# Tidal deformability with magnetic field for neutron star and quark star

**Zhenyu Zhu**

**Supervisors: Luciano Rezzolla, Ang Li**

*Institute for Theoretical Physics, Goethe University*

*Department of Astronomy, Xiamen University*

## Introduction

Tidal deformability is a quantity to describe the star's quadrupole deformation in response to the companion's perturbing tidal field. Consider a static spherical star is placed in a external quadrupole tidal field $\varepsilon_{ij}$, this star will be deformed and generate a quadrupole moment $Q_{ij} = -\lambda\varepsilon_{ij}$. The coefficient λ is defined as the tidal deformability. A widely used dimensionless quantities $k_2$ is defined as $k_2 = \frac{3}{2}\lambda R^{-5}$.

The tidal deformability is a parameter in the calculation of gravitational wave. It can significantly affect the waveform. The first gravitational wave event from binary neutron star merger GW170817 has given a constraint of the tidal deformability.

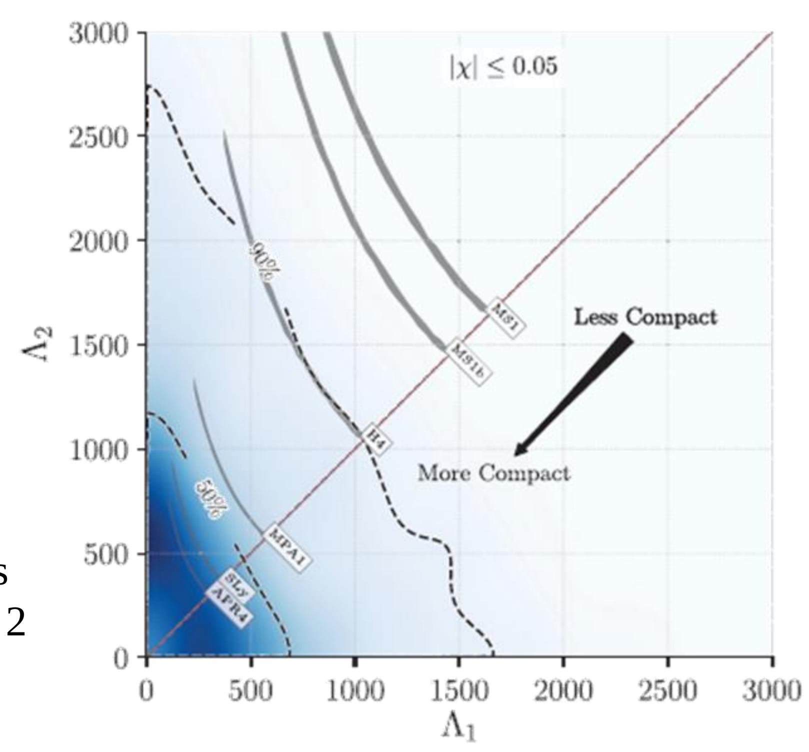


Fig I. The constraint of tidal deformability from GW170817. The horizontal axis and vertical axis represent the tidal deformability of star 1 and star 2 of binary, respectively.

B. P. Abbott *et al*, PRL **119**, 161101 (2017)

The tidal deformability depend on the equation of state of the star (E. Annala et al, PRL 120, 172703 (2018); E. Most et al, PRL 120, 261103 (2018); Z. Zhu et al, APJ 862, 98 (2018); F. J. Fattoyev et al, PRL 120, 172702 (2018) ), spin (P. Pani et al, PRD 92, 024010 (2015); P. Landry et al, PRD 95, 124058 (2017) ) and magnetic field (present work). Some properties or matter like anisotropic pressure (B. Biswas et al, PRD 99, 104002 (2019) ) or dark matter hole around the star (A. Nelson et al, arXiv:1803.03266 ) can also affect the tidal deformability.

## Theoretical Method

The study is based on the perturbation theory. Both the tidal field and magnetic field are treated as a perturbation of TOV equations. Because of the linearity of first order, we have to do the second order perturbation to work out the coupling of tidal field and magnetic field. The metric perturbations of the first order are

$$h^m_{\mu\nu} = \begin{pmatrix} -2e^{\nu}(h^m_0 + h^m_2 p_2(\cos\theta)) & 0 & 0 & 0 \\ 0 & 2e^{2\lambda}(m^m_0 + m^m_2 p_2(\cos\theta))/r & 0 & 0 \\ 0 & 0 & 2r^2 k^m_2 p_2(\cos\theta) & 0 \\ 0 & 0 & 0 & 2r^2 k^m_2 \sin^2(\theta) p_2(\cos\theta) \end{pmatrix} \qquad h^T_{\mu\nu} = \sum_l \begin{pmatrix} -e^{\nu}H^l_0 & 0 & 0 & 0 \\ 0 & e^{\lambda}H^l_2 & 0 & 0 \\ 0 & 0 & r^2K_l & 0 \\ 0 & 0 & 0 & r^2\sin^2(\theta)K_l \end{pmatrix} p_l(\cos\theta).$$

Plug these metric perturbation into Einstein equation and magnetohydrodynamic equations, The first order equations can be obtained. The equation of magnetic field is

$$e^{-\lambda}a_1'' + \frac{\nu' - \lambda'}{2}e^{-\lambda}a_1' + \left(\zeta^2 e^{-\nu} - \frac{2}{r^2}\right)a_1 = 4\pi(\rho + p)r^2 c_0\,,$$

$$B_{(r)} = -\frac{2a_1}{r^2}\cos\theta$$
$$B_{(\theta)} = \frac{e^{-\lambda/2}a_1'}{r}\sin\theta$$
$$B_{(\phi)} = \zeta\frac{e^{-\nu/2}a_1}{r}\sin\theta\,.$$

The equations of metric from magnetic field are written as

$$h_2' + \frac{4e^{\lambda}}{\nu' r^2}y_2 + \left[\nu' - \frac{8\pi e^{\lambda}}{\nu'}(p^{(0)} + \rho^{(0)}) + \frac{2}{r^2\nu'}(e^{\lambda} - 1)\right]h_2 = \frac{\nu'}{3}e^{-\lambda}a_1'^2 + \frac{4}{3r^2}a_1 a_1' + \frac{1}{3}\left(-\nu' + \frac{2}{\nu' r^2}e^{\lambda}\right)\zeta^2 e^{-\nu}(a_1)^2 - \frac{16\pi}{3\nu' r^2}e^{\lambda}j_1 a_1$$

$$y_2' + \nu' h_2 = \frac{\nu'}{2}e^{-\lambda}a_1'^2 + \frac{1}{3}\left[\frac{e^{-\lambda}}{r}\left(\nu' + \lambda' + \frac{2}{r}\right) + e^{-\nu}\zeta^2 - \frac{2}{r^2}\right]a_1 a_1' - \frac{\nu'}{3}e^{-\nu}\zeta^2 a_1^2 - \frac{4\pi}{3}j_1\left(a_1' + \frac{2}{r}a_1\right),$$

The metric equation of tidal field is written as

$$H_0^{l\prime\prime} + \left[\frac{2}{r} + \frac{2m}{r^2}e^{\lambda} + 4\pi r(P - \rho)e^{\lambda}\right]H_0^{l\prime} + \left[4\pi e^{\lambda}\left(4\rho + 8P + (P + \rho)(1 + \frac{1}{c_s^2})\right) - \frac{6e^{\lambda}}{r^2} - \nu'^2)\right]H_0^l\Big\} = 0$$

For The second order, we fix the magnetic field and plug a modification of tidal metric from magnetic field into Einstein equation

$$\delta h^T_{\mu\nu} = \sum_l \begin{pmatrix} -e^{\nu}\delta H^l_0 & 0 & 0 & 0 \\ 0 & e^{\lambda}\delta H^l_2 & 0 & 0 \\ 0 & 0 & r^2\delta K_l & 0 \\ 0 & 0 & 0 & r^2\sin^2(\theta)\delta K_l \end{pmatrix} p_l(\cos\theta).$$

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$$\delta H_0^{l\prime\prime} + \left[\frac{2}{r} + \frac{2m}{r^2}e^{\lambda} + 4\pi r(P - \rho)e^{\lambda}\right]\delta H_0^{l\prime} + \left[4\pi e^{\lambda}\left(4\rho + 8P + (P + \rho)(1 + \frac{1}{c_s^2})\right) - \frac{6e^{\lambda}}{r^2} - \nu'^2)\right]\delta H_0^l\Big\} = S(r)$$

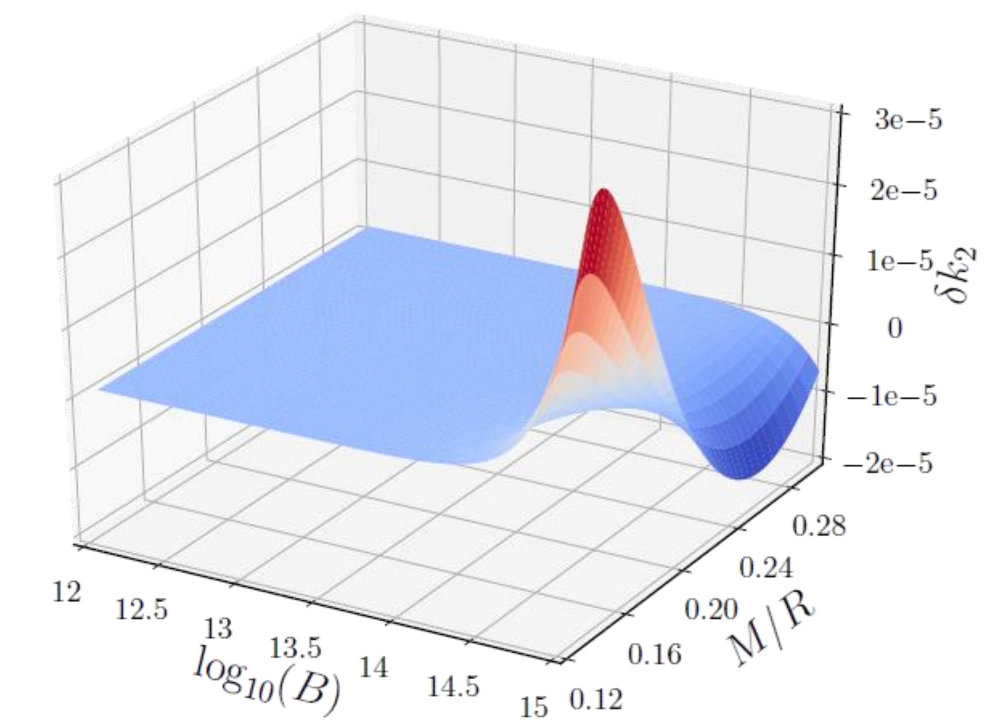
Numerically integral the differential equations with arbitrary initial condition, we can write down both the general interior and exterior solutions

$$H^l_0|_{\rm int} = C_t H^h_0 \qquad \delta H^l_0|_{\rm int} = C_{\Delta t}H^h_0 + C_t\delta\hat{H}_0$$
$$H^l_0|_{\rm ext} = c^e_1 Q^2_2(r/M - 1) + c^e_2 P^2_2(r/M - 1) \qquad \delta H^l_0|_{\rm ext} = d^e_1 Q^2_2(r/M - 1) + c^e_1\delta\hat{H}_1 + c^e_2\delta\hat{H}_2$$

$C_t$, $C_{\Delta t}$, $c_1^e$, $c_2^e$, $d_1^e$ and $d_2^e$ are integral constants, they can be determined by matching interior and exterior solutions at star surface. The final tidal love numbers are written as

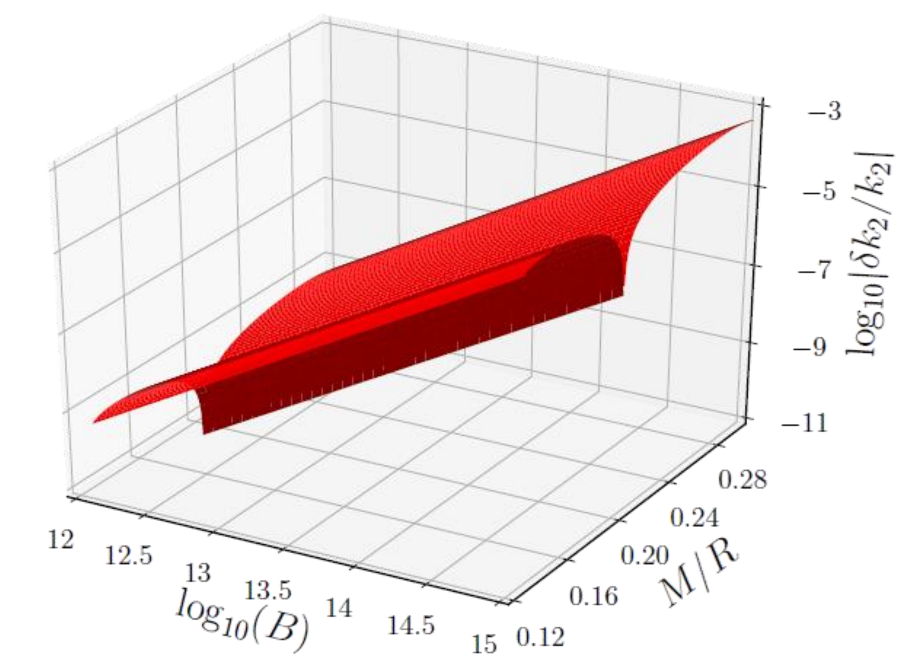
$$k_2 = \frac{4}{15}\frac{c^e_1}{c^e_2}\frac{M^5}{R^5} \qquad \delta k_2 = \frac{4}{15}\left(\frac{M}{R}\right)^5\frac{d^e_1}{c^e_2} + \frac{c_2}{M}\left(\frac{M}{R}\right)^5\frac{c^e_1}{c^e_2}$$

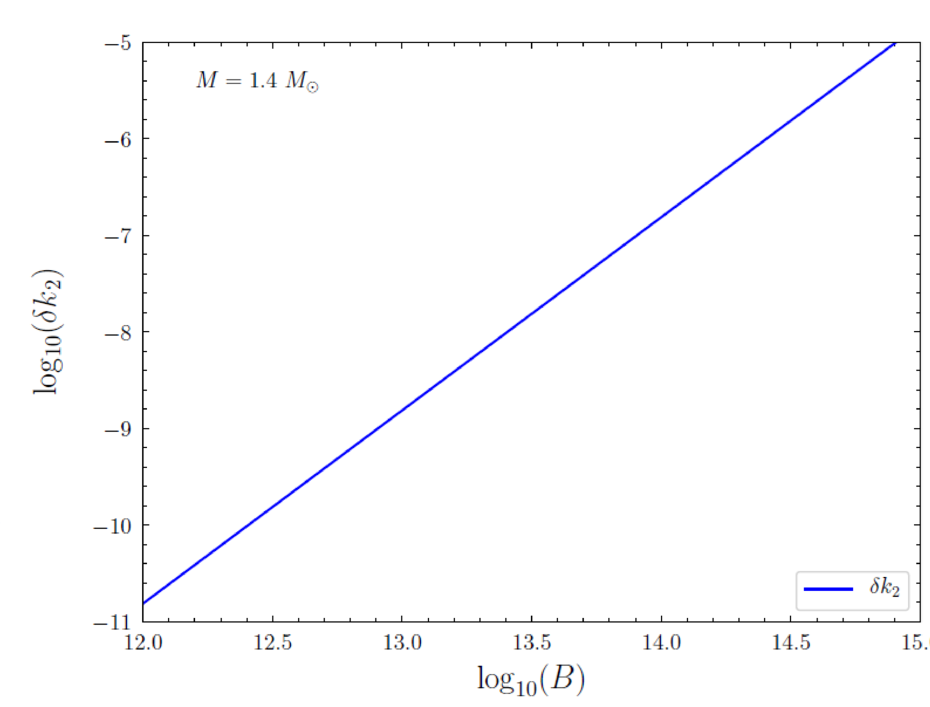
## Results

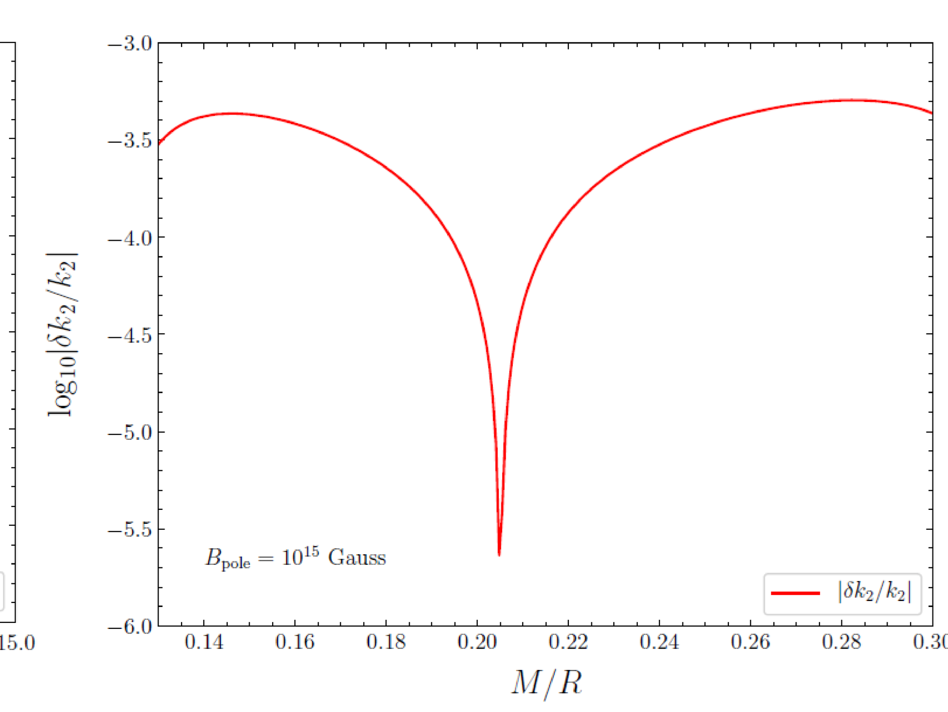



The following results are all base on the APR equation of state if not specify. This 3D figure displays the second order correction of tidal love number $\delta k_2$ as a function of compactness *M/R* and magnetic field strength at pole of the star *B*. It shows that the $\delta k_2$ achieve its extremum when the compactness close to 0.14 and mass close to 1.1 $M_\odot$. The second order tidal love number goes to negative when compactness larger than 0.2.

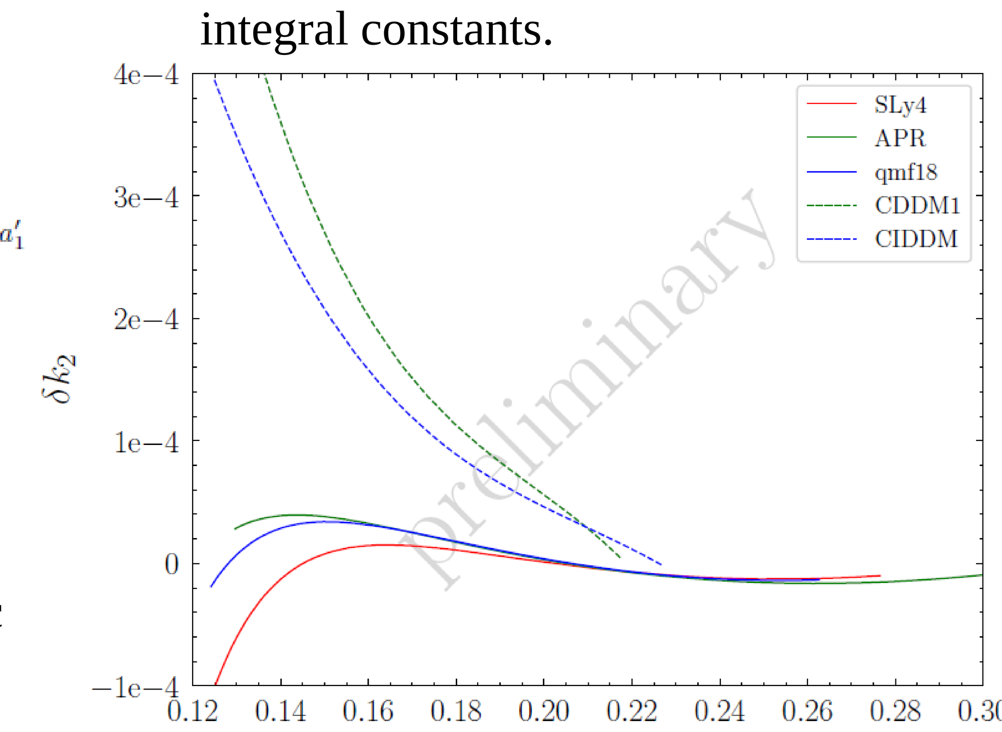
For comparing the second order with first order of love number, we plot the 3D figure for $\log_{10}|\delta k2/k_2|$. Because $\delta k2$ equal to 0 at some specific value of compactness, the value goes to infinity at this logarithmic figure. This figure shows that the maximum value for $\delta k2/k_2$ is $10^{-3}$, which is a very small value.

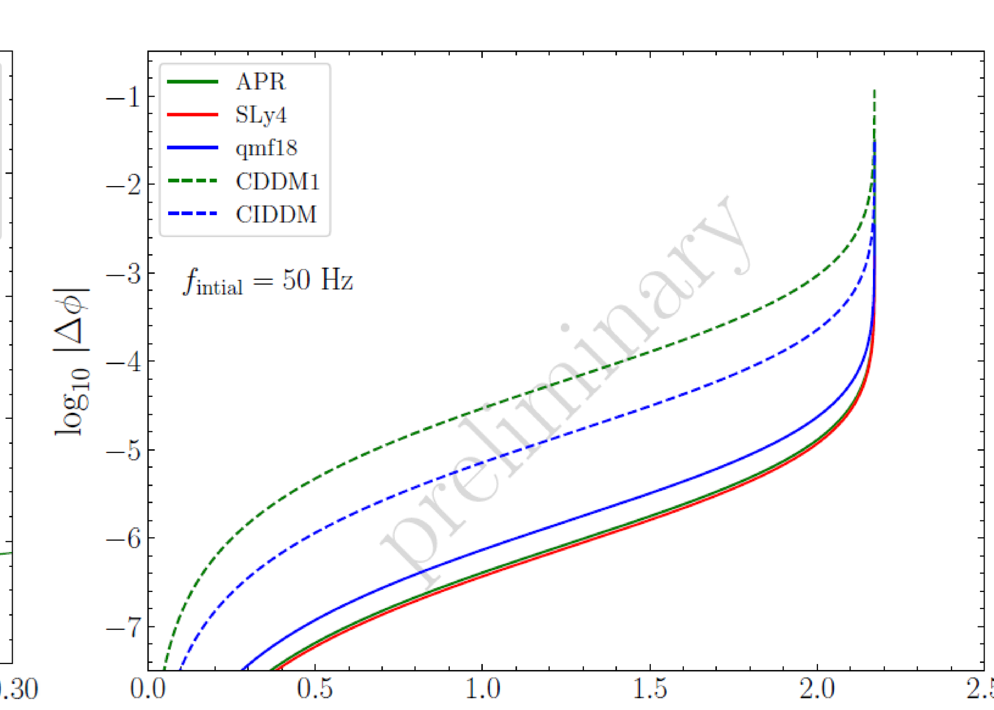



In order to make the relations more clear, the 2D figures $\log_{10}(\delta k_2)$ vs $\log_{10}(B)$ and $\log_{10}|\delta k_2/k_2|$ vs compactness *M/R* are plotted. The left figure shows $\log_{10}(\delta k_2)$ has a linear relation with $\log_{10}(B)$. This is because $\log_{10}(B)$ is linear correlated with those integral constants.




The results for different EOSs both for neutron star and quark star are take into account. Noting that quark star have a total different trend with neutron star and it is much more stronger. The right figure shows the phase difference of gravitational wave. The quark stars have a much stronger effects on phase difference.

## Summary

The effects of magnetic field on tidal deformability are studied in the framework of perturbation theory.

The magnetic field could make the star easier to be deformed at low compactness and harder at high compactness.

Different EoSs are taken into account and our results shows $\delta k_2$ of quark stars have a different trend with neutron stars.

This difference in the tidal deformability could result a significant phase difference for quark star, but neutron star's phase difference is weaker.