

# Integrability in gauge and string theory

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Motivation: **new tools** for quantum field theory and gravity

**Holographic duality**: gauge theory  $\leftrightarrow$  gravity

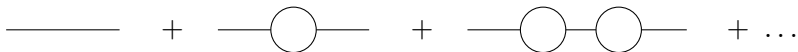
**Exact** methods from **integrability**

An **interacting** solvable QFT

# Plan

- Maximally supersymmetric **gauge theory in 4D**
- **String theory** on  $AdS_5 \times S^5$
- **Deformations** preserving integrability

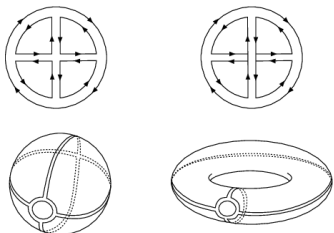
# Gauge theory



## $\mathcal{N}=4$ super Yang-Mills

gauge group  $SU(N)$ , coupling constant  $g_{\text{YM}}$   
 scalar+fermions+gauge bosons with **maximal susy in 4D**

**Conformal:**  $\beta(g_{\text{YM}}) = 0$



**Planar limit:**

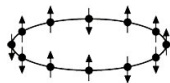
$g_{\text{YM}} \rightarrow 0$ ,  $N \rightarrow \infty$   
 while 't Hooft coupling  
 $\lambda \equiv Ng_{\text{YM}}^2$  is fixed

picture stolen from Alfonso's  
 review [\[arXiv:1310.4319\]](https://arxiv.org/abs/1310.4319)

# The spin chain

$\mathfrak{su}(2)$  sector  $\supset$  scalar fields  $\Phi, \bar{\Phi}$  of  $\mathcal{N} = 4$  SYM

$$\mathcal{O}(x) = \text{Tr}[\Phi\Phi\bar{\Phi}\bar{\Phi}\Phi\bar{\Phi}\Phi \dots \Phi\Phi\bar{\Phi}\Phi\Phi]$$



[Minahan, Zarembo 02]

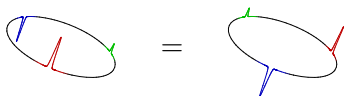
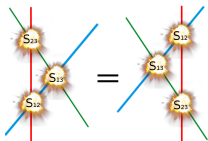
**Anomalous dimension at 1-loop:** operators mix and the mixing matrix is the Hamiltonian of **Heisenberg's XXX spin chain!**

**Higher-loop** corrections  $\implies$  **long-range** interactions

For similar methods applied to QCD see [\[arXiv:1012.4000\]](https://arxiv.org/abs/1012.4000)



Magnon excitations interact with **factorised S-matrix**



$$e^{ip_k L} \prod_{j \neq k} S(p_k, p_j) = 1 \quad \text{Bethe eqs}$$

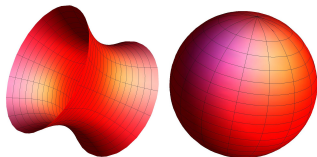
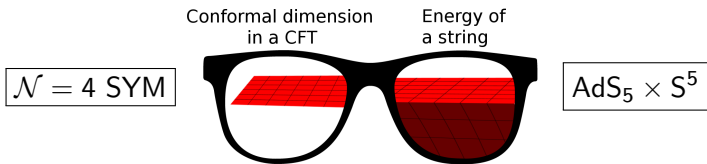
S-matrix fixed at **all loops** from supersymmetry and analyticity

**Exact spectrum** in  $\lambda$  and  $L$  (size of the chain) from  
 “Thermodynamic Bethe Ansatz” or “Quantum Spectral Curve”

# String theory



[Maldacena 97]



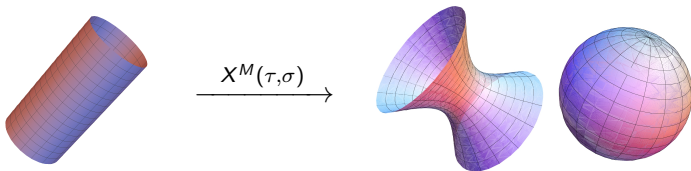
$$-X_0^2 + \sum_{i=1}^4 X_i^2 - X_5^2 = -1$$

$$\sum_{i=1}^6 Y_i^2 = 1$$

$\lambda \ll 1$  weakly-coupled gauge theory /  $\lambda \gg 1$  classical string

$$S = -\frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma \gamma^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N G_{MN} + \text{fermions}$$

$$ds^2 = G_{MN} dX^M dX^N = ds_{\text{AdS}_5}^2 + ds_{S^5}^2$$



Hamiltonian in light-cone gauge for 8 bosons + 8 fermions

$$\mathbf{H} = \mathbf{H}_2 + \frac{1}{\lambda} \mathbf{H}_4 + \frac{1}{\lambda^2} \mathbf{H}_6 + \dots$$

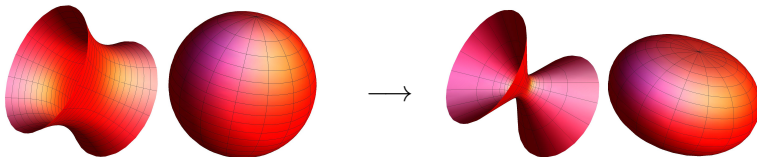
**Same S-matrix** of spin-chain but expanded at  $\lambda \sim \infty$

(Classical integrability)

# Deformations

# Integrability **beyond** spectrum of $AdS_5/CFT_4$

- **Higher point-functions**
- **Lower dimensional** dualities  
e.g.  $AdS_4/CFT_3$ ,  $AdS_3/CFT_2$
- **Deformations** of  $AdS_5/CFT_4$



**Break isometries** of target space of string

Some deformations  $\sim$  twisted boundary conditions for the string

Deformations of the **gauge theory**?

On the gauge theory we can break e.g. **supersymmetry**,  
**conformal invariance**

In certain cases, deformations correspond to **non-commutative**  
gauge theories

**Extension of the integrability methods** to the deformed models?

