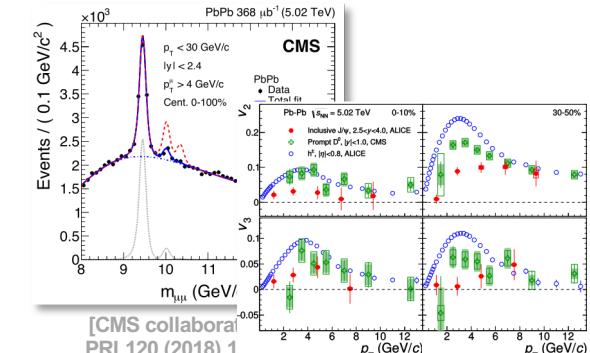
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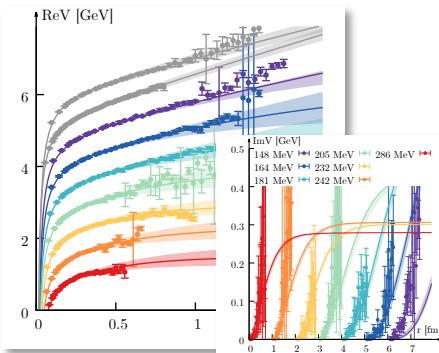


[ALICE collaboration]
JHEP 1902 (2019) 012

Heavy Quarkonium

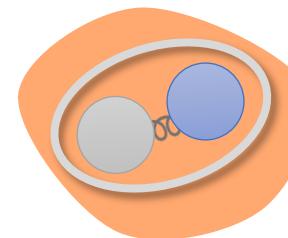
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Y.Burnier, O.Kaczmarek, A.R. PRL114 (2015) 082001
D. Lafferty, A.R., arXiv:1906.00035v3

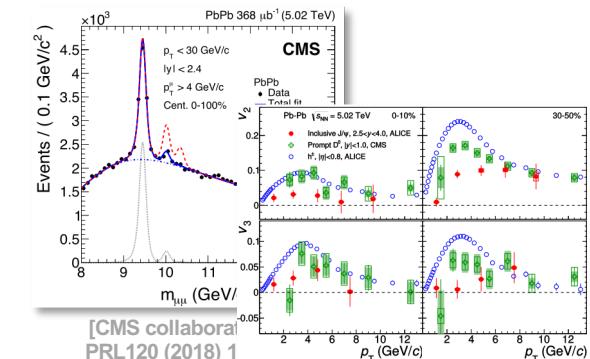
1st principles + intuitive non-relativistic language
(e.g. potential and transport coefficients)



$$\frac{\Lambda_{\text{QCD}}}{m_Q} \ll 1 \quad \frac{\epsilon_{\text{env}}}{m_Q} \ll 1$$

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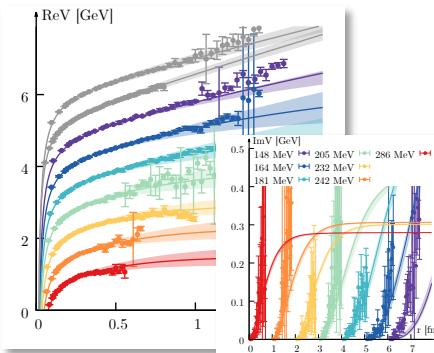


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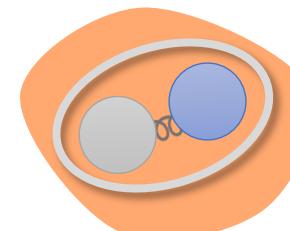
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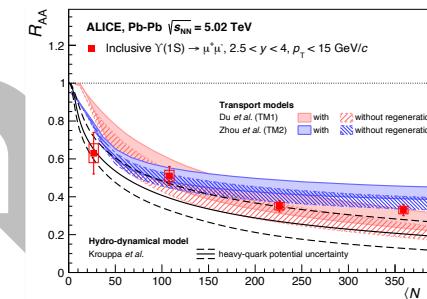


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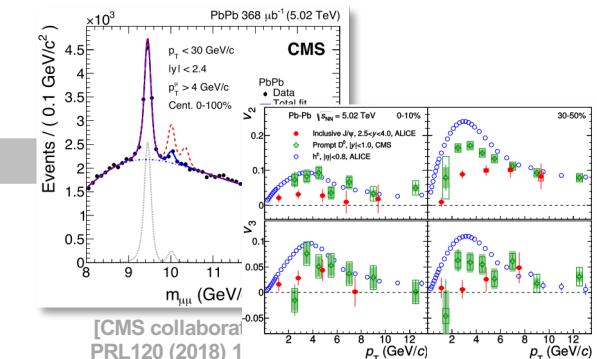
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gain a quantitative understanding of QCD in-medium bound states & QCD medium properties

a precision probe
in HIC studies

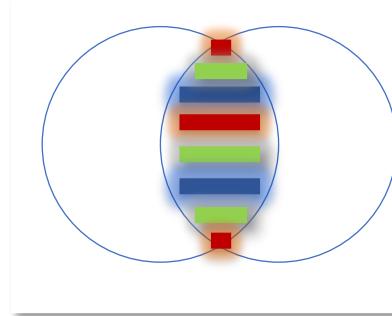
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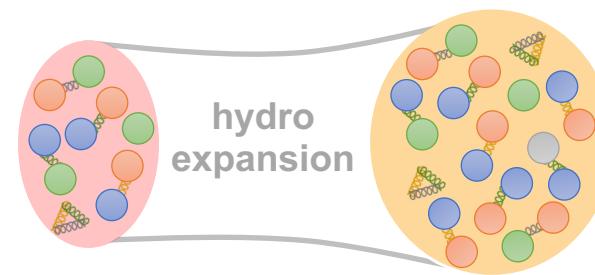
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Quarkonium in HIC challenge

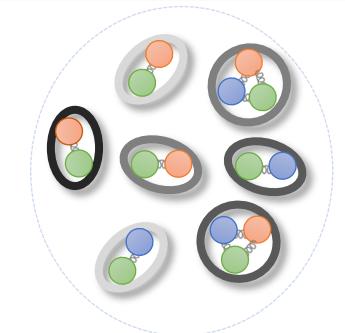
bulk: pre-thermalization



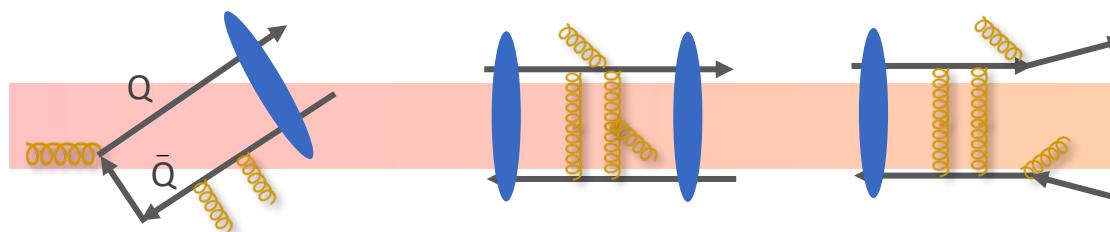
Quark-Gluon-Plasma



hadronization

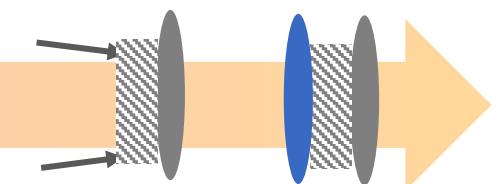


$Q\bar{Q}$: production /formation



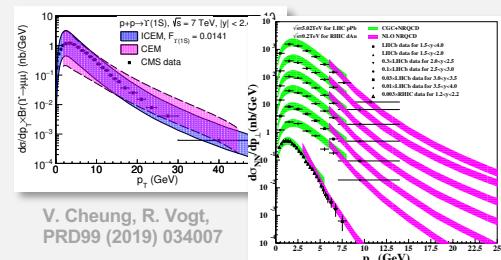
medium interaction

freeze-out



Quarkonium in HIC challenge

Production in pp & pA

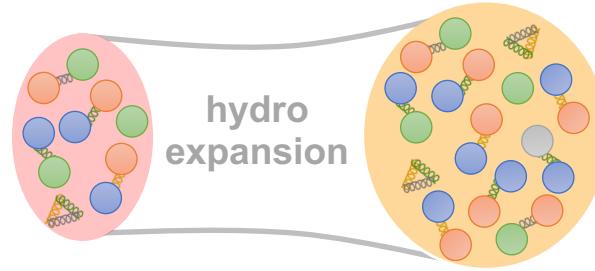


V. Cheung, R. Vogt,
PRD99 (2019) 034007

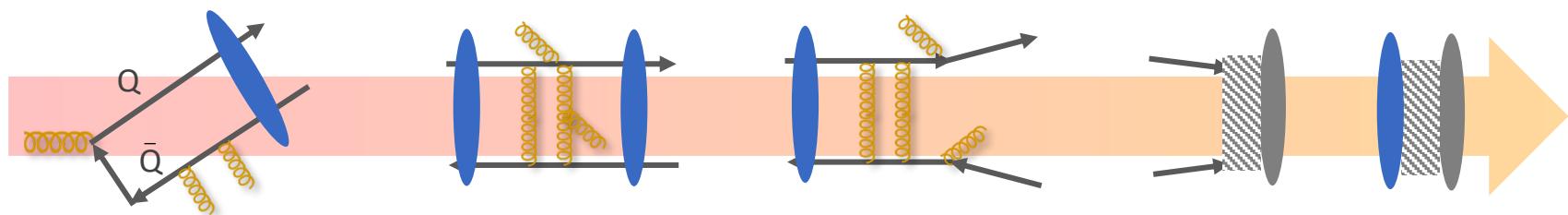
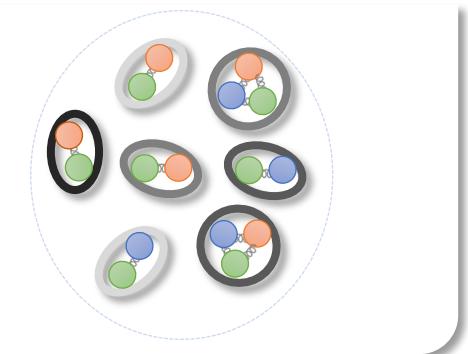
Y.Q. Ma, R. Venugopalan, H.F.
Zhang, PRD92 (2015) 071901

■ factorization &
NRQCD + CGC
ICEM

Quark-Gluon-Plasma



hadronization



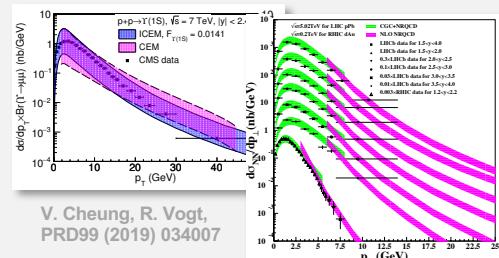
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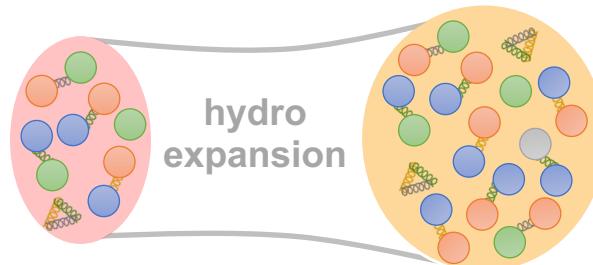


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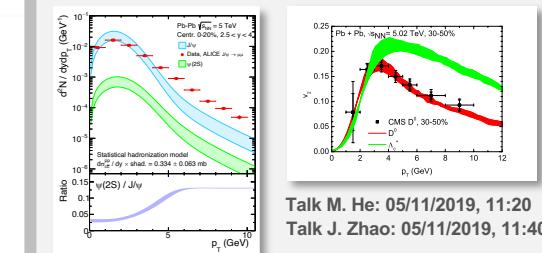
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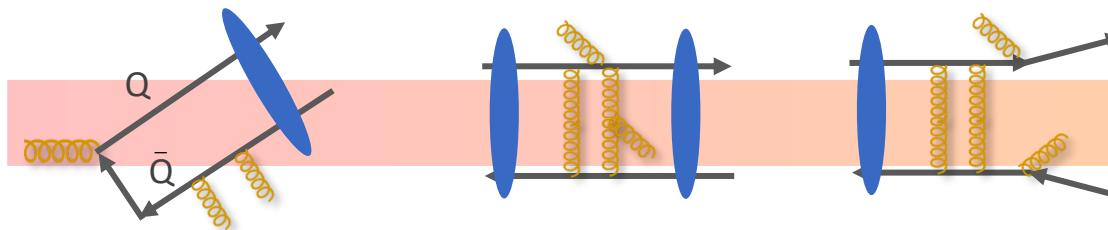
Hadronization



A. Andronic et. al., PLB797 (2019) 134836

Talk M. He: 05/11/2019, 11:20
Talk J. Zhao: 05/11/2019, 11:40

■ statistical hadronization &
coalescence for open heavy flavor



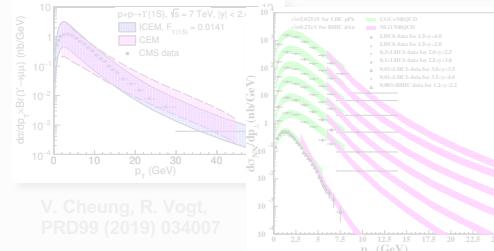
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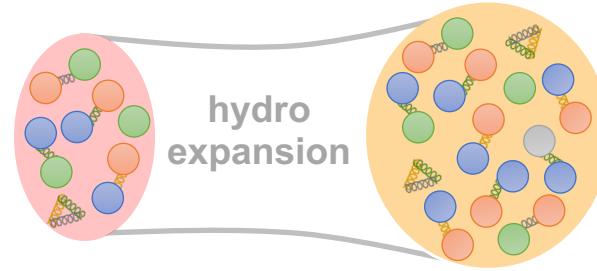


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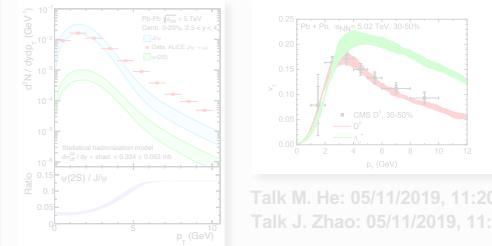
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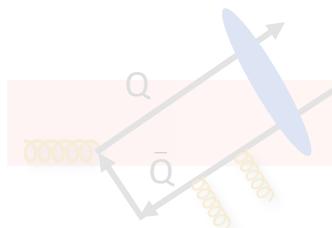
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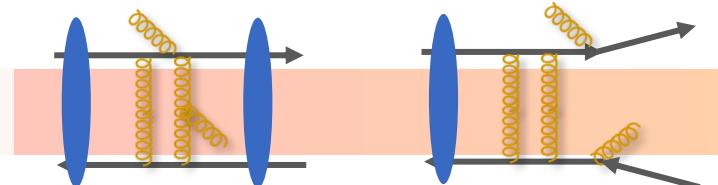
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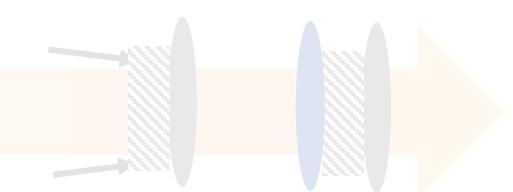
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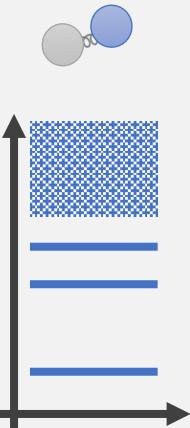


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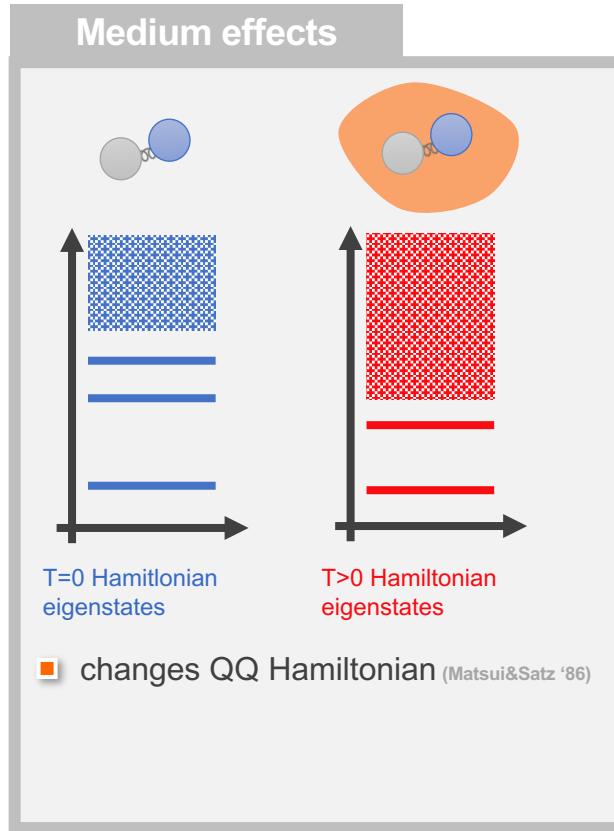
An intuitive non-relativistic picture

University
of StavangerThe Research Council
of Norway

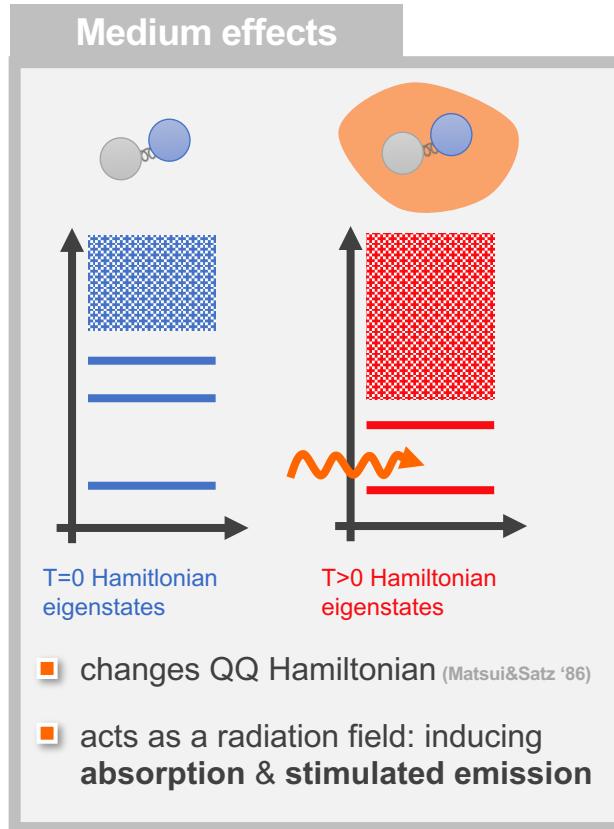
Medium effects

T=0 Hamiltonian
eigenstates

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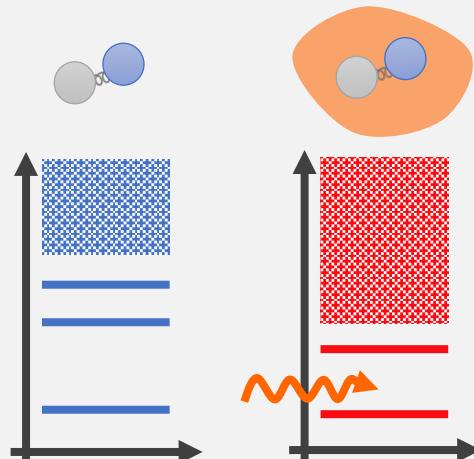


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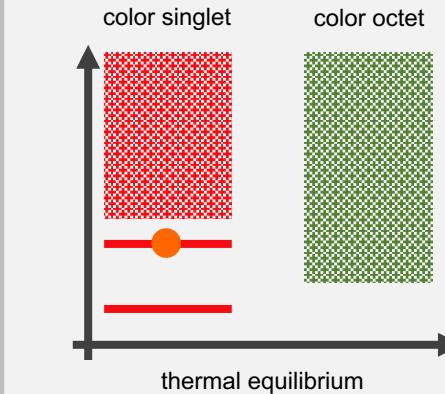


T=0 Hamiltonian
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T>0 Hamiltonian
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- changes QQ Hamiltonian (Matsui&Satz '86)
- acts as a radiation field: inducing **absorption & stimulated emission**

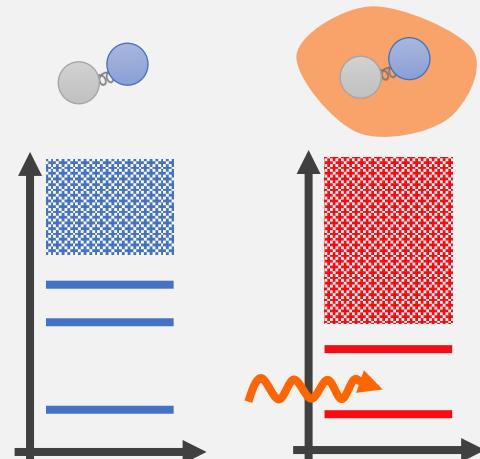
Quarkonium in-medium physics



thermal equilibrium

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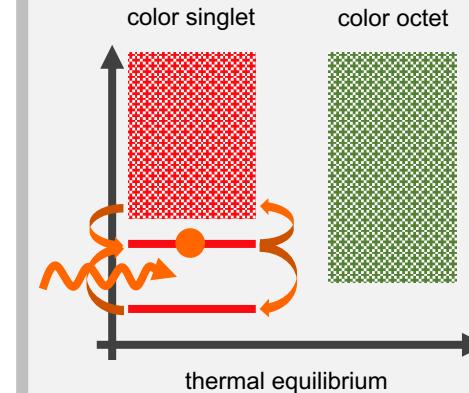


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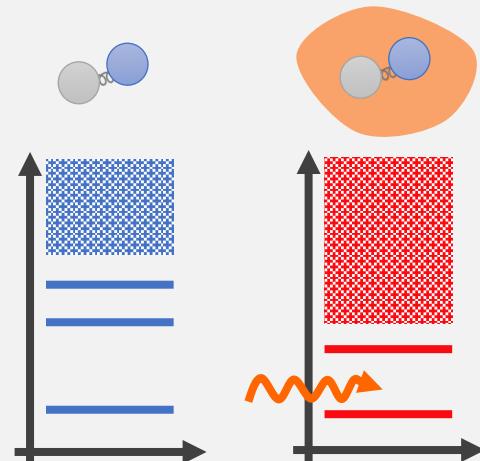
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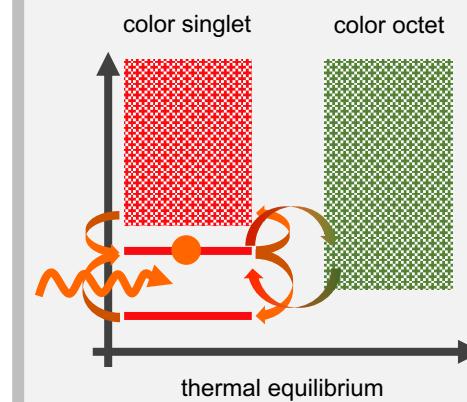


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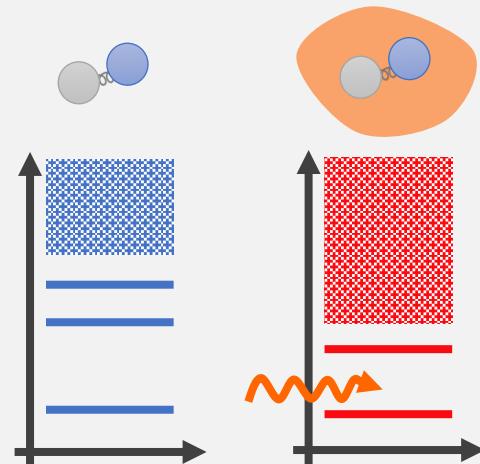
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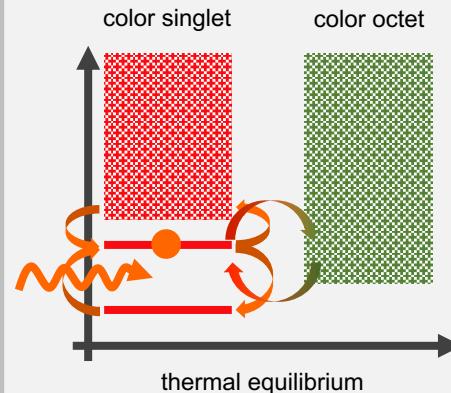


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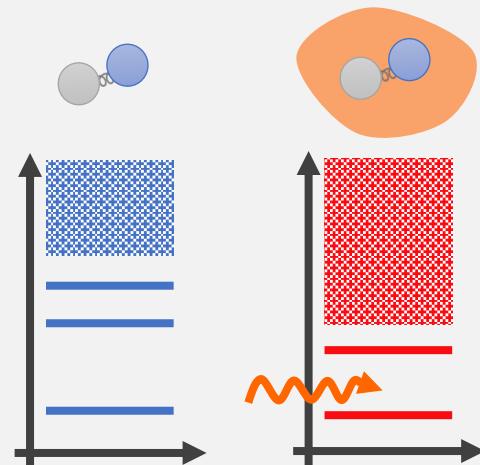


$\rho_{\text{singlet}}(\omega)$

■ lattice QCD **spectral function**
(fully non-perturbative)

An intuitive non-relativistic picture

Medium effects

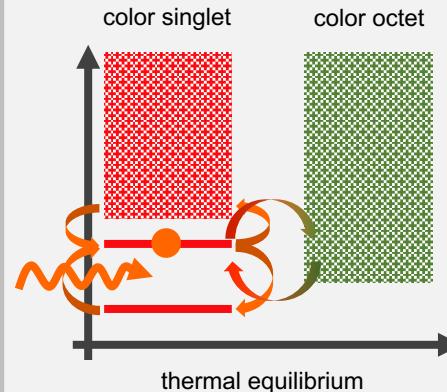


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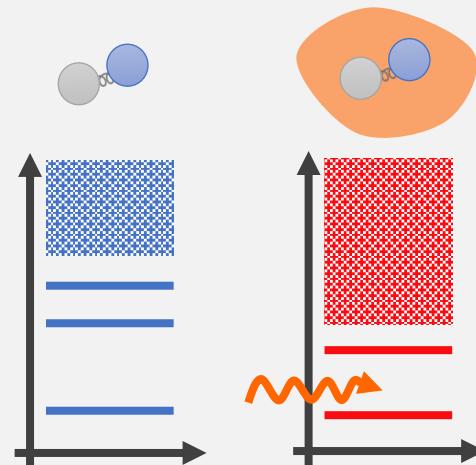
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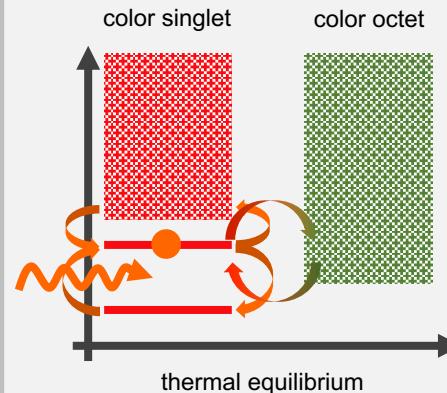


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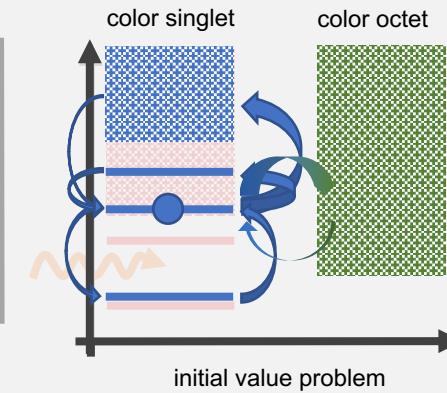
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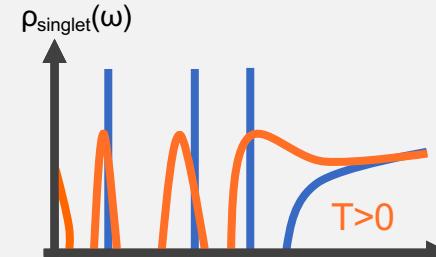
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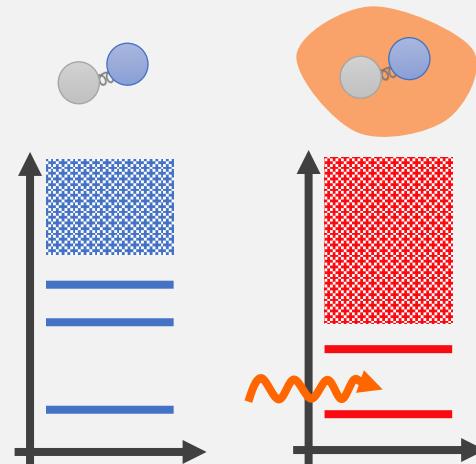
initial value problem



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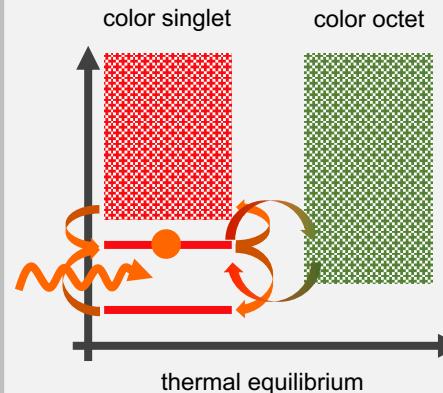


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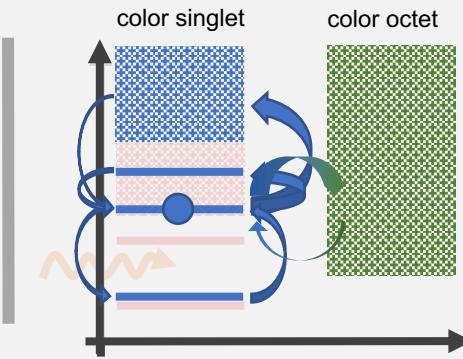
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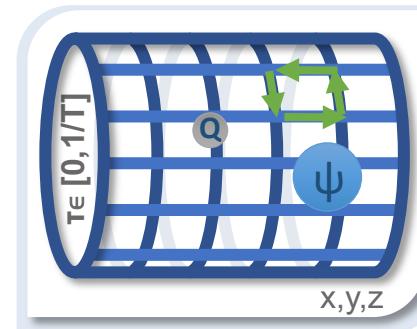
■ lattice QCD **spectral function**
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P_{singlet}

t

■ **density matrix master equation**
(non-perturbative in progress)

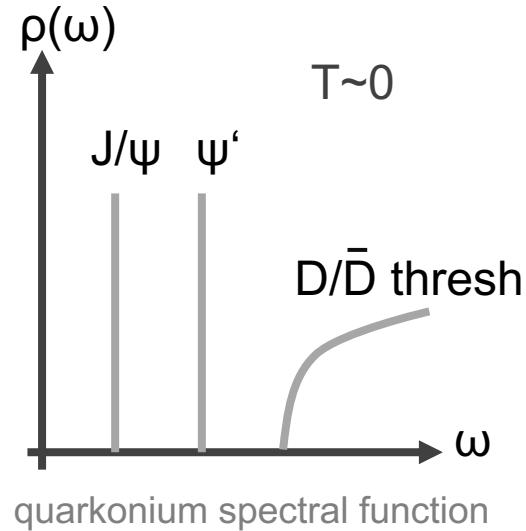
Equilibrium Q \bar{Q} from lattice QCD



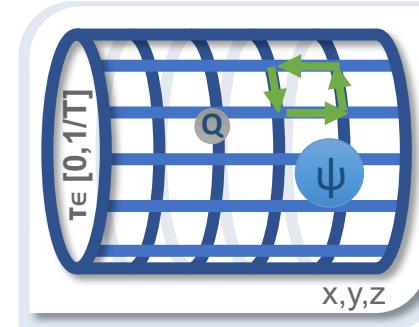
simulations in artificial Euclidean time &
temporal extent related to temperature

- Lattice QCD acts as a very imperfect detector for spectral functions: **ill-posed unfolding** problem

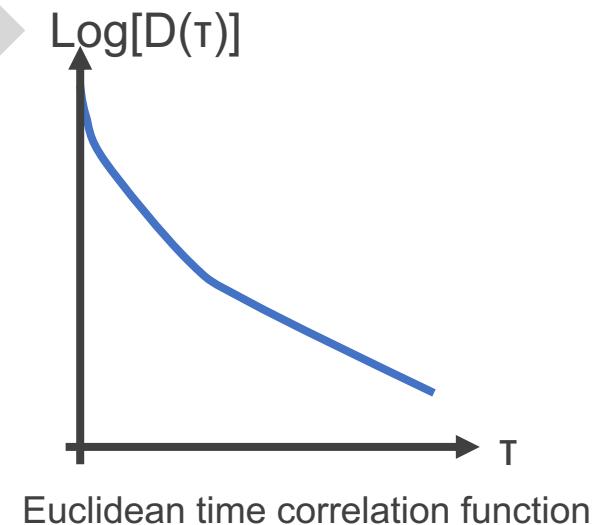
Equilibrium Q \bar{Q} from lattice QCD



$$D(\tau) = \int_{\pi/N_a}^{\pi/a} d\omega \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} \rho(\omega)$$



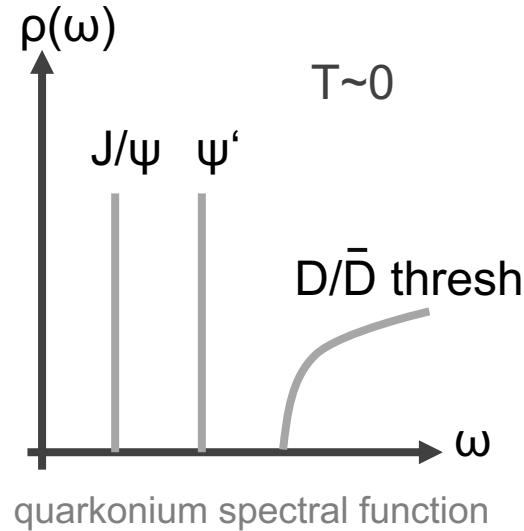
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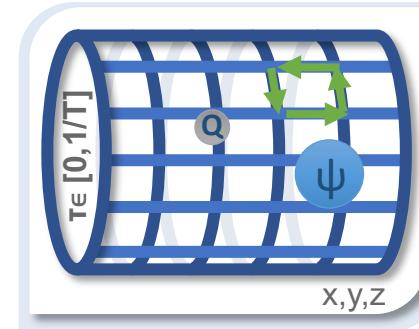
Euclidean time correlation function

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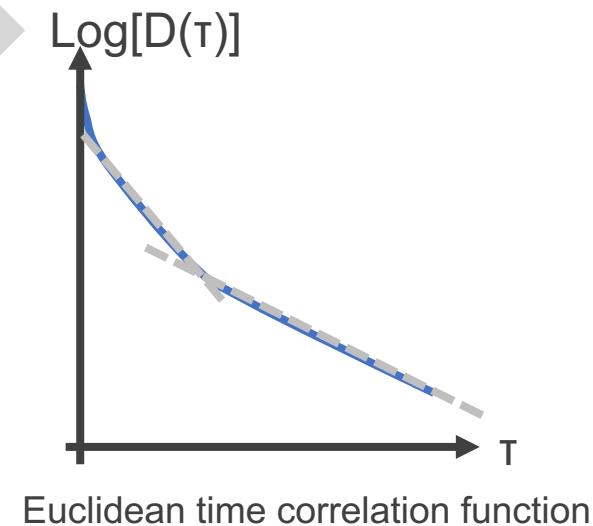
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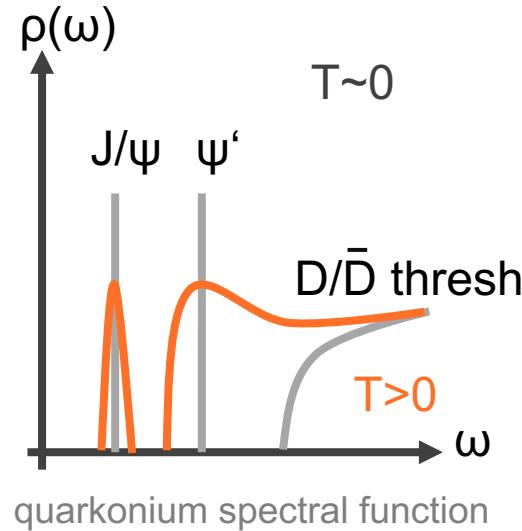
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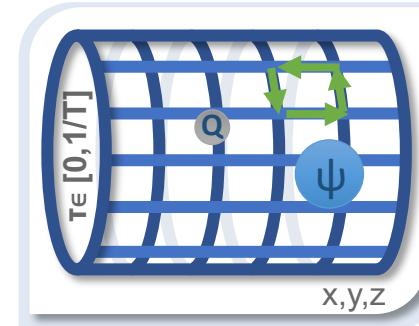
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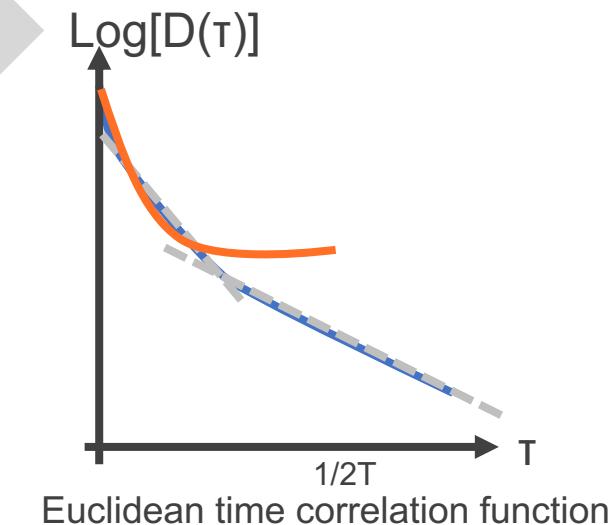
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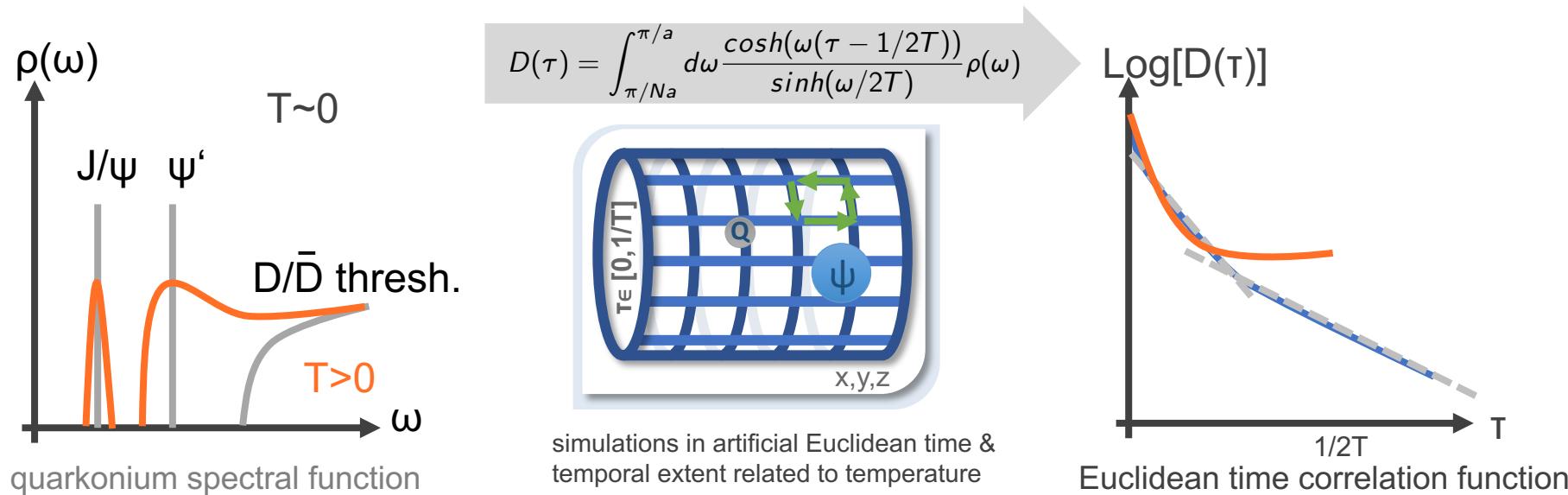


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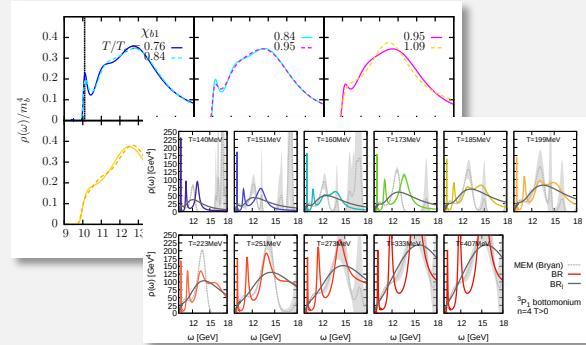
Equilibrium Q \bar{Q} from lattice QCD



- Lattice QCD acts as a very imperfect detector for spectral functions: **ill-posed unfolding** problem
- Exploit generic QCD **domain knowledge** (e.g. positivity) to regularize inversion: **Bayesian inference (MEM,BR)**
prior knowledge does not include specific structures but require very high quality simulation data
- **Model the spectral function** using EFT and perturbation theory and extract parameters
gives access to excited states and transport physics but introduces model dependence

Recent progress from the lattice

Bayesian reconstructions

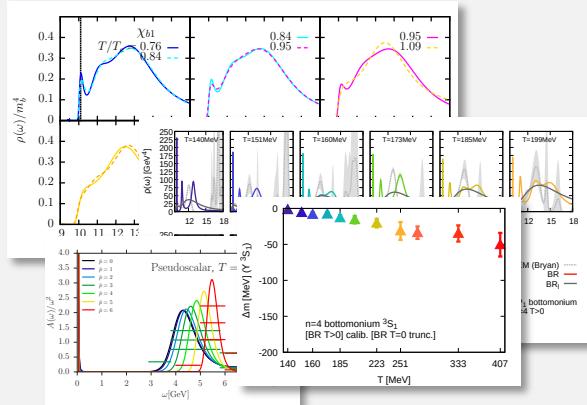


FASTSUM collaboration JHEP 1407 (2014) 097, S. Kim, P. Petrecy, A.R. JHEP 1811 (2018) 088, A. Ikeda et. al. PRD95 (2017) 014504

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stability estimate & sequential melting
(beware melting \neq suppression)
- recent NRQCD: negative mass shifts
- No excited states result: exponentially hard

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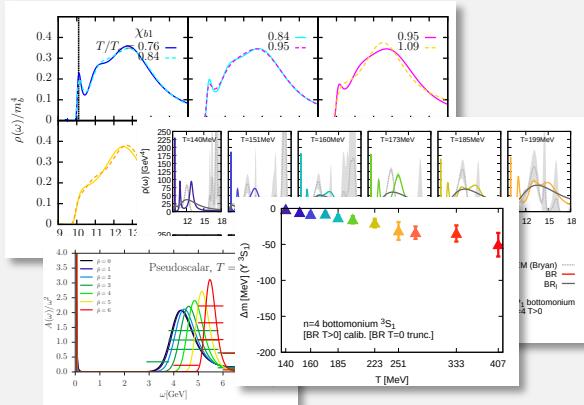


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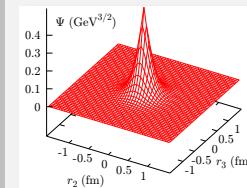
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Spectral modelling

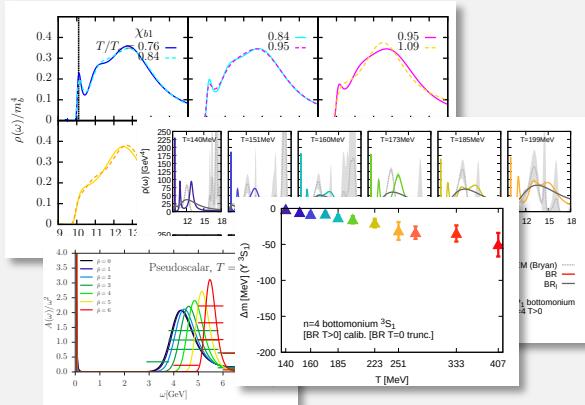


Talk: R. Larsen 06/11/2019, 17:00

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- extend techniques from T=0 lattice QCD to finite T.

Recent progress from the lattice

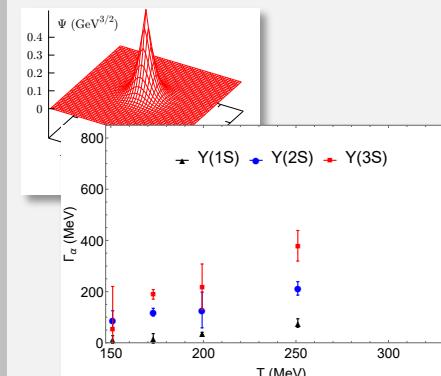
Bayesian reconstructions



FASTSUM collaboration JHEP 1407 (2014) 097, S. Kim, P. Petrecy, A.R. JHEP 1811 (2018) 088, A. Ikeda et al. PRD 95 (2017) 014504

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- recent NRQCD: negative mass shifts
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Spectral modelling



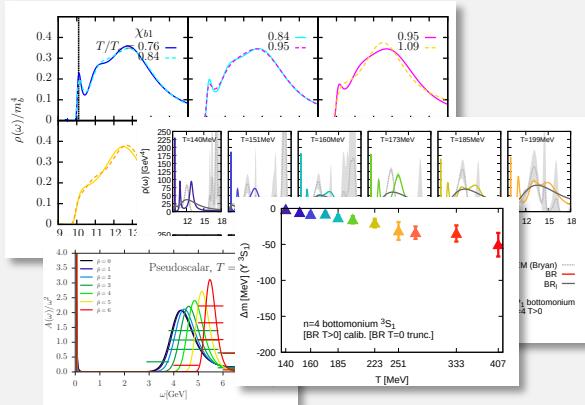
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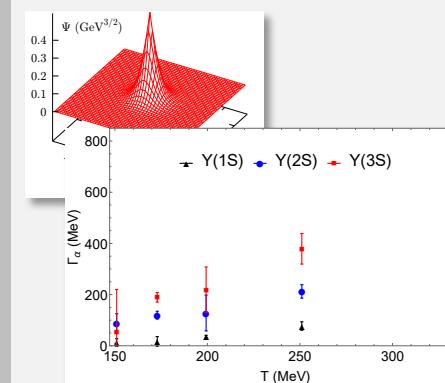
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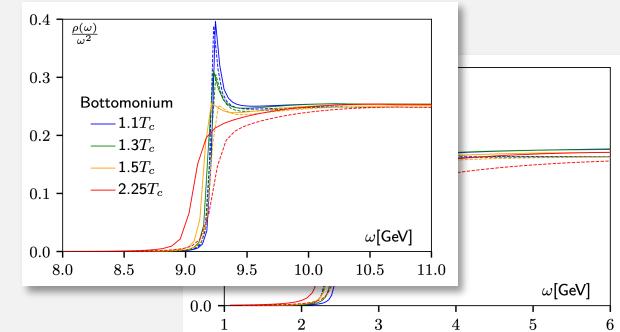
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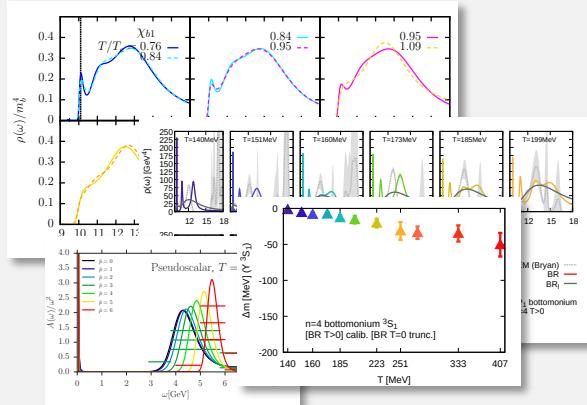
Talk: O. Kaczmarek 05/11/2019, 14:40

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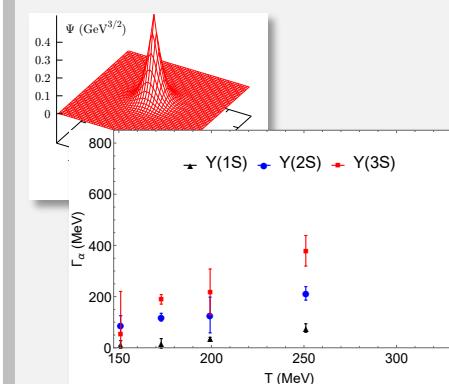
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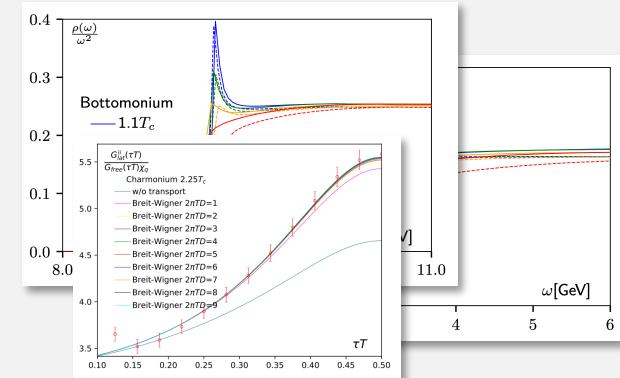
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- relativistic correlator gives access to transport physics ($D > 2/2\pi T$)

In-medium heavy-quark potential

University
of Stavanger The Research Council
of Norway

EFT potential

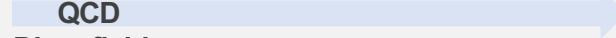
- systematic expansion of QCD heavy quark Lagrangian

$$\frac{\Lambda_{\text{QCD}}}{m_Q} \ll 1$$

Brambilla et.al. Rev.Mod.Phys. 77 (2005) 1423 m

QCD**Dirac fields**

$$\frac{\varepsilon_{\text{env}}}{m_Q} \ll 1$$

 $Q(x), \bar{Q}(x)$ 

In-medium heavy-quark potential

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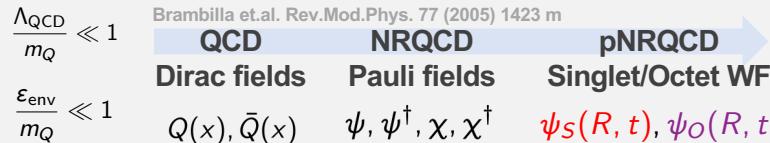
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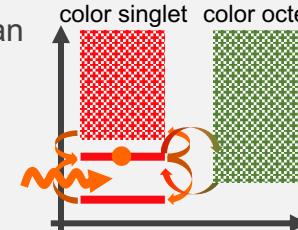
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Dirac fields	Pauli fields	Singlet/Octet WF
$Q(x), \bar{Q}(x)$	$\psi, \psi^\dagger, \chi, \chi^\dagger$	$\psi_s(R, t), \psi_o(R, t)$



$$\begin{aligned} L_{\text{pNRQCD}} = & \int d^3 r \text{Tr} \left[\psi_s^\dagger \left[i\partial_0 - \left(\frac{\mathbf{P}^2}{2M} + V_s^{(0)} + \mathcal{O}\left(\frac{1}{m_Q}\right) \right) \right] \psi_s + \psi_o^\dagger \left[iD_0 - \left(\frac{\mathbf{P}^2}{2M} + V_o^{(0)} + \mathcal{O}\left(\frac{1}{m_Q}\right) \right) \right] \psi_o \right] \\ & + V_A(r) \text{Tr} \left[\psi_o^\dagger \mathbf{r} g \mathbf{E} \psi_s + \psi_s^\dagger \mathbf{r} g \mathbf{E} \psi_o \right] + \mathcal{O}(r^2, \frac{1}{m_Q}) \end{aligned}$$

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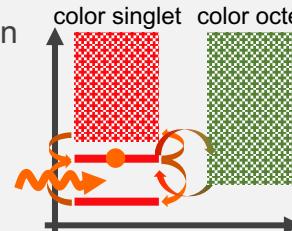
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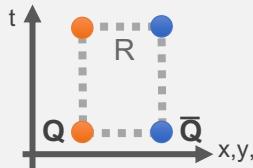
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$$V(R) = \lim_{t \rightarrow \infty} \frac{i\partial_t W_\square(R, t)}{W_\square(R, t)}$$



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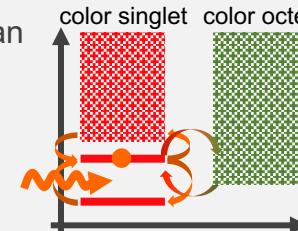
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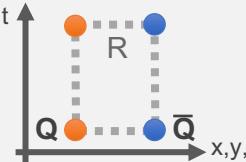


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University
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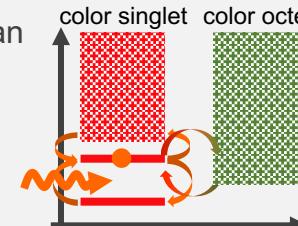
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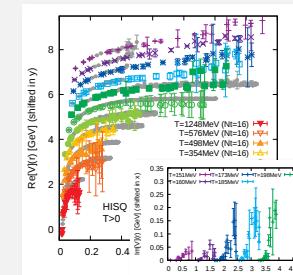
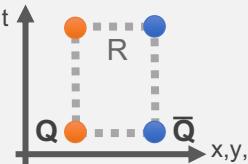
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NPA982 (2019) 735

■ $V(R)$ non-perturbatively via lattice QCD spectral functions: unique QCD result

A.R., T.Hatsuda, S.Sasaki PRL108 (2012) 162001, Y.Burnier, O.Kaczmarek, A.R. PRL114 (2015) 082001 (see also poster: N. Barnard ($V(R)$ in AdS/CFT))

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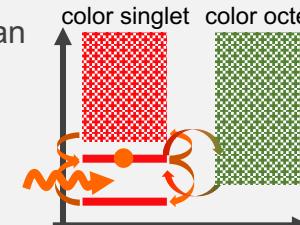
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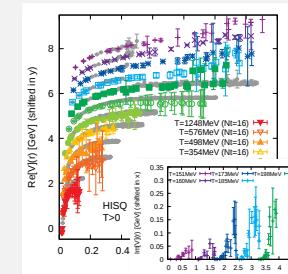
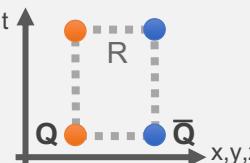


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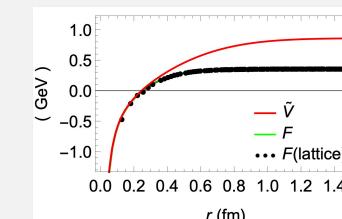
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T-Matrix potential

- sets out to describe physics of light and heavy quarks

$$\begin{aligned} H = & \sum \epsilon_i(\mathbf{p}) \psi_i^\dagger(\mathbf{p}) \psi_i(\mathbf{p}) + \\ & \frac{1}{2} \psi_i^\dagger \left(\frac{\mathbf{P}}{2} - \mathbf{p} \right) \psi_j^\dagger \left(\frac{\mathbf{P}}{2} + \mathbf{p} \right) V_{ij}^a \psi_j \left(\frac{\mathbf{P}}{2} + \mathbf{p}' \right) \psi_i \left(\frac{\mathbf{P}}{2} - \mathbf{p}' \right) \\ & \boxed{\tau} = \boxed{\tau} + \boxed{\tau} + \boxed{\tau} + \dots \end{aligned}$$

- $\text{Re}[V^\tau]$ fitted by comparison to e.g. lattice QCD free energy

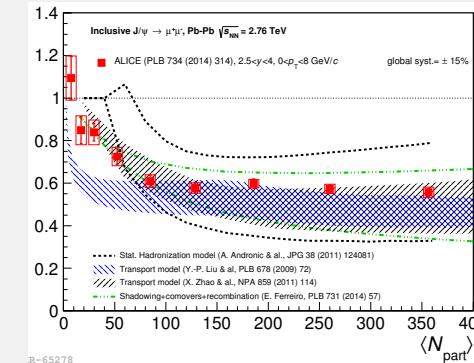


S. Y. F. Liu, R. Rapp, PRC97 (2018) 034918

- deviation from EFT potential OK, two different quantities

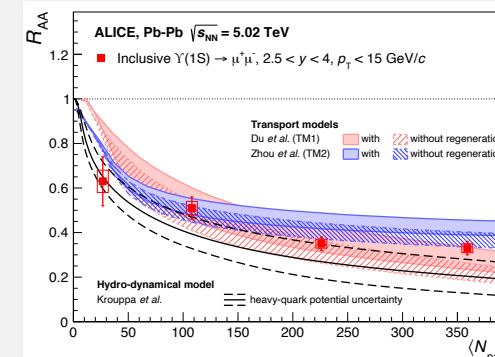
Necessity of 1st principles dynamics

Transport models - Charmonium



$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T) [N_\psi - N_\psi^{eq}(T)]$$

Schrödinger Eq. - Bottomonium



$$i\partial_t \psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R, T) - iIm[V](R, T) \right] \psi$$

- For quantitative inference of physics: need first principles based control on range of validity and parameters of models
- **Theory goal:** derive more accurate real-time descriptions from QCD

Langevin for heavy quarks

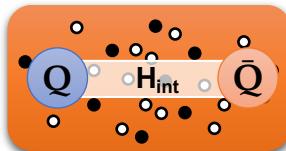
$$dx_i = \frac{p_i}{E} dt, \quad dp_i = -\Gamma p_i dt + C_{ij} \eta_j \sqrt{dt}$$

Open-quantum-systems

- Require **general real-time approach** for quarkonium coupled to a thermal medium

$$H = H_{Q\bar{Q}} \otimes I_{med} + I_{Q\bar{Q}} \otimes H_{med} + H_{int}$$

overall system is closed, hermitean Hamiltonian



Open-quantum-systems

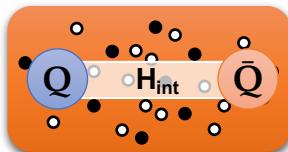
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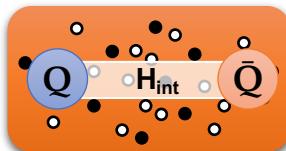
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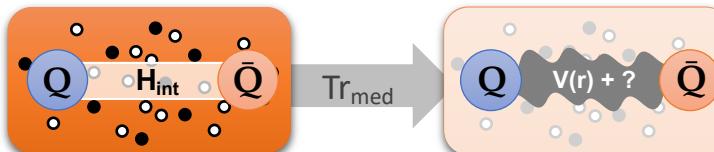
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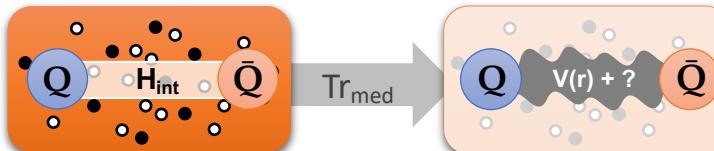
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Goal: Dynamics of reduced density matrix

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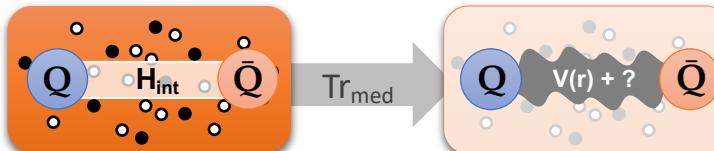
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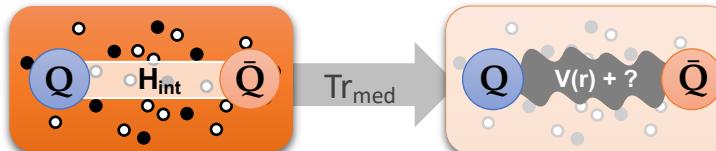
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coarse graining leads to **non-reversible dynamics** from QCD

- Separation of time-scales** determines the nature of the e.o.m. :

Environment relaxation scale τ_E :

$$\langle \Xi_m(t) \Xi_m(0) \rangle \sim e^{-t/\tau_E}$$

$Q\bar{Q}$ system scale τ_S :

$$\tau_S \sim 1/|\omega - \omega'|$$

$Q\bar{Q}$ relaxation scale τ_{rel} :

$$\langle \rho(t) \rangle \propto e^{-t/\tau_{rel}}$$

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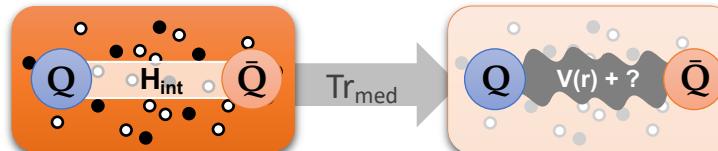
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von Neumann equation



$$\rho_{Q\bar{Q}} = \text{Tr}_{med} [\rho]$$

$$\frac{d}{dt} \rho_{Q\bar{Q}} = ?$$

Goal: Dynamics of reduced density matrix

coarse graining leads to **non-reversible dynamics** from QCD

- Separation of time-scales** determines the nature of the e.o.m. :

Environment relaxation scale τ_E :	$Q\bar{Q}$ system scale τ_S :	$Q\bar{Q}$ relaxation scale τ_{rel} :
$\langle \Xi_m(t) \Xi_m(0) \rangle \sim e^{-t/\tau_E}$	$\tau_S \sim 1/ \omega - \omega' $	$\langle \rho(t) \rangle \propto e^{-t/\tau_{rel}}$

- In case of Markovian time evolution ($\tau_E \ll \tau_{rel}$) leads to a **Lindblad equation**:

$$\frac{d}{dt} \rho_{Q\bar{Q}} = -i[\tilde{H}_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \sum_k \gamma_k \left(L_k \rho_{Q\bar{Q}} L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho_{Q\bar{Q}} - \frac{1}{2} \rho_{Q\bar{Q}} L_k^\dagger L_k \right)$$

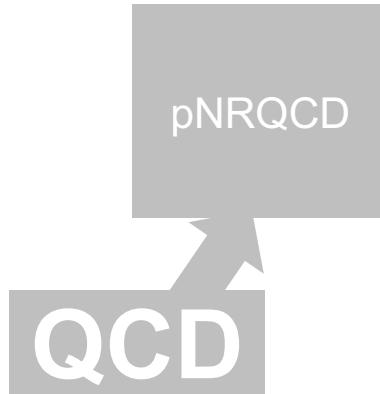
$$\langle n | \rho_{Q\bar{Q}} | n \rangle > 0, \forall n$$

$$\rho_{Q\bar{Q}}^\dagger = \rho_{Q\bar{Q}}, \quad \text{Tr}[\rho_{Q\bar{Q}}] = 1$$

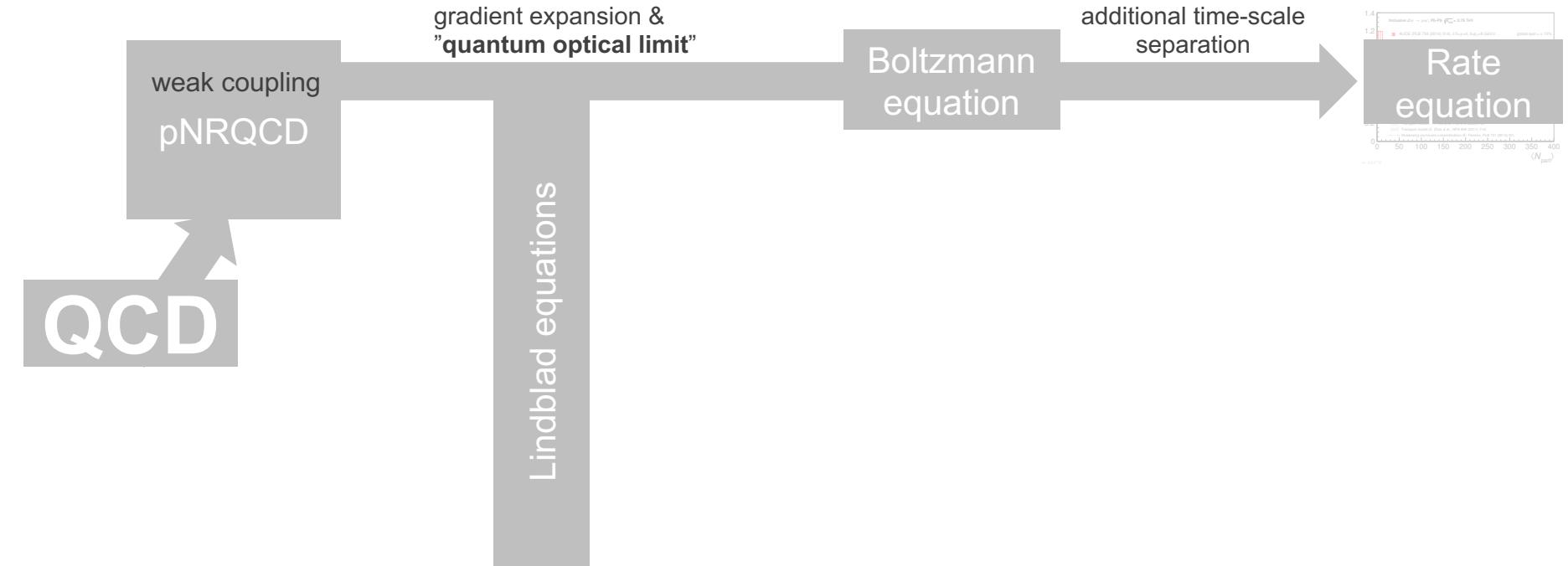
Current real-time approaches

QCD

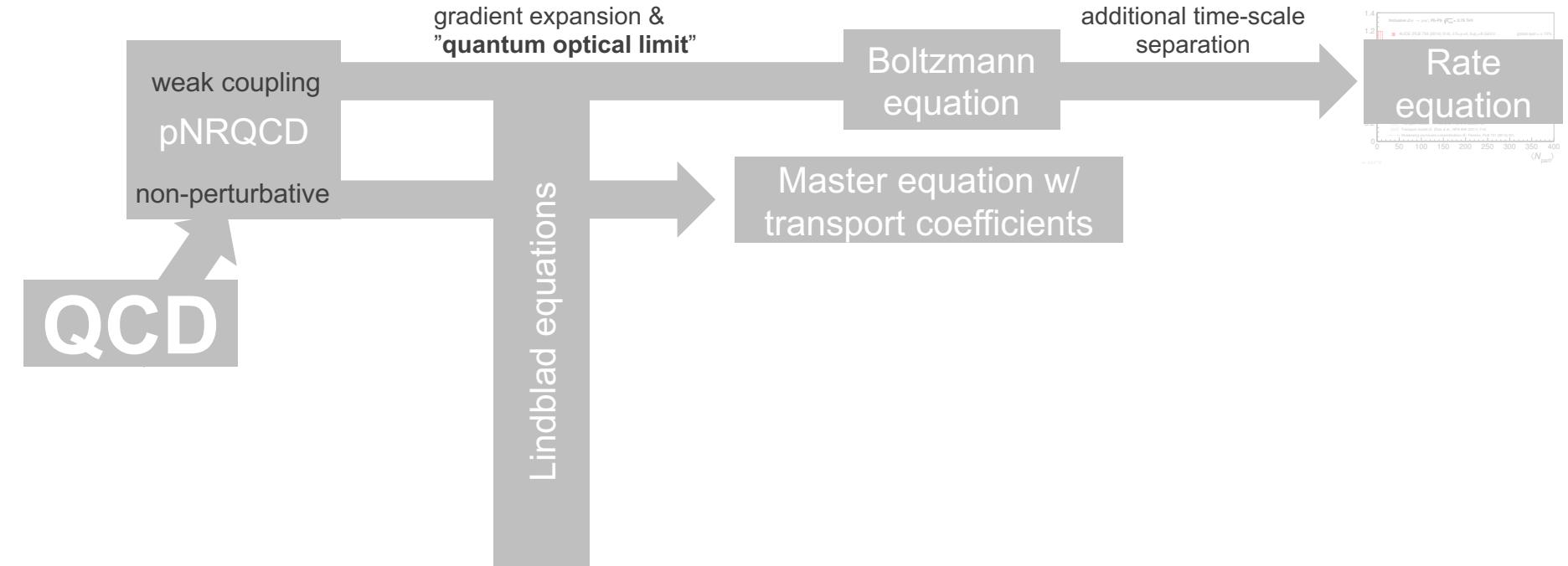
Current real-time approaches



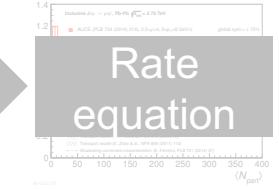
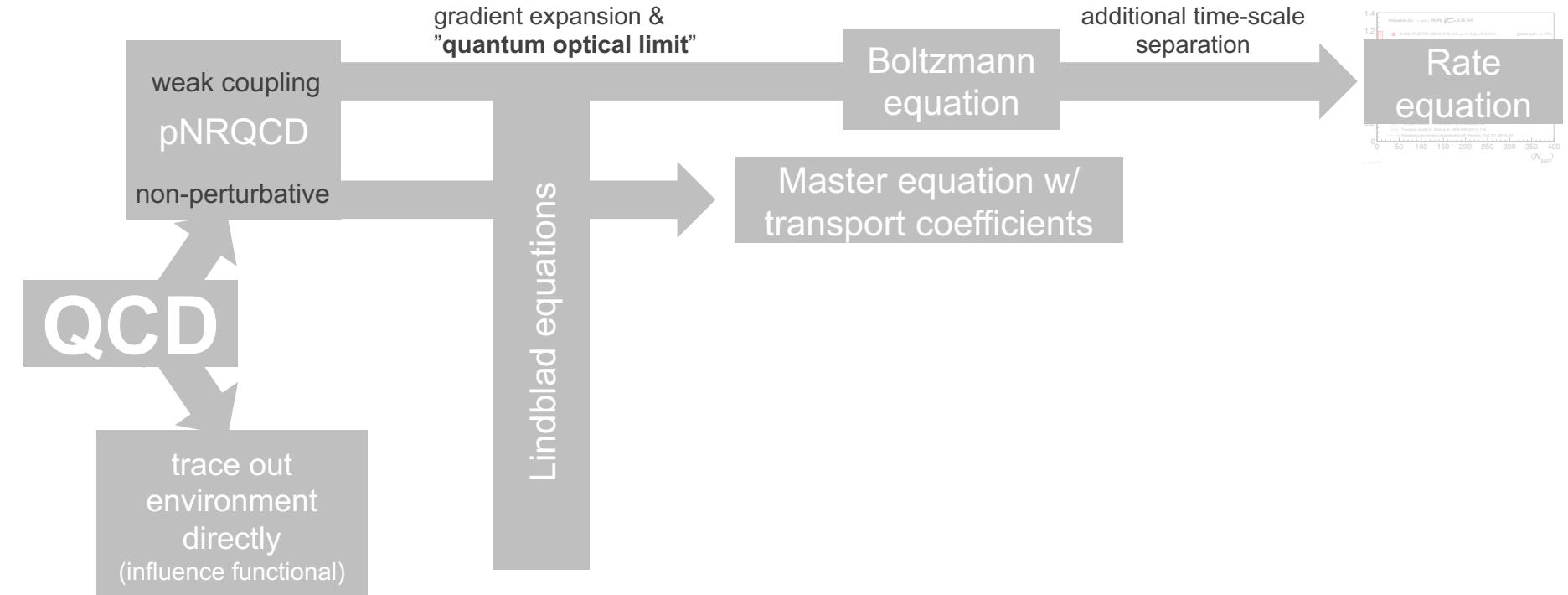
Current real-time approaches



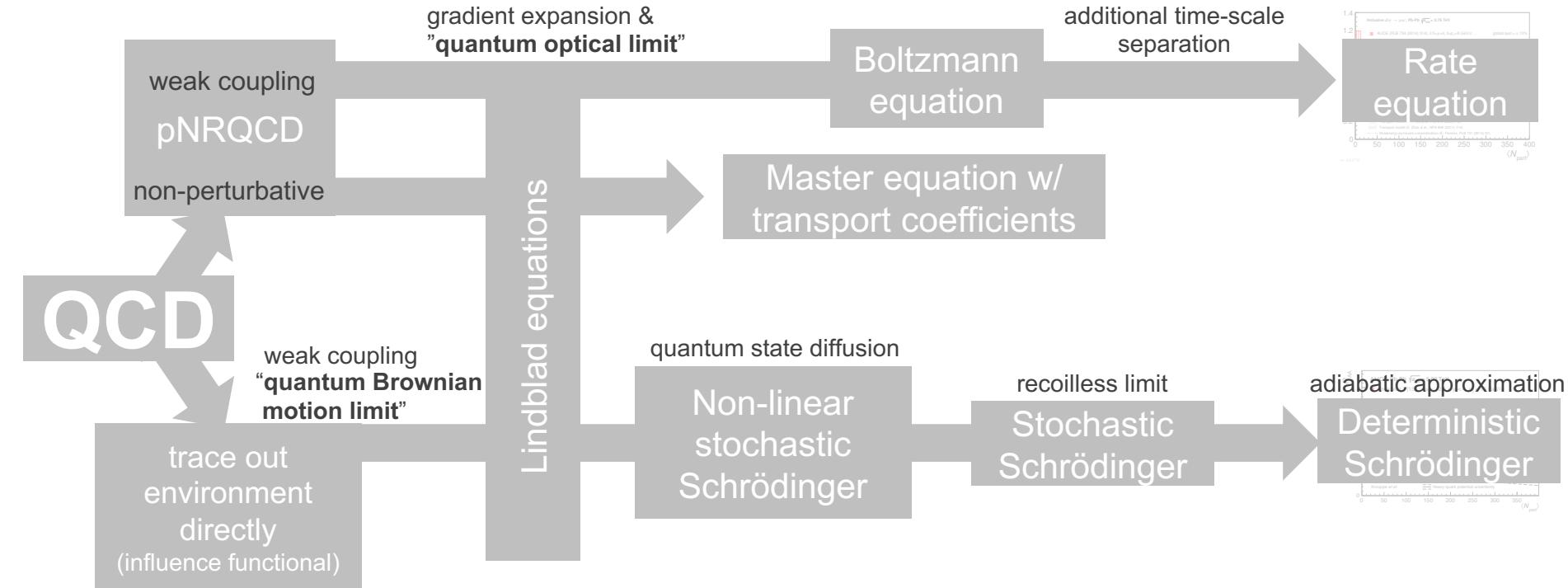
Current real-time approaches



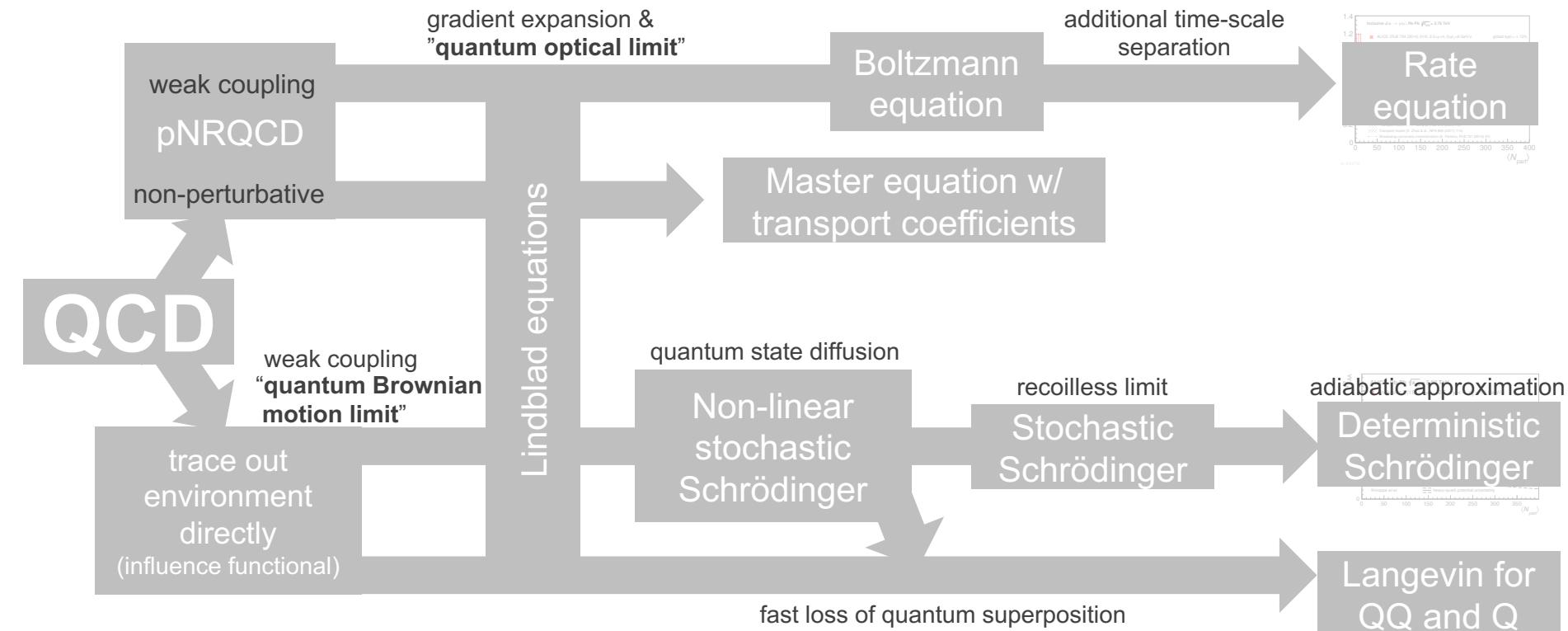
Current real-time approaches



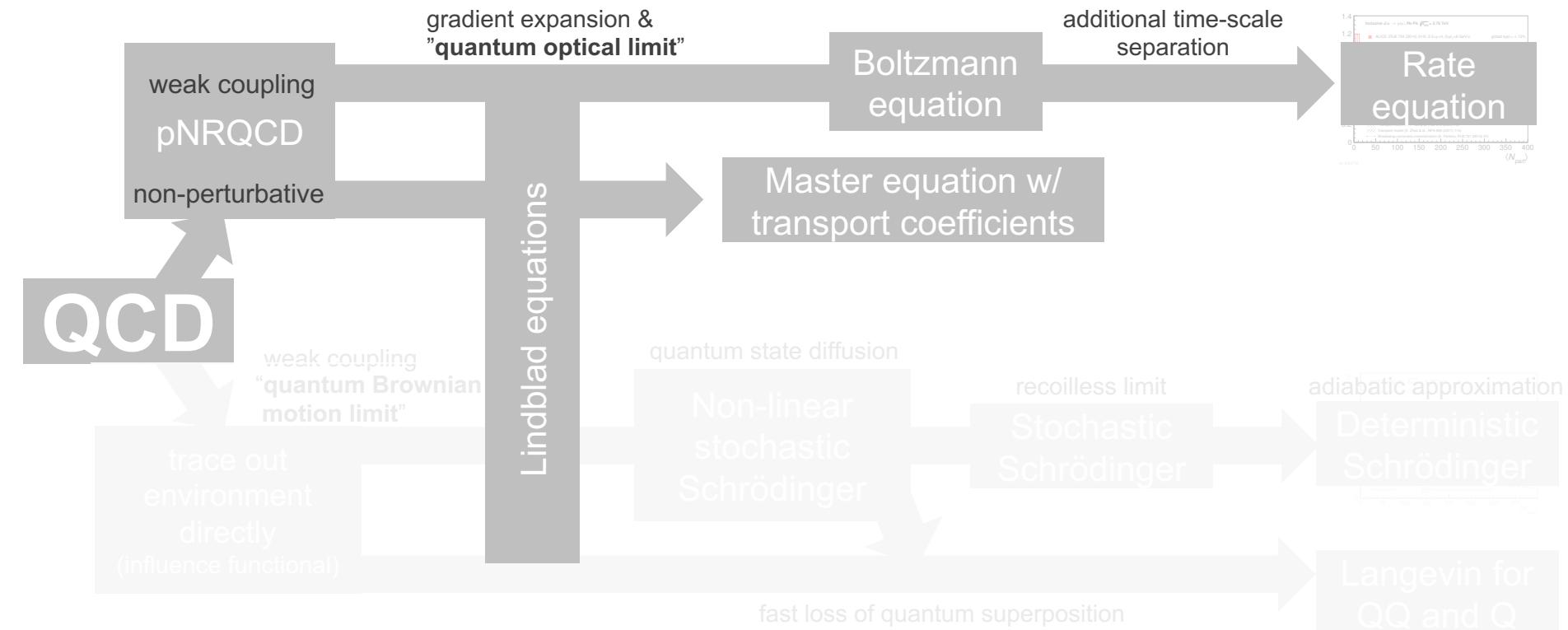
Current real-time approaches



Current real-time approaches



Current real-time approaches



pNRQCD based approaches

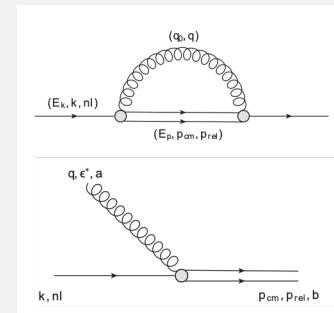
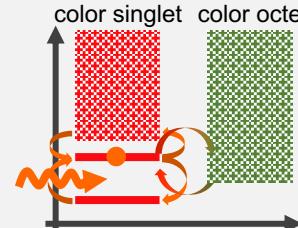
pNRQCD & weakly coupled medium

$$\text{Tr}[\psi_O^\dagger \mathbf{r} g \mathbf{E} \psi_S + \psi_S^\dagger \mathbf{r} g \mathbf{E} \psi_O]$$

$$\Sigma_m \equiv \text{Re}[\langle \psi_S(\mathbf{R}, t) | r_i | \psi_O^a(\mathbf{R}, t) \rangle] \quad \Xi_m \equiv \sqrt{\frac{T_F}{N_C}} g E_i^a(\mathbf{R}, t)$$

X. Yao, T. Mehen PRD99 (2019) 096028

Talk: X. Yao 05/11/2019, 17:20



pNRQCD based approaches

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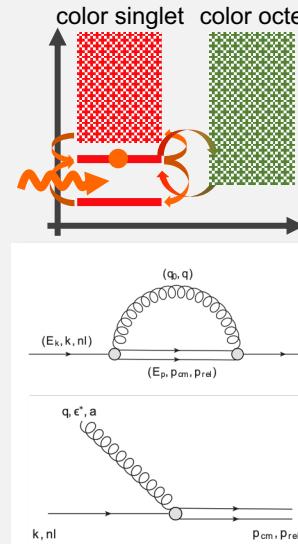
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Wigner transform: from density matrix to singlet distribution function



pNRQCD based approaches

pNRQCD & weakly coupled medium

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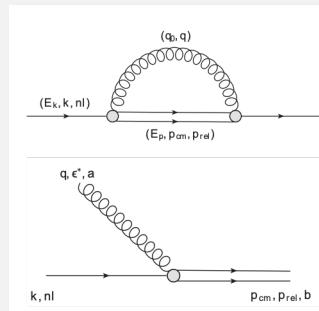
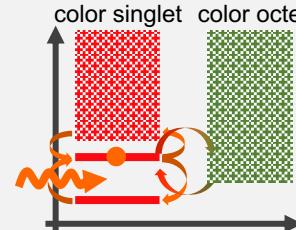
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pNRQCD based approaches

pNRQCD & weakly coupled medium

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X. Yao, T. Mehen PRD99 (2019) 096028

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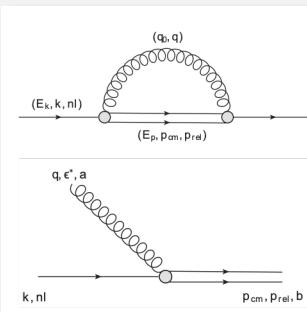
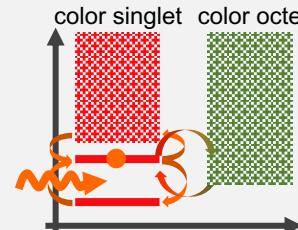
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need to keep track of octet distribution

only needs singlet distribution



pNRQCD based approaches

pNRQCD & weakly coupled medium

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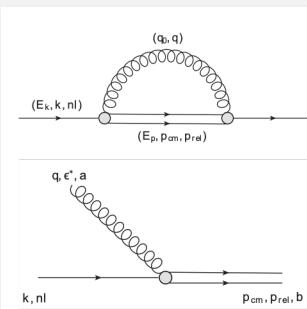
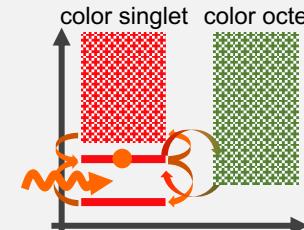
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Implemented as Langevin diffusion for heavy quarks

X. Yao, B. Müller PRC97 (2018) 014908,
PRD100 (2019), 014008

pNRQCD based approaches

pNRQCD & weakly coupled medium

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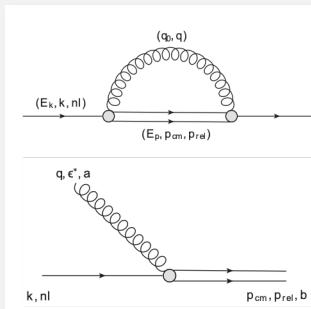
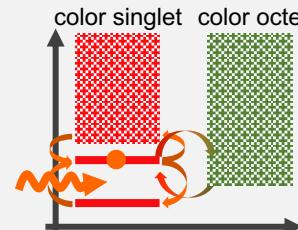
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PRD100 (2019), 014008

pNRQCD & strongly coupled medium

Non-perturbative medium but Coulombic bound states

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_O \end{pmatrix} + \frac{r^2}{2} \gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2 - 2}{2(N_c^2 - 1)} \end{pmatrix}$$

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N. Brambilla et. al. PRD100 (2019), 054025

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X. Yao, T. Mehen PRD99 (2019) 096028

Talk: X. Yao 05/11/2019, 17:20

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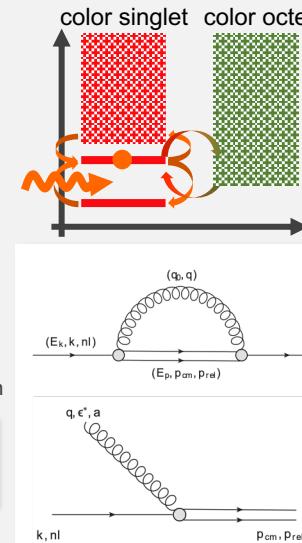
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N. Brambilla et. al. PRD100 (2019), 054025

governed by two (static) transport coefficients:

$$\kappa \propto \frac{1}{6N_c} \int_0^\infty dt \langle \{ E^{a,i}(t, \mathbf{0}), E^{a,i}(t, \mathbf{0}) \} \rangle \quad \text{heavy quark diffusion constant}$$

$$\gamma \propto -\frac{i}{6N_c} \int_0^\infty dt \langle [E^{a,i}(t, \mathbf{0}), E^{a,i}(t, \mathbf{0})] \rangle \quad \text{potential correction}$$

see also A. Eller, J. Ghiglieri, G. Moore, PRD99 (2019) 094042

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X. Yao, T. Mehen PRD99 (2019) 096028

Talk: X. Yao 05/11/2019, 17:20

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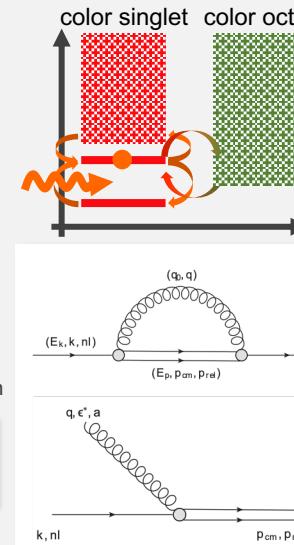
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N. Brambilla et. al. PRD100 (2019), 054025

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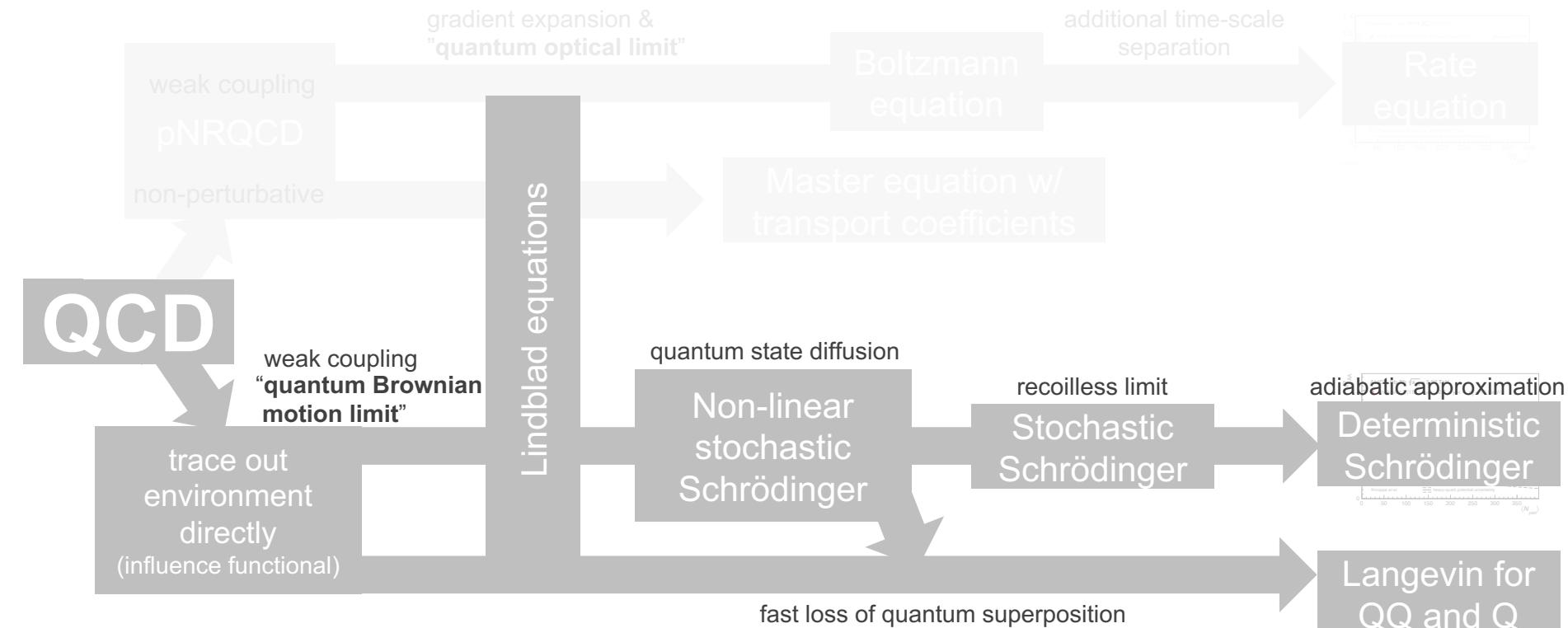
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see also A. Eller, J. Ghiglieri, G. Moore, PRD99 (2019) 094042

Caveat: pNRQCD assumes a hierarchy of scales that needs to be ascertained for all involved states.

Current real-time approaches



Lindblad equation & potential

■ Lindblad equation for the quantum Brownian motion regime at high temperature

Y. Akamatsu, PRD87 (2013) 4, 045016 and arXiv:1403.5783

$$\frac{d}{dt} \rho_{Q\bar{Q}}(t) = -i[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \sum_{i=1}^{N_{LB}} \gamma_i \left(\hat{L}_i \rho_{Q\bar{Q}} \hat{L}_i^\dagger - \frac{1}{2} \hat{L}_i \hat{L}_i^\dagger \rho_{Q\bar{Q}} - \frac{1}{2} \rho_{Q\bar{Q}} \hat{L}_i \hat{L}_i^\dagger \right)$$

$$L_{\mathbf{k},a} = \sqrt{\frac{D(\mathbf{k})}{2}} \left[1 - \frac{\mathbf{k}}{4m_Q T} \cdot \left(\frac{1}{2} \mathbf{P}_{CM} + \mathbf{p} \right) \right] e^{i\mathbf{k}\cdot\mathbf{r}/2} (T^a \otimes 1) - \sqrt{\frac{D(\mathbf{k})}{2}} \left[1 - \frac{\mathbf{k}}{4m_Q T} \cdot \left(\frac{1}{2} \mathbf{P}_{CM} - \mathbf{p} \right) \right] e^{-i\mathbf{k}\cdot\mathbf{r}/2} (1 \otimes T^a)$$

Lindblad equation & potential

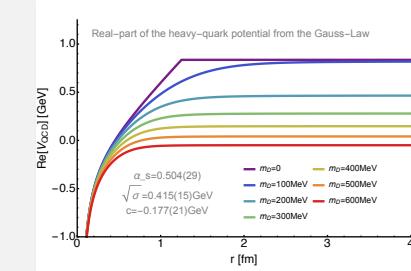
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$$Re[V] = -\alpha_S \frac{e^{-m_D r}}{r}$$



Lindblad equation & potential

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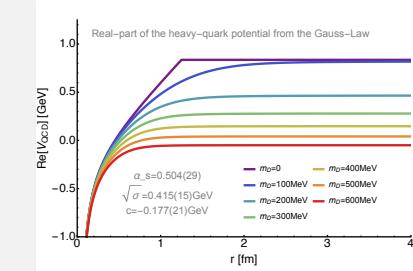
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fluctuations dissipation fluctuations dissipation

$$Re[V] = -\alpha_S \frac{e^{-m_D r}}{r}$$



Lindblad equation & potential

■ Lindblad equation for the quantum Brownian motion regime at high temperature

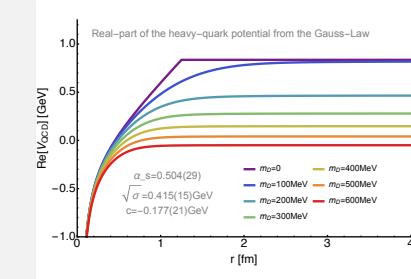
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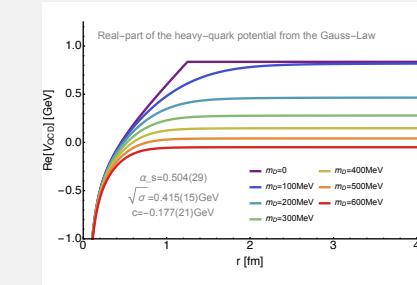
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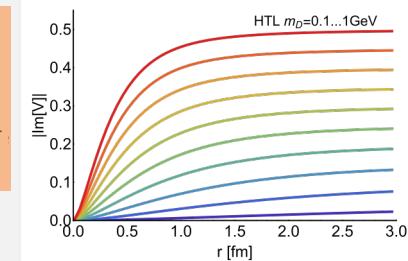
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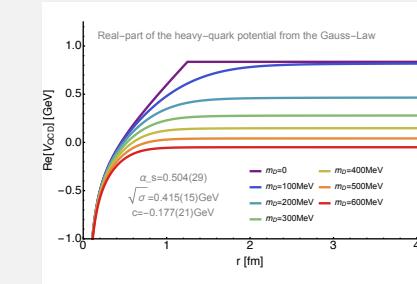
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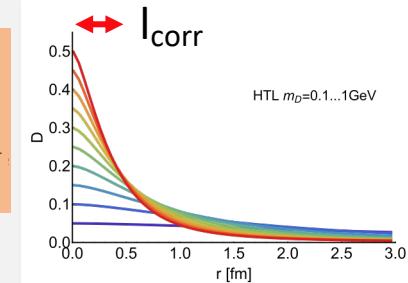
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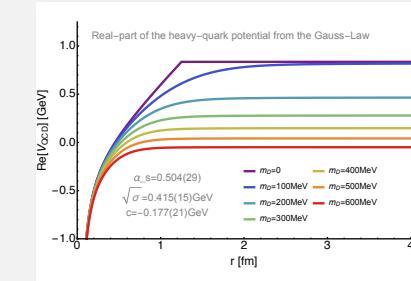
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Interplay between screening (m_D) & decoherence (l_{corr})

see discussion in S. Kajimoto, Y. Akamatsu, M. Asakawa, A.R., PRD97 (2018), 014003

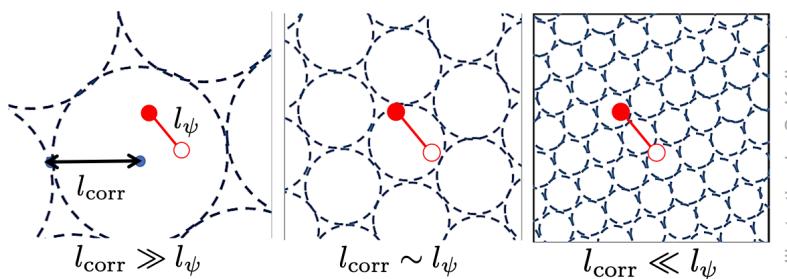
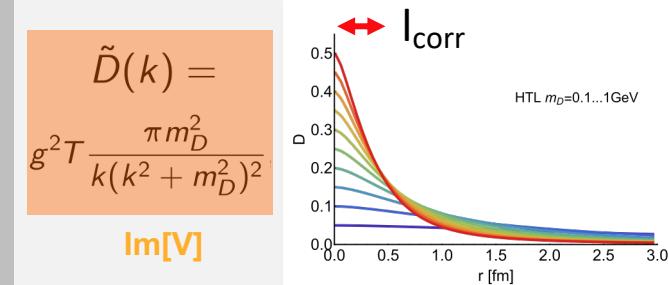


Illustration by S. Kajimoto



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Lindblad equation & potential

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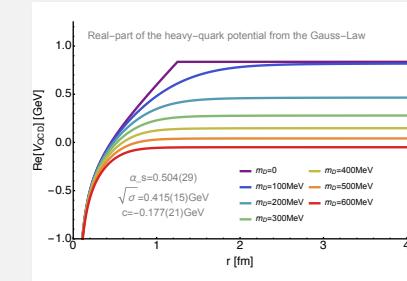
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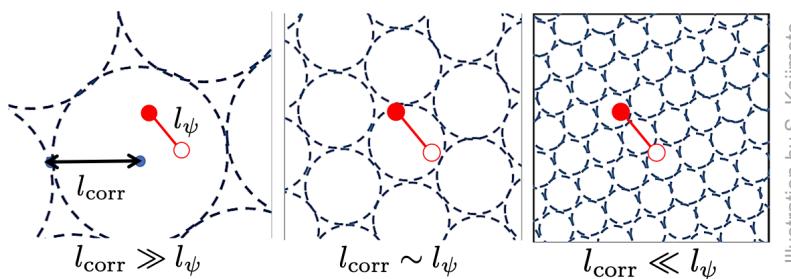
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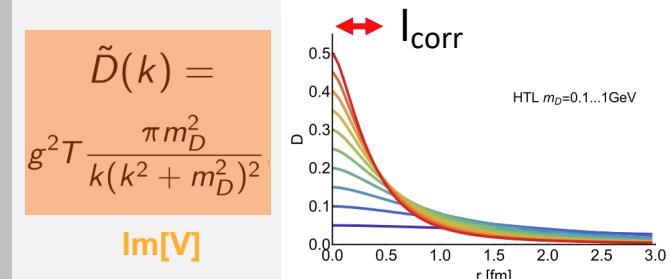
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$$\rho(\mathbf{x}_1, \mathbf{x}_2, t) \sim \rho(\mathbf{x}_1, \mathbf{x}_2, 0) \exp \left[- (D(0) - D(\mathbf{x}_1 - \mathbf{x}_2)) t \right]$$

decay of off-diagonal terms = decoherence



$$D(r) = Im[V](r) - Im[V](\infty)$$

Decoherence at work

Semi classical approximation?

If decoherence is efficient: classicalization possible

$$\rho(\mathbf{x}, \mathbf{y}) \approx \rho(\mathbf{R} + \mathbf{r}/2, \mathbf{R} - \mathbf{r}/2)\delta(\mathbf{r})$$

If color d.o.f. thermalizes quickly: similar to Abelian

$$\frac{M}{2}\ddot{\mathbf{r}}^i = -\gamma_{ij}\mathbf{v} - \nabla^i V(\mathbf{r}) + \theta^i(\mathbf{r}, t)$$

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J.P. Blaizot, M. Escobedo JHEP 1806 (2018) 034 & PRD98 (2018) 074007

Straight forward generalization to many Q \bar{Q} pairs

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Color Stochastic Potential

Stochastic SE in the **recoilless limit** including SU(3) color d.o.f.:
(poster: S. Kajimoto & see also poster: A. Tiwari)

$$\begin{aligned} H(\mathbf{r}, t) &= -\frac{1}{2\mu} \nabla_{\mathbf{r}}^2 + V(\mathbf{r})(t^a \otimes t^{a*}) + \Theta(\mathbf{r}, t) \\ \Theta(\mathbf{r}, t) &= \theta^a(\mathbf{R} + \mathbf{r}/2, t)(t^a \otimes 1) - \theta^a(\mathbf{R} - \mathbf{r}/2, t)(1 \otimes t^{a*}) \\ \langle \theta^a(\mathbf{x}, t) \theta^b(\mathbf{x}', t') \rangle &= D(\mathbf{x} - \mathbf{x}') \delta(t - t') \delta^{ab} \end{aligned}$$

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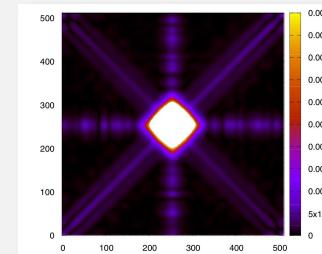
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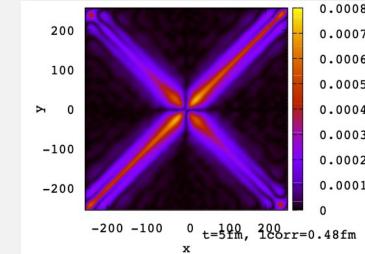
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color singlet density matrix



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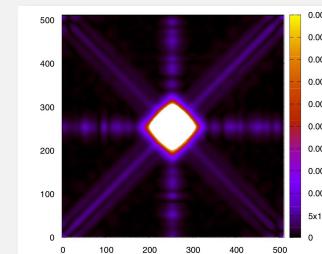
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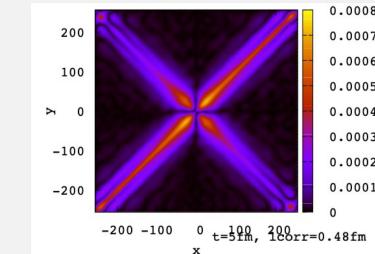
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color octet density matrix



$$W(R, p) = \int dr \rho(x, y) e^{-ipr}$$

Wigner distribution around origin shows **non-positive contributions**

Genuine **quantum nature** of quarkonium **remains relevant**

The full dissipative dynamics

- Lindblad equation **equivalent** to stochastic wavefunction dynamics: **Quantum State Diffusion**

T. Miura, Y. Akamatsu, M. Asakawa, A.R., arXiv:1908.06293

$$|d\psi\rangle = -iH|\psi(t)\rangle dt + \sum_n \begin{pmatrix} 2\langle L_n^\dagger \rangle_\psi L_n - L_n^\dagger L_n \\ -\langle L_n^\dagger \rangle_\psi \langle L_n \rangle_\psi \end{pmatrix} |\psi(t)\rangle dt + \sum_n (L_n - \langle L_n \rangle_\psi) |\psi(t)\rangle d\xi_n,$$

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- QCD derivation of stochastic non-linear Schrödinger equation, used in pheno. studies of dissipative dynamics**

c.f. e.g. R. Katz, P. Gossiaux Annals Phys. 368 (2016) 267

The full dissipative dynamics

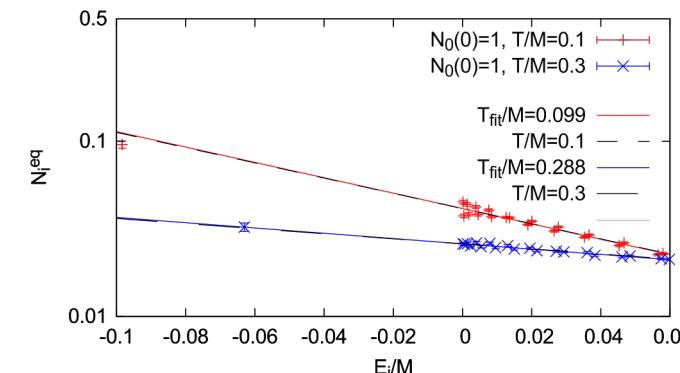
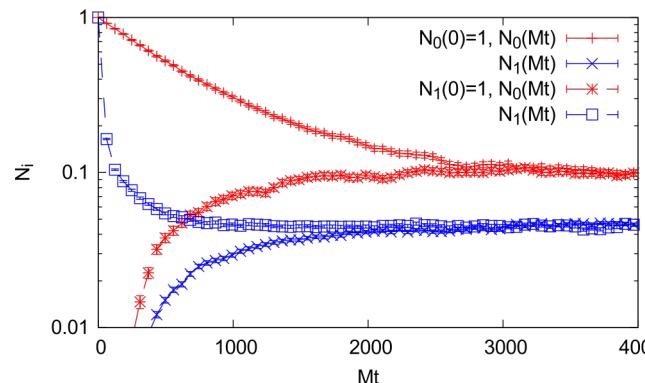
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- First potential based computation thermalizing quantum quarkonium system in one dimension (poster: T. Miura)

The full dissipative dynamics

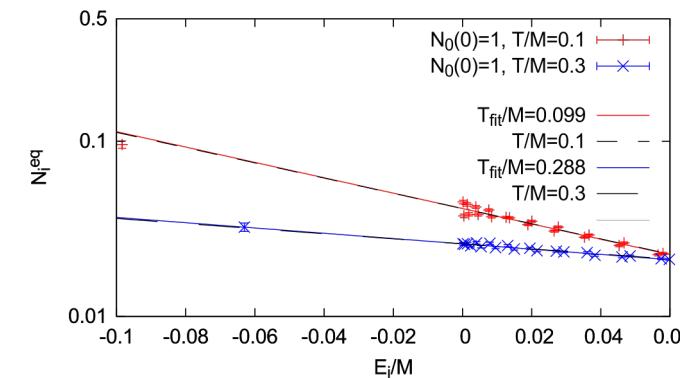
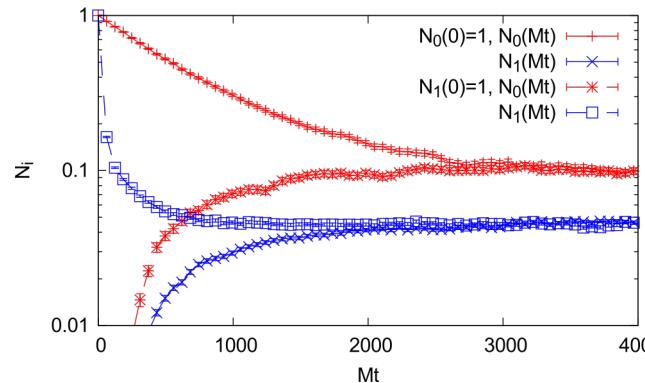
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- First potential based computation **thermalizing quantum quarkonium** system in one dimension (poster: T. Miura)
- Unified framework: dynamics of a **single quark** governed by the **same function D** in its Lindblad operator ($\text{Im}V$)

Y. Akamatsu, M. Asakawa, S. Kajimoto, A.R., JHEP 1807 (2018) 029

Conclusion

- Progress on **in-medium quarkonium** both in **equilibrium & its real-time dynamics**
 - intricate interplay of **color screening** and **decoherence** governs quarkonium evolution
(dynamical reshuffling of states instead of static melting)
 - Quarkonium in HIC tell us about QCD screening properties (m_D) and correlation length (ℓ_{corr})
- equilibrium lattice QCD progress towards **excited states** spectra & **transport** properties
(Talk: R. Larsen 06/11/2019, 17:00) (Talk: O. Kaczmarek 05/11/2019, 14:40)
- QCD derivation of **Boltzmann/rate & Schrödinger** equation: range of validity & parameters
(talk: X. Yao 05/11/2019, 17:20)
- new **Lindblad** type **master equations** for accurate real-time simulations (stochastic NLSE)
(poster: T. Miura)
- improved understanding of the **role of the complex heavy-quark potential** in evolution
(deterministic SE only crude approximation to dissipative dynamics)
- ToDo: **non-perturbative & colored OQS + early time quarkonium formation**
(poster: S. Kajimoto & poster: A. Tiwari) (see e.g. talk: I. Vitev 06/11/2019, 9:00)