Sphere partition function and the refined swampland distance conjecture

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with David Erkinger: arXiv:1905.05225 [hep-th]

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CERN, June 13, 2019

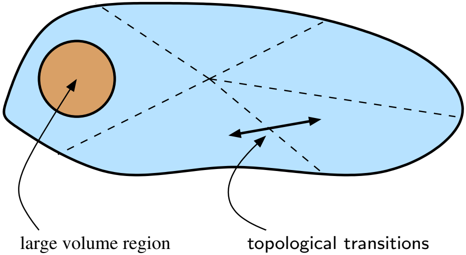
Outline

Swampland conjectures

- Swampland conjectures aim to give deciding criteria whether a theory of quantum gravity is UV complete. [Ooguri-Vafa 06][Palti 19]
- Today's conjecture:
  Refined swampland distance conjecture (RSDC)
  - The length parameters of geodesics in a moduli space are of order $\mathcal{O}(1)$ in Planck units. [Klaewer-Palti 16,Blumenhagen et al. 18]
- Test in concrete settings in string theory.
- This talk: test the RSDC for the stringy Kähler moduli space of type II compactifications on CY 3-folds.

The stringy Kähler moduli space $\mathcal{M}_K$

- Structure of $\mathcal{M}_K$



- geodesics crossing chamber boundaries
- exotic regions
- Kähler potential $K(t, \bar{t})$ on $\mathcal{M}_K$ receives worldsheet instanton corrections
- RSDC: all distance parameters involved should be $O(1)$

Gauged linear sigma model

- GLSM: [Witten 93]
  - 2D ($N = (2,2)$) supersymmetric gauge theory
  - Calabi-Yaus $\leftrightarrow$ vacuum configurations
  - Coupling constants $t \leftrightarrow$ Kähler parameters
  - Phases: different solutions (CYs) depending on the values of the couplings $\leftrightarrow$ chamber structure
- Sphere partition function Z_{S^2}

[Benini-Cremonesi 12][Doroud et al 12][Jockers et al 12][Gomis-Lee 12]

$$Z_{S^2} = e^{-K(t,\bar{t})}$$

- Metric on M_K

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K(t, \bar{t})$$

RSDC on $\mathcal{M}_K$

- Consider models with $\dim_{\mathbb{C}}\mathcal{M}_K = 1$.
- Geodesics crossing chamber boundaries
    - Start: point at finite distance
      $\Theta_0 \ldots$ distance to chamber boundary
    - End: large volume point (infinite distance)
      $\lambda^{-1} \ldots$ length parameter
- Behavior at large volume
    - Scalar masses: $M \sim M_0 e^{-\lambda\Theta(p,p_0)}$
    - $p_0 \ldots$ chamber boundary, $p \ldots$ large volume
- Determine Θ_0, λ^{-1} by solving the geodesic equation numerically (parametrization $z(t) = re^{i\varphi}$)

$$\begin{aligned}
\ddot{\varphi} &= \frac{1}{2}g^{\varphi\varphi}\partial_\varphi g_{rr}\dot{r}^2 - g^{\varphi\varphi}\partial_r g_{\varphi\varphi}\dot{r}\dot{\varphi} - \frac{1}{2}g^{\varphi\varphi}\partial_\varphi g_{\varphi\varphi}\dot{\varphi}^2, \\
\ddot{r} &= -\frac{1}{2}g^{rr}\partial_r g_{rr}\dot{r}^2 - g^{rr}\partial_\varphi g_{rr}\dot{r}\dot{\varphi} + \frac{1}{2}g^{rr}\partial_r g_{\varphi\varphi}\dot{\varphi}^2.
\end{aligned}$$

Examples

- Geodesics from Landau-Ginzburg points to large volume in toric hypersurfaces satisfy the RSDC [Blumenhagen et al 18]
  - also multi-parameter examples
- Pseudo-hybrid models [Aspinwall-Plesser 09]
  - at finite distance in $\mathcal{M}_K$
  - no field-theoretic description known
  - singular CFT
  - no decoupling limit of gravity (?)
  - also in the context of non-abelian GLSMs

[Hori-JK 13,16][Caldararu-JK-Sharpe 17]
- Result: The RSDC also holds for pseudo-hybrids [Erkinger-JK 19]

GLSM data

- G ... a compact Lie group (gauge group)
- V ... space of chiral fields $\phi_i \in V$
- $\rho_V : G \to GL(V)$ ... faithful complex representation
  - CY condition: $G \to SL(V)$
- $R : U(1) \to GL(V)$ ... R-symmetry
  - R_i ... R-charges
- $T \subset G$ ... maximal torus
  - Lie algebras: $\mathfrak{g} = Lie(G)$, $\mathfrak{t} = Lie(T)$
  - $Q_i^a \in \mathfrak{t}_{\mathbb{C}}^*$ ... gauge charges of chiral fields

GLSM Data (ctd.)

- $t \in \mathfrak{g}_{\mathbb{C}}^*$ . . . FI-theta parameter
  - $t^a = \zeta^a - i\theta^a \qquad \zeta$: real, $\quad \theta$: 2π-periodic
  - $t^a \leftrightarrow$ Kähler moduli of the CY
- $\sigma \in \mathfrak{g}_{\mathbb{C}}$ . . . scalar component of the vector multiplet
- $W(\phi)$ . . . superpotential
  - G-invariant
  - R-charge 2
  - non-zero for compact CYs
- D-terms: $\mu(\phi) = \zeta$ ($\mu : V \to \mathfrak{g}^*$ . . . moment map)
- F-terms: $dW(\phi) = 0$

Phases of the GLSM

- Classical Vacua

$$X_{\zeta} = \{dW^{-1}(0)\} \cap \mu^{-1}(\zeta)/G$$

- Phases: parameter space gets divided into chambers
- There are singularities at the phase boundary
  - Determined by $\mathcal{W}_{eff}$ on the Coulomb branch
  - $$\mathcal{W}_{eff} = -t(\sigma) - \sum_i Q_i(\sigma)(\log(Q_i(\sigma)) - 1) + \pi i \sum_{\alpha>0} \alpha(\sigma)$$

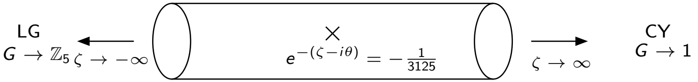
    α ... positive roots
  - Geodesics must avoid singularities

Quintic - $G = U(1)$

- Field content: $\phi = (p, x_1, \ldots, x_5) \in \mathbb{C}(-5) \oplus \mathbb{C}(1)^{\oplus 5}$
- Potential: $W = pG_5(x_1, \ldots, x_5)$
- D-term: $-5|p|^2 + \sum_{i=1}^{5} |x_i|^2 = \zeta$
- F-terms: $G_5(x_1, \ldots, x_5) = 0$, $p\frac{\partial G_5}{\partial x_i} = 0$
- $\zeta > 0$: $p = 0$, Quintic $G_5 = 0$ in $\mathbb{P}^4$
- $\zeta < 0$: Landau-Ginzburg orbifold with potential G_5
- Landau-Ginzburg/CY correspondence

[Witten 93][Herbst-Hori-Page 08][Kontsevich,Orlov]

- Moduli space



Sphere partition function Z_{S^2}

- Sphere partition function

$$Z_{S^2} = C \sum_{m \in \mathbb{Z}} \int_\gamma d^{\mathrm{rk}G}\sigma \prod_{\alpha>0} (-1)^{\alpha(m)} \left(\frac{\alpha(m)^2}{4} + \alpha(\sigma)^2 \right) \times$$

$$\prod_{i=1}^{\dim V} \frac{\Gamma\left(-iQ_i(\sigma) - \frac{Q_i(m)}{2} + \frac{R_i}{2}\right)}{\Gamma\left(1 + iQ_i(\sigma) - \frac{Q_i(m)}{2} - \frac{R_i}{2}\right)} e^{4\pi i \zeta(\sigma) + i\theta(m)}$$

  - γ ...integration contour (s.t. integral is convergent)
- Can be evaluated in any phase
- Compute the exact metric on $\mathcal{M}_K$ via Z_{S^2}

$U(1)$-model with pseudo-hybrid phase

- Field content [Aspinwall-Plesser 09]

ϕ	p_1	p_2	p_3	$x_1,\ldots,x_7$
$U(1)$	-3	-2	-2	1
R	$2-6q$	$2-4q$	$2-4q$	$2q$

- Superpotential

$$W = p_1 f_1(x_1,\ldots,x_7) + p_2 f_2(x_1,\ldots,x_7) + p_3 f_3(x_1,\ldots,x_7)$$

- Phases
  - $\zeta \gg 0$: $X^{\zeta\gg 0} = \mathbb{P}^6[3,2,2]$
  - $\zeta \ll 0$: pseudo-hybrid
    solution to D-terms and F-terms have several components
  - Singular point at $e^{-t} = -\frac{1}{432}$

Z_{S^2} and metric

- Sphere partition function

$$Z_{S^2} = \sum_{m\in\mathbb{Z}} \int_{-\infty}^{\infty} \frac{d\sigma}{2\pi} e^{(-4\pi i\zeta\sigma - i\theta m)}$$

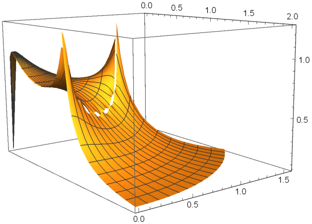
$$\frac{\Gamma\left(q - i\sigma - \frac{m}{2}\right)^7}{\Gamma\left(1 - q + i\sigma - \frac{m}{2}\right)^7} \frac{\Gamma(1 - 2q + 2i\sigma + m)^2}{\Gamma(2q - 2i\sigma + m)^2} \frac{\Gamma\left(1 - 3q + 3i\sigma + \frac{3}{2}m\right)}{\Gamma\left(3q - 3i\sigma + \frac{3}{2}m\right)}$$

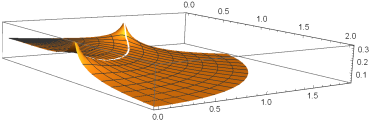
- Metric

$$g^{\zeta\ll 0} = -\frac{2^8 7^3 \sqrt{3}\pi^7}{3^3 \Gamma\left(\frac{1}{6}\right)^4 \Gamma\left(\frac{1}{3}\right)^{10}} r^{1/3}\log(r) + \dots, \quad g^{\zeta\gg 0} = \frac{3}{4r^2\log(r)^2} + \dots.$$

Testing the RSDC

- Pseudo-hybrid example vs. quintic





- Numerical evaluation of length parameters for different starting values of φ
- Result: mean values

$$\Theta_0 \approx 0,8937, \quad \lambda^{-1} \approx 0,9608, \quad \Theta_c = \Theta_0 + \lambda^{-1} \approx 1,8545.$$

  - In agreement with the RSDC

$U(2)$-model with pseudo-hybrid phase

- Field content [Hori-JK 13]

ϕ	$p^1,\ldots,p^5$	p^6,p^7	x_1,x_2	$x_3,\ldots,x_5$
$U(2)$	$\det^{-1}$	$\det^{-2}$	$\det\otimes\square$	$\square$
R	$4q$	$8q$	$1-6q$	$1-2q$

- Superpotential

$$W=\sum_{i,j=1}^{5}A^{ij}(p)[x_i x_j]$$

- Phases
    - $\zeta \gg 0$: pseudo-hybrid
    - $\zeta \ll 0$: Pfaffian CY [Kanazawa 10]

    strongly coupled!

Singularities

- There are two singular points at the phase boundary.

$$e^{-t_\pm} = (540 \pm 312\sqrt{3}).$$

- Region between the two points
  - Does one use $Z^{\zeta \gg 0}$ or $Z^{\zeta \ll 0}$? - Neither has good convergence properties there.
  - What is a good coordinate to parametrize $\mathcal{M}_K$?
- Geodesics from the pseudo-hybrid point to the nearest singularity at ζ_-

$$\Theta_0 \approx 0,5303$$

  - In agreement with the RSDC

Conclusions and open questions

- We have shown that the RSDC holds for exotic hybrid CYs.
- New issues for geodesics in non-abelian models .
- What more can we learn about the properties of pseudo-hybrids?
- What is "$\mathcal{O}(1)$" and how does it depend on the model data?
- Swampland distance conjecture (SDC) and hemisphere partition function?