Tidal deformability with magnetic field for neutron star and quark star

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Introduction

Tidal deformability is a quantity to describe the star’s quadrupole deformation in response to the companion’s perturbing tidal field. Consider a static spherical star placed in a external quadrupole tidal field $\delta B_\mu$. This star will be deformed and generate a quadrupole moment $Q_\mu = \lambda \delta B_\mu$. The coefficient $\lambda$ is defined as the tidal deformability. A widely used dimensionless quantities $k_2$ is defined as:

$$k_2 = \frac{\lambda}{2GM^3}.$$  

The tidal deformability is a parameter in the calculation of gravitational wave. It can significantly affect the waveform. The first gravitational wave event from binary neutron star merger GW170817 has given a constraint of the tidal deformability.

The tidal deformability depend on the equation of state of the star (E. Annala et al, PRL 120, 172703 (2018); E. Most et al, PRL 120, 261103 (2018); Z. Zhu et al, APJ 862, 98 (2018); F.J. Fatteyev et al, PRL 120, 172702 (2018)), spin (P. Pani et al, PRD 92, 024040 (2015); P. Landry et al, PRD 95, 124058 (2017) and magnetic field (present work). Some properties or matter like anisotropic pressure (B. Biswas et al, PRD 99, 104002 (2019)) or dark matter hole around the star (A. Nelson et al, arXiv:1803.03266) can also affect the tidal deformability.

![Image](Image 1)

Fig. 1. The constraint of tidal deformability from GW170817. The horizontal axis and vertical axis represent the tidal deformability of star 1 and star 2 of binary, respectively.

The tidal deformability of neutron star and quark star are different. The tidal deformability of neutron star is bigger than quark star. This difference in the tidal deformability could result a significant phase difference. The following results are all based on the APR equation of state if not specify. This 3D figure displays the second order correction of tidal love number $\delta k_2$ as a function of compactness $M/R$ and magnetic field strength at pole of the star $B$. It shows that the $\delta k_2$ achieve its extremum when the compactness close to 0.14 and mass close to 1.1 $M_\odot$. The second order tidal love number goes to negative when compactness larger than 0.2.

![Image](Image 2)

Fig. 2. The relation between $\delta k_2$ and $\log_{10}(B)$ for different EOS. The left figure shows $\log_{10}(\delta k_2)$ has a linear relation with $\log_{10}(B)$. This is because $\log_{10}(B)$ is linear correlated with those integral constants.

Numerically integral the differential equations with arbitrary initial condition, we can specify the tidal deformability of a neutron star and quark star. The tidal deformability of both neutron and quark stars is taken into account and our results shows $\delta k_2$ is $10^{-4}$ for neutron star and $10^{-3}$ for quark star. This difference in the tidal deformability could result a significant phase difference for quark star, but neutron star’s phase difference is weaker.

Theoretical Method

The study is based on the perturbation theory. Both the tidal field and magnetic field are treated as a perturbation of TOV equations. Because of the linearity of first order, we have to do the second order perturbation to work out the coupling of tidal field and magnetic field. The metric perturbations of the first order are:

$$\mathcal{E} = \sum_{ij} \mathcal{E}^{ij}_{\mu} \partial_{\mu} \mathcal{E}_{ij}.$$  

Plug these metric perturbation into Einstein equation and magnetohydrodynamic equations, The first order can be obtained.

The metric equation of field is written as:

$$\mathcal{E}^{ij}_{\mu} = \sum_{ij} \mathcal{E}^{ij}_{\mu} \partial_{\mu} \mathcal{E}_{ij} = \frac{\delta k_2}{2GM} \epsilon_{ij}. $$

The equations of metric from magnetic field are written as:

$$k_2 \epsilon_{ij} = \frac{1}{2} \left[ \frac{\delta k_2}{2GM} \epsilon_{ij} \right] \frac{1}{2} \left[ \frac{\delta k_2}{2GM} \epsilon_{ij} \right] = \frac{1}{2} \left[ \frac{\delta k_2}{2GM} \epsilon_{ij} \right] \frac{1}{2} \left[ \frac{\delta k_2}{2GM} \epsilon_{ij} \right]. $$

The metric equation of tidal field is written as:

$$\delta k_2 = \frac{1}{2} \left[ \frac{\delta k_2}{2GM} \epsilon_{ij} \right] \frac{1}{2} \left[ \frac{\delta k_2}{2GM} \epsilon_{ij} \right] = \frac{1}{2} \left[ \frac{\delta k_2}{2GM} \epsilon_{ij} \right] \frac{1}{2} \left[ \frac{\delta k_2}{2GM} \epsilon_{ij} \right]. $$

In order to make the relations more clear, the 2D figures $\log_{10}(\delta k_2)$ vs $\log_{10}(B)$ and $\log_{10}(\delta k_2)M_\odot$ vs compactness $M/R$ are plotted. The left figure shows $\log_{10}(\delta k_2)$ has a linear relation with $\log_{10}(B)$. This is because $\log_{10}(B)$ is linear correlated with those integral constants.

Results

The results for different EOSs both for neutron star and quark star are take into account. Noting that quark star have a total different trend with neutron star and it is much more stronger. The right figure shows the phase difference of gravitational wave. The quark stars have a much stronger effects on phase difference.

Summary

The effects of magnetic field on tidal deformability are studied in the framework of perturbation theory. The magnetic field could make the star easier to be deformed at low compactness and harder at high compactness. Different EOSs are taken into account and our results shows $\delta k_2$ of quark stars have a different trend with neutron stars. This difference in the tidal deformability could result a significant phase difference for quark star, but neutron star’s phase difference is weaker.