Naturalness vs Unitarity and BFB: a comparative study in Higgs Triplet Models

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Plan

- Type II Seesaw Model (HTM2)
- Higgs Triplet Model (HTM0)

- Model Description
- The Higgs spectrum
- From Unitarity to Naturalness
- Veltman Conditions (VC)

- RESULTS
- Model parameter space
- Model Higgs masses

Implement a scalar field Δ with hypercharge $y = 2$ in Standard Model (Perez et al. PRD 78 (2008) & Arhrib et al. PRD 84 (2011)) :

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) + Tr(D_\mu \Delta)^\dagger (D^\mu \Delta) - V(H, \Delta) + \mathcal{L}_{\text{Yukawa}}$$

The potential $V(H, \Delta)$ reads as,

$$V(H,\Delta) = -m_H^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 + M_\Delta^2 Tr(\Delta^\dagger \Delta) + [\mu(H^T i\sigma^2 \Delta^\dagger H) + \text{h.c}]$$
$$+\lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr\Delta^\dagger \Delta)^2 + \lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H$$

The Triplet Δ and Higgs doublet H are represented by :

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

with $\delta^0 = \frac{1}{\sqrt{2}}(v_t + \xi^0 + iZ_2)$ and $\phi^0 = \frac{1}{\sqrt{2}}(v_d + h + iZ_1)$.

After EWSB, HTM2 Higgs spectrum consists of : two CP_{even} neutral scalars (h^0, H^0), one neutral pseudoscalar A^0 and a pair of simply-, doubly- charged Higgs bosons $(H^{\pm}, H^{\pm\pm})$,

$$m_{h^0} = \frac{1}{2}(A + C - \sqrt{(A-C)^2 + 4B^2})$$

$$m_{H^0} = \frac{1}{2}(A + C + \sqrt{(A-C)^2 + 4B^2})$$

$$m^2_{H^{\pm\pm}} = \frac{\sqrt{2}\mu v_d^2 - \lambda_4 v_d^2 v_t - 2\lambda_3 v_t^3}{2v_t}$$

$$m^2_{H^{\pm}} = \frac{(v_d^2 + 2v_t^2)\,[2\sqrt{2}\mu - \lambda_4 v_t]}{4v_t}$$

$$m^2_{A^0} = \frac{\mu(v_d^2 + 4v_t^2)}{\sqrt{2}v_t}$$

$$A = \frac{\lambda}{2}v_d^2, \quad B = v_d(-\sqrt{2}\mu + (\lambda_1 + \lambda_4)v_t), \quad C = \frac{\sqrt{2}\mu\, v_d^2 + 4(\lambda_2 + \lambda_3)v_t^3}{2v_t}$$

HTM2 Higgs potential parameters must obey several constraints originating from theoretical requirements :

- BFB :

$$\lambda \geq 0 \;\;\&\;\; \lambda_2 + \lambda_3 \geq 0 \;\;\&\;\; \lambda_2 + \frac{\lambda_3}{2} \geq 0$$
$$\&\;\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \;\;\&\;\; \lambda_1 + \lambda_4\sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0$$
$$\&\;\; \lambda_3\sqrt{\lambda} \leq |\lambda_4|\sqrt{\lambda_2 + \lambda_3} \;\; \text{or} \;\; 2\lambda_1 + \lambda_4 + \sqrt{(\lambda - \lambda_4^2)(2\frac{\lambda_2}{\lambda_3} + 1)} \geq 0$$

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$$\&\;\; \lambda_1 + \sqrt{\lambda(\lambda_2+\lambda_3)} \geq 0 \;\;\&\;\; \lambda_1 + \lambda_4\sqrt{\lambda(\lambda_2+\lambda_3)} \geq 0$$

$$\&\;\; \lambda_3\sqrt{\lambda} \leq |\lambda_4|\sqrt{\lambda_2+\lambda_3} \;\; \text{or} \;\; 2\lambda_1 + \lambda_4 + \sqrt{(\lambda - \lambda_4^2)(2\frac{\lambda_2}{\lambda_3}+1)} \geq 0$$

- Unitarity :

$$|\lambda_1+\lambda_4| \leq 8\pi,\; |\lambda_1| \leq 8\pi,\; |2\lambda_1+3\lambda_4| \leq 16\pi,\; |\lambda| \leq 16\pi, |\lambda_2| \leq 4\pi$$

$$|\lambda + 4\lambda_2 + 8\lambda_3 \pm \sqrt{(\lambda - 4\lambda_2 - 8\lambda_3)^2 + 16\lambda_4^2}\;| \leq 32\pi,$$

$$|3\lambda + 16\lambda_2 + 12\lambda_3 \pm \sqrt{(3\lambda - 16\lambda_2 - 12\lambda_3)^2 + 24(2\lambda_1+\lambda_4)^2}\;| \leq 32\pi$$

$$|2\lambda_1 - \lambda_4| \leq 16\pi, \quad |2\lambda_2 - \lambda_3| \leq 8\pi, |\lambda_2+\lambda_3| \leq 4\pi$$

Other constraints :

- ρ parameter : $\rho \simeq 1 - 2\frac{v_t^2}{v_d^2}$.
  From Precision measurements : $|\delta\rho| \leq 10^{-3} \rightarrow v_t \leq 5\ GeV$.
- μ constraints :$\mu > 0$,..
- Experimental mass limits for the Heavy Higgs bosons.

Veltman conditions means that the quadratic divergencies of the two possible tadpoles T_{h^0} and T_{H^0} of the CP-even neutral scalar fields vanish.

- Combinations $s_\alpha T_{h^0} + c_\alpha T_{H^0}$ and $c_\alpha T_{h^0} - s_\alpha T_{H^0}$ induce nice simplification, that ends up with short expressions :

$$T_d = -2Tr(I_n)\Sigma_f \frac{m_f^2}{v_d^2} + 3(\lambda + 2\lambda_1 + \lambda_4) + 2\frac{m_W^2}{v_{sm}^2}(\frac{1}{c_w^2} + 2)$$

$$T_t = 4\frac{m_W^2}{v_{sm}^2}(\frac{1}{c_w^2} + 1) + (2\lambda_1 + 8\lambda_2 + 6\lambda_3 + \lambda_4)$$

Here $c_w = cos(\theta_{Weinberg})$, $v_{SM}^2 = 246$ GeV .

$$T_d = -2Tr(I_n)\Sigma_f \frac{m_f^2}{v_d^2} + 3(\lambda + 2\lambda_1 + \lambda_4) + 2\frac{m_W^2}{v_{sm}^2}(\frac{1}{c_w^2} + 2)$$

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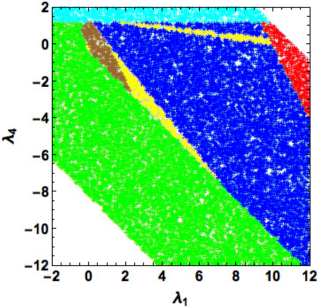
Veltman Condition in SM : remove λ_1 and λ_4 in formula T_d (M.C. Peyranere et al. PLB 260 (1991)).

We show the impact of VC on the parameter space : Allowed regions in (λ_1, λ_4) plan.

(cyan) : Excluded by μ constraints.
(red) : Excluded by μ+Unitarity.
(green) : Excluded by μ+Unitarity+BFB.
(blue) : Excluded by μ+Unitarity+BFB+$R_{\gamma\gamma}$
(yellow) : Excluded by μ+Unitarity+BFB $R_{\gamma\gamma}$& $T_d = 0 \wedge T_t = 0$.
(Inputs : $-2 \leq \lambda_1 \leq 12$, $\lambda_2 = -\frac{1}{6}$, $\lambda_3 = \frac{3}{8}$, $-12 \leq \lambda_4 \leq 2$, $v_t = 1$ GeV and $\mu = 1$ GeV.)

Only brown area obeys ALL constraints .



Inclusion of VC clearly reduces the (λ_1, λ_4) parameter space.

As consequence, tadpole additions have strong impact on the Higgs masses.

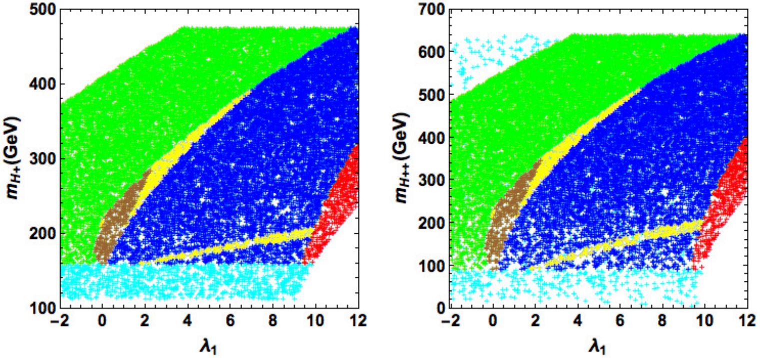
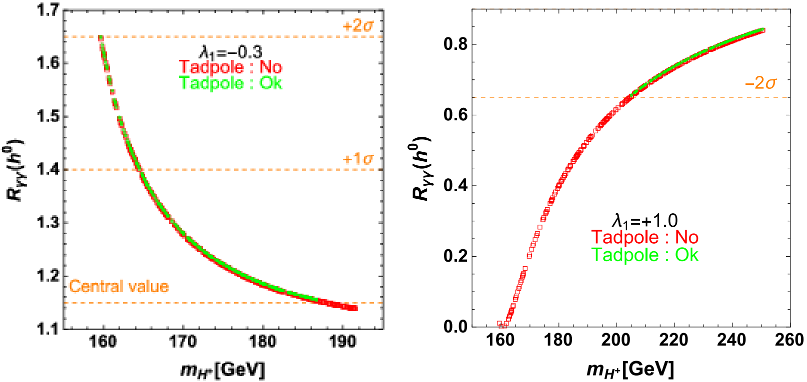
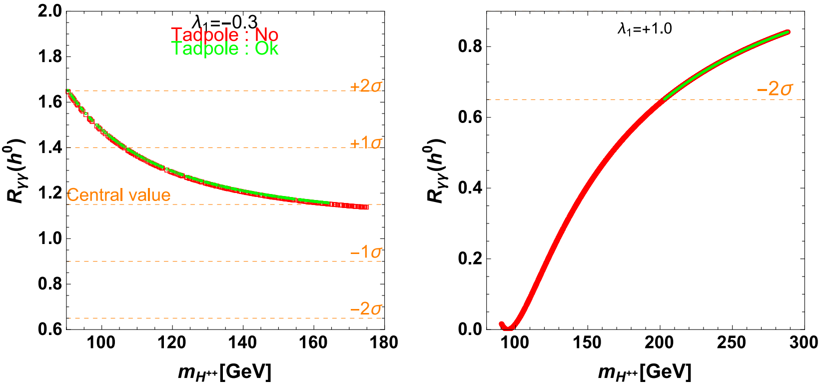


FIGURE – Allowed regions in $(\lambda_1, m_{H^\pm})$ (left) and $(\lambda_1, m_{H^{\pm\pm}})$ (right) plans. Inputs : $-2 \leq \lambda_1 \leq 12$, $\lambda_2 = -\frac{1}{6}$, $\lambda_3 = \frac{3}{8}$, $-12 \leq \lambda_4 \leq 2$, $v_t = 1$ GeV and $\mu = 1$ GeV.


1.7
1.6
1.5
1.4
1.3
1.2
1.1
160
170
180
190
$R_{\gamma\gamma}(h^0)$
$m_{H^{+}}$[GeV]
$+2\sigma$
$+1\sigma$
Central value
λ_1=−0.3
Tadpole : No
Tadpole : Ok
0.8
0.6
0.4
0.2
0.0
160
180
200
220
240
260
-2σ
λ_1=+1.0
Tadpole : No
Tadpole : Ok


λ_1=−0.3
Tadpole : No
Tadpole : Ok
+2σ
+1σ
Central value
−1σ
−2σ
$R_{\gamma\gamma}(h^0)$
$m_{H^{++}}$[GeV]
λ_1=+1.0
−2σ
$R_{\gamma\gamma}(h^0)$
$m_{H^{++}}$[GeV]

m_Φ	**Unitarity**	**Unitarity + BFB**	**Unitarity + BFB +** $R_{\gamma\gamma}$	**Unitarity + BFB +** $R_{\gamma\gamma}$ **+ mVC**
H^0	$[206.8 - 207.3]$ GeV	$[206.8 - 207]$ GeV	$[206.8 - 207]$ GeV	206.8 GeV
A^0	206.8 GeV	206.8 GeV	206.8 GeV	206.8 GeV
$H^\pm$	$[160 - 474]$ GeV	$[160 - 474]$ GeV	$[160 - 392]$ GeV	$[161 - 288]$ GeV
$H^{\pm\pm}$	$[90 - 637]$ GeV	$[90 - 637]$ GeV	$[90 - 513]$ GeV	$[90 - 351]$ GeV

TABLE I. Higgs bosons masses allowed intervals in the Higgs triplet model resulting from various constraints, including the modified Veltman conditions

Lagrangian of the scalar sector is given by,

$$\mathcal{L} \quad = \quad (D_\mu H)^\dagger (D^\mu H) + Tr(D_\mu \Delta)^\dagger (D^\mu \Delta) - V(H, \Delta) + \mathcal{L}_{\text{Yukawa}}$$

with the potential $V(H, \Delta)$,

$$\begin{aligned} V(H, \Delta) \quad = \quad & -m_H^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 - M_\Delta^2 Tr(\Delta^\dagger \Delta) + \mu H^\dagger \Delta H \\ & + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr \Delta^\dagger \Delta)^2 + \lambda_3 Tr(\Delta^\dagger \Delta)^2 \\ & + \lambda_4 H^\dagger \Delta^\dagger \Delta H \end{aligned} \tag{1}$$

The two Higgs multiplets in components as :

$$\Delta = \frac{1}{2}\begin{pmatrix} \delta^0 & \sqrt{2}\delta^+ \\ \sqrt{2}\delta^- & -\delta^0 \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2}$$

with

$$\phi^0 = \frac{1}{\sqrt{2}}(v_d + h_1 + i\,z_1) \quad \text{and} \quad \delta^0 = v_t + h_2 \tag{3}$$

After EWSB, denoting the corresponding VEVs as,

$$\langle\Delta\rangle = \frac{1}{2}\begin{pmatrix} v_t & 0 \\ 0 & -v_t \end{pmatrix} \quad \text{and} \quad \langle H\rangle = \begin{pmatrix} 0 \\ v_d/\sqrt{2} \end{pmatrix} \tag{4}$$

one finds the conditions :

$$M_\Delta^2 = \frac{\lambda_a}{2}v_d^2 - \frac{\mu v_d^2}{4v_t} + \lambda_b v_t^2 \quad (5)$$

$$m_H^2 = \frac{\lambda}{4}v_d^2 - \frac{\mu v_t}{2} + \frac{\lambda_a}{2}v_t^2 \quad (6)$$

with $\lambda_a = \lambda_1 + \frac{\lambda_4}{2}$ and $\lambda_b = \lambda_2 + \frac{\lambda_3}{2}$.

$$m_{H^\pm}^2 = \frac{(v_d^2 + 4v_t^2)}{4v_t}\mu \quad (7)$$

Main difference with HTM2 is the absence of A^0 and $H^{\pm\pm}$ from HTM0 spectrum.

The phenomenological analysis is performed in the parameter space scanned by the potential parameters obeying the standard theoretical constraints.

BFB :

As usual, consider the scalar potential at large field values,

$$\begin{aligned} V^{(4)}(H,\Delta) &= \lambda(H^\dagger H)^2/4 + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) \\ &+ \lambda_2 (Tr\Delta^\dagger \Delta)^2 + \lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta^\dagger \Delta H \end{aligned} \tag{8}$$

Again, BFB requires that $V^{(4)} > 0$ *for all* directions, consequently translated to :

$$\lambda \geq 0 \;\; \& \;\; \lambda_b \geq 0 \tag{9}$$

$$\& \;\; \lambda_a + \sqrt{\lambda \lambda_b} \geq 0 \tag{10}$$

Unitarity :

$$|\lambda_a| \leq \kappa\pi \tag{11}$$

$$|\lambda| \leq 2\kappa\pi \tag{12}$$

$$|\lambda_b| \leq \frac{\kappa}{2}\pi \tag{13}$$

$$|3\lambda + 10\lambda_b \pm \sqrt{(3\lambda - 10\lambda_b)^2 + 48\lambda_a^2}\,| \leq 4\kappa\pi \tag{14}$$

Note that $\kappa = 8$ since the formula $|Re(a_0)| \leq \frac{1}{2}$ has been adopted..

Collect the quadratic divergencies of the CP-neutral Higgs H^0 and h^0 tadpoles, then derive mVC :

$$T_d = v_d \Big(-2Tr(I_n)\Sigma_f \frac{m_f^2}{v_d^2} + 3(\lambda + \lambda_a) + 2\frac{m_W^2}{v_{sm}^2}(\frac{1}{c_w^2} + 2)\Big)$$

and for the triplet :

$$T_t = v_t \Big(8\frac{m_W^2}{v_{sm}^2} + 2\lambda_a + 5\lambda_b\Big)$$

with $v = \sqrt{v_d^2 + v_t^2}$
In the parameter space, they vary within the reduced conservative range from 0.1 to 10 GeV.

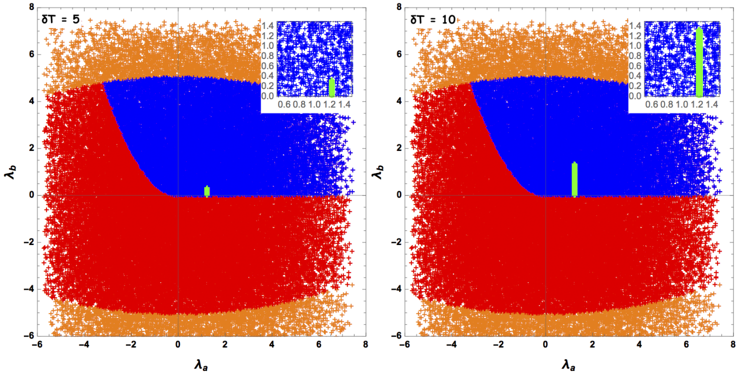


figure
Allowed regions in (λ_a, λ_b) for $\delta T = 5\,,10$.
orange : Excluded by Unitarity.
red : Excluded by Unitarity & BFB..
blue : Excluded by Unitarity & BFB & VC.
Green area obeys ALL constraints.
(Inputs are : $-5 \leq \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \leq 5$, $v_t = 1$ GeV and $2 \leq \mu \leq 5$ (GeV))

- The parameter space dramatically reduced... its extent depends on the value given to the deviation δT.
- Naturalness constraint is stronger than the other theoretical conditions.

This figure displays the deficit of $R_{\gamma\gamma}(h^0)$ as a function of $H^{\pm}$ mass for various values of λ_a, and with $m_{H^0} \geq 140$ GeV.

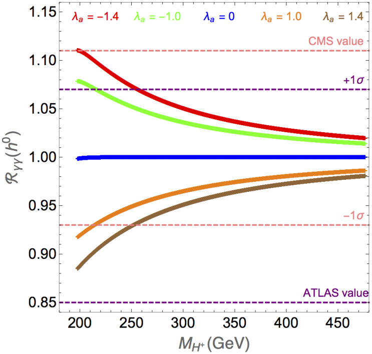


FIGURE – $R_{\gamma\gamma}(h^0)$ vs $m_{H\pm}$
(Inputs : $2.5 \leq \mu \leq 15$ (GeV) ($m_{h^0} \approx 125$ GeV), $\lambda_b = 1$ and $v_t = 1$ GeV)

- As can be seen, a mass about 255 GeV and above is allowed for $H^{\pm}$ within $+1\sigma$ of **ATLAS** data for $\lambda_a = -1.4$. Once λ_a increases, this lower bound decreases consistently to reach its lowest value around $\sim$ 197 GeV.
- For **CMS**, this situation is exactly the opposite : only the range $200 \leq m_{H^{\pm}} \leq 250$ (GeV) is excluded for $\lambda_a = 1.4$. Besides, $R_{\gamma\gamma}(h^0)$ tends towards its standard values for $\lambda_a \neq 0$, and to 1 for large $m_{H^{\pm}}$ whatever the variation of λ_a.

- Naturalness is stronger than the other theoretical constraints.
- **Drastic reduction** of the parameter space due to mVC.
- Important effect for the **upper bound** on $m_{H^{\pm\pm}}$ which is reduced to 288 GeV (HTM2).
- Similarly, the **upper bound** on $m_{H^{\pm}}$ decreased to 351 GeV.
- In HTM0, $m_{H^{\pm}}$ **lower bound** varies between 197 and 255 GeV.

Thank you