# Compton Scattering of Polarized Electrons


**Mohamed Jakha, Souad Taj and Bouzid Manaut**

Department of Physics, Polydisciplinary Faculty,
Sultan Moulay Slimane University,
Beni Mellal, Morocco.



## Abstract

*In this work, we present a study of Compton scattering of polarized electrons using the helicity formalism. First, we begin with unpolarized Compton scattering analyzed in the Laboratory frame, to show the known Klein-Nishina formula and its classical version called the Thomson formula. Then, we treat the Compton scattering by polarizing the electrons in the Center-of-Mass frame. A well-known concept of spin-flip differential cross section (DCS) as well as spin-non flip DCS is introduced. An important consistency check that has been carried out successfully is that the sum of the two spin-flip and spin-non flip DCSs always gives the unpolarized DCS. It should be noted that we used the REDUCE [1] software to compute the complicated traces and MATHEMATICA to obtain the graphs of the DCSs .*


## 1. Introduction

THE THEORETICAL study of scattering processes, based on the mathematical formalism of QED, can be considered as an useful way to test the validity of this theory, and to ensure the compatibility between its predictions and the experimental results. Generally, the study of collisions is very important in particle physics, because first of all, historically, the discovery of the quantum world and the knowledge of the fundamental properties of particles were made by studying scattering processes. The Compton scattering, which is the subject of this work, is one of the basic processes in QED. It is the inelastic scattering of a photon with an electrically charged particle, first discovered in 1923 by Arthur Compton. Furthermore, the Compton scattering is a process that can be described with a high level of precision by the theory of QED. The main objective of this work is to treat the Compton scattering of polarized electrons, i.e. to treat the process taking into account that only the electrons are in well defined states of polarization.

## 2. Unpolarized Compton scattering in the Laboratory frame

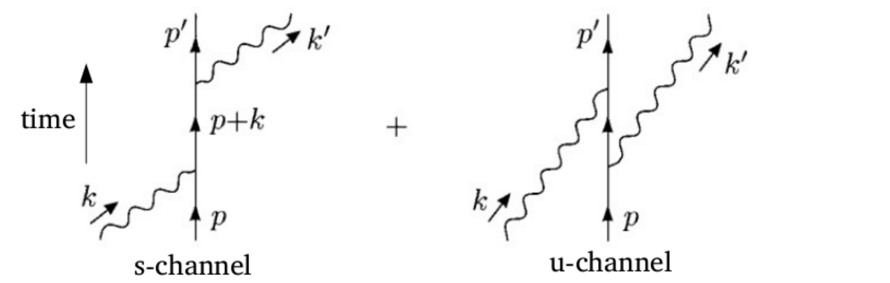


**Figure 1:** *Feynman diagrams for Compton scattering:* $e^- + \gamma \longrightarrow e^- + \gamma$.

As usual, the Feynman rules allow us to write the unpolarized squared matrix element as follows:

$$|\overline{\mathcal{M}_{fi}}|^2 = \frac{1}{4}\, e^4\, Tr\Big[(\not{p}' + m)\Big(\frac{\gamma^\mu \not{k}\gamma^\nu + 2\gamma^\mu p^\nu}{2p.k} + \frac{\gamma^\nu \not{k}'\gamma^\mu - 2\gamma^\nu p^\mu}{2p.k'}\Big) \times (\not{p} + m)\Big(\frac{\gamma_\nu \not{k}\gamma_\mu + 2\gamma_\mu p_\nu}{2p.k} + \frac{\gamma_\mu \not{k}'\gamma_\nu - 2\gamma_\nu p_\mu}{2p.k'}\Big)\Big]. \quad (1)$$

$$|\overline{\mathcal{M}_{fi}}|^2 = 2e^4\Big[\frac{p.k'}{p.k} + \frac{p.k}{p.k'} + 2m^2\Big(\frac{1}{p.k} - \frac{1}{p.k'}\Big) + m^4\Big(\frac{1}{p.k} - \frac{1}{p.k'}\Big)^2\Big]. \quad (2)$$

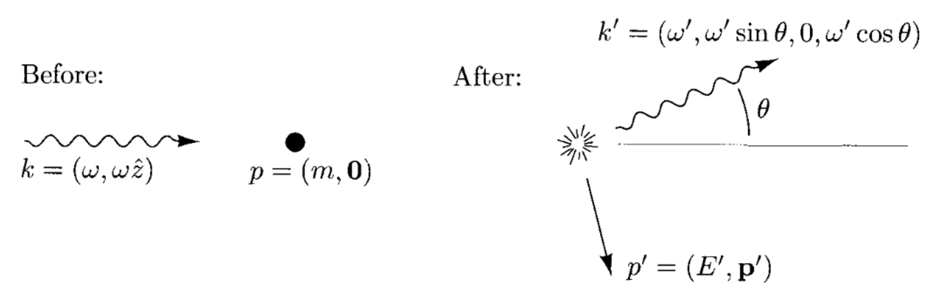


**Figure 2:** *Compton scattering in the laboratory frame [2].*

The general expression of the cross section for unpolarized scattering involving two initial particles and $n-2$ final particles is given by:

$$\overline{d\sigma} = \frac{1}{\phi}|\overline{\mathcal{M}_{fi}}|^2 \prod_{f=3}^{n} \frac{d^3\overrightarrow{p}_f}{(2\pi)^3 2E_f}(2\pi)^4\delta^4(P_f - P_i). \quad (3)$$

Where $\phi = 4\sqrt{(p_1.p_2)^2 - m_1^2 m_2^2}$ is the flux of the incident particles.
For our process, $\phi$ is written in the Lab frame as follows:

$$\phi = 4\sqrt{(k.p)^2}, \quad = 4mw. \quad (4)$$

The **Klein-Nishina** formula which is first derived in 1929:

$$\left(\frac{\overline{d\sigma}}{d\Omega_{k'}}\right)_{Lab} = \frac{\alpha^2}{2m^2}\left(\frac{w'}{w}\right)^2\left[\frac{w'}{w} + \frac{w}{w'} - \sin^2(\theta_L)\right]. \quad (5)$$

The **Thomson** formula: In the limit of low photon energies ($w \longrightarrow 0$), then $\frac{w'}{w} \longrightarrow 1$ and the DCS takes the following form:

$$\left(\frac{\overline{d\sigma}}{d\Omega_{k'}}\right)^{Lab}_{w\to 0} = \frac{\alpha^2}{2m^2}(1 + \cos^2(\theta_L)) \quad , \quad \sigma_{tot} = \frac{8\pi\alpha^2}{3m^2}. \quad (6)$$

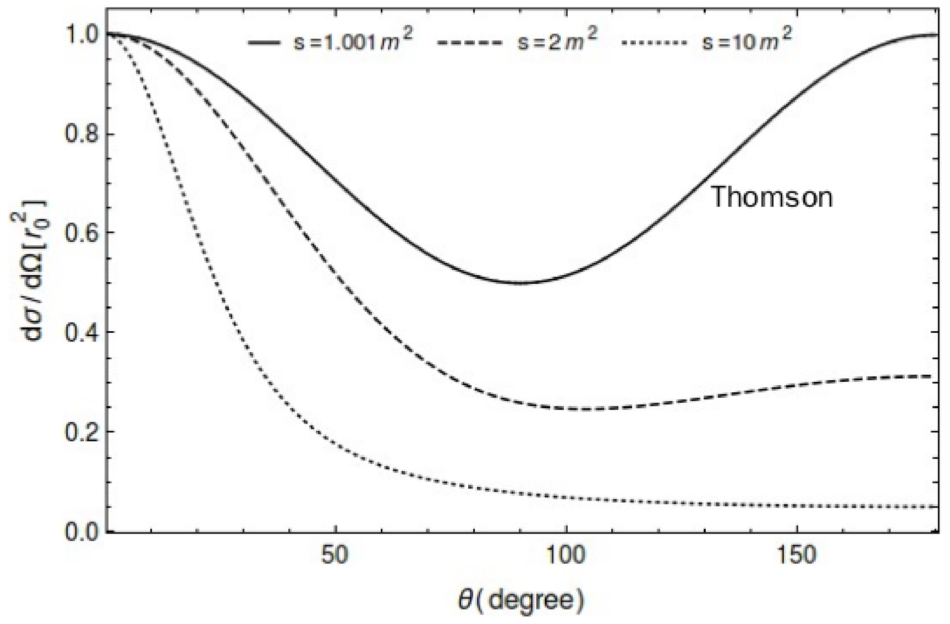


**Figure 3:** *Unpolarized DCS as a function of the angle* $\theta_L$ *in the Lab frame, drawn for different values of energy* $s$.

## 3. Compton scattering of polarized electrons in the Center-of-Mass frame

An ensemble of electrons is said to be polarized if the electron spins have a preferential orientation so that there exists a direction for which the two possible spin states are not equally populated [3].
Let us apply the polarization of electrons on one of the most important processes of QED namely the Compton scattering.
Using the Feynman rules, we can get the evaluation of the polarized squared matrix element as follows:

$$|\mathcal{M}_{fi}|^2 = \frac{1}{2}\, e^4\, Tr\Big[\Big(\frac{1+\lambda'\gamma_5\not{s}'}{2}\Big)(\not{p}' + m)\Big(\frac{\gamma^\mu \not{k}\gamma^\nu + 2\gamma^\mu p^\nu}{2p.k} + \frac{\gamma^\nu \not{k}'\gamma^\mu - 2\gamma^\nu p^\mu}{2p.k'}\Big) \times \Big(\frac{1+\lambda\gamma_5\not{s}}{2}\Big)(\not{p} + m)\Big(\frac{\gamma_\nu \not{k}\gamma_\mu + 2\gamma_\mu p_\nu}{2p.k} + \frac{\gamma_\mu \not{k}'\gamma_\nu - 2\gamma_\nu p_\mu}{2p.k'}\Big)\Big]. \quad (7)$$

In Compton scattering in the center-of-mass frame, we have:

$$p = (E, -\mathbf{k}) \quad , \quad k = (w, \mathbf{k}) \qquad p' = (E', -\mathbf{k}') \quad , \quad k' = (w', \mathbf{k}') \quad (8)$$

where $w = |\mathbf{k}|$ and $w' = |\mathbf{k}'|$. The energy conservation $E + w = E' + w'$ allows us to conclude that $w' = w$ and then $E' = E$, which means that the collision is elastic in this frame.
In this case, the four-vectors spin $s$ and $s'$ are expressed by [4]:

$$s^\mu = \left(\frac{w}{m}, -\frac{E}{mw}\mathbf{k}\right) \quad , \quad s'^\mu = \left(\frac{w}{m}, -\frac{E}{mw}\mathbf{k}'\right). \quad (9)$$

Generally, the polarized DCS for Compton scattering in the center-of-mass frame is expressed by:

$$\frac{d\sigma}{d\Omega_{k'}}(\lambda, \lambda')\Big|_{CM} = \frac{1}{64\pi^2 s}|\mathcal{M}_{fi}|^2. \quad (10)$$

$$\frac{d\sigma}{d\Omega_{k'}}(\lambda, \lambda')\Big|_{CM} = \frac{f_1(s, \theta, \lambda, \lambda')}{g_1(s, \theta)}. \quad (11)$$

Where

$$\begin{aligned} f_1 = \alpha^2\big(&\cos^3(\theta)s^5 - 5\cos^3(\theta)s^4m^2 + 10\cos^3(\theta)s^3m^4 - 10\cos^3(\theta)s^2m^6 \\ &+ 5\cos^3(\theta)sm^8 - \cos^3(\theta)m^{10} + 3\cos^2(\theta)s^5 - \cos^2(\theta)s^4m^2 - 10\cos^2(\theta)s^3m^4 \\ &+ 14\cos^2(\theta)s^2m^6 - 9\cos^2(\theta)sm^8 + 3\cos^2(\theta)m^{10} + 7\cos(\theta)s^5 - 15\cos(\theta)s^4m^2 \\ &+ 6\cos(\theta)s^3m^4 + 2\cos(\theta)s^2m^6 + 3\cos(\theta)sm^8 - 3\cos(\theta)m^{10} + 5s^5 - 11s^4m^2 \\ &+ 10s^3m^4 - 6s^2m^6 + sm^8 + m^{10} + \lambda\lambda'\big(\cos^3(\theta)s^5 + \cos^3(\theta)s^4m^2 - 2\cos^3(\theta)s^3m^4 \\ &- 2\cos^3(\theta)s^2m^6 + \cos^3(\theta)sm^8 + \cos^3(\theta)m^{10} + 3\cos^2(\theta)s^5 - 6\cos^2(\theta)s^3m^4 \\ &- 3\cos^2(\theta)s^4m^2 + 6\cos^2(\theta)s^2m^6 + 3\cos^2(\theta)sm^8 - 3\cos^2(\theta)m^{10} + 7\cos(\theta)s^5 \\ &- 13\cos(\theta)s^4m^2 + 2\cos(\theta)s^3m^4 + 10\cos(\theta)s^2m^6 - 9\cos(\theta)sm^8 + 3\cos(\theta)m^{10} \\ &+ 5s^5 - 17s^4m^2 + 22s^3m^4 - 14s^2m^6 + 5sm^8 - m^{10}\big)\big). \end{aligned} \quad (12)$$

And

$$\begin{aligned} g_1(s,\theta) = 8s^2\big(&\cos^2(\theta)s^4 - 4\cos^2(\theta)s^3m^2 + 6\cos^2(\theta)s^2m^4 - 4\cos^2(\theta)sm^6 + \cos^2(\theta)m^8 \\ &+ 2\cos(\theta)s^4 - 4\cos(\theta)s^3m^2 + 4\cos(\theta)sm^6 - 2\cos(\theta)m^8 + s^4 - 2s^2m^4 + m^8\big). \end{aligned} \quad (13)$$

**Spin Flip DCS:**

$$\lambda = -\lambda' = \pm 1, \quad \Longrightarrow \quad \lambda\lambda' = -1. \quad (14)$$

$$\left(\frac{d\sigma}{d\Omega_{k'}}\right)^{CM}_{flip} = \frac{f_2(s,\theta)}{g_2(s,\theta)}. \quad (15)$$

Where

$$\begin{aligned} f_2(s,\theta) = \alpha^2m^2\big(&-3\cos^3(\theta)s^4 + 6\cos^3(\theta)s^3m^2 - 4\cos^3(\theta)s^2m^4 + 2\cos^3(\theta)sm^6 \\ &- \cos^3(\theta)m^8 + \cos^2(\theta)s^4 - 2\cos^2(\theta)s^3m^2 + 4\cos^2(\theta)s^2m^4 - 6\cos^2(\theta)sm^6 \\ &+ 3\cos^2(\theta)m^8 - \cos(\theta)s^4 + 2\cos(\theta)s^3m^2 - 4\cos(\theta)s^2m^4 + 6\cos(\theta)sm^6 \\ &- 3\cos(\theta)m^8 + 3s^4 - 6s^3m^2 + 4s^2m^4 - 2sm^6 + m^8\big). \end{aligned} \quad (16)$$

And

$$\begin{aligned} g_2(s,\theta) = 4s^2\big(&\cos^2(\theta)s^4 - 4\cos^2(\theta)s^3m^2 + 6\cos^2(\theta)s^2m^4 - 4\cos^2(\theta)sm^6 + \cos^2(\theta)m^8 \\ &+ 2\cos(\theta)s^4 - 4\cos(\theta)s^3m^2 + 4\cos(\theta)sm^6 - 2\cos(\theta)m^8 + s^4 - 2s^2m^4 + m^8\big). \end{aligned} \quad (17)$$

**Spin Non Flip DCS:**

$$\lambda = \lambda' = \pm 1, \quad \Longrightarrow \quad \lambda\lambda' = 1, \quad (18)$$

$$\left(\frac{d\sigma}{d\Omega_{k'}}\right)^{CM}_{non\ flip} = \frac{f_3(s,\theta)}{g_3(s,\theta)}. \quad (19)$$

Where

$$\begin{aligned} f_3(s,\theta) = \alpha^2\big(&\cos^3(\theta)s^4 - 2\cos^3(\theta)s^3m^2 + 4\cos^3(\theta)s^2m^4 - 6\cos^3(\theta)sm^6 + 3\cos^3(\theta)m^8 \\ &+ 3\cos^2(\theta)s^4 - 2\cos^2(\theta)s^3m^2 - 8\cos^2(\theta)s^2m^4 + 10\cos^2(\theta)sm^6 - 3\cos^2(\theta)m^8 \\ &+ 7\cos(\theta)s^4 - 14\cos(\theta)s^3m^2 + 4\cos(\theta)s^2m^4 + 6\cos(\theta)sm^6 - 3\cos(\theta)m^8 \\ &+ 5s^4 - 14s^3m^2 + 16s^2m^4 - 10sm^6 + 3m^8\big). \end{aligned} \quad (20)$$

And

$$\begin{aligned} g_3(s,\theta) = 4s\big(&\cos^2(\theta)s^4 - 4\cos^2(\theta)s^3m^2 + 6\cos^2(\theta)s^2m^4 - 4\cos^2(\theta)sm^6 + \cos^2(\theta)m^8 \\ &+ 2\cos(\theta)s^4 - 4\cos(\theta)s^3m^2 + 4\cos(\theta)sm^6 - 2\cos(\theta)m^8 + s^4 - 2s^2m^4 + m^8\big). \end{aligned} \quad (21)$$

Of course, the sum of the spin flip DCS and the spin non flip DCS must give the unpolarized DCS.

$$\left(\frac{\overline{d\sigma}}{d\Omega_{k'}}\right) = \left(\frac{d\sigma}{d\Omega_{k'}}\right)_{flip} + \left(\frac{d\sigma}{d\Omega_{k'}}\right)_{non\ flip}. \quad (22)$$

**Degree of Polarization:**

$$\mathcal{P} = \frac{\left(\frac{d\sigma}{d\Omega_{k'}}\right)_{non\ flip} - \left(\frac{d\sigma}{d\Omega_{k'}}\right)_{flip}}{\left(\frac{d\sigma}{d\Omega_{k'}}\right)_{non\ flip} + \left(\frac{d\sigma}{d\Omega_{k'}}\right)_{flip}}. \quad (23)$$

For our process, this degree of polarization can be written as

$$\mathcal{P}_{CM} = \frac{f_4(s,\theta)}{g_4(s,\theta)}. \quad (24)$$

Where

$$\begin{aligned} f_4(s,\theta) = &\cos^3(\theta)s^5 + \cos^3(\theta)s^4m^2 - 2\cos^3(\theta)s^3m^4 - 2\cos^3(\theta)s^2m^6 + \cos^3(\theta)sm^8 \\ &+ \cos^3(\theta)m^{10} + 3\cos^2(\theta)s^5 - 3\cos^2(\theta)s^4m^2 - 6\cos^2(\theta)s^3m^4 + 6\cos^2(\theta)s^2m^6 \\ &+ 3\cos^2(\theta)sm^8 - 3\cos^2(\theta)m^{10} + 7\cos(\theta)s^5 - 13\cos(\theta)s^4m^2 + 2\cos(\theta)s^3m^4 \\ &+ 10\cos(\theta)s^2m^6 - 9\cos(\theta)sm^8 + 3\cos(\theta)m^{10} + 5s^5 - 17s^4m^2 + 22s^3m^4 - 14s^2m^6 \\ &+ 5sm^8 - m^{10}. \end{aligned} \quad (25)$$

And

$$\begin{aligned} g_4(s,\theta) = &\cos^3(\theta)s^5 - 5\cos^3(\theta)s^4m^2 + 10\cos^3(\theta)s^3m^4 - 10\cos^3(\theta)s^2m^6 + 5\cos^3(\theta)sm^8 \\ &- \cos^3(\theta)m^{10} + 3\cos^2(\theta)s^5 - \cos^2(\theta)s^4m^2 - 10\cos^2(\theta)s^3m^4 + 14\cos^2(\theta)s^2m^6 \\ &- 9\cos^2(\theta)sm^8 + 3\cos^2(\theta)m^{10} + 7\cos(\theta)s^5 - 15\cos(\theta)s^4m^2 + 6\cos(\theta)s^3m^4 \\ &+ 2\cos(\theta)s^2m^6 + 3\cos(\theta)sm^8 - 3\cos(\theta)m^{10} + 5s^5 - 11s^4m^2 + 10s^3m^4 - 6s^2m^6 \\ &+ sm^8 + m^{10}. \end{aligned} \quad (26)$$

In the low-energy scattering $s \approx m^2$ (exactly $s = 1.001m^2$), the degree of polarization reduces to

$$\mathcal{P}_{CM} \simeq \cos(\theta). \quad (27)$$

## 4. Results and discussions

Before presenting the results and their physical interpretation, we would like to note that the all graphs of DCSs are plotted in units of $r_0^2 = \alpha^2/m^2$ where $r_0 \simeq 2.8 \times 10^{-13}$cm is the classical radius of the electron.

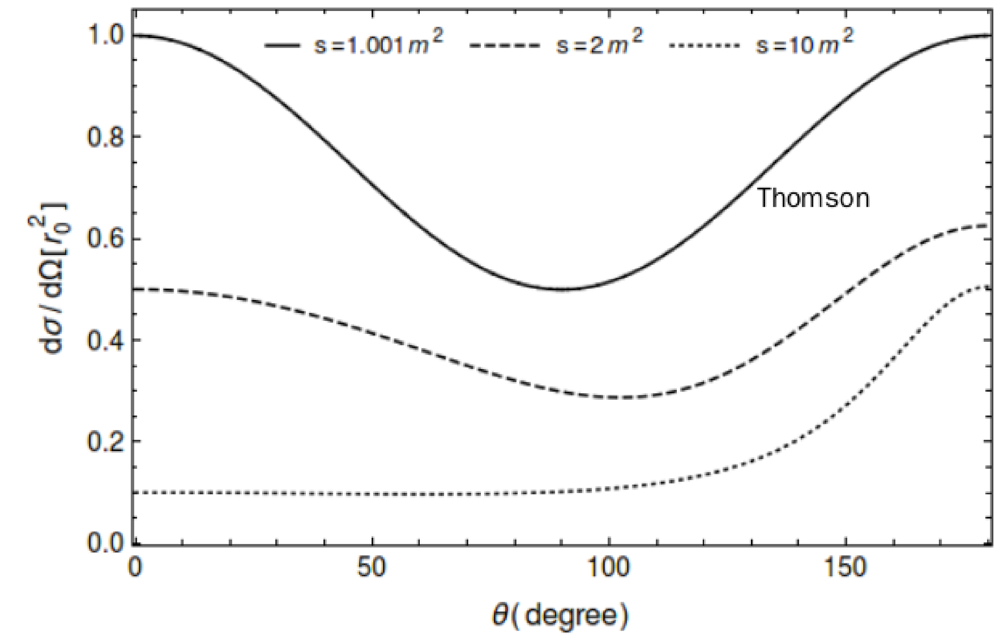


**Figure 4:** *Unpolarized DCS as function of the angle* $\theta_{CM}$ *in the CM frame, drawn for different values of energy* $s$.

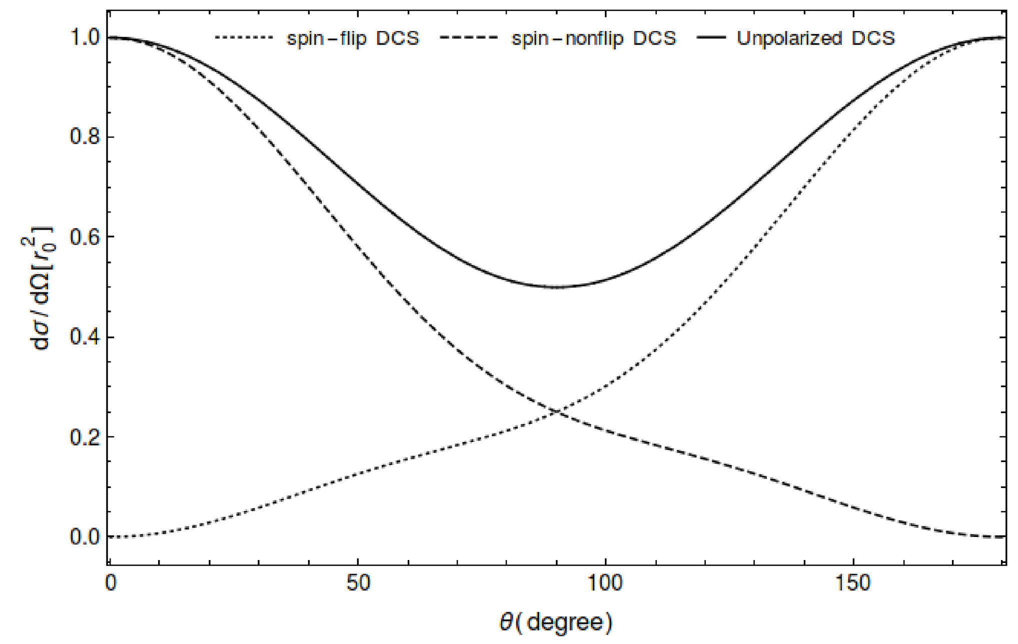


**Figure 5:** *The various DCSs as a function of the angle* $\theta_{CM}$ *in the CM frame for* $s = 1.001m^2$.

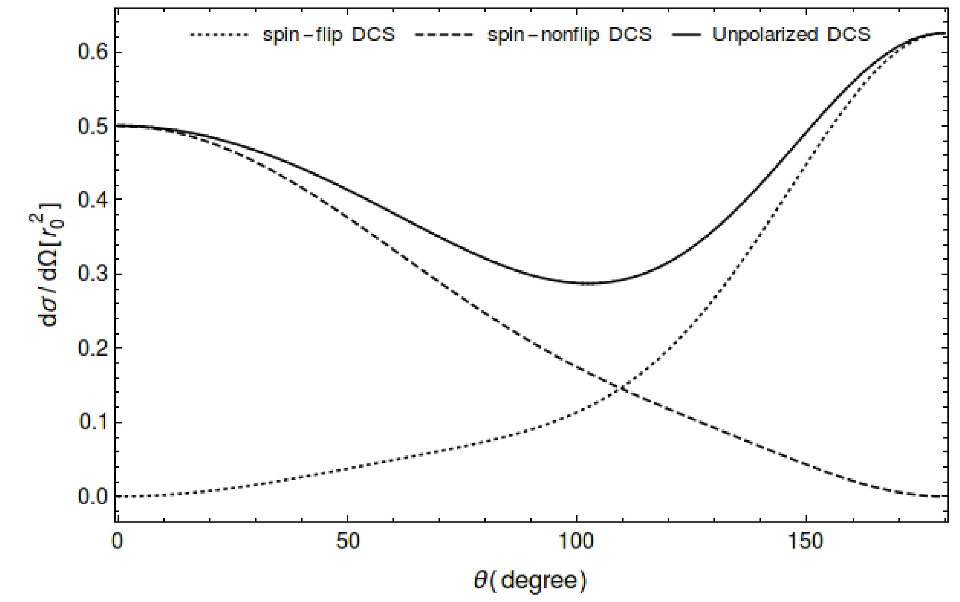


**Figure 6:** *The various DCSs as a function of the angle* $\theta_{CM}$ *in the CM frame for* $s = 2m^2$.

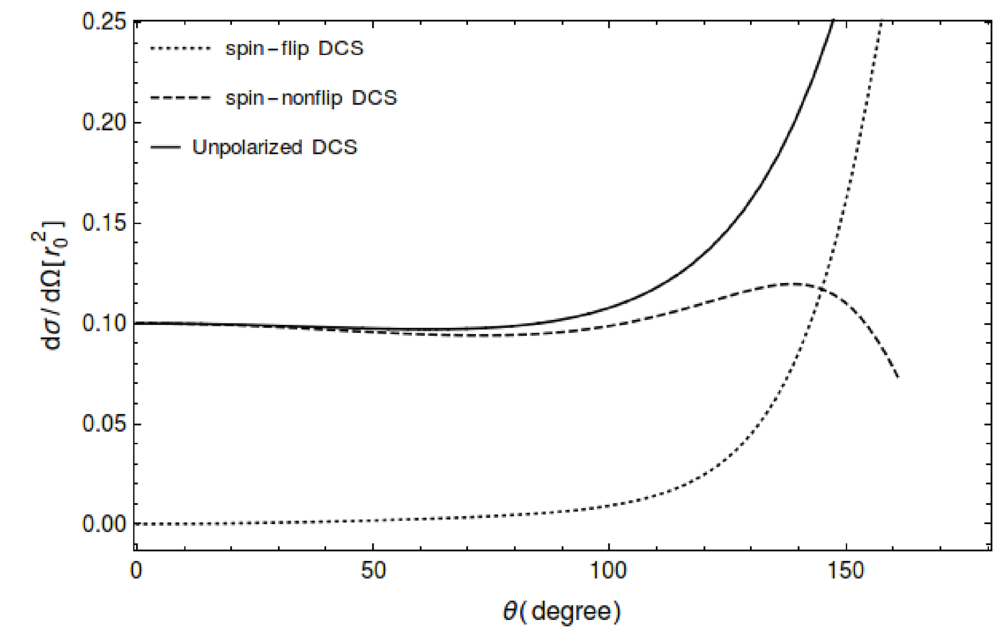


**Figure 7:** *The various DCSs as a function of the angle* $\theta_{CM}$ *in the CM frame for* $s = 10m^2$.

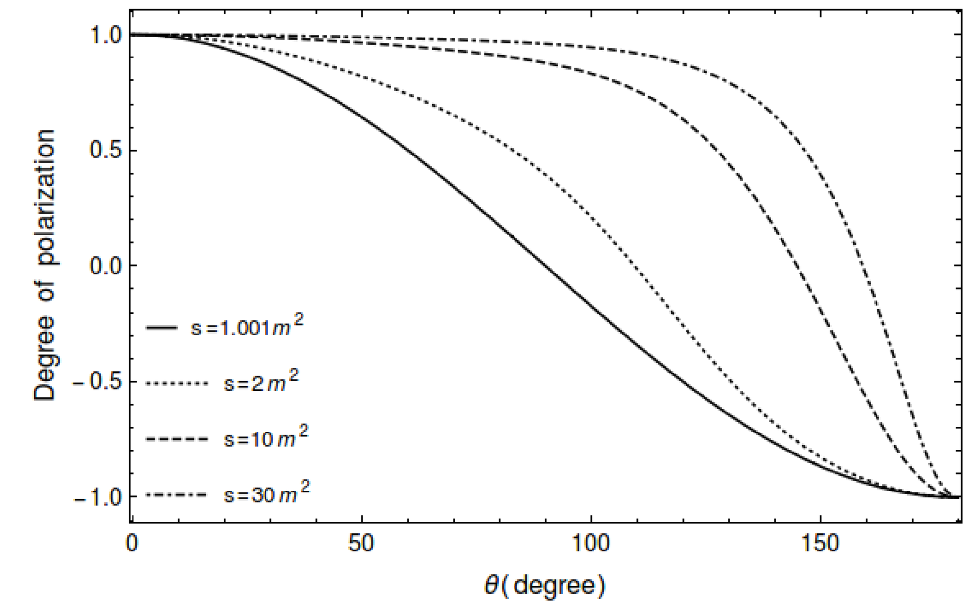


**Figure 8:** *Degree of polarization* $\mathcal{P}$ *as a function of the angle* $\theta_{CM}$ *in the CM frame, plotted for different values of energy* $s$.

## 5. Conclusion

In this work, we have studied the Compton scattering of polarized electrons. We started with checking the results already known for unpolarized electrons. Then we turn to the case in which we applied the concept of polarization of electrons in the CM frame. The expressions of the three DCSs (unpolarized DCS, spin flip DCS and spin non flip DCS) and the degree of polarization were derived. We have studied their behavior in both low and high energies. It is concluded that in Compton scattering at high energy, the electron's probability for changing its spin is zero. A consistency check that is successfully achieved is that the sum of the two spin-flip and spin-non flip DCSs always gives the unpolarized DCS.

## References


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