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Analogy between classical Higgs Mechanism and Photon interaction with Plasma waves

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Introduction

In Physics, the photon is described as massless, that is to say, the rest mass of a photon is zero but as it moves at the speed of light, it does have relativistic mass. In this article and by resolving the Maxwell equations in a neutral plasma (composed of ionized atoms and their removed electrons) we obtain the plasma Frequency which indicates the oscillation of plasma particles. This oscillation of the plasma can be shown to give the photon(the quantized particle of the electromagnetic (EM) wave) a mass in a mechanism analogue to the Higgs Mechanism.

Maxwell's equations and Electromagnetic waves

The Maxwell equations are given as follows :

$$\nabla.\vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla.\vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

By combining these equations all together, we obtain the main equation which will be used to generate the electric and current field for different mediums :

$$\nabla^2 \vec{E} - \frac{\nabla \rho}{\epsilon_0} = \mu_0 \frac{\partial \vec{J}}{\partial t} + \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t} \qquad \textbf{(1)}$$

in free space, $\rho = J = 0$ then : (1)$\Leftrightarrow \nabla^2 \vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t}$

Which is a wave equation with a solution :

$$\vec{E} = E_0 e^{i(\omega t - kx)} \ \vec{y}$$

where ω is the Frequency of the wave and k is the wavenumber. To determine the plasma wave and the wavenumber, two cases should be studied: spatial independence and spatial dependence.

Spatial Independence

In neutral Plasma the charges and the current are :

$\boldsymbol{\rho = \rho(t)}$ and $\boldsymbol{\vec{J} = \vec{J}(t)}$

$$\textbf{(1)} \Leftrightarrow 0 = \mu_0 \frac{\partial \vec{J}}{\partial t} + \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t}$$

and from inspection, one can obtain the expressions of the fields :

$$\vec{E} = E_0 e^{i\omega t} \vec{x} \text{ and } \vec{J} = -iJ_0 e^{i\omega t} \vec{x}$$

and by substituting into **(5)** we find :

$$J_0 = \omega \epsilon_0 E_0 \qquad \textbf{(2)}$$

Lorentz force in the absence of Magnetic Field is given by

$$\vec{F} = \mathrm{q}\vec{E}$$

From this equation, and by applying the dynamic principle, one can find the expression for the velocity of the charged particles within plasma :

$$m \frac{dv}{dx} = \mathrm{q} E_0 e^{i\omega t}$$

$$\Leftrightarrow \vec{v} = \frac{qE_0}{im\omega} e^{i\omega t} \ \vec{x} \qquad \textbf{(3)}$$

The current density $\vec{J}$ can also be expressed as a function of the velocity :

$$\vec{J} = \sum_i n_{v_i} q_i \vec{v_i}$$

From **(2)** and **(3)** we can obtain :

$$-iJ_0 e^{i\omega t} = \sum_i \left(\frac{n_{v_i} q_i^2 E_0}{im_i\omega} e^{i\omega t}\right) \qquad \textbf{(4)}$$

One can find then the Plasma Frequeny :

$$\omega_p = \frac{1}{\sqrt{\epsilon_0}} \sqrt{\sum \frac{n_{v_i} q_i^2}{m_i}}$$

Spatial dependence

The electric and current density depends on time as well as space coordinates, thus the expression of the fields are :

$$\vec{E} = E_0 e^{i(\omega t - kx)} \ \vec{x} \text{ and } \vec{J} = J_0 e^{i(\omega t - kx)} \ \vec{x}$$

By applying **(1)** on the fields, we get the dispersion relation :

$$-k^2 \vec{E} = -\epsilon_0 \mu_0 \omega^2 \vec{E} + \mu_0 i\omega \vec{J} \qquad \textbf{(5)}$$

From (4) and Ohm's law which states that $\vec{J} = \sigma \vec{E}$, **(5)** became :

$$-k^2 \vec{E} = -\epsilon_0 \mu_0 \omega^2 \vec{E} + \mu_0 \ i\omega \ \frac{1}{i\omega} \sum \left(\frac{n_{v_i} q_i^2}{m_i}\right) \vec{E}$$

From which one can conclude that the wavenumber equals :

$$k = \frac{2\pi}{\lambda} \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

Energy, momentum and Mass of a Photon in Plasma

The energy and the momentum of an electromagnetic wave are given by :

$$E = h\omega \quad \text{and} \quad p = h\,k$$

And the mass-energy equation by :

$$E^2 - p^2 c^2 = m^2 c^4$$

$$\Leftrightarrow \quad h^2 \omega^2 - h^2 c^2 \frac{4\pi^2}{\lambda^2} \left(1 - \frac{\omega_p^2}{\omega^2}\right) = m^2 c^4$$

$$\Leftrightarrow \quad \frac{h}{c^2} \omega_p = m$$

$$\text{Finally : } m = \frac{1}{\sqrt{\epsilon_0}} \frac{h}{c^2} \sqrt{\sum \frac{n_{v_i} q_i^2}{m_i}}$$

With m_i is the mass of charged particles in the plasma,whilst the m is the mass the photon has when traveling through the plasma.

Conclusion

when traveling through a plasma, a photon gains a mass due to the oscillation of the plasma. The mass is only influenced by the properties of the plasma itself namely the number densities, charges, and masses of the plasma's constituent components. This mechanism can be seen as an analogy to the Higgs Mechanism through which the massive particles in our universe gain their mass.

References :

[1] Albert Shadowitz. *The Electromagnetic Field*. Dover, first edition, 1988.

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