IDM at one-loop and the Relic Density of Dark Matter

IDM: Inert Doublet Model

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Work done with Shankha Banerjee,Nabarun Chakrabarty, Guillaume Chalons and Hao Sun

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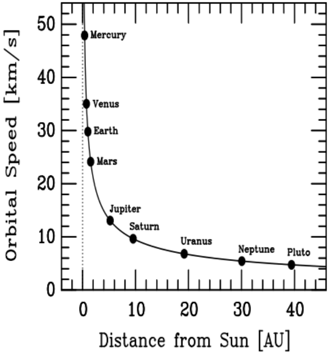
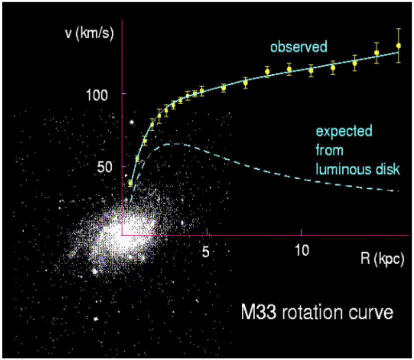
- how relic at one-loop reveals the dynamics of the dark sector
- Conclusions

Evidence for Dark Matter: DM

Newton's law $\longrightarrow v_{\text{rot.}}^2/r = G_N M(r)/r^2$ (tracer star at a distance r from centre of mass distribution)





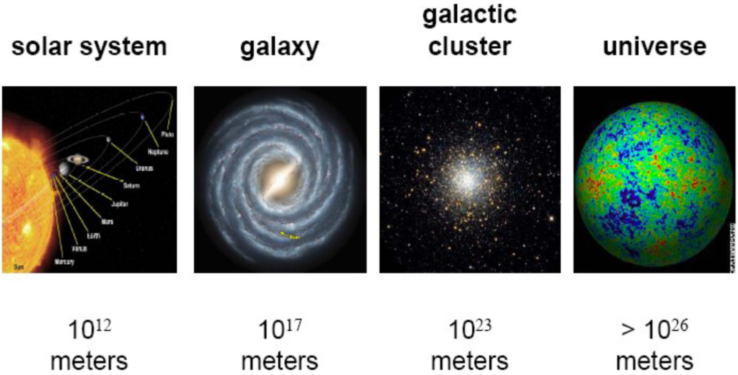
We are not in the centre of the universe

Dark Matter= New Physics

we are not made up of the same stuff as most of our universe

DM is “seen"at all scale

DM “seen" on many different scales



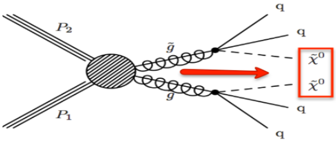
In our neighbourhood, search: Direct Detection

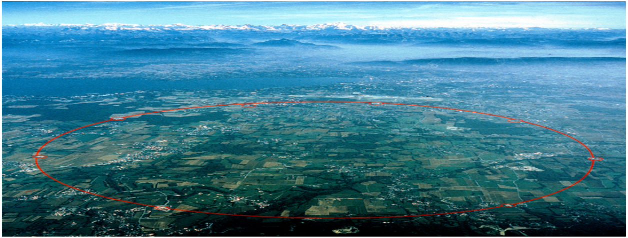
Galaxy, extragalactic: in indirect detection

Universe: relic density

DM at the LHC

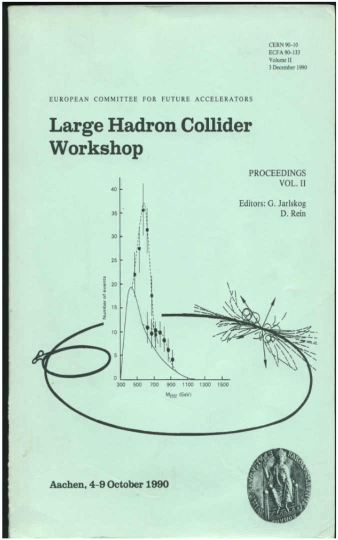
And may be created at the LHC





The new paradigm: Higgs and DM?

LHC Dark Matter Connection is new: The new paradigm the Aachen Proceedings



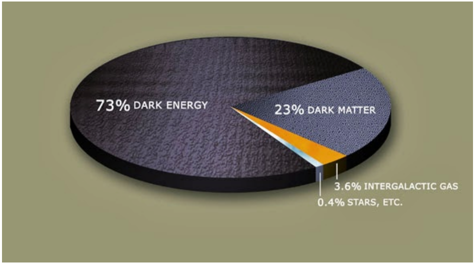
• No mention of a connection between the LHC and Dark Matter, despite a SUSY WG. There is a mention of LSP to be stable/neutral because of cosmo reason, but no attempt at identifying it or **weighing the universe at the LHC**

• LHC: Symmetry breaking and Higgs

• New Paradigm, Dark Matter is New Physics. Dark Matter is being looked for everywhere

• New Paradigm, Particle Physics to match the precision of recent cosmological measurements

Dark Matter Budget



DM: properties

DM: What is it? Properties

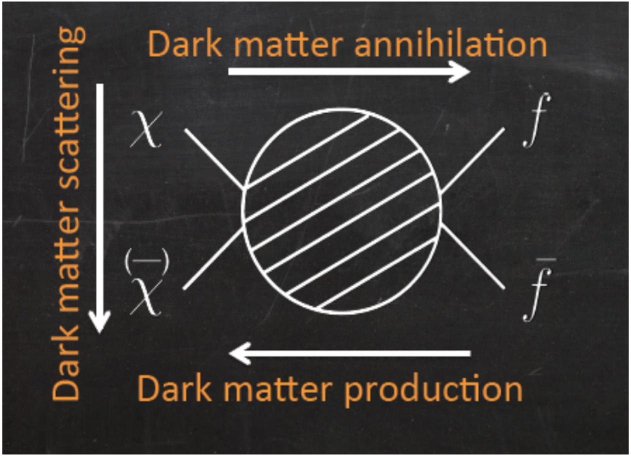
Microscopic Level: interaction, couplings, masses $\Longrightarrow$ We don't know

Macroscopic level: How is it distributed? $\Longrightarrow$ We don't know *really*, but we somehow know

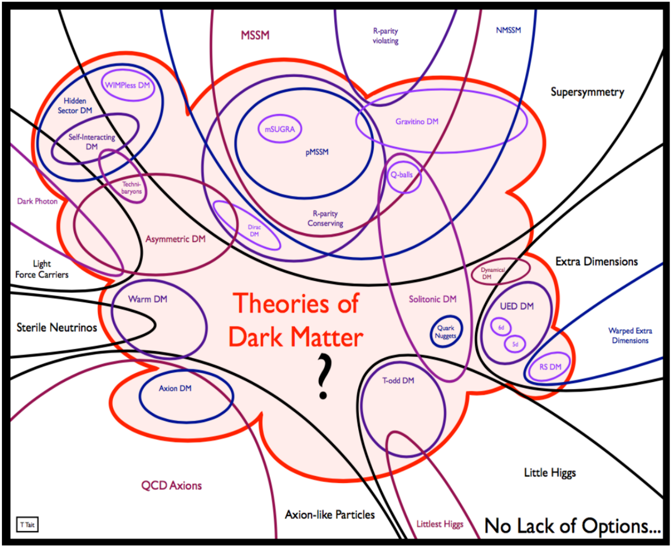
Apart from being

- NEUTRAL
- STABLE

DM: properties (2)



New Physics Models/DM from Tm Tait



The relic density. 1

- In a universe which expands, at each epoch a mixture of different particles in thermal contact with each other maintaining a temp. which evolves with time.
- To maintain equilibrium interactions had to be frequent enough, interaction rate is

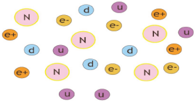
$$\Gamma = n\sigma v$$

- The critical time scale is set by the expansion of the Universe, Hubble parameter,

$$H$$

The relic density. 2

formation of DM: Very basics of decoupling



- At first all particles in thermal equilibrium, frequent collisions. Particles are trapped in the cosmic soup

The relic density. 2

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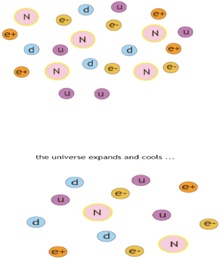

- At first all particles in thermal equilibrium, frequent collisions. Particles are trapped in the cosmic soup
- universe cools and expands: interaction rate too small or not efficient to maintain equilibrium

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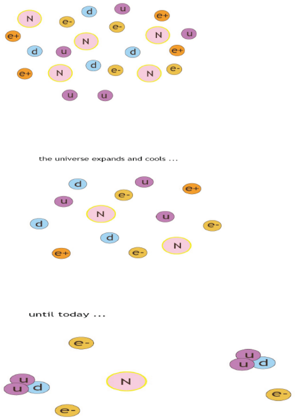
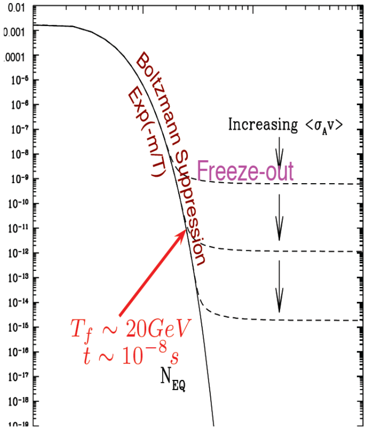


- At first all particles in thermal equilibrium, frequent collisions. Particles are trapped in the cosmic soup
- universe cools and expands: interaction rate too small or not efficient to maintain equilibrium
- (stable) particles can not find each other: freeze out and get free and leave the soup, their number density is locked giving the observed relic density
- from then on total number $(n \times a^3) = cste$
- Condition for equilibrium: mean free path smaller than distance traveled: $l_{\rm m.f.p} < vt \quad l_{\rm m.f.p} = 1/n\sigma$ $t \sim 1/H$ or Equilbrium: $\Gamma = n\sigma v > H$

freeze out/decoupling occurs at $T = T_D = T_F : \Gamma = H$ and $\Omega_{\tilde{\chi}_1^0} h^2 \propto 1/\sigma_{\tilde{\chi}_1^0}$

The relic density. 3

Relic Density: Boltzman transport equation



based on $\hat{L}[f] = \hat{C}[f]$

dilution due to expansion

$dn/dt = -3Hn - <\sigma v> (n^2 - n_{eq}^2)$

$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to X$

$X \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$

- at early times $\Gamma \gg H \to \sim n_{eq}$
- $T \sim m$ X not enough energy to give $X \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ n drops and so does Γ

$T_f \simeq m/25$

$\Omega_{\tilde{\chi}_1^0} h^2 \propto 1/\sigma_{\tilde{\chi}_1^0}$

Relic density: co-annihilation

Co-annihilations

$$\chi_i^0 \chi_j^0 \to X_{SM} Y_{SM},\ \chi_1^0 \tilde{f}_1 \to X_{SM} Y_{SM}, \ldots$$

$$< \sigma v > = \frac{\sum_{i,j} g_i g_j \int_{(m_i+m_j)^2} ds \sqrt{s} K_1(\sqrt{s}/T)\, p_{ij}^2 \sigma_{ij}(s)}{2T\left(\sum_i g_i m_i^2 K_2(m_i/T)\right)^2},$$

p_{ij} is the momentum of the incoming particles in their center-of-mass frame.

$$p_{ij} = \frac{1}{2}\left[\frac{(s-(m_i+m_j)^2)(s-(m_i-m_j)^2)}{s}\right]^{\frac{1}{2}} \to v$$

$v = 0$

$v \times \sigma v$

co-annihilation, only if particle close thermodynamically to DM particle, close in small otherwise suffers very large Boltzmann suppression

Relic density: co-annihilation

Co-annihilations: Thermal average

$$\chi_i^0 \chi_j^0 \to X_{SM} Y_{SM},\ \chi_1^0 \tilde{f}_1 \to X_{SM} Y_{SM}, \ldots$$

$v \times \sigma v$

$$< \sigma v >= \frac{\sum_{i,j} g_i g_j \int_{(m_i+m_j)^2} ds \sqrt{s}\, K_1(\sqrt{s}/T)\, p_{ij}^2 \sigma_{ij}(s)}{2T\left(\sum_i g_i m_i^2 K_2(m_i/T)\right)^2},$$

Origin of Boltzman factor $exp(-\delta M/T)$

$$B_f = \frac{K_1((m_i+m_j)/T_f)}{K_1(2m_{\tilde{\chi}_1^0}/T_f)} \approx e^{-X_f \frac{(m_i+m_j-2m_{\tilde{\chi}_1^0})}{m_{\tilde{\chi}_1^0}}} \qquad \textit{if } B_f < B_\epsilon \text{ do not compute}$$

In micrOMEGAs $B_\epsilon = 10^{-6}$ by default. Most often $B_\epsilon = 10^{-2}$ enough for 1% accuracy, only a few processes computed.

Relic density: recap

All in all...in the standard freeze-out

$$\Omega_{\tilde{\chi}_1^0} h^2 \simeq \frac{10^9}{M_P} \frac{x_f}{\sqrt{g_\star}} \frac{1}{<\sigma_{\tilde{\chi}_1^0} v>}$$

$$\Omega_{\tilde{\chi}_1^0} h^2 \sim 0.1 \to < \sigma_{\tilde{\chi}_1^0} v > \sim 1pb$$

order of magnitude of LHC cross sections

$$< \sigma_{\tilde{\chi}_1^0} v > = \pi\alpha^2/m^2$$

$$\Omega_{\tilde{\chi}_1^0} h^2 \sim (m/TeV)^2 \to m \sim G_F^{-1/2} \sim 300\text{GeV}$$

Alternatively to get $\Omega h^2 \sim 0.1$ then typical $< \sigma v > \sim 3.10^{-26} cm^2 s^{-1}$ for astro minded people.

but with the precision on the relic that we have now (2%) and the many possibilities from the particle physics perspectives, need more than just an order of magnitude calculation.

The Inert Doublet Model at the classical level

To the Higgs doublet Φ_1 of the SM, a doublet Φ_2 is added.

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An unbroken $\mathbb{Z}_2$ symmetry is imposed under which Φ_2 is odd while all other fields (of the SM) are even.

The immediate consequence is that Φ_2 cannot couple to fermions to any order (in perturbation theory) and guarantees the stability of the lightest inert particle, thus providing a **possible dark matter candidate.**

$$\mathcal{L}_{IDM} = \mathcal{L}_{SM} + (D^{\mu}\Phi_2)^{\dagger} D_{\mu}\Phi_2 + \mathscr{V}_{IDM}(\Phi_1, \Phi_2),$$

Potential

$$\mathscr{V}_{IDM}(\Phi_1, \Phi_2) = \mu_1^2|\Phi_1|^2 + \mu_2^2|\Phi_2|^2 + \lambda_1|\Phi_1|^4 + \lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4(\Phi_2^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \left(\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \mathrm{h.c}\right).$$

Additional Scalars

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + iG\right) \end{pmatrix} \text{ and } \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(H + iA\right) \end{pmatrix},$$

where v is the SM vacuum expectation value (vev) with $v \simeq 246$ GeV, defined from the measurement of the W (M_W) and Z (M_Z) masses. We have

$$s_W^2 \equiv \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}, \quad M_W = \frac{1}{2}\frac{e}{s_W}v,$$

e is the electromagnetic coupling (the $SU(2)$ gauge coupling g is then $g = e/s_W$), h is the SM 125 GeV Higgs boson, $G, G^\pm$ are the Goldstone bosons, H, A are the new neutral physical scalars

Since these additional scalars do not couple to the fermions (of the SM), we can not assign them definite CP numbers. By an abuse of language, we will call A the pseudo-scalar. and $H^\pm$ is the charged physical scalar. H and A are the possible DM candidates. These scalars have gauge couplings to the SM gauge bosons, controlled by the SM gauge coupling. For example, for the tri-linear couplings we have

$$(H^+H^-\gamma,\ H^+H^-Z,\ HH^\pm W^\mp,\ iAH^\pm W^\mp,\ iAHZ) = i\frac{g}{2}(2s_W,\ c_{2W}/c_W,\ \mp 1,\ -1,\ -1/c_W).$$

Minimisation of the Potential

$$\frac{T}{v} = \mu_1^2 + \lambda_1 v^2 \equiv 0.$$

There is no corresponding tadpole term for Φ_2 because of the unbroken $\mathbb{Z}_2$ symmetry.

The no-tadpole condition will be maintained at all orders.

Mass spectrum

$$\begin{aligned}
M_h^2 &= \frac{T}{v} + 2\lambda_1 v^2, \\
M_{H^\pm}^2 &= \mu_2^2 + \lambda_3 \frac{v^2}{2}, \\
M_H^2 &= \mu_2^2 + \lambda_L \frac{v^2}{2} = M_{H^\pm}^2 + (\lambda_4 + \lambda_5)\frac{v^2}{2}, \\
M_A^2 &= \mu_2^2 + \lambda_A \frac{v^2}{2} = M_{H^\pm}^2 + (\lambda_4 - \lambda_5)\frac{v^2}{2} = M_H^2 - \lambda_5 v^2,
\end{aligned}$$

$$\lambda_{L/A} = \lambda_3 + \lambda_4 \pm \lambda_5.$$

Self-coupling

$$\lambda_{hHH} = \lambda_L\, v, \quad \lambda_{hAA} = \lambda_A\, v \qquad \lambda_{hH^+H^-} = \lambda_3 v.$$

The quartic couplings between the SM Higgs and the new scalar are set by $\lambda_{3,L,A}$,

$$\lambda_{hhHH,hhAA,hhH^+H^-} = \lambda_L, \lambda_A, \lambda_3.$$

On the other hand, λ_2 controls all the quartic couplings solely within the dark sector ($HHHH$, $HHAA$, HHH^+H^-, $AAAA$, AAH^+H^- and $H^+H^-H^+H^-$).

Counting parameters: Independent Parameters

The IDM requires 5 extra parameters,

$$(\mu_2, \lambda_2, \lambda_3, \lambda_4, \lambda_5)\,.$$

Trade 3 of these parameters with the physical masses of the new scalars through

$$(\mu_2, \lambda_3, \lambda_4, \lambda_5; \lambda_2) \to \left(M_H, M_A, M_{H^\pm}, \lambda_{L/A}; \lambda_2\right) \text{ or } \left(M_H, M_A, M_{H^\pm}, \mu_2; \lambda_2\right).$$

λ_2: describes couplings solely between the additional scalars and not involving the SM Higgs. At tree-level for example and for $2 \to 2$ annihilation processes, λ_2 is irrelevant. This would mean that at one-loop order, for annihilation processes, a renormalisation for λ_2 is not necessary.

reconstruction

However, λ_4 and λ_5 can be reconstructed from a combination of the additional scalar masses

$$\lambda_4 = \frac{1}{v^2}\left(M_H^2 + M_A^2 - 2M_{H^\pm}^2\right),$$
$$\lambda_5 = \frac{1}{v^2}\left(M_H^2 - M_A^2\right).$$

The extraction of λ_3 not only requires a knowledge of at least one scalar mass but also either a value of λ_L (or equivalently the *hHH* coupling) or the mass parameter μ_2,

$$\lambda_3 = \frac{2}{v^2}\left(M_{H^\pm}^2 - \mu_2^2\right) = \frac{2}{v^2}\left(M_{H^\pm}^2 - M_H^2\right) + \lambda_L.$$

Renormalisation at one-loop: As much as possible a On-shell Scheme

• $\mathbb{Z}_2$ symmetry means no mixing between the new scalars

• $H, A, H^{\pm}$ masses are (physical) input parameters (instead of the parameters of the scalar potential)

• λ_2 not needed for scattering involving SM particles

• An extra parameter needs renormalisation (μ_2 or $\lambda_{L,A}$ or a combination; $\lambda_{L/A}$ has, at tree-level, a simple physical interpretation as the portal coupling hHH/hAA.)

Counterterms

All bare quantities (X_0), are decomposed into renormalised quantities (X) and counterterms (δX) as

$$X_0 \to X + \delta X, \quad X = \mu_2, \lambda_2, \lambda_3, \lambda_4, \lambda_5,$$

and wave function renormalisation is imposed

$$\phi_0 \to \phi + \frac{1}{2}\delta Z_\phi, \quad \phi = (h, H, A, H^\pm),$$

The OS conditions on the physical scalars require that their masses are defined as pole masses of the renormalised one-loop propagator and that the residue at the pole be unity. With $\Sigma_{\phi\phi}(p^2)$ being the scalar two-point function with momentum p we have $(\phi = h, H, A, H^{\pm})$,

$$
\begin{aligned}
\delta M_\phi^2 &= \Sigma_{\phi\phi}(M_\phi^2) \\
\delta Z_\phi &= - \left. \frac{\partial \Sigma_{\phi\phi}(p^2)}{\partial p^2} \right|_{p^2 = M_\phi^2} .
\end{aligned}
$$

hHH (hAA to define $\lambda_{L,A}$)

at tree-level $\mathcal{A}^0_{hHH} = -\lambda_L v$,

hHH (hAA to define $\lambda_{L,A}$)

Full one-loop renormalised amplitude for $h(p^2) \to H(p_1^2)H(p_2^2)$

$$\mathcal{A}_{hHH}^{\text{ren},1-\text{loop}}(p^2,p_1^2,p_2^2) = -\lambda_L v \left(\frac{\delta\lambda_L}{\lambda_L} + \frac{\delta v}{v} + \frac{1}{2}\delta Z_h + \delta Z_H \right) + \mathcal{A}_{HHh}^{\text{1PI}}(p^2,p_1^2,p_2^2),$$

$\mathcal{A}_{HHh}^{\text{1PI}}(p_1^2,p_2^2,p)$: full one-loop 1 particle irreducible vertex.

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$\mathcal{A}^{\mathrm{1PI}}_{HHh}(p_1^2,p_2^2,p)$: full one-loop 1 particle irreducible vertex. When the threshold is open: $p^2=M_h^2$, $p_1^2=p_2^2=M_H^2$, $p^2>(p_1+p_2)^2$, a gauge invariant OS counterterm for λ_L is

$$\frac{\delta^{\mathrm{OS}}\lambda_L}{\lambda_L} = \frac{\mathcal{A}^{\mathrm{1PI}}_{HHh}(m_h^2,M_H^2,M_H^2)}{\lambda_L v}-\frac{\delta v}{v}-\frac{1}{2}\delta Z_h-\delta Z_H.$$

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Another gauge invariant but scale dependent scheme is to use a $\overline{\rm MS}$ definition where only the (mass independent term) ultraviolet divergent part is kept

$$\frac{\delta^{\overline{\rm MS}}\lambda_L}{\lambda_L} = \left(\frac{\mathcal{A}_{HHh}^{\rm 1PI}(m_h^2,M_H^2,M_H^2)}{\lambda_L v} - \frac{\delta v}{v} - \frac{1}{2}\delta Z_h - \delta Z_H \right)_\infty .$$

A general scheme

A general scheme can be defined as

$$\frac{\delta\lambda_L}{\lambda_L} = \beta_{\lambda_L}\left(C_{\rm UV} + \ln(\bar{\mu}^2/Q_\lambda^2)\right), \quad C_{\rm UV} = \frac{2}{\varepsilon} - \gamma_E + \ln(4\pi)$$

Q_λ is an effective scale that depends on the external momenta and the internal masses introduced to define the counterterm, and $\bar{\mu}$ is the scale introduced by dimensional reduction. For $\overline{\rm MS}$, $Q_\lambda = \bar{\mu}$.

For $m_h < 2M_H$, difficult to come up with a straightforward OS scheme for λ_L

A *formal* OS extraction that would work for any configuration of H and h masses could use the cross-section that builds up direct detection, namely $Hq \to Hq$ in the limit of zero Q^2 transfer, in effect isolating the $H(M_H^2) \to H(M_H^2)h(Q^2 \to 0)$ vertex. But direct detection involves uncertainties through the introduction of parameters from nuclear matrix elements. Moreover, the λ_L contribution to direct detection can be swamped by the pure gauge contribution (which we discuss later).

SLOOPS

N. Baro, FB, G. Chalons, G. Drieu La Rochelle, S. Hao, Ninh Le Duc, A. Semenov, (D. Temes)

- Need for an automatic tool for susy calculations
- handles large numbers of diagrams both for tree-level
- and loop level
- able to compute loop diagrams at $v = 0$: dark matter, LSP, move at galactic velocities, $v = 10^{-3}$
- ability to check results: UV and IR finiteness but also gauge parameter independence for example
- ability to include different models easily and switch between different renormalisation schemes
- Used for SM one-loop multi-leg: new powerful loop libraries (with Ninh Le Duc, Sun Hao)

Strategy: Exploiting and interfacing modules from different codes

Lagrangian of the model defined in LanHEP

- particle content
- interaction terms
- shifts in fields and parameters
- ghost terms constructed by BRST

↓ ↓

Generic Model
-kinematical structures

Classes Model
-Feynman rules, including CT

⇓

Evaluation via FeynArts-FormCalc

LoopTools modified!!
tensor reduction inappropriate for small relative velocities
(Zero Gram determinants)

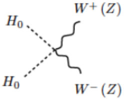
⇑

Renormalisation scheme

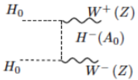
- definition of renorm. const. in the classes model

Non-Linear gauge-fixing constraints, gauge parameter dependence checks

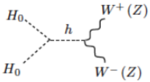
Annihilation processes for the relic density



g^2



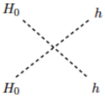
g^2



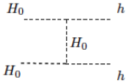
$\lambda_L g$

annihilation into gauge bosons

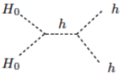
$$\lambda_L = \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)$$



λ_L

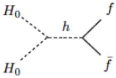


λ_L^2

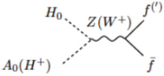


$\lambda_L \lambda_1$

annihilation into Higgs



$\lambda_L y_f$



g^2

annihilation into fermions

Benchmark Point: Only 2 regimes pass the test

I. Higgs Funnel

$M_H \sim M_h/2$

$HH \to WW^\star, b\bar{b}$

$2 \to 3$ processes at 1-loop

II. High Mass

$M_H > 500$GeV

EWPO imply $M_H \sim M_A \sim M_H^\pm$

Co-annihilation: $\to VV'$

low $\lambda_{L,A}$

Characteristics of the Heavy Mass Benchmark

$$M_H = 550 \text{ GeV}, \quad M_A = 551 \text{ GeV}, \quad M_H^{\pm} = 552 \text{ GeV},$$

$$\lambda_L = 0.0193 \qquad , \quad \lambda_2 = 0.01$$

$$(\lambda_3 = 0.0926, \quad \lambda_4 = -0.0545, \quad \lambda_5 = -0.0181 \text{ and } \mu_2 = 549.45 \text{ GeV}).$$

Characteristics of the Heavy Mass Benchmark

$$M_H = 550\ \mathrm{GeV}, \quad M_A = 551\ \mathrm{GeV}, \quad M_H^{\pm} = 552\ \mathrm{GeV},$$

$$\lambda_L = 0.0193\ \text{small!}, \quad \lambda_2 = 0.01 \mathit{irrelevant\ at\ tree-level}$$

$$(\lambda_3 = 0.0926, \quad \lambda_4 = -0.0545, \quad \lambda_5 = -0.0181\ \text{and}\ \mu_2 = 549.45\ \mathrm{GeV}).$$

Characteristics of the Heavy Mass Benchmark

- $\Omega h^2 \simeq 0.117$: at tree-level . OK
- stability: OK
- Minimum is associated with the inert vacuum. OK
- Mass degeneracy means that EWPO . OK
- Most stringent test (apart from relic density) is Direct detection

$$\sigma_{HN}^{(\lambda_L)} = f^2 \frac{\lambda_L^2}{4\pi} \left(\frac{m_N^2}{m_H m_h^2} \right)^2,$$

where m_N is the nucleon mass, and $f \sim 1/3$ is the nucleon form factor. With $M_H \gg M_W$, one can write

$$\mathcal{R}(\lambda_L) = \frac{\sigma_{HN}^{(g)}}{\sigma_{HN}^{(\lambda_L)}} \sim \left(6\pi \frac{\alpha^2}{\lambda_L s_W^4}\right)^2 \left(\frac{M_H}{8M_W}\right)^2 \left(1 + \frac{M_h^2}{M_W^2}\right)^2$$

$$\sim 9 \qquad \text{for } \lambda_L = 0.019 \text{ and } M_H = 550 \text{ GeV}.$$

Annihilation processes that contribute more than 5% to the relic density

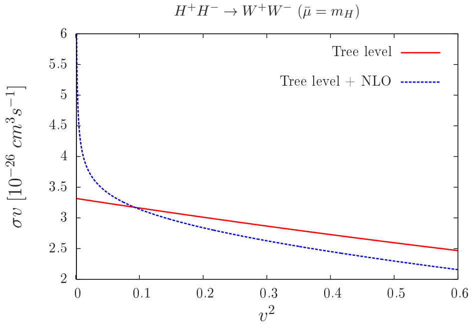
$$\begin{cases} HH \to W^+W^- \quad (18\%), \\ HH \to ZZ \quad (14\%), \\ H^+H^- \to W^+W^- \quad (13\%), \\ AA \to W^+W^- \quad (9\%), \\ H^+H \to W^+\gamma \quad (8\%), \Longrightarrow \textit{pure gauge} \\ AA \to ZZ \quad (7\%), \\ H^+A \to W^+\gamma \quad (6\%) \Longrightarrow \textit{pure gauge}. \end{cases}$$

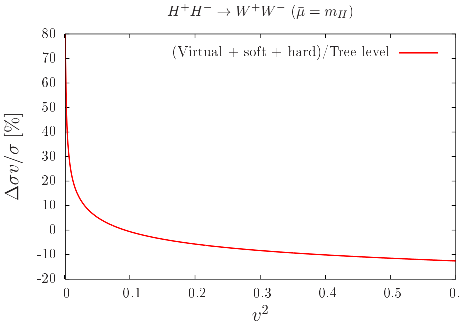
Because $\lambda_L \ll 1$,

$$\sigma_{HH\to W^+W^-} \sim \sigma_{AA\to W^+W^-} \sim \sigma_{H^+H^-\to W^+W^-} = 2c_W^4 \sigma_{HH\to ZZ} = 2c_W^4 \sigma_{AA\to ZZ}$$

Processes at one-loop. Will show results for $\bar{\mu} = M_H$ taken to define λ_L

$H^+H^- \to W^+W^-$

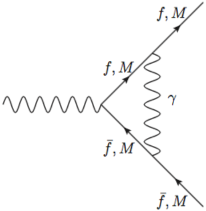




Sharp rise for $v \to 0$: Sommerfeld effect

Sommerfeld Effect

- Singularities arise in scalar triangle C_0 and box D_0 loop integrals when $\beta \to 0$.



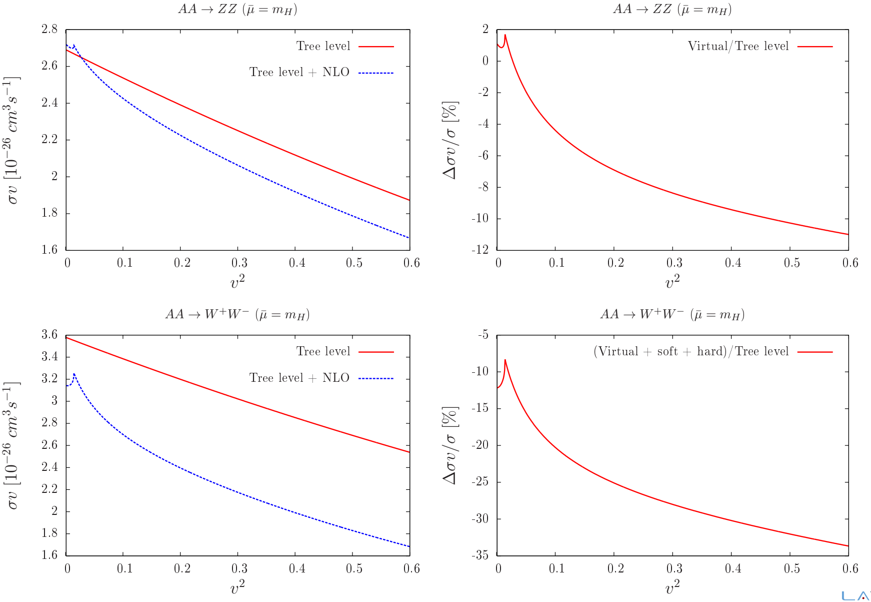
$$C_0 \xrightarrow{\beta \to 0} -\frac{\pi^2}{Q^2 \beta}$$

$$\beta = v/2 = \sqrt{1 - 4M^2/Q^2}$$

- D_0 has the same infrared behavior because for $\beta = 0$ it can be split into a sum of triangle integrals.
- This effect can be resummed to all orders.
- $S_{1L} = \frac{\pi\alpha}{v} \times \sigma_0 Q_i Q_j$
- $S_{nr} = X_{nr}/(1 - e^{-X_{nr}}) \times \sigma_0 \quad X_{nr} = 2\pi\alpha Q_i Q_j / v$

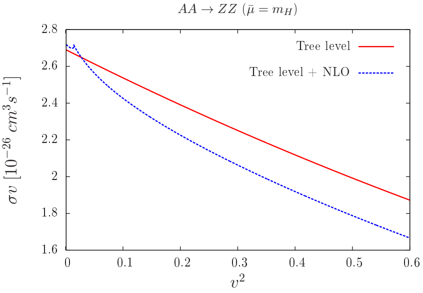
Since characteristic velocities for the calculation of the relic density are typically in the range $v \sim 0.2 - 0.3$, the Sommerfeld enhancement taken either at one-loop or resummed to all orders does not have much of an impact on the relic density.

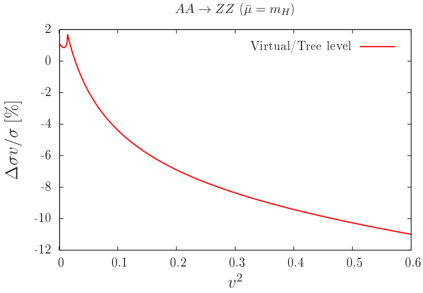
$AA \to ZZ, W^+W^-$



$AA \to ZZ\ (\bar{\mu} = m_H)$
Tree level
Tree level + NLO
$\sigma v\ [10^{-26}\ cm^3 s^{-1}]$
v^2
$AA \to ZZ\ (\bar{\mu} = m_H)$
Virtual/Tree level
$\Delta\sigma v/\sigma$ [%]
v^2
$AA \to W^+W^-\ (\bar{\mu} = m_H)$
Tree level
Tree level + NLO
$\sigma v\ [10^{-26}\ cm^3 s^{-1}]$
v^2
$AA \to W^+W^-\ (\bar{\mu} = m_H)$
(Virtual + soft + hard)/Tree level
$\Delta\sigma v/\sigma$ [%]
v^2

$AA \to ZZ, W^+W^-$



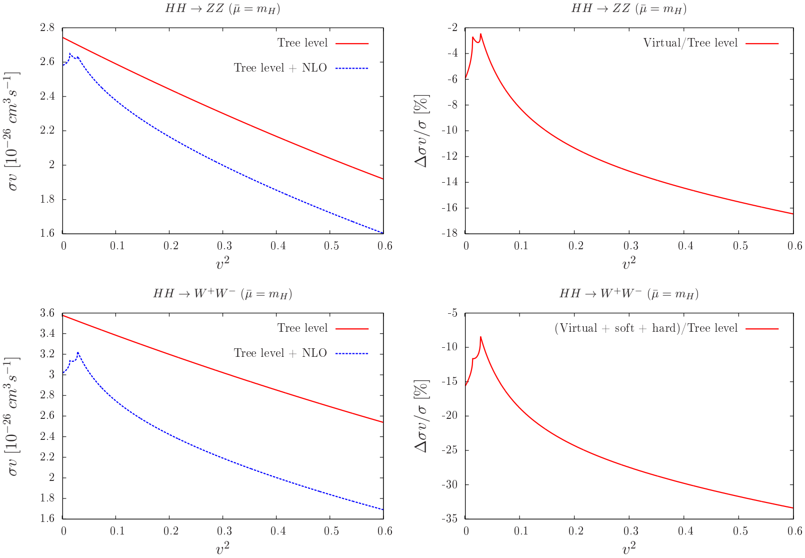


Dent at small v:

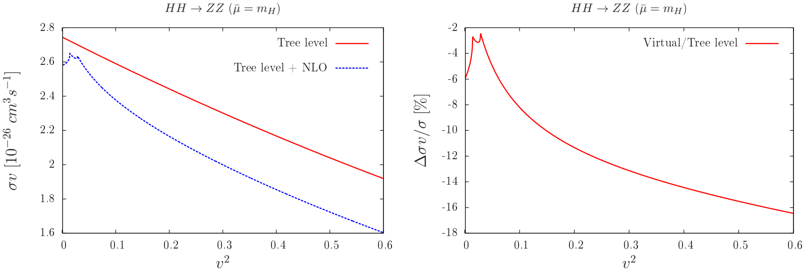
Electroweak Sommeferld Rescattering (exchange of W, coupling $AH^{\pm}W^{\mp}$.)

No exchange of Z. No AAZ coupling).

$HH \to ZZ, W^+W^-$

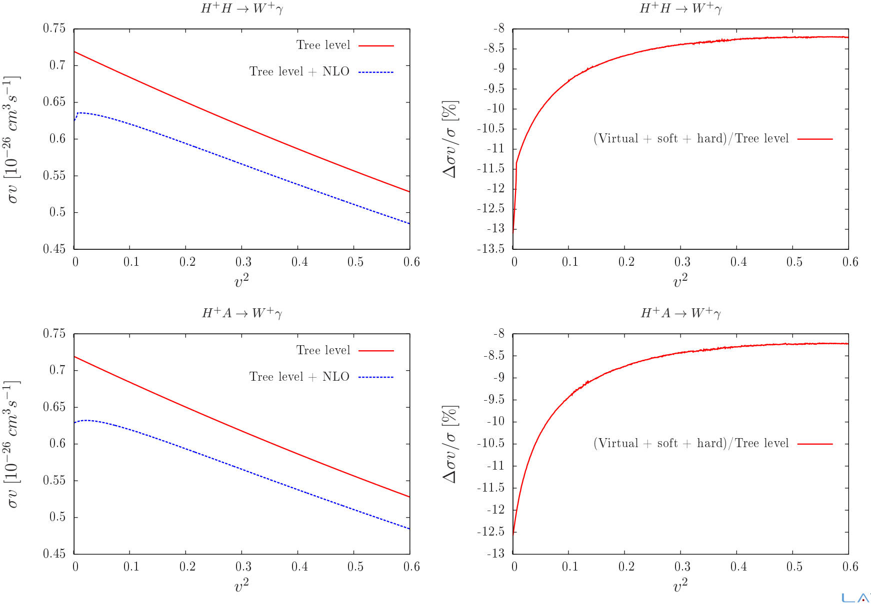

$HH \to ZZ$ ($\bar{\mu} = m_H$)
Tree level
Tree level + NLO
σv [10^{-26} $cm^3 s^{-1}$]
v^2
$HH \to ZZ$ ($\bar{\mu} = m_H$)
Virtual/Tree level
$\Delta\sigma v/\sigma$ [%]
v^2
$HH \to W^+W^-$ ($\bar{\mu} = m_H$)
Tree level
Tree level + NLO
σv [10^{-26} $cm^3 s^{-1}$]
v^2
$HH \to W^+W^-$ ($\bar{\mu} = m_H$)
(Virtual + soft + hard)/Tree level
$\Delta\sigma v/\sigma$ [%]
v^2

$HH \to ZZ, W^+W^-$

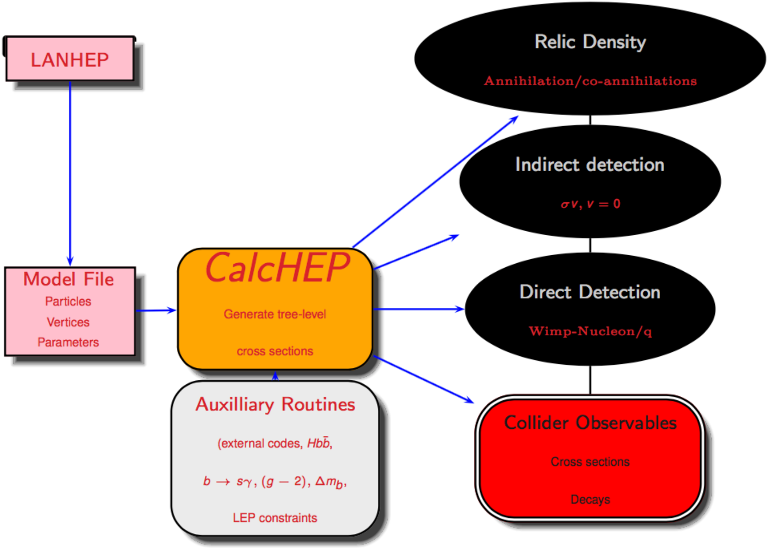


Double dents at small v: Sommeferld Rescattering (exchange of W and Z)

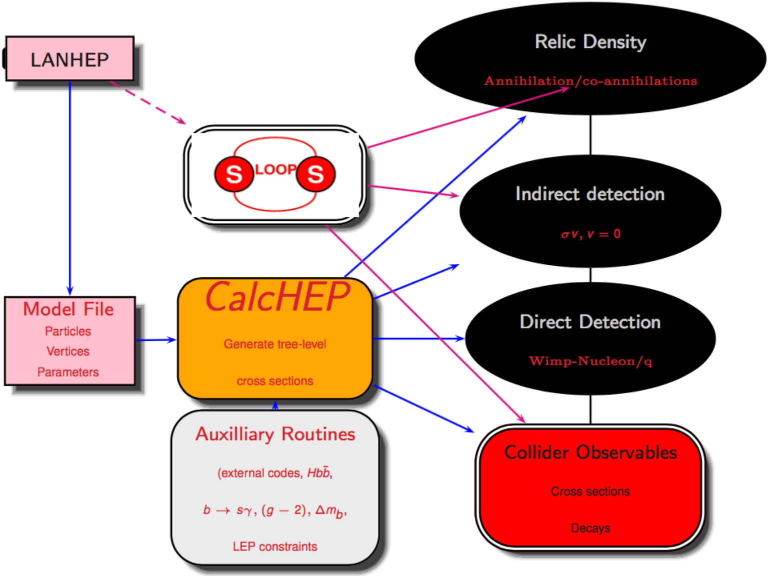
$H^+H \to W^+\gamma$ and $H^+A \to W^+\gamma$: since λ_L independent, $\bar{\mu}$ independent.



microMEGAs


LANHEP
Model File
Particles
Vertices
Parameters
CalcHEP
Generate tree-level
cross sections
Auxilliary Routines
(external codes, $Hb\bar{b}$,
$b \to s\gamma$, $(g-2)$, Δm_b,
LEP constraints
Relic Density
Annihilation/co-annihilations
Indirect detection
$\sigma v, v = 0$
Direct Detection
Wimp-Nucleon/q
Collider Observables
Cross sections
Decays

SloopS+micrOMEGAs



Relic density at one-loop

Process	LO	$\mu = m_X$	$\mu = m_X/2$	$\mu = 2 \times m_X$
$HH \to W^+W^-$	18%	16%	18%	14%
$HH \to ZZ$	14%	14%	14%	14%
$H^+H^- \to W^+W^-$	13%	15%	15%	15%
$AA \to W^+W^-$	9%	8%	9%	7%
$H^+H \to W^+\gamma$	8%	7%	7%	8%
$AA \to ZZ$	7%	7%	7%	8%
$H^+A \to W^+\gamma$	6%	6%	6%	7%
$\bullet H^+H^- \to \gamma\gamma$	5%	5%	5%	6%
$\bullet H^+H^- \to \gamma Z$	4%	5%	4%	5%
$\bullet H^+H \to ZW^+$	3%	3%	3%	3%
$\bullet H^+A \to ZW^+$	3%	3%	2%	3%
$\bullet H^+H^- \to ZZ$	2%	2%	2%	2%

Relative contributions to the relic abundance with and without corrections. Note that although the cross-sections for the last 5 processes (identified with $\bullet$) are not loop corrected, their relative contribution could change.

News from the dark sector: Impact of λ_2

λ_2 does not enter the calculation of the annihilation cross-sections at tree-level, but at one-loop rescattering of $HH \to HH$ involves the self-coupling λ_2.

λ_2	$\bar{\mu} = M_H$	$\bar{\mu} = M_H/2$	$\bar{\mu} = 2M_H$
0.01	0.12494 (6.9%)	0.11652 (-0.3%)	0.13469 (15.3%)
0.1	0.12210 (4.5%)	0.11843 (1.3%)	0.12601 (7.8%)
1	0.09950 (-14.9%)	0.14163 (21.2%)	0.07683 (-34.3%)

Summary λ_2

- small change when λ_2 is increased from 0.01 to 0.1, the scale variation is reduced and the total electroweak corrections to the relic density are below 7.8%.

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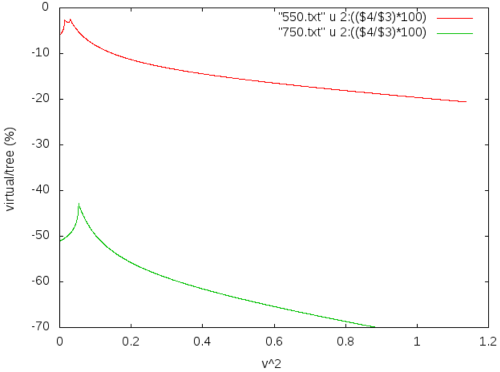
Summary λ_2

- small change when λ_2 is increased from 0.01 to 0.1, the scale variation is reduced and the total electroweak corrections to the relic density are below 7.8%.
- The case $\lambda_2 = 1$ is much more interesting. The corrections are now quite large for each of the three renormalisation scales $M_H/2$, M_H and $2M_H$. For all of these three scales, the tree-level benchmark point would be ruled out.
- However, we note that the large scale uncertainty with corrections ranging between +21.2% for $\mu = M_h/2$ and -34.3% for $\mu = 2M_H$ means that a judicious scale choice, within the range $M_H/2$ to $2M_H$, can minimise the corrections. A more thorough one-loop analysis is in order by studying other scenarios with a larger range of values for the other quartic couplings.

TeV scale DM



Very large corrections

Needs resummations

Matching perturbative and non perturbative calculations to be set up

Conclusions

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- One could find points not allowed by a tree-level analysis that could be validated by a one-loop analysis.
- although the virtual effect of λ_2 is not at all negligible it (fortunately or unfortunately) introduces also a non-negligible scale variation to the corrections.
- More importantly, compared to a tree-level treatment, one loop corrections introduce not only a scale uncertainty which is manageable for small values of λ_2 but also a parametric dependence (dependence on λ_2) which is not caught by a tree-level treatment.

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- More importantly, compared to a tree-level treatment, one loop corrections introduce not only a scale uncertainty which is manageable for small values of λ_2 but also a parametric dependence (dependence on λ_2) which is not caught by a tree-level treatment.
- Improve TeV scale treatment