QCD and Monte Carlo tools

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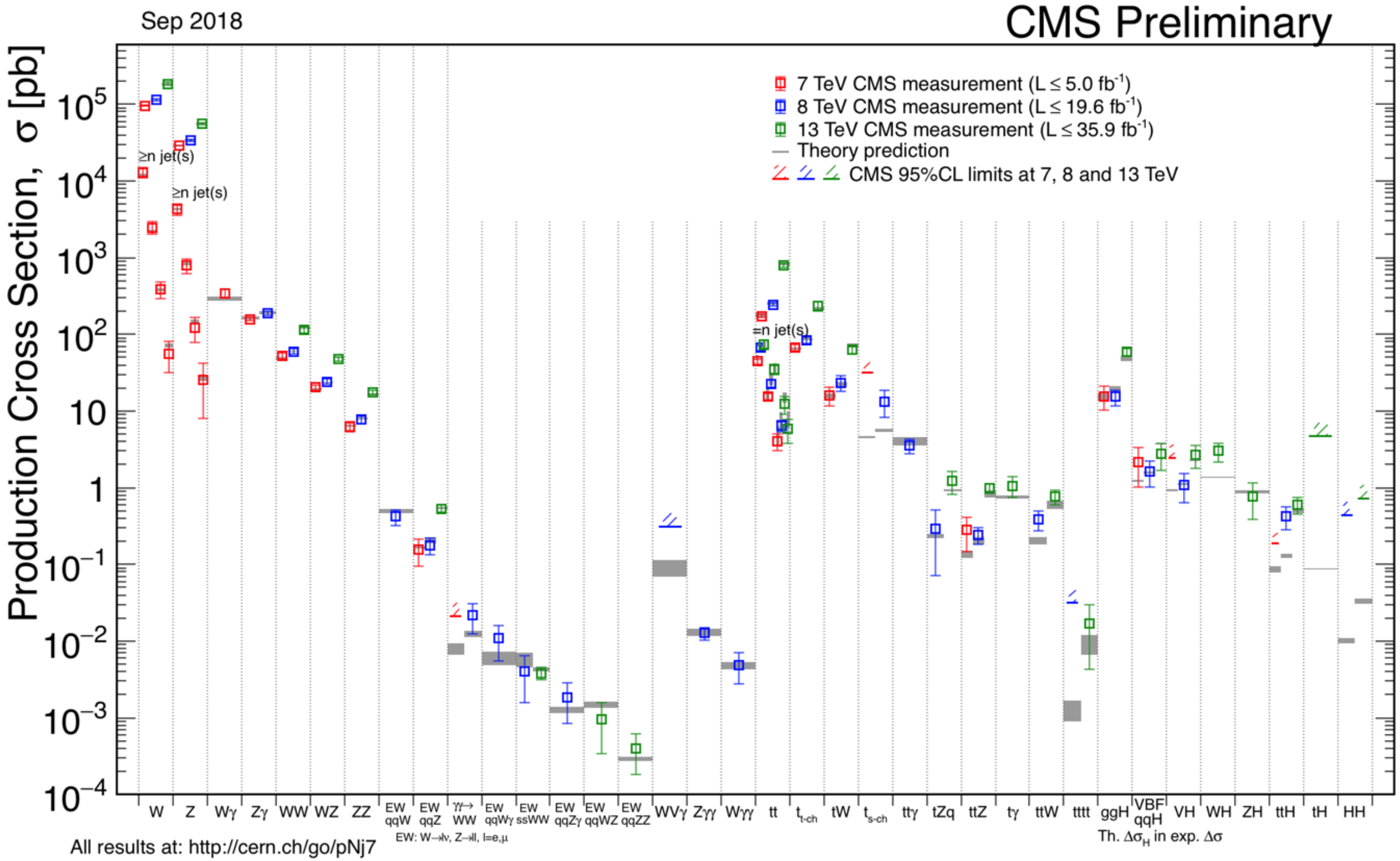
HADRON COLLIDER PHYSICS
Summer School

How do we get here ?



All quantitative predictions for hard processes at hadron colliders are based on QCD

- Test the Standard Model
- Reliably predict BSM signals and the corresponding backgrounds

Outline

- Quick introduction to QCD
  - Lagrangian, Feynman rules, Colour algebra
- Infrared divergences and the factorisation theorem
  - infrared safety, collinear factorization, DGLAP

Lecture 1

- QCD at hadron colliders
  - Parton distributions functions, Drell-Yan
  - Jets

Lecture 2

Outline

- QCD at higher orders
  - NLO corrections to Drell-Yan
  - NLO, NNLO and beyond

Lecture 3

- Beyond fixed order: analytic resummations
  - transverse-momentum resummation
- Event generators
  - parton showers, colour coherence,
  - NLO matching, merging

Lecture 4

Outline

General references:

- R.K.Ellis, W.J.Stirling, B.R. Webber, "QCD and Collider Physics", Cambridge, 1996

- J.Campbell, J.Huston, F.Krauss, "The Black book of Quantum Chromodynamics: a primer for the LHC era", Oxford, 2018

More specific:

- G.Salam, "Towards Jetography", arXiv:0906.1833, EPJC67 (2010), 637

The QCD Lagrangian

QCD ⟷ Quantum - Chromo - Dynamics

Quantum theory of colour:
It is a non abelian gauge theory with $SU(N_c)$ gauge group

Completely specified given the number of colours N_c

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \sum_{f=1}^{N_f} \bar{\psi}^i_f \left[i\gamma^\mu (D_\mu)_{ij} - m_f \delta_{ij} \right] \psi^j_f$$

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - i g t^a_{ij} A^a_\mu$$ Covariant derivative $i, j = 1 \ldots\ldots N_c$

$SU(N_c)$ color matrices

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$ Gluon field tensor

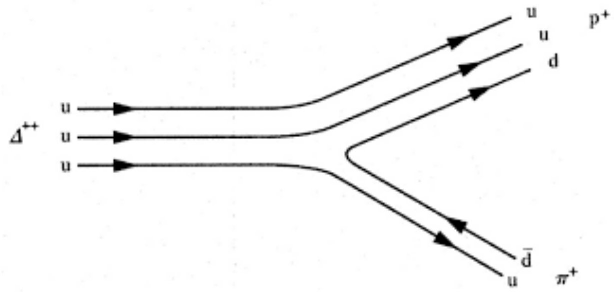
$SU(N_c)$ structure constants

Difference with QED: gluons are charged and interact among themselves

Colour

Within the quark model, the additional quantum number of color was initially introduced to accommodate the existence of the barion Δ^{++}

➡ Complete symmetry of Δ^{++} three-quark state requires additional quantum number: colour



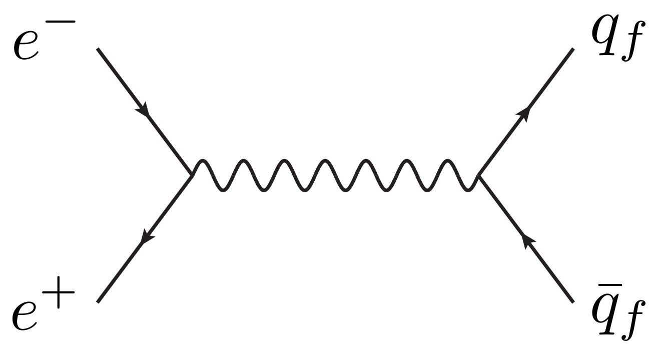
Colour quantum number not observed (hadrons are colour singlet)

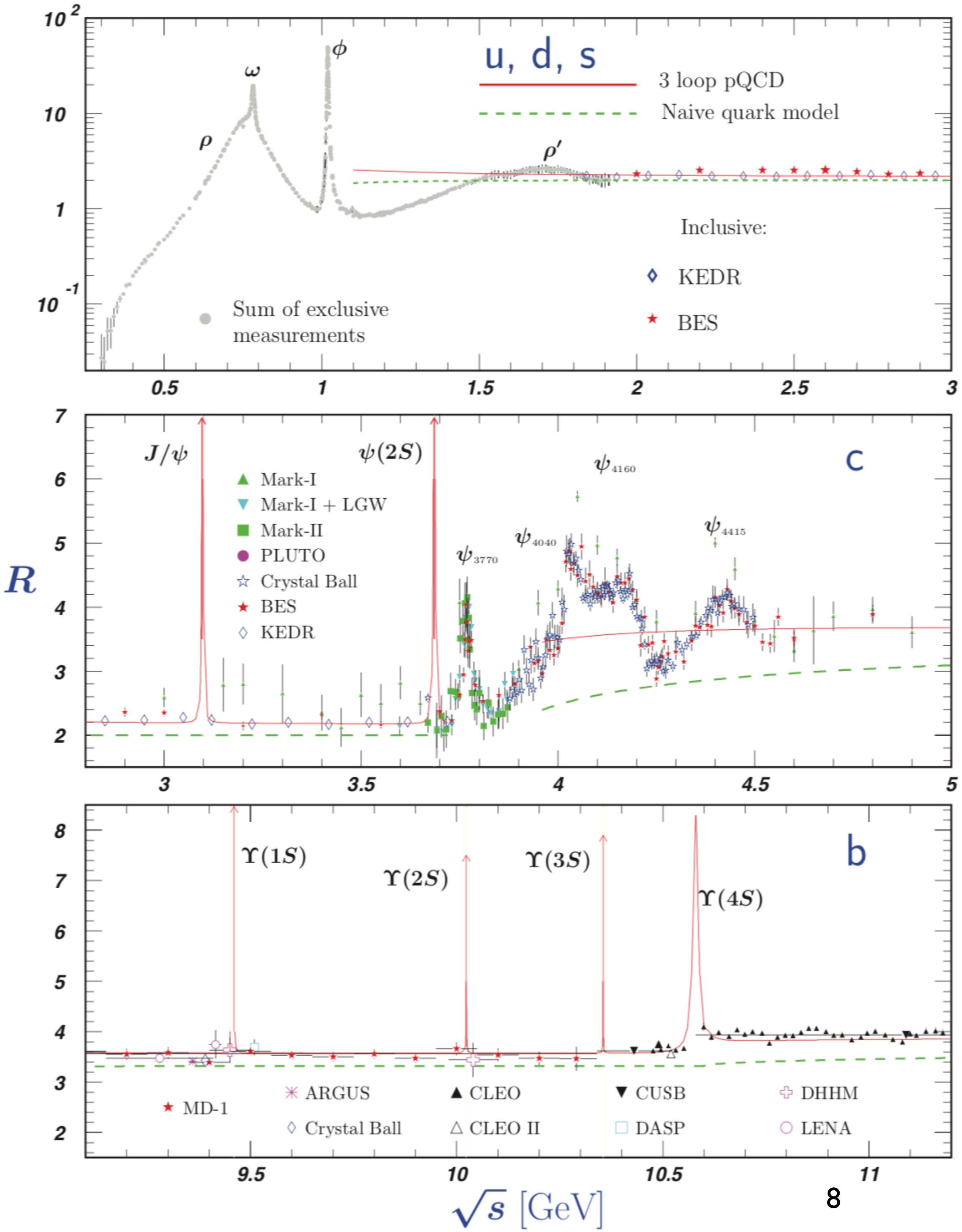
Evidence of colour in e⁺e⁻ collisions

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_f Q_f^2$$

Assume each quark has N_c colours

Quark electric charge





$$R_{uds} = 3 \times \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right]$$

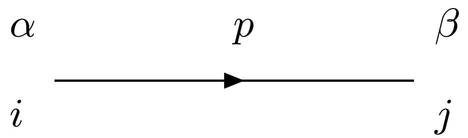
$$= 2$$

$$R_{udcs} = R_{uds} + 3 \times \left(\frac{2}{3}\right)^2 = \frac{10}{3}$$

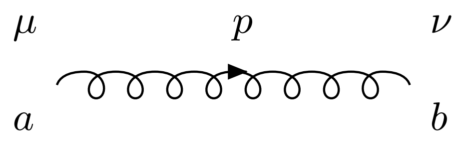
$$R_{udcsb} = R_{udcs} + 3 \times \left(-\frac{1}{3}\right)^2 = \frac{11}{3}$$

Feynman rules

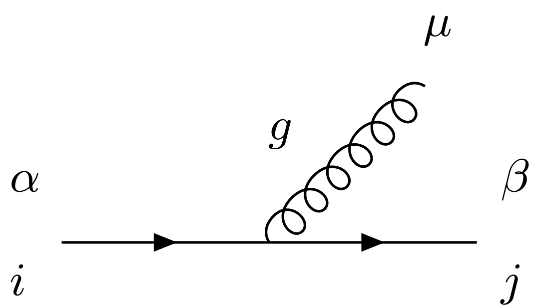
Gluon spin pol. tensor: gauge dependent



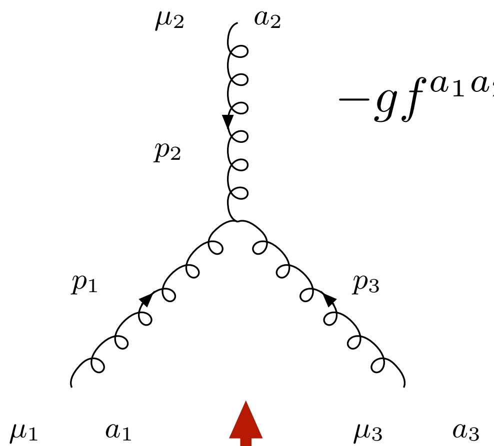
$$\frac{i(\not{p}+m)_{\alpha\beta}}{p^2-m^2+i\epsilon}\,\delta^{ij}$$



$$\frac{i}{p^2+i\epsilon}\,d^{\mu\nu}(p)\,\delta^{ab}$$

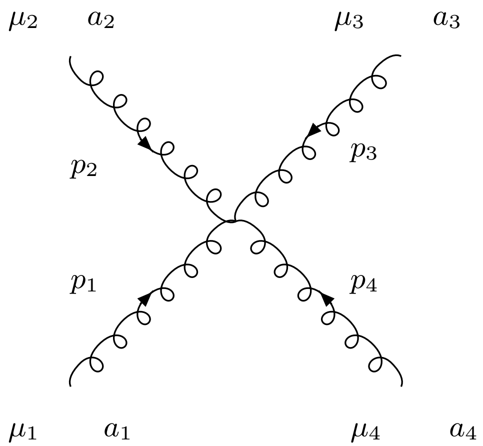


$$-ig(t^a)_{ij}(\gamma^\mu)_{\alpha\beta}$$



$$-gf^{a_1a_2a_3}\Big[g^{\mu_1\mu_2}(p_1-p_2)^{\mu_3}$$
$$+g^{\mu_2\mu_3}(p_2-p_3)^{\mu_1}$$
$$+g^{\mu_3\mu_1}(p_3-p_1)^{\mu_2}\Big]$$

Three-gluon vertex



Four-gluon vertex

$$-ig^2\Big[f^{ba_1a_2}f^{ba_3a_4}(g^{\mu_1\mu_3}g^{\mu_2\mu_4}-g^{\mu_1\mu_4}g^{\mu_2\mu_3})$$
$$+(2\leftrightarrow 3)+(2\leftrightarrow 4)\Big]$$

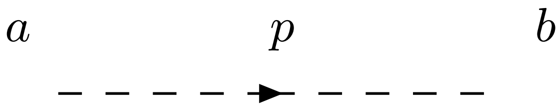
The spin polarization tensor is $$d^{\mu\nu}(p) = \sum_\lambda \varepsilon^\mu_{(\lambda)}(p)\varepsilon^\nu_{(\lambda)}(p)$$

and depends on the gauge choice

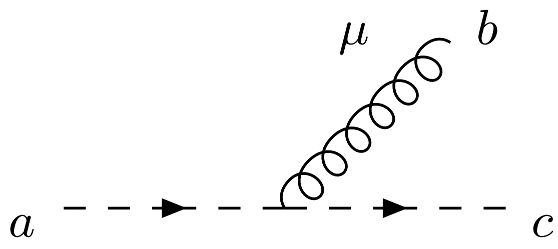
$$d^{\mu\nu}(p) = \begin{cases} -g^{\mu\nu} + (1-\alpha)\dfrac{p^\mu p^\nu}{p^2+i\epsilon} & \text{covariant gauges} \\ -g^{\mu\nu} + \dfrac{p^\mu n^\nu + p^\nu n^\mu}{p\cdot n} - n^2 \dfrac{p^\mu\, p^\nu}{(p\cdot n)^2} & \text{axial gauges} \end{cases}$$

In covariant gauges Lorentz invariance is manifest but ghosts must be included to cancel effect of unphysical gluon polarizations

(we'll never need them in these lectures !)



$$\frac{i}{p^2+i\epsilon}\delta^{ab}$$



$$g f^{abc} p^\mu$$

In physical gauges (e.g. $n_\mu A^\mu=0$, n arbitrary direction) only two tranverse polarizations are present

→ more transparent physical picture: for lowest order or approximated calculations physical gauges make life simpler

Colour algebra

The calculation of Feynman diagrams is similar to QED: just keep into account colour factors

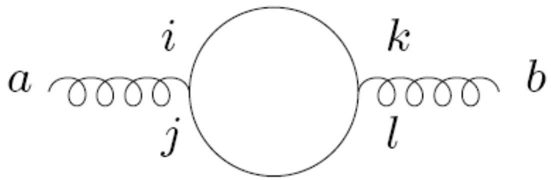
$$[T^a, T^b] = i f^{abc} T^c$$

The explicit form of colour matrices is not important in practice

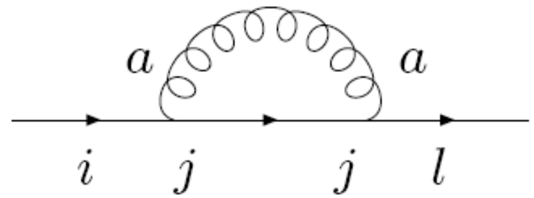
$(T^a)_{ij} = t^a_{ij}$ fundamental $\qquad (T^a)_{bc} = i f_{abc}$ adjoint $\qquad Tr(T^a) = 0$

Useful relations:

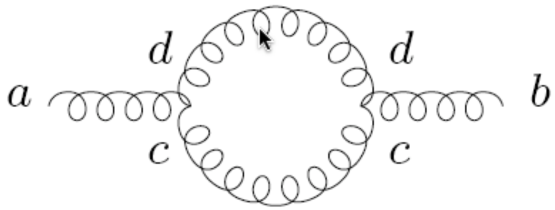
$$Tr(t^a t^b) = T_R \delta_{ab} \quad T_R = 1/2$$



$$(t^a t^a)_{il} = C_F \delta_{il} \qquad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$



$$f^{adc} f^{bdc} = C_A \delta^{ab} \qquad C_A = N_c = 3$$



Exercise: prove the above expressions for C_F and C_A
(hint: t^a and I form a basis for N_c x N_c hermitian matrices)

The strong coupling

QCD is a renormalizable gauge theory

→ Ultraviolet (UV) singularities appear in loop diagrams but they can be removed by the renormalization procedure

- Regularization: allows to make sense of divergent loop integrals
- Subtraction: redefine the coupling $\alpha_S = g^2/(4\pi)$ → $\alpha_S(\mu^2)$

Renormalisation scale

The theory does not predict the absolute value of the coupling but its dependence on the scale can be predicted

$$\frac{d\alpha_S(\mu^2)}{d\ln\mu^2} = \beta(\alpha_S(\mu^2))$$

it is one the renormalization group equations

if for a given scale μ_0 we have $\alpha_S(\mu_0) << 1$

→ a perturbative solution of the renormalization group equation can be given

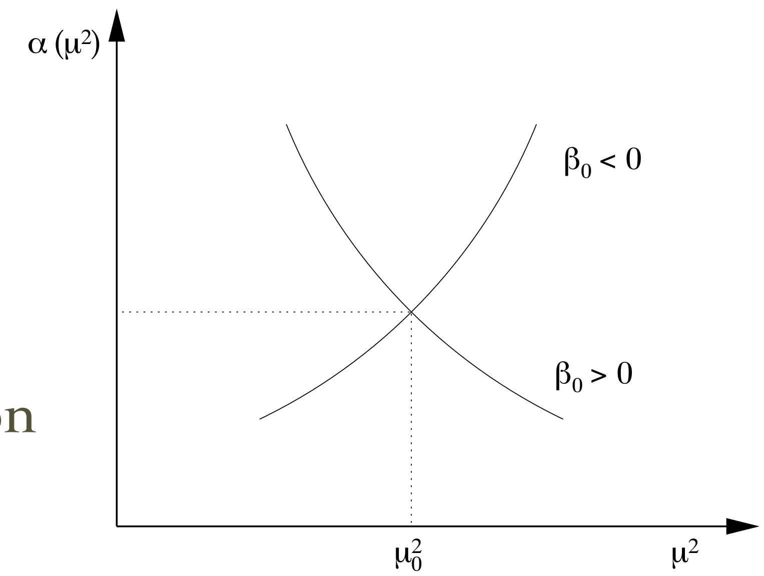
Asymptotic freedom

At one-loop order we have $\beta(\alpha_S) = -\beta_0 \alpha_S^2 + \mathcal{O}(\alpha_S^3)$

$$\frac{d\alpha_S(\mu^2)}{d\ln\mu^2} = -\beta_0 \alpha_S^2 \quad \Rightarrow \quad \alpha_S(Q^2) = \frac{\alpha_S(Q_0^2)}{1 + \alpha_S(Q_0^2)\beta_0 \ln Q^2/Q_0^2}$$

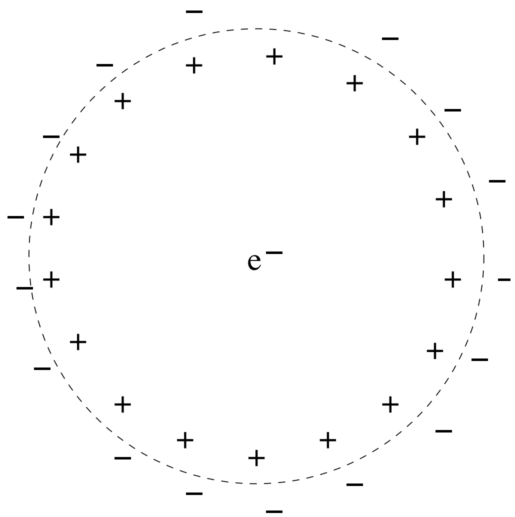
The behaviour crucially depends on the sign of the coefficient β_0

In QED the dependence of the coupling on the scale has a simple physical interpretation



Asymptotic freedom

The vacuum around a pointlike charge becomes polarized due to the emission of e^+e^- pairs and produces a screening effect



$\beta_0<0$: the effective coupling decreases with the distance

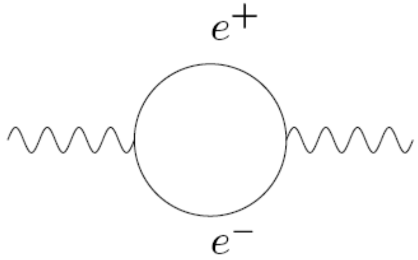
But:

In QCD gluons are charged and provide a positive contribution to β_0

$$\beta_0 = \frac{11N_c - 2N_F}{12\pi} > 0$$

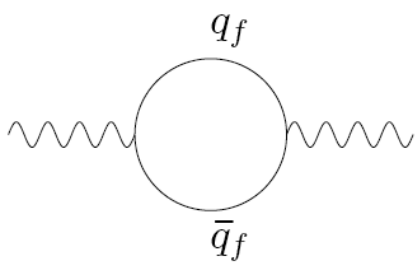
for $N_F < 16$

QED

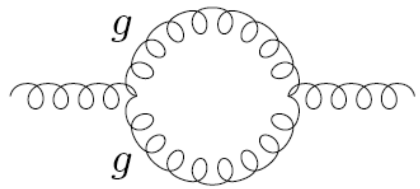


$\beta_0 = -\frac{1}{3\pi} < 0$

QCD



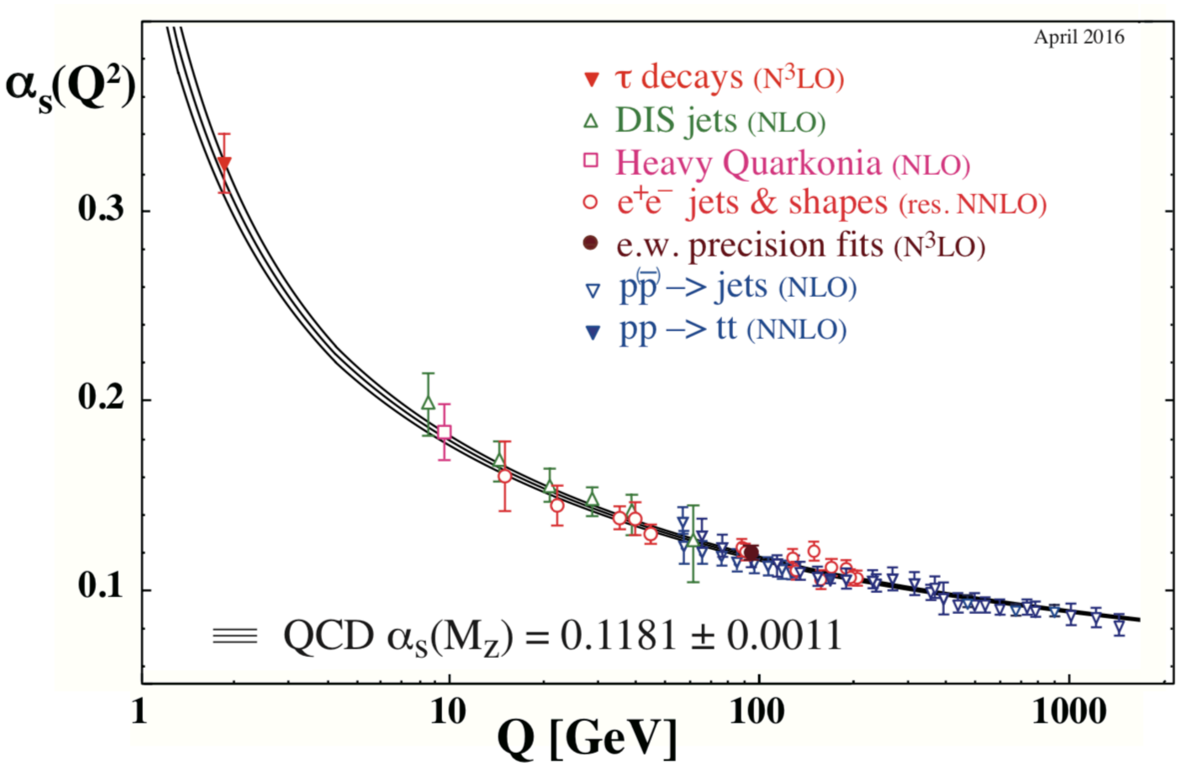
$\beta_0^{quark} = -\frac{1}{3\pi} T_R N_F < 0$



$\beta_0^{gluon} = \frac{11}{12\pi} C_A$ positive !

Asymptotic freedom

Intuitively: gluons are charged and spread colour charge over larger distances: anti-screening effect



ASYMPTOTIC FREEDOM

In processes with large momentum transfer we can use perturbation theory even if we have not solved the full theory

Λ_{QCD} ~ scale at which the coupling becomes strong

Asymptotic freedom

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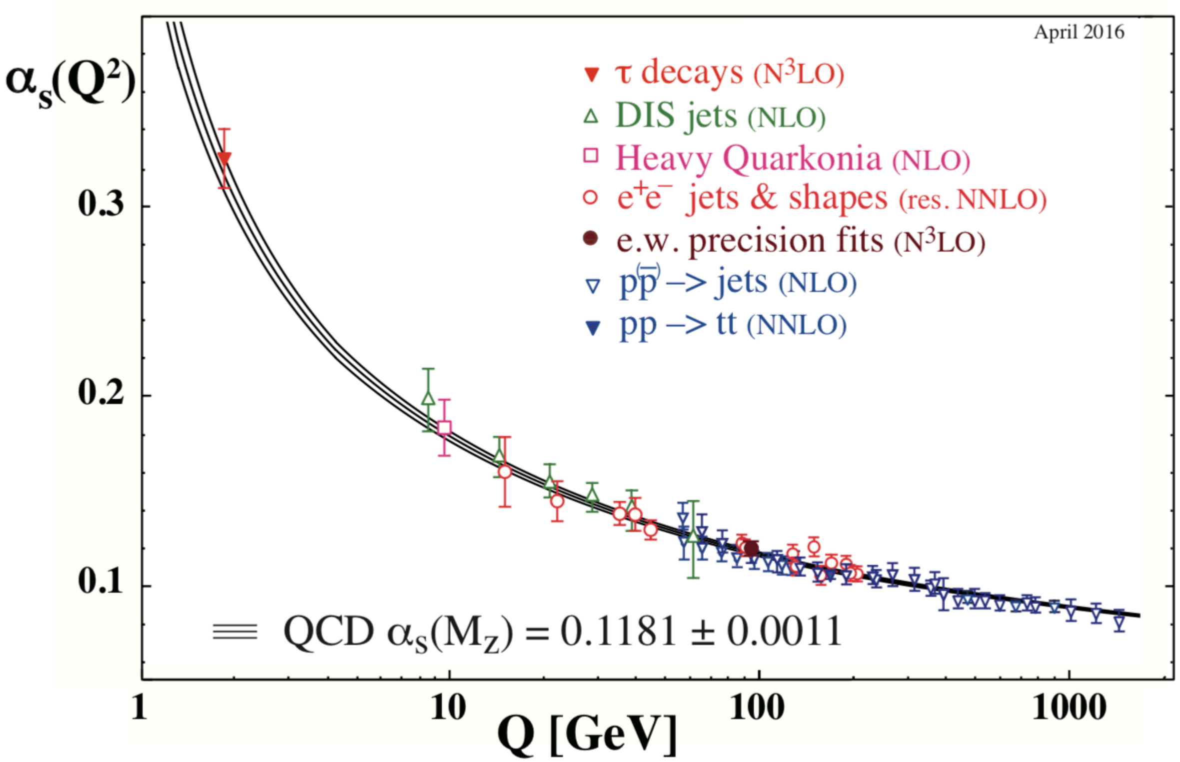


→ **ASYMPTOTIC FREEDOM**

In processes with large momentum transfer we can use perturbation theory even if we have not solved the full theory

Nobel prize in Physics 2004

D. Gross, H.D. Politzer, F. Wilczek

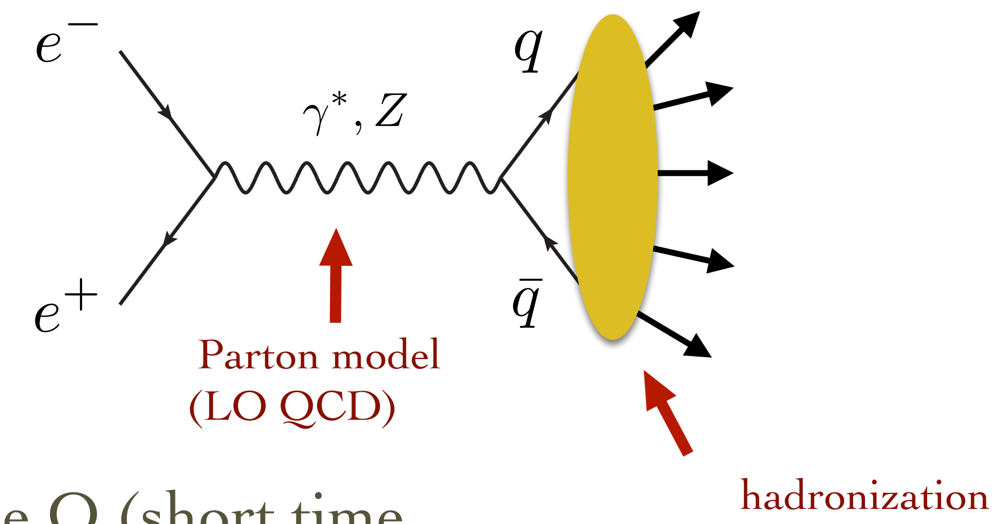
Λ_{QCD} ~ scale at which the coupling becomes strong

Parton model

Asymptotic freedom → at large tranferred momenta hadrons behave as collections of free (weakly-interacting) partons

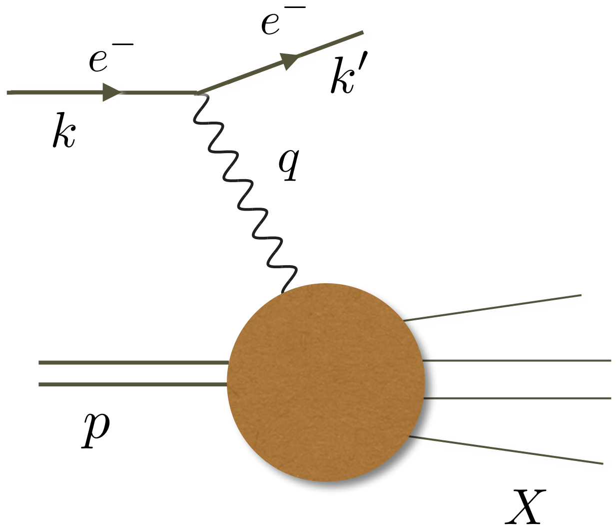
at small scales the interaction becomes strong but if we are not interested in the details of hadronic processes (consider inclusive enough final states) → we can use the parton picture

$e^+ e^-$ annihilation



produce a hard $q\bar{q}$ pair at scale Q (short time scale $\tau=1/Q$) which travel far apart as free

Deep inelastic scattering



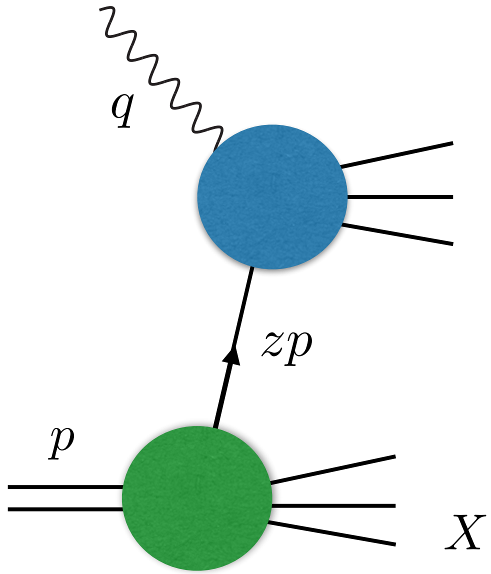
Let us consider the process ep→eX

$Q^2 = -q^2 = (k-k')^2 \gg \Lambda^2_{QCD}$

If $Q^2 < m^2_Z$ the cross section is dominated by one-photon exchange

The photon acts as a probe of the proton structure

Parton interactions in the proton characterized by a large time scale $\tau \sim 1/\Lambda_{QCD}$ with respect to $\tau_{hard} \sim 1/Q$



→ **Scattering is incoherent on the single partons**

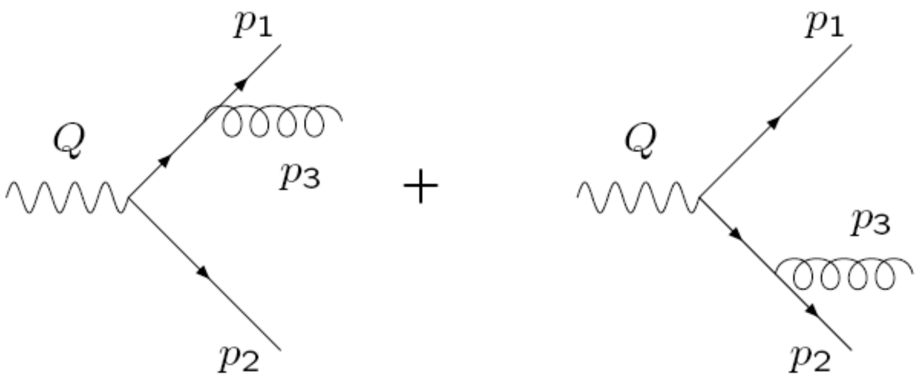
$$\sigma(p) \sim \int dz\, f(z)\, \hat{\sigma}(zp)$$

The question:
does the parton picture survive when radiative corrections are included ?

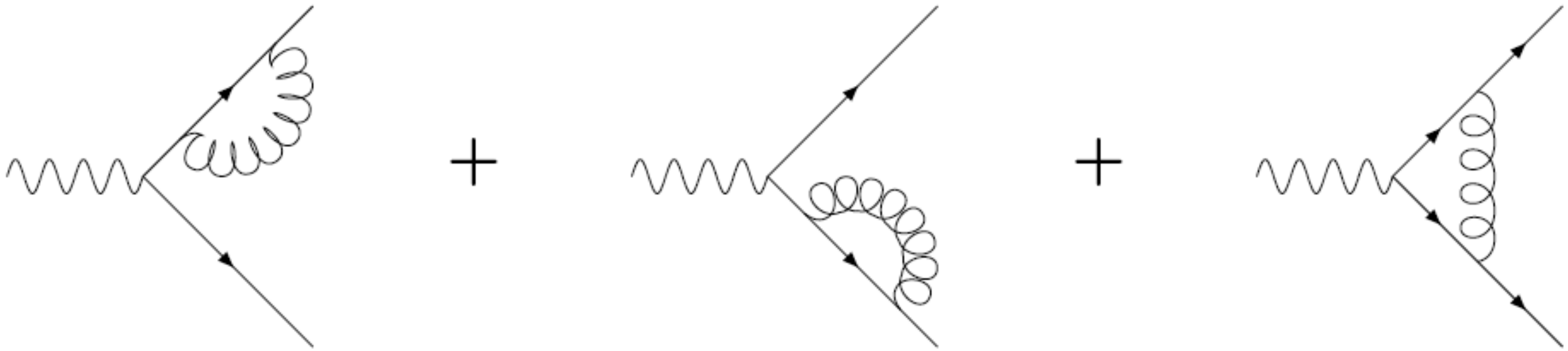
$e^+ e^-$ annihilation

Consider the $O(\alpha_S)$ corrections to the total cross section

Real:

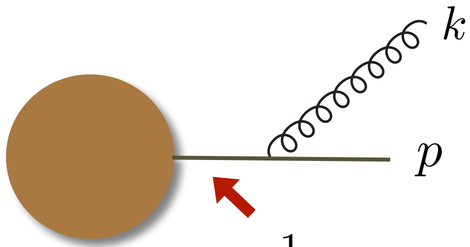


Virtual:



Real and virtual contributions are separately divergent !

Infrared divergences

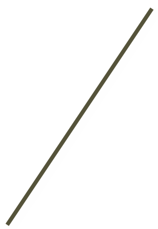


$$\frac{1}{(p+k)^2} = \frac{1}{2E_pE_k(1-\cos\theta)}$$

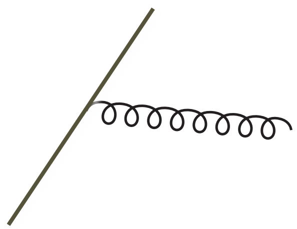
Originate in theories with massless particles and are of two kinds

- Soft: the energy of a gluon vanishes
- Collinear: two partons become parallel

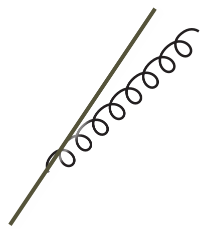
Physically a hard parton plus a soft gluon or two very close partons are indistinguishable ➡ They correspond to degenerate states



hard parton



hard parton +soft gluon



2 collinear partons

Infrared divergences are a manifestation of long-distance effects

Even in QED we cannot separate an electron from an electron + a soft photon, or an electron from an electron plus a collinear photon

→ **Kinoshita-Lee-Naumberg (KLN) theorem:**
if we limit ourself to considering quantities inclusive over initial and final states soft and collinear (degenerate) configurations infrared divergences cancel out

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The answer:
intuitive parton picture survives to the computation of radiative corrections provided we consider inclusive enough processes

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NOTE:

Phase space is flat
Matrix elements are enhanced in soft and collinear regions

→ The hadronic final state is typically formed by jets of collinear particles plus soft particles

In order to cancel the divergences it is not necessary to integrate over the full phase space

According to the KLN theorem we can define a wide class of IR safe observables

Leading Order (LO):

$$\sigma_{LO} = \int_m |\mathcal{M}^{(0)}(\{p_i\})|^2 F_m(\{p_i\}) d\mathrm{PS}_m$$

Tree level matrix element — Measurement function — Phase space

Next-to-Leading Order (LO):

$$\sigma_{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

m number of partons at LO (e.g. 2jets m=2, 3jets m=3...)

Here one more parton — Here same number of partons but one-loop matrix element

To be sure that the presence of F does not spoil the cancellation we should have:

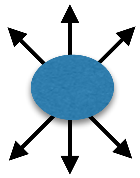
$$F_{m+1}(.....p_i, ...p_j......) \sim F_m(....p_i + p_j....) \text{ if } p_i \parallel p_j \text{ or } p_j \to 0$$

Example: thrust

$$F_m(\{p_i\}) = \delta\left(T - T_m(p_1, ...p_m)\right)$$

Where $$T_m = \max_{\mathbf{n}} \frac{\sum_i^m |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i^m |\mathbf{p}_i|}$$

T=1: two jet limit



T=1/2: spherical event

- Suppose that p_{m+1} is the momentum of a soft gluon

  ➡ It drops out from numerator and denominator, so $T_{m+1} \to T_m$

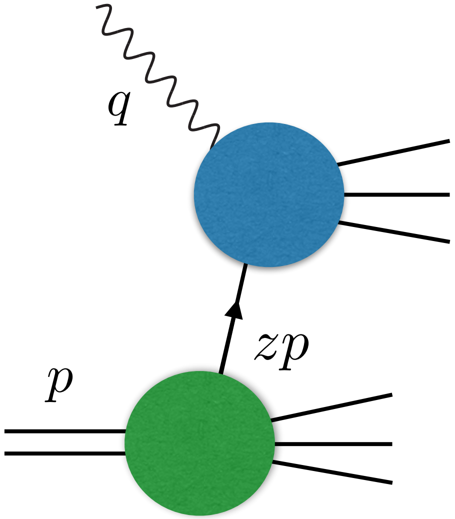
- Suppose that $\mathbf{p}_i \parallel \mathbf{p}_j$ that is $\mathbf{p}_i = z\,\mathbf{p}$ and $\mathbf{p}_j = (1-z)\,\mathbf{p}$

  - In the numerator $|\mathbf{p}_i \cdot \mathbf{n}| + |\mathbf{p}_j \cdot \mathbf{n}| = |\mathbf{p} \cdot \mathbf{n}|$
  - In the denominator $|\mathbf{p}_i| + |\mathbf{p}_j| = |\mathbf{p}|$

  ➡ $T_{m+1} \to T_m$ ➡ **IR safe !**

Crucial ingredient: linearity

Hard processes with initial state hadrons: deep inelastic scattering

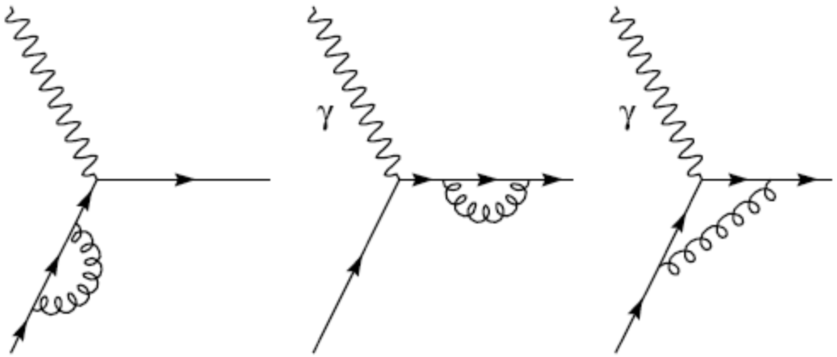


Partonic cross section

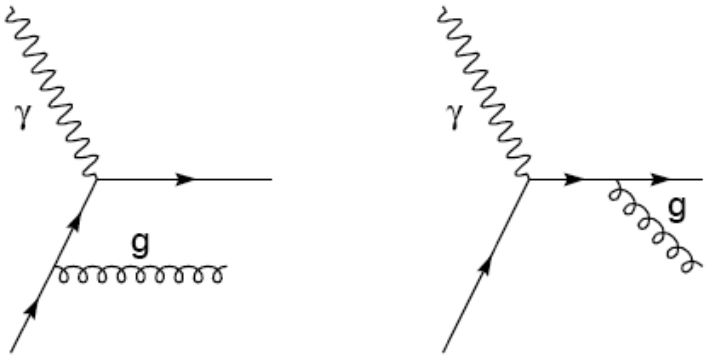
Parton density

Consider O(α_S) corrections to the partonic cross section

Virtual:



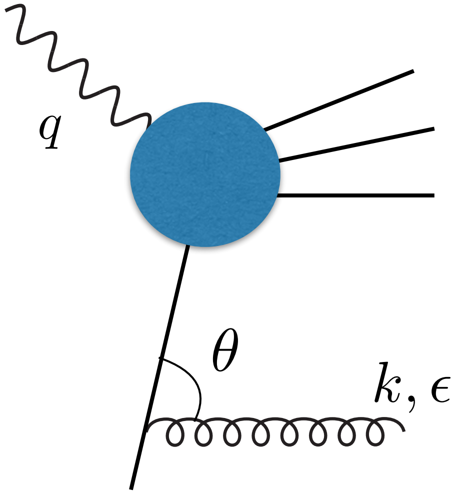
Real:



In the calculation we find infrared divergences

Inclusive final state → KLN cancellation of final state singularities

However in the initial state we have only one parton and we cannot sum over degenerate states → uncancelled collinear singularity



$$\sim \alpha_S(Q^2)\int_0^1 \frac{d\theta^2}{\theta^2} \sim \int_0^{Q^2} \frac{dk_T^2}{k_T^2}$$

Actually the collinear divergence would be regularized by a physical cutoff Q_0 of the order of the typical hadronic scale

→ The singularity implies the existence of long distance effects

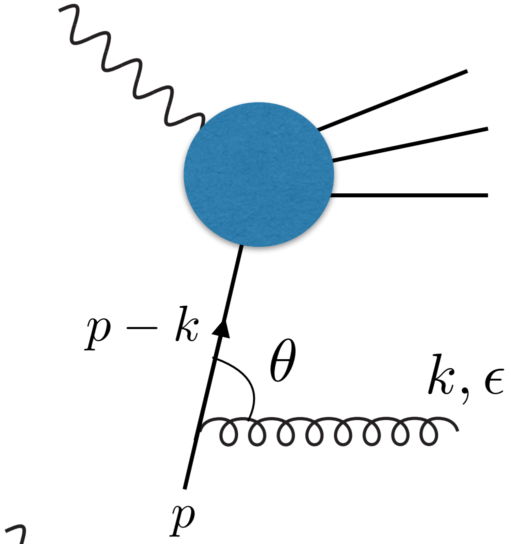
Multiple emission → $\alpha_S^n \log^n Q^2/Q_0^2$ to be resummed to all orders

Both problems are solved by the FACTORIZATION THEOREM

IN SHORT: Collinear singularities can be absorbed in bare parton densities $f_0(x) \to f(x, Q^2)$

Physical parton densities thus become scale dependent

This is possible only if this redefinition is independent on the hard process

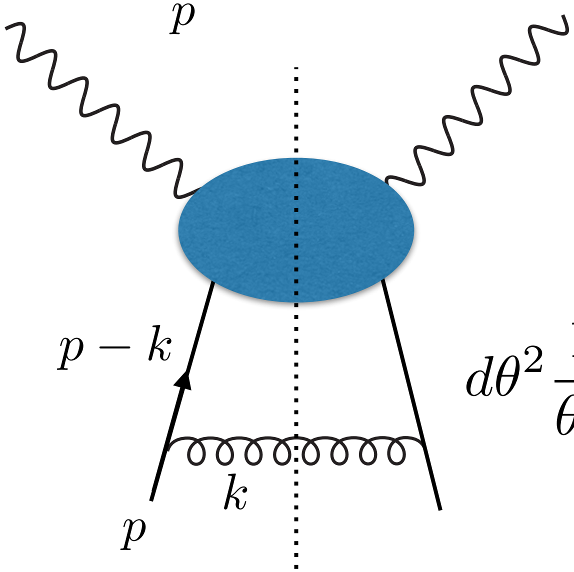


Phase space: $\frac{d^3k}{2k_0} \sim k_0 dk_0 d\cos\theta \sim d\theta^2$

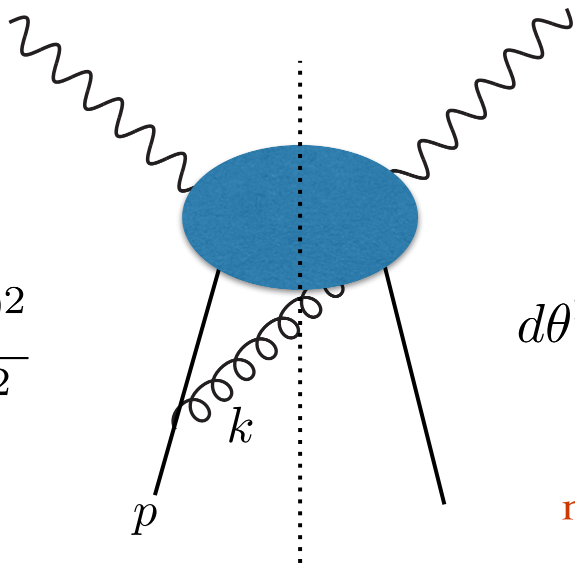
Propagator: $\frac{1}{(p-k)^2} \sim \frac{1}{p_0 k_0 (1-\cos\theta)} \sim \frac{1}{\theta^2}$

Vertex: $(\not{p}-\not{k})\not{\epsilon}(k)\not{p} \sim \theta$

in a physical gauge (helicity conservation in the quark gluon vertex !)



$$d\theta^2 \frac{1}{\theta^2}\frac{1}{\theta^2}\theta\theta \sim \frac{d\theta^2}{\theta^2}$$



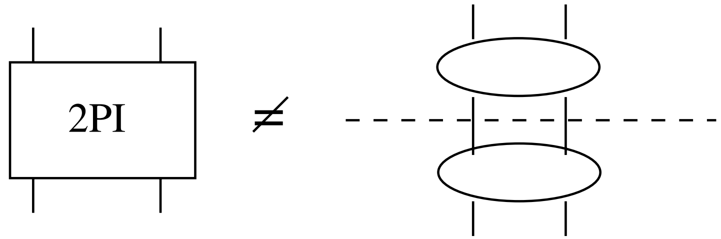
$$d\theta^2 \frac{1}{\theta^2}\theta$$

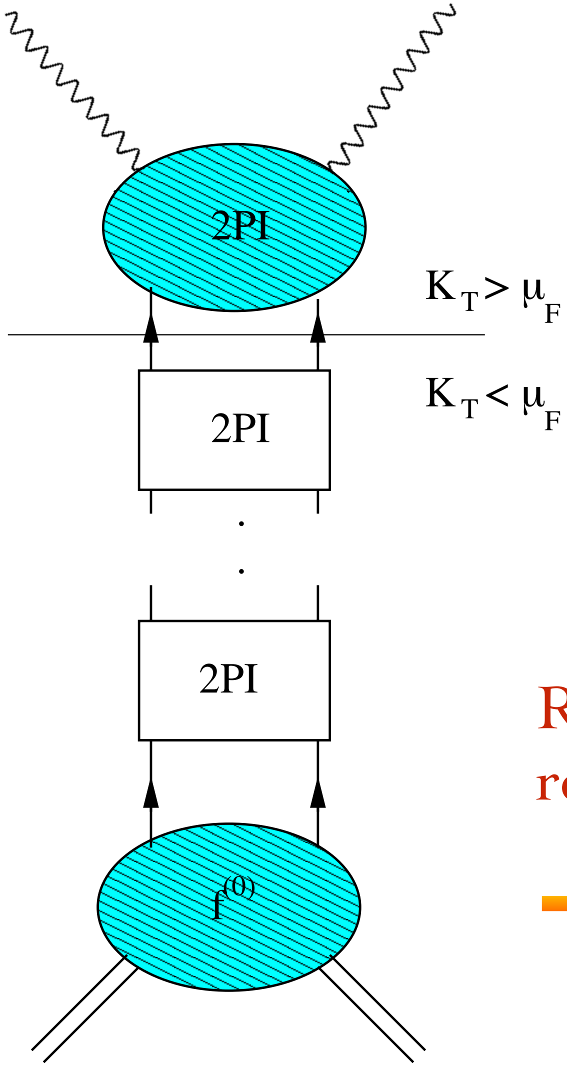
not singular enough !

→ **In a physical gauge interferences can be neglected**

General strategy: decompose diagrams in 2PI blocks (such that they cannot be disjoint by cutting only two lines)

2PI blocks are free of collinear singularities in a physical gauge





Introduce an arbitrary separation scale μ_F

Process dependent but finite

Universal but divergent

Reabsorb the divergent part in the redefinition of f(x)

analogy with renormalization

$$\sigma(p,Q) = \sum_a \int_0^1 dz f_a(z,\mu_F^2)\hat{\sigma}_a(zp,\alpha_S(Q^2);\mu_F^2)$$

Introduction of an arbitrary scale μ_F

→ Physical cross sections cannot depend on μ_F

The choice of μ_F is arbitrary but if μ_F is too different from the hard scale Q → log Q/ μ_F terms reappear that spoil the perturbative expansion

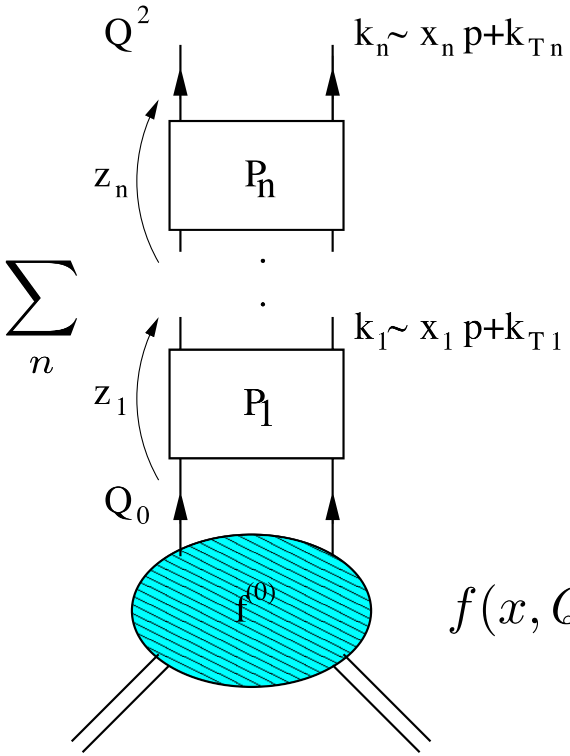
Parton densities become scale dependent

Such scale dependence is associated with the resummation of large collinear logs

The scale dependence is perturbatively computable

→ DGLAP equations

DGLAP equations



$$f(\mu_F^2) = f_0 \otimes E(\mu_F^2/Q_0^2)$$

The important region is: $Q^2 > k_{Tn}^2 > ...k_{T1}^2 > Q_0^2$

where the maximum power of $\log Q^2/Q_0^2$

is generated → iterative structure

$$f(x,Q^2) = f_0(x) + \int_{Q_0^2}^{Q^2} \frac{dk_{Tn}^2}{k_{Tn}^2} \int_x^1 \frac{dz_n}{z_n} P_n(\alpha_S(k_{Tn}^2), z_n) f(x/z_n, k_{Tn}^2)$$

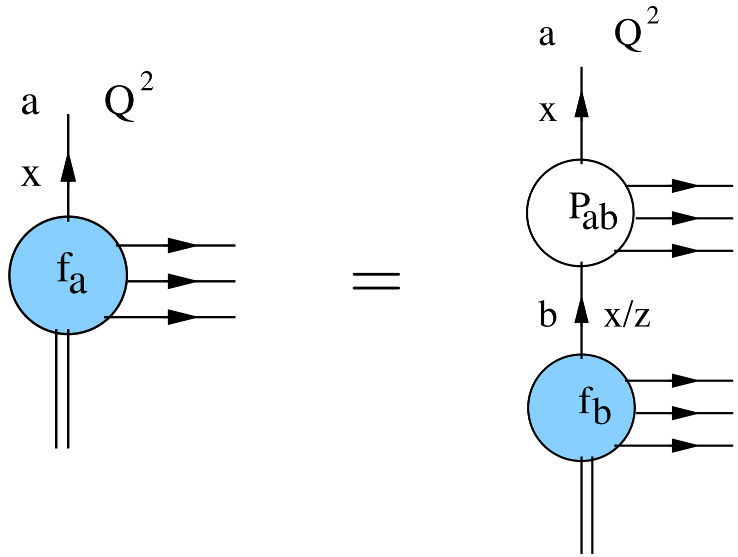
Taking derivative with respect to Q²

$$Q^2 \frac{\partial f(x,Q^2)}{\partial Q^2} = \int_x^1 \frac{dz}{z} P(\alpha_S(Q^2), z) f(x/z, Q^2)$$

First order differential equation: can be solved if $f(x,Q^2)$ is known at a reference scale Q_0

Probabilistic interpretation:

$$Q^2 \frac{\partial}{\partial Q^2}$$



$$P_{ab}(\alpha_S(Q^2), z)$$

Probability to find parton a in parton b at scale Q^2

- $P_{ab}(\alpha_S, z) = \frac{\alpha_S}{2\pi} P_{ab}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ab}^{(1)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^3 P_{ab}^{(2)}(z) + \ldots..$

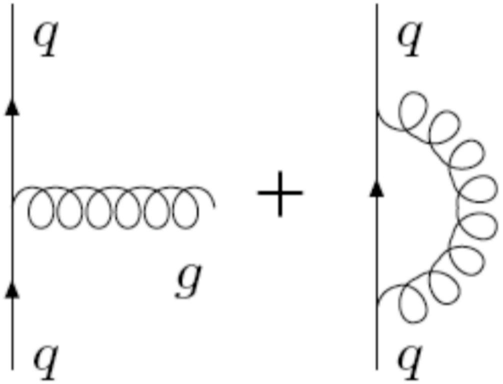
Dokshitzer (1977)
Gribov, Lipatov (1972)
Altarelli-Parisi (1977)

Curci, Furmanski, Petronzio (1980)

Moch, Vermaseren, Vogt (2004)

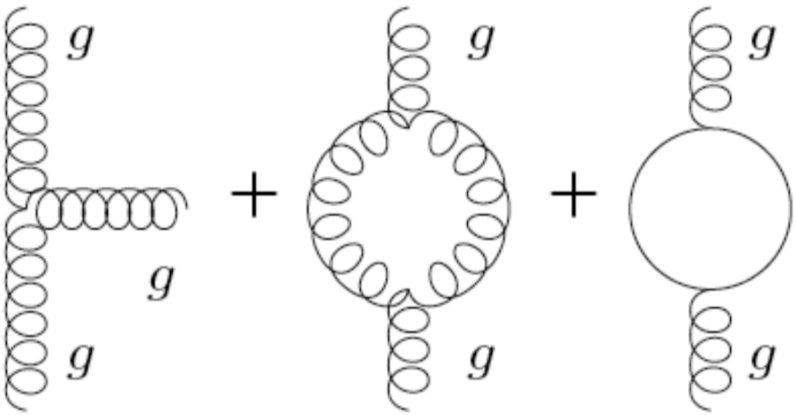
- Solving DGLAP equation using $P_{ab}^{(0)}$ allows us to resum Leading Logarithmic (LL) contributions $\alpha_S^n \log^n Q^2/Q_0^2$

  With $P_{ab}^{(1)}$ we resum Next-to-leading terms (NLL) and so on
- Convolution ⟷ conservation of longitudinal momentum

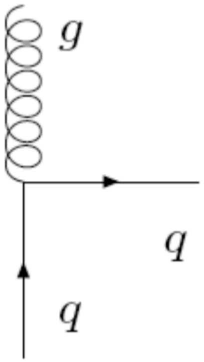


$$P_{qq}^{(0)}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

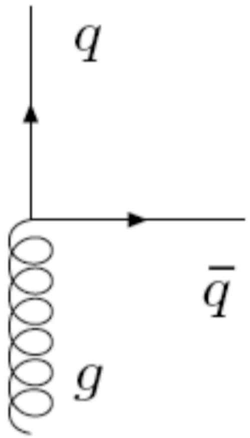
+ distribution defined as $\displaystyle\int_0^1 \frac{f(z)}{(1-z)_+} \equiv \int_0^1 \frac{f(z)-f(1)}{1-z}$



$$P_{gg}^{(0)}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \left(\frac{11}{6}C_A - \frac{2}{3}T_R\, n_F\right)\delta(1-z)$$



$$P_{gq}^{(0)}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right]$$



$$P_{qg}^{(0)}(z) = T_R\left[z^2 + (1-z)^2\right]$$

Solving DGLAP equation

Consider for simplicity the case of non-singlet (q-$\bar{q}$) → DGLAP equation is diagonal

$$Q^2 \frac{\partial f(x,Q^2)}{\partial Q^2} = \int_x^1 \frac{dz}{z} P_{qq}(\alpha_S(Q^2), z) f(x/z, Q^2)$$

Define Mellin moments

$$(f \otimes g)(x) = \int_x^1 \frac{dz}{z} f(z) g(\frac{x}{z})$$

$$f_N = \int_0^1 f(x) x^{N-1} dx$$

$$(f \otimes g)_N = \int_0^1 (f \otimes g)(x)\, x^{N-1} dx$$

$$= \int_0^1 dx x^{N-1} \int_x^1 \frac{dz}{z} f(z) g\left(\frac{x}{z}\right)$$

$$= \int z^{N-1}\; t^{N-1} dt\, dz\, f(z)\, g(t) = f_N\; g_N$$

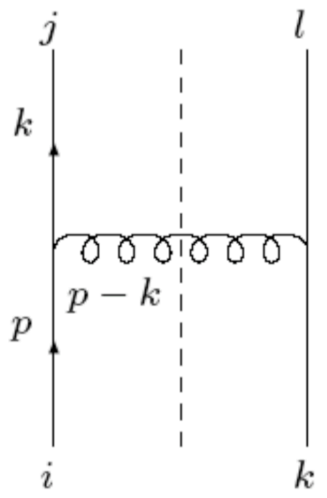
In Mellin space DGLAP becomes

Anomalous dimension ↙

$$Q^2 \frac{\partial f_N(Q^2)}{\partial Q^2} = \gamma_{N,qq}\left(\alpha_S(Q^2)\right) f_N(Q^2)$$

Exercise: compute $P_{qq}^{(0)}$

Real:



Work in axial gauge

Use Sudakov parametrization:

$$k^\mu = zp^\mu + k_T^\mu + \frac{k^2 - k_T^2}{2zp\cdot n} n^\mu$$ ← gauge vector

$$(p-k)^\mu = (1-z)p^\mu - k_T^\mu - \frac{k_T^2}{2(1-z)p\cdot n} n^\mu$$

$$k_T \cdot p = k_T \cdot n = 0$$

Extract the leading contribution in $1/k^2$

and check that $P_{qq}^{\text{real}}(z) = C_F \dfrac{1+z^2}{1-z}$

Virtual:

must be of the form $P_{qq}^{\text{virt}} = A\,\delta(1-z)$

→ Compute A from real by using fermion number conservation

$$\int_0^1 P_{qq}(x)dx = 0$$

integral of valence (q-$\bar{\text{q}}$) quark density (N=1 moment) cannot depend on Q^2

Scaling violations

$$\gamma_{ab}(N) = \int_0^1 P_{ab}(x) x^{N-1}$$

The small x region corresponds to small N, whereas x→1 selects large N

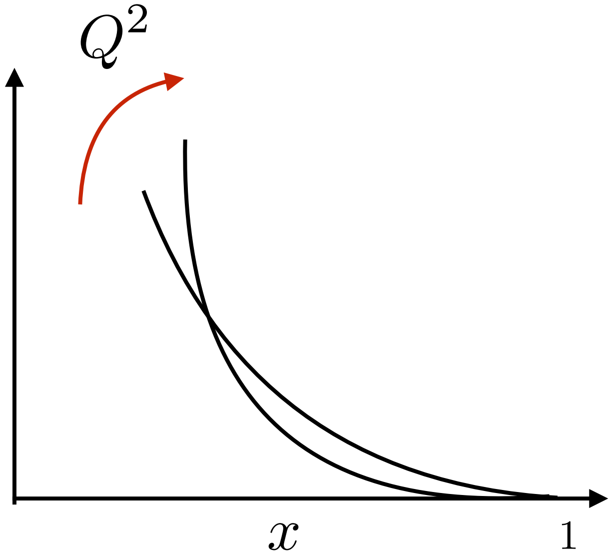
$$\gamma_{gg} \sim \frac{2C_A}{N-1}$$ N small

N large: $x^{N-1} \sim \Theta(1/N - (1-x))$

→ $\gamma_{aa} \to -2C_a \log N$ as N→∞

Scaling violations are:

- Positive at small x
- Slightly negative at large x



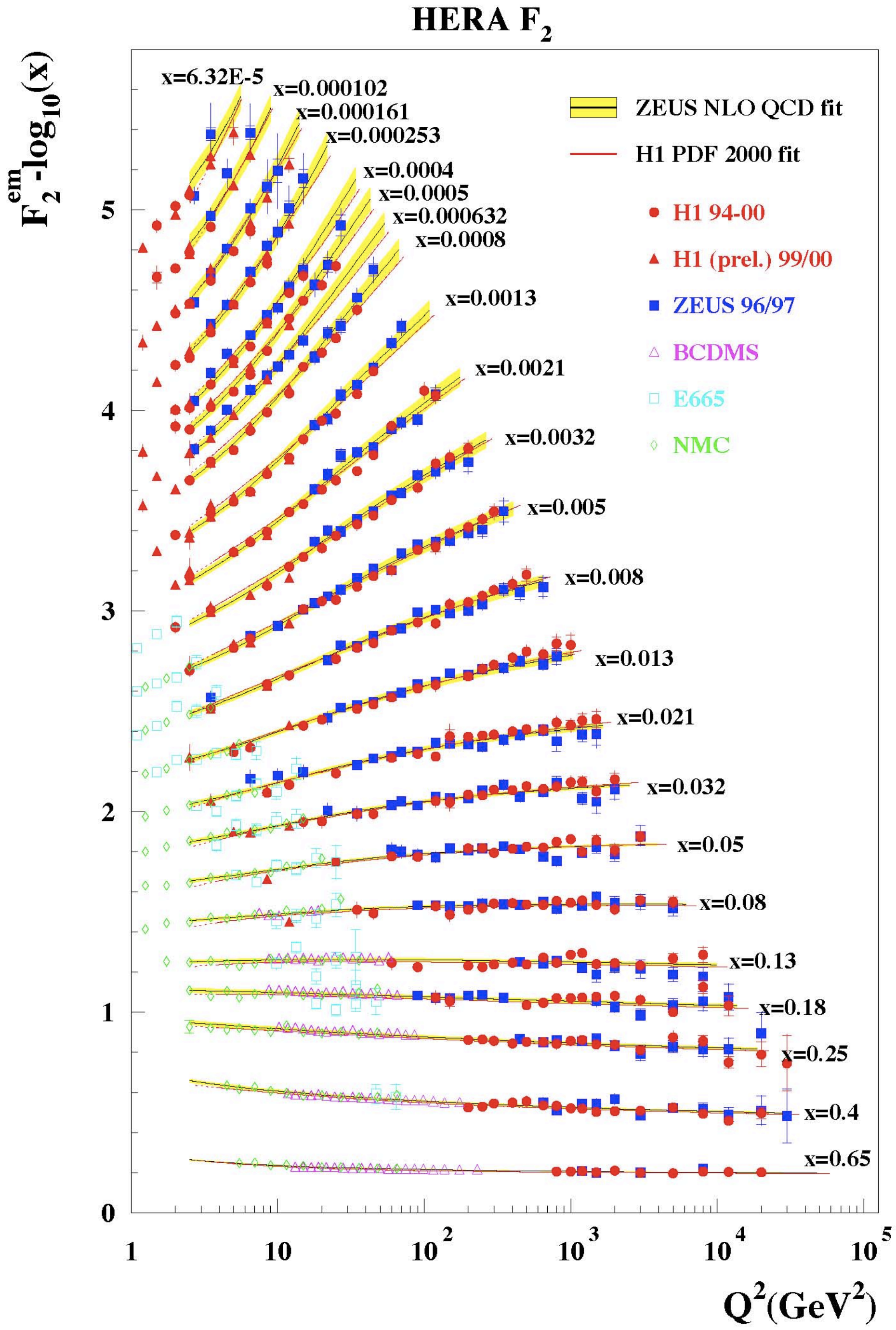
Main effect of increase in Q² is shift of partons from larger to smaller x

"Structure function" $F_2(x,Q^2)$ measured at HERA

The Parton model would predict F_2 to depend only on $x=Q^2/2pq$

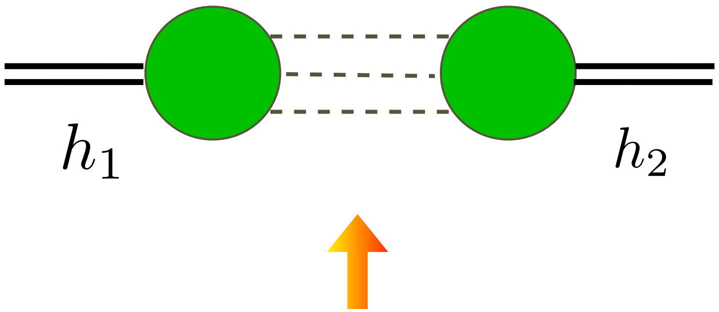
→ Bjorken scaling

Scaling violations nicely consistent with DGLAP picture



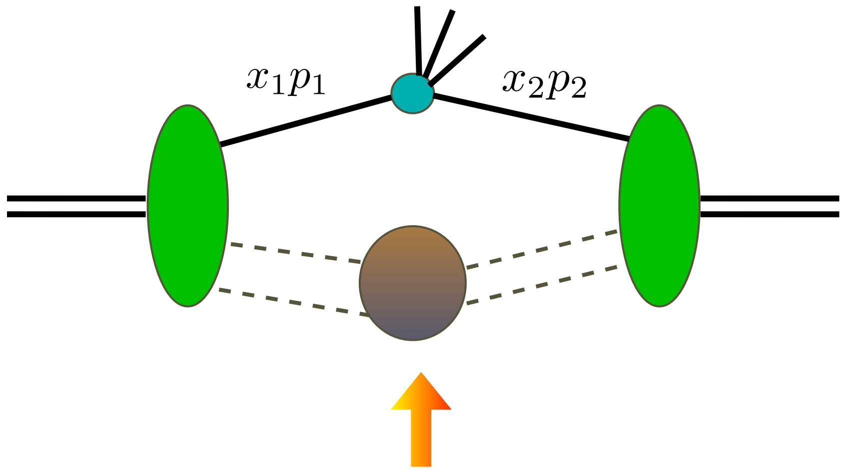
QCD at hadron colliders

In hadron collisions all phenomena are QCD related but we must distinguish between hard and soft processes



Production of low pt hadrons: most common events

Hard subprocess



Soft underlying event

Only hard scattering events can be controlled via the factorization theorem

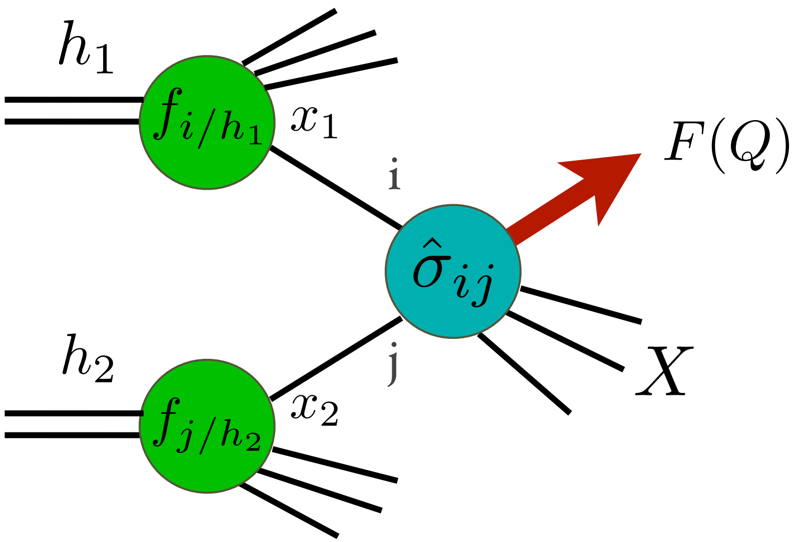
Hard processes are identified by the presence of a hard scale Q

This can be for example the invariant mass of a lepton pair, the transverse momentum of a jet or of a heavy quark...

The corresponding cross section can be written as

$$\sigma(P_1,P_2) = \sum_{i,j}\int dx_1 dx_2 f_{i/h_1}(x_1,\mu_F^2) f_{j/h_2}(x_2,\mu_F^2)\hat{\sigma}_{ij}(p_1,p_2,\alpha_S(\mu_R),Q^2;\mu_F^2,\mu_R^2)$$

$$p_1 = x_1 P_1 \qquad p_2 = x_2 P_2$$



$f_{i/h}(x,\mu_F^2)$ — Parton distributions: universal but not perturbatively computable

$\hat{\sigma}_{ij}$ — Hard partonic cross section: process dependent but perturbatively computable

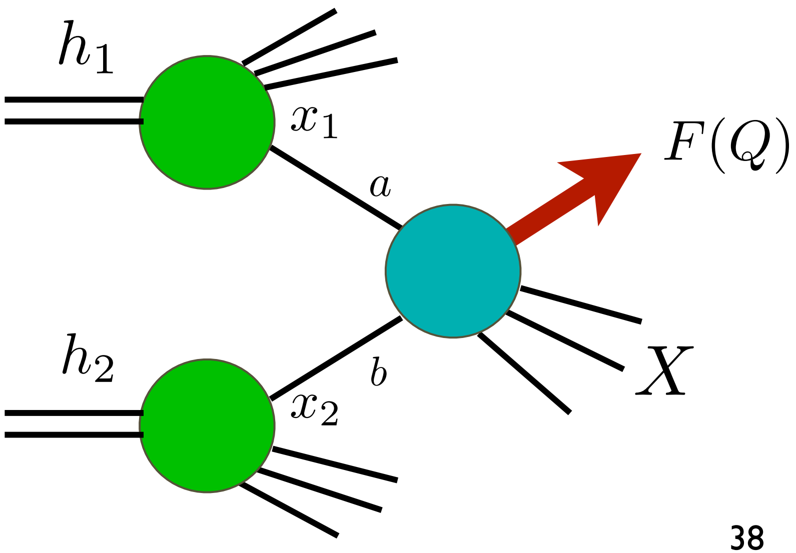
The partonic cross section can be computed in QCD perturbation theory as

$$\hat{\sigma}_{ij} = \alpha_S^k \sum_n \left(\frac{\alpha_S}{\pi}\right)^n \sigma_{ij}^n$$

Different hard processes will contribute with different leading powers k:

- Vector boson production: k=0
- Jet production, heavy quark production: k=2

According to the factorization theorem, the initial state collinear singularities can be absorbed in the parton distribution functions as in the case of DIS



Note that the generally speaking the factorization theorem in hadron collisions does not have a solid proof as in DIS (where Operator Product Expansion can be advocated)

Kinematics

The spectrum of the two hadrons provides two beams of incoming partons

The spectrum of longitudinal momenta is determined by the parton distributions

The centre of mass of the partonic interaction is normally boosted with respect to the laboratory frame

→ **It is useful to classify the final state according to variables that transform simply under longitudinal boosts**

We introduce the rapidity y and the azimuthal angle ϕ

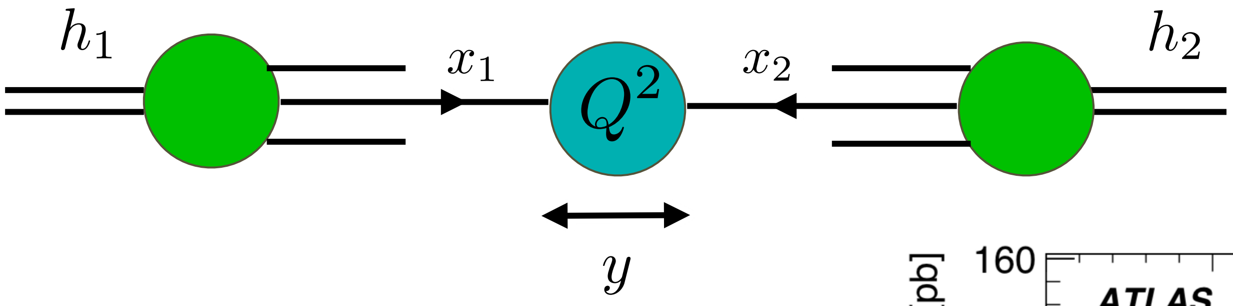
$$p^\mu = (E, p_x, p_y, p_z) = (m_T \cosh y, p_T \sin\phi, p_T \cos\phi, m_T \sinh y)$$

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \qquad m_T = \sqrt{m^2 + p_T^2}$$

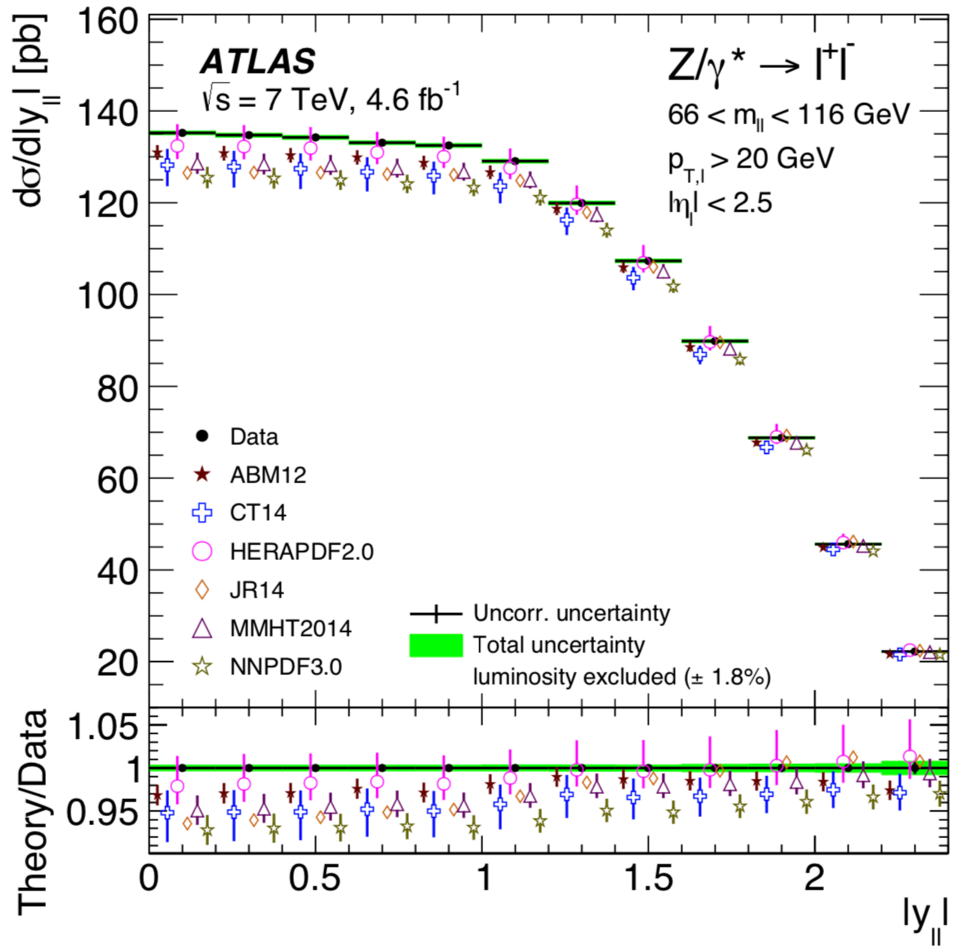
Rapidity differences are boost invariant

Varying Q and y → Sensitivity to different x_1 and x_2

$$x_1 x_2 S = Q^2 \qquad x_{1,2} = Q/\sqrt{S}\, e^{\pm y} \qquad y_{\max} = \ln Q/\sqrt{S}$$



At large rapidities we have two competitive effects:
- small x enhancement of gluon and sea quark distributions
- large x suppression



The large x suppression always "wins":

→ The bulk of the events is concentrated in the central rapidity region (y not too large)

In practice the rapidity is often replaced by the pseudorapidity

$$\eta = -\ln\tan(\theta/2)$$

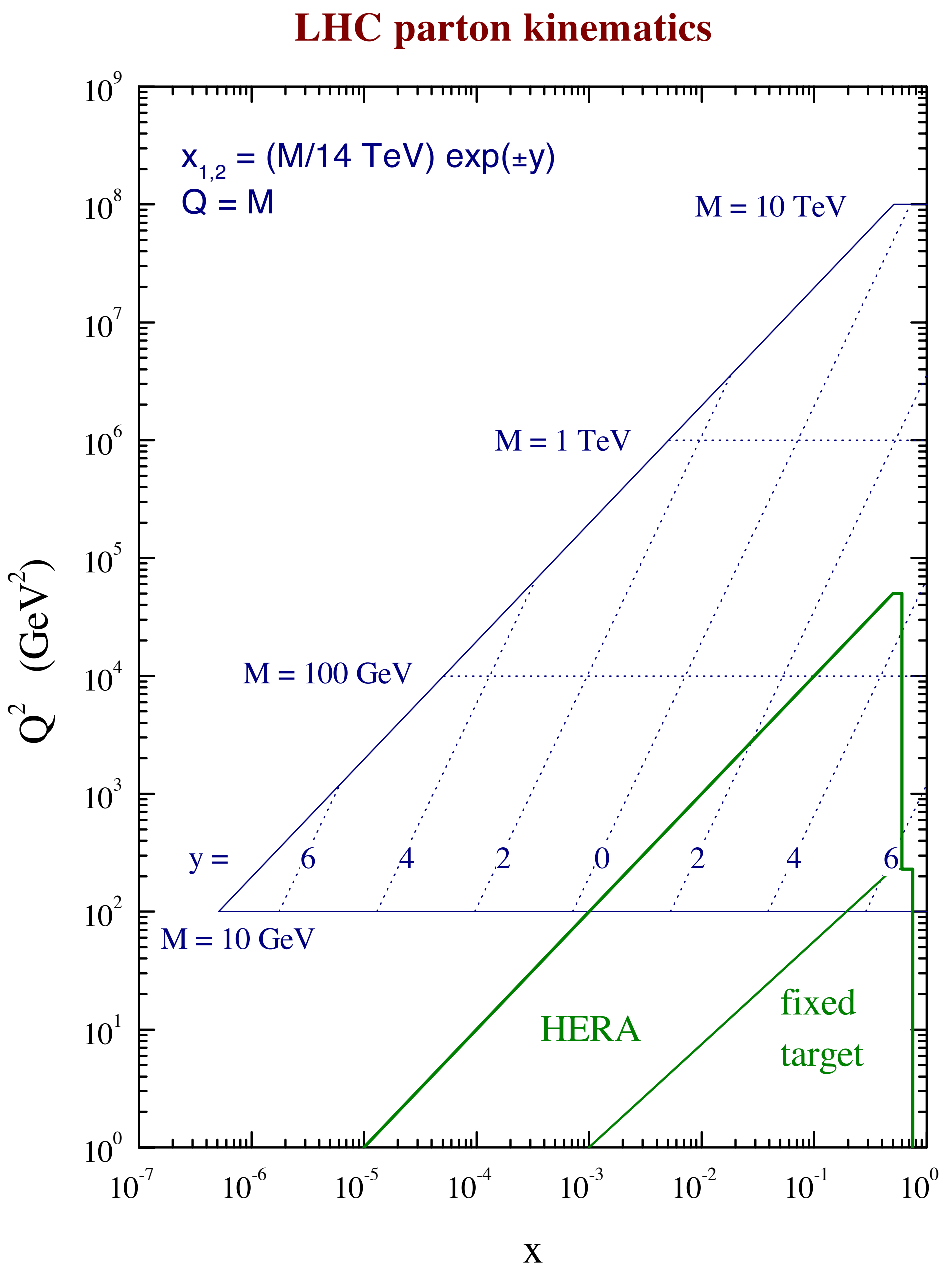
It coincides with the rapidity in the massless limit

Varying Q and y

➡ Sensitivity to different x_1, x_2

$$x_1 x_2 S = Q^2 \qquad x_{1,2} = Q/\sqrt{S}\, e^{\pm y}$$

LHC probes a kinematical region never reached before



Parton Distribution Functions

Determined by global fits to different data sets

Standard procedure:

- Parametrize at input scale $Q_0 = 1 - 4\ \mathrm{GeV}$

$$xf(x, Q_0^2) = Ax^{\alpha}(1-x)^{\beta}(1 + \epsilon\sqrt{x} + \gamma x + ...)$$

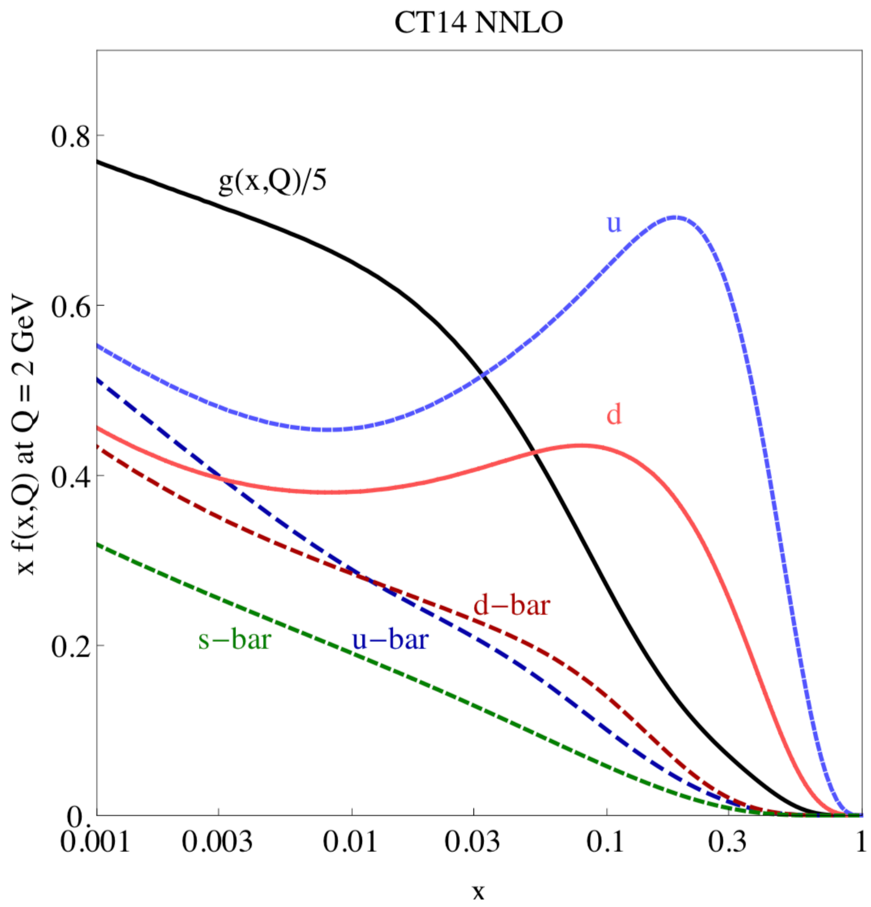
- Impose momentum sum rule: $\sum_a \int_0^1 dx x f_a(x, Q_0^2) = 1$
- Evolve to desired Q^2 and compute physical observables
- Then fit to data to obtain the parameters

Main groups: MSTW (now MMHT), CTEQ, NNPDF

and also: HERA, ABM….

Typical behaviour of parton densities in the proton: Q=2 GeV

u and d quarks peaked at x=0.2-0.3



- All densities vanish as x→1 the gluon vanishing fastest

- At x→0

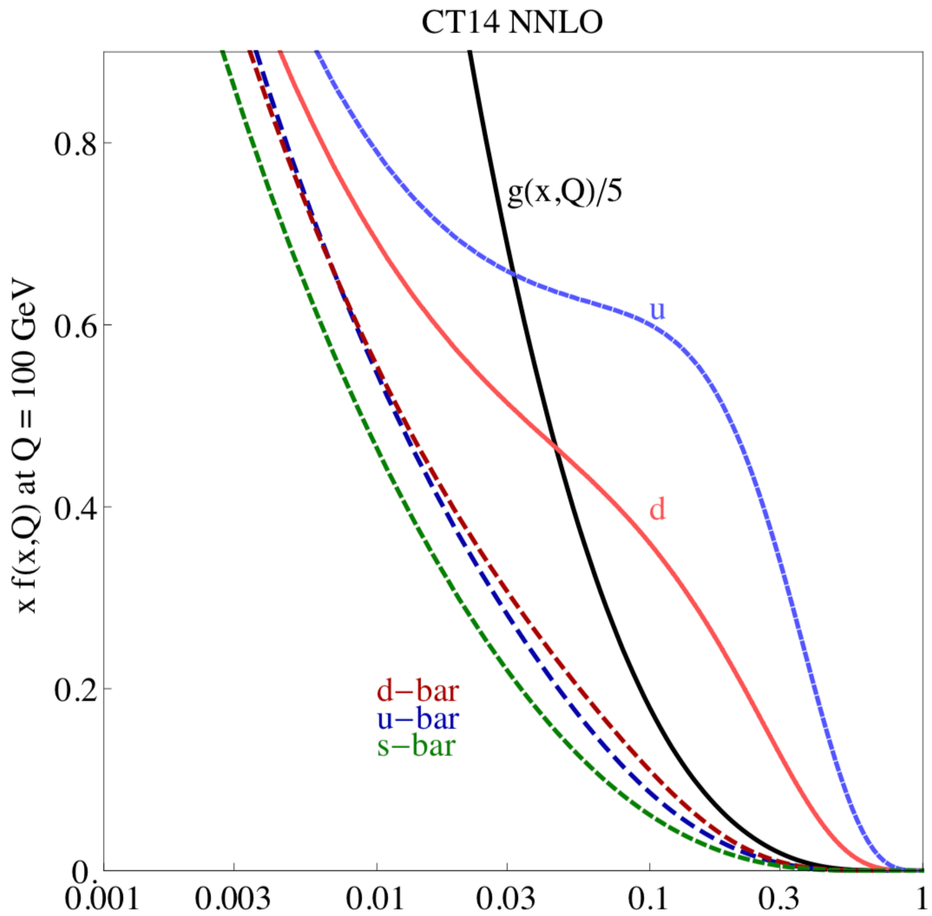
  - Valence quarks vanish

  - Strong rise of the gluon, which becomes dominant

  - Also sea quarks increase ➡ driven by the gluon through $g\to q\bar{q}$ splitting

Typical behaviour of parton densities in the proton: Q=100 GeV

u and d quarks peaked at smaller x, gluon and sea dominant



- All densities vanish as x→1 the gluon vanishing fastest
- At x→0

  - Valence quarks vanish
  - Strong rise of the gluon, which becomes dominant
  - Also sea quarks increase ➡ driven by the gluon through $g \to q\bar{q}$ splitting

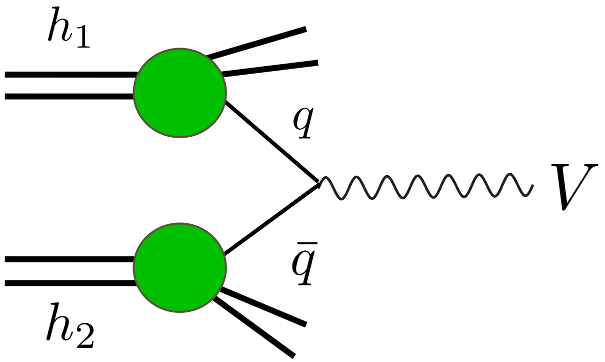
Typical processes:

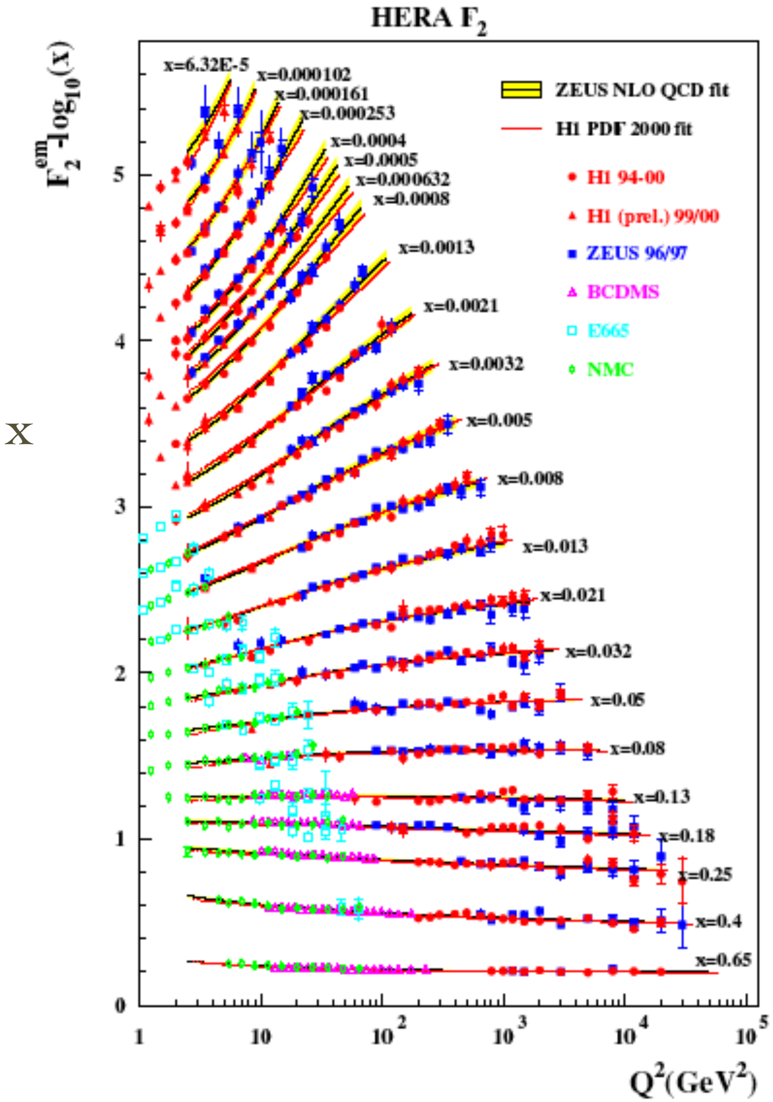
- DIS:

Fixed target: valence quark densities ($u-\bar{u}$, $d-\bar{d}$)

HERA: Gluon and see quarks at small x

- Drell-Yan → quark densities





pp collisions: sensitive to antiquarks and sea densities

$p\bar{p}$ collisions: sensitive to flavour asymmetries of valence quarks

The NNPDF approach

The fitting procedure relies on the choice of the functional form, which introduces a bias in the fit

The classical approach to PDF fitting is based on the choice of a (relatively) simple parametrization

The NNPDF approach generates Monte Carlo replicas of the experimental data

→ Fit PDFs by using a set of neural networks on each replica

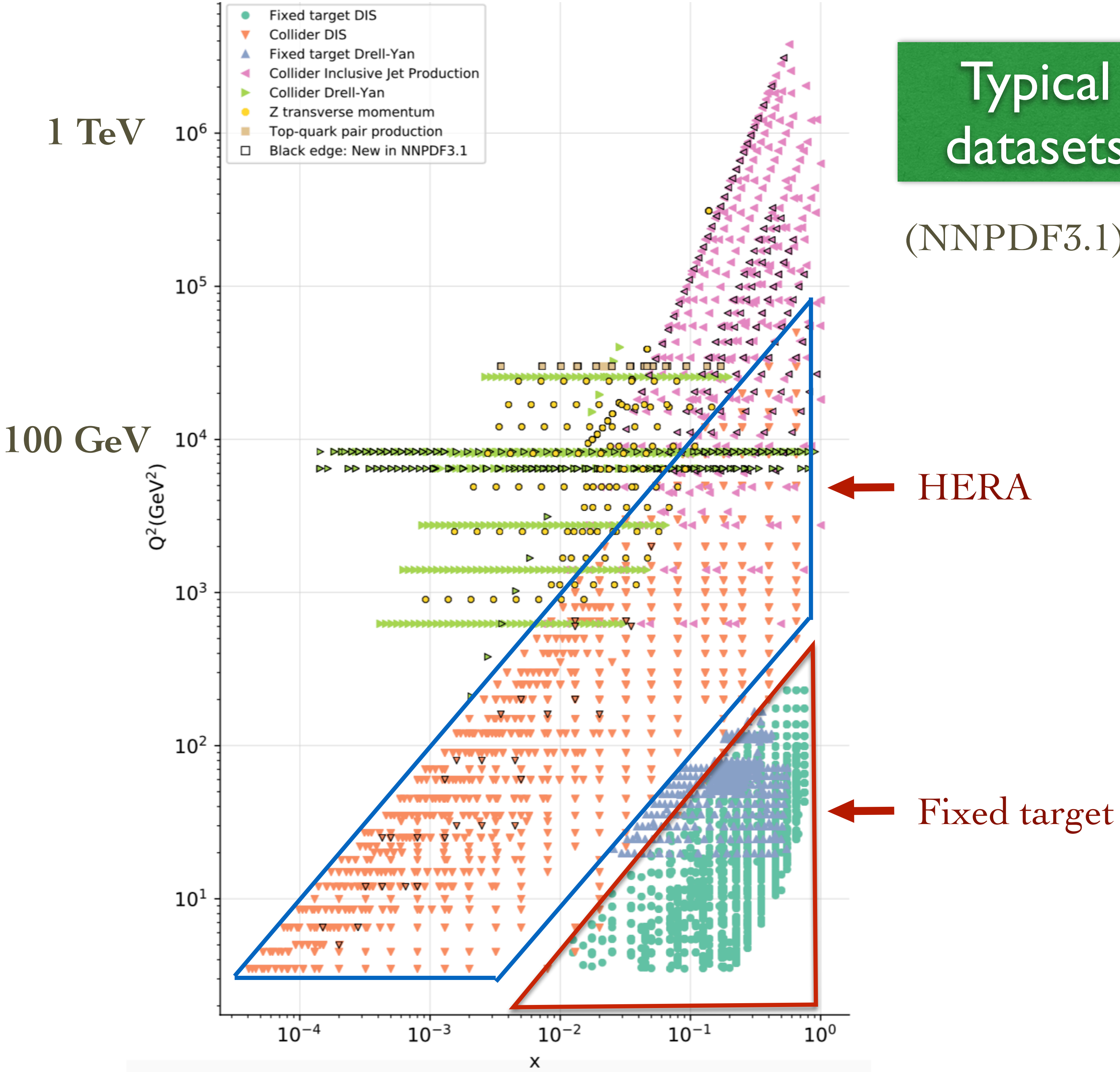
No need to rely on standard error propagation

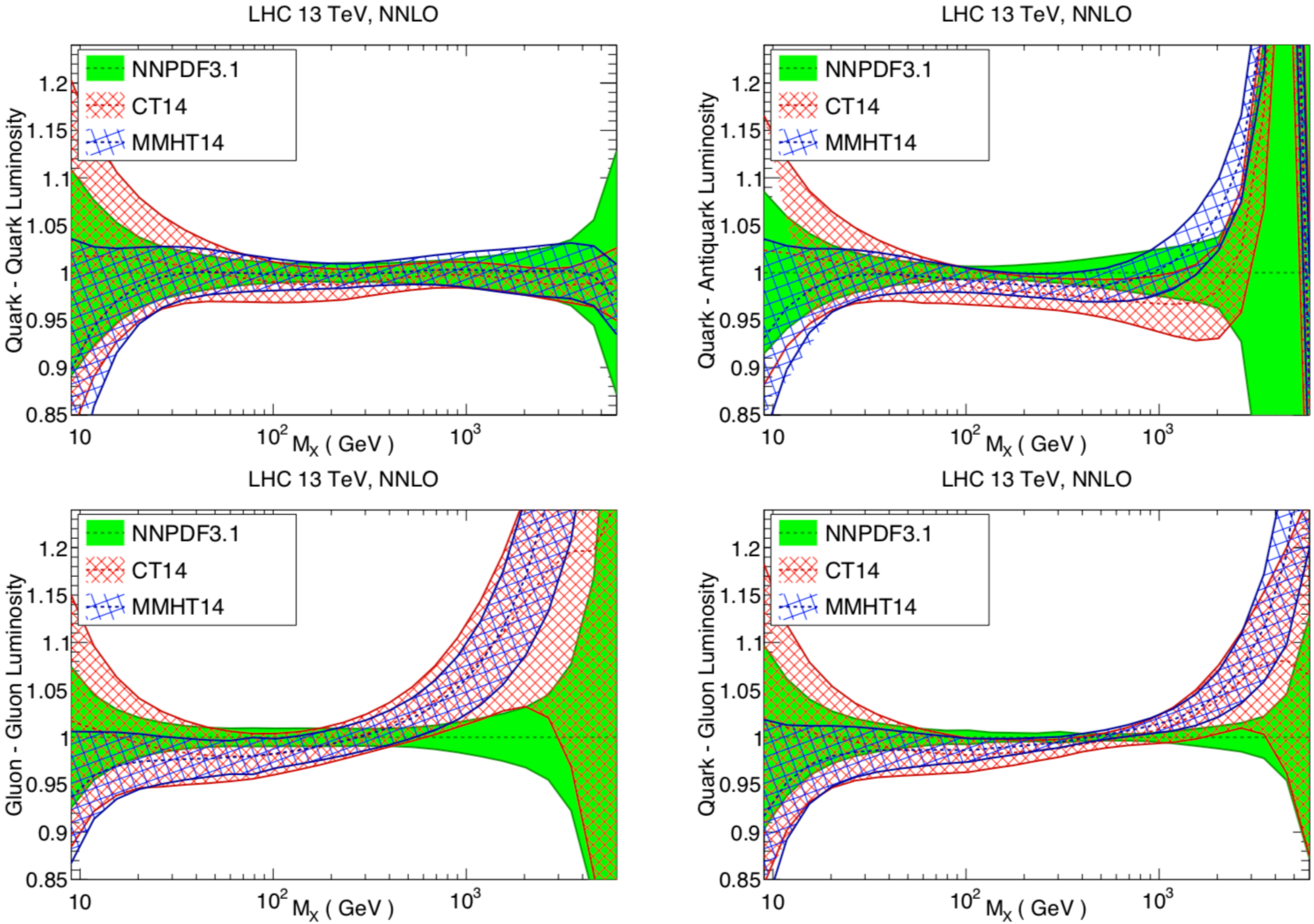
More realistic error estimate

Most recent efforts devoted to understand theory uncertainties

Typical datasets

(NNPDF3.1)



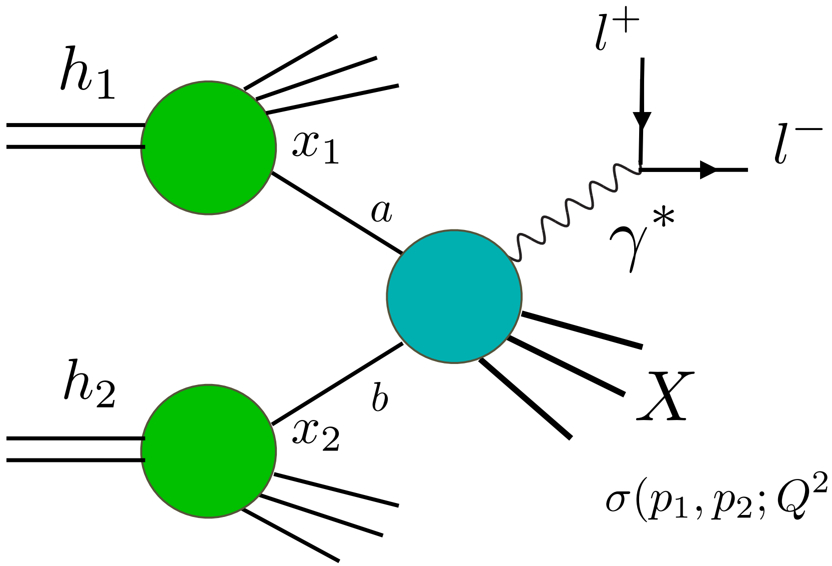


Good consistency in the well constrained region 50-500 GeV

The Drell-Yan process

The Drell-Yan mechanism was historically the first process where parton model ideas developed for DIS were applied to hadron collisions

Drell, Yan (1970)



$$\sigma(p_1, p_2; Q^2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{h_1,a}(x_1, \mu_F^2)\, f_{h_2,b}(x_2, \mu_F^2)$$

$$\times \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2, \alpha_S(Q^2), \mu_F^2)$$

The hard scale is given by the invariant mass Q^2 of the lepton pair

It lead to the discovery of W and Z bosons at CERN !

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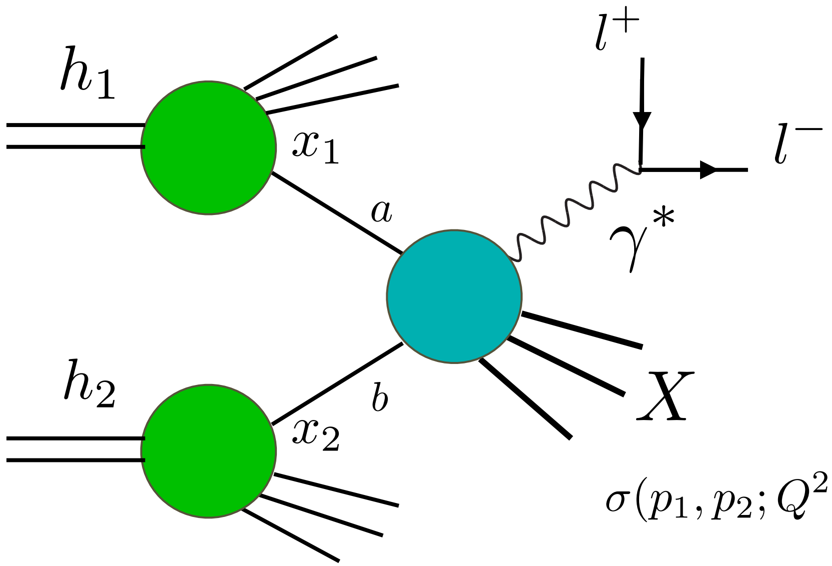


Same parton densities measured in DIS !

$$\sigma(p_1, p_2; Q^2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{h_1,a}(x_1, \mu_F^2)\, f_{h_2,b}(x_2, \mu_F^2)$$

$$\times \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2, \alpha_S(Q^2), \mu_F^2)$$

The hard scale is given by the invariant mass Q^2 of the lepton pair

It lead to the discovery of W and Z bosons at CERN !

Kinematics

$$\sigma(q(p_1)\bar{q}(p_2) \to l^+l^-) = \frac{4}{3}\pi\frac{\alpha^2}{\hat{s}}\frac{1}{N_c}Q_q^2$$

QED limit: $\sigma = \frac{4}{3}\pi\frac{\alpha^2}{\hat{s}}$

Average over number of colours

Quark electric charge

$$\frac{d\hat{\sigma}_{q\bar{q}}}{dQ^2} = \frac{\sigma_0}{N_c}Q_q^2\,\delta(\hat{s} - Q^2)$$

$$\sigma_0 = \frac{4}{3}\pi\frac{\alpha^2}{Q^2}$$

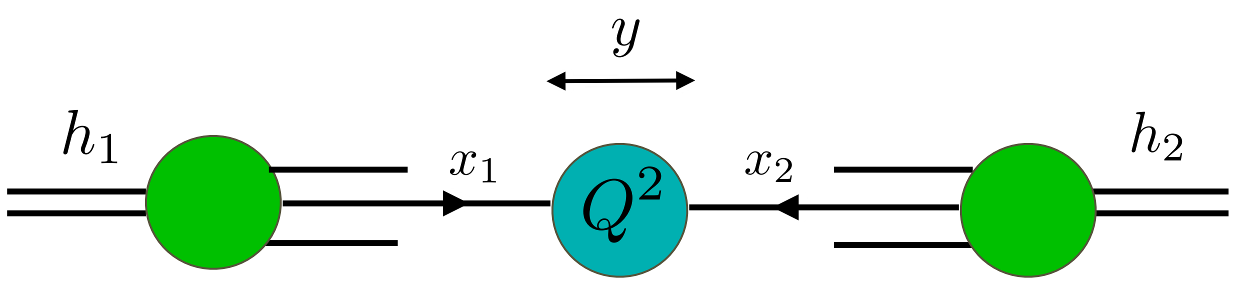
$$p_1 = \frac{\sqrt{s}}{2}(x_1, 0, 0, x_1)$$

$$p_2 = \frac{\sqrt{s}}{2}(x_2, 0, 0, -x_2)$$

$$y = \frac{1}{2}\ln\frac{E + p_z}{E - p_z} = \frac{1}{2}\ln\frac{x_1}{x_2}$$

$$x_1 = Q/\sqrt{s}\,e^{y}$$

$$x_2 = Q/\sqrt{s}\,e^{-y}$$



Scaling

In the parton model the parton distributions functions are independent of the scale

➡ by constructing an adimensional quantity the Drell-Yan cross section exhibits scaling in the variable $\tau = Q^2/s$

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{4}{3}\pi \frac{\alpha^2}{N_c}\tau \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) \sum_q Q_q^2 (f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q})) = \frac{4}{3}\pi \frac{\alpha^2}{N_c}\tau \mathcal{F}(\tau)$$

This scaling is completely analogous to the Bjorken scaling of DIS structure functions and is verified experimentally to a good approximation

Note that to test it one has to study $Q^4 \frac{d\sigma}{dQ^2}$ at fixed τ

$$\frac{d^2\sigma}{dQ^2 dy} = \frac{\sigma_0}{N_c\, s}\left[\sum_q Q_q^2 (f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q}))\right] \qquad \begin{aligned} x_1 &= \sqrt{\tau}\exp(y) \\ x_2 &= \sqrt{\tau}\exp(-y) \end{aligned}$$

The parton model neglects parton transverse momenta

→ **Lepton pair has zero transverse momentum in LO QCD**

Assume:

$$dx\, f(x) \to dk_T^2\, dx\, P(\mathbf{k_T}, x) \qquad \text{with} \qquad \int d^2 k_T P(\mathbf{k_T}, x) = f(x)$$

Consider a simple model in which: $P(\mathbf{k_T}, x) = h(\mathbf{k_T}) f(x)$

$$\frac{1}{\sigma}\frac{d^2\sigma}{d^2 p_T} = \int d^2 k_{T1} d^2 k_{T2}\, \delta^{(2)}(\mathbf{k_{T1}} + \mathbf{k_{T2}} - \mathbf{p_T}) h(\mathbf{k_{T1}}) h(\mathbf{k_{T2}})$$

Assuming a Gaussian distribution

$$h(\mathbf{k_T}) = \frac{b}{\pi}\exp(-b\,k_T^2)$$

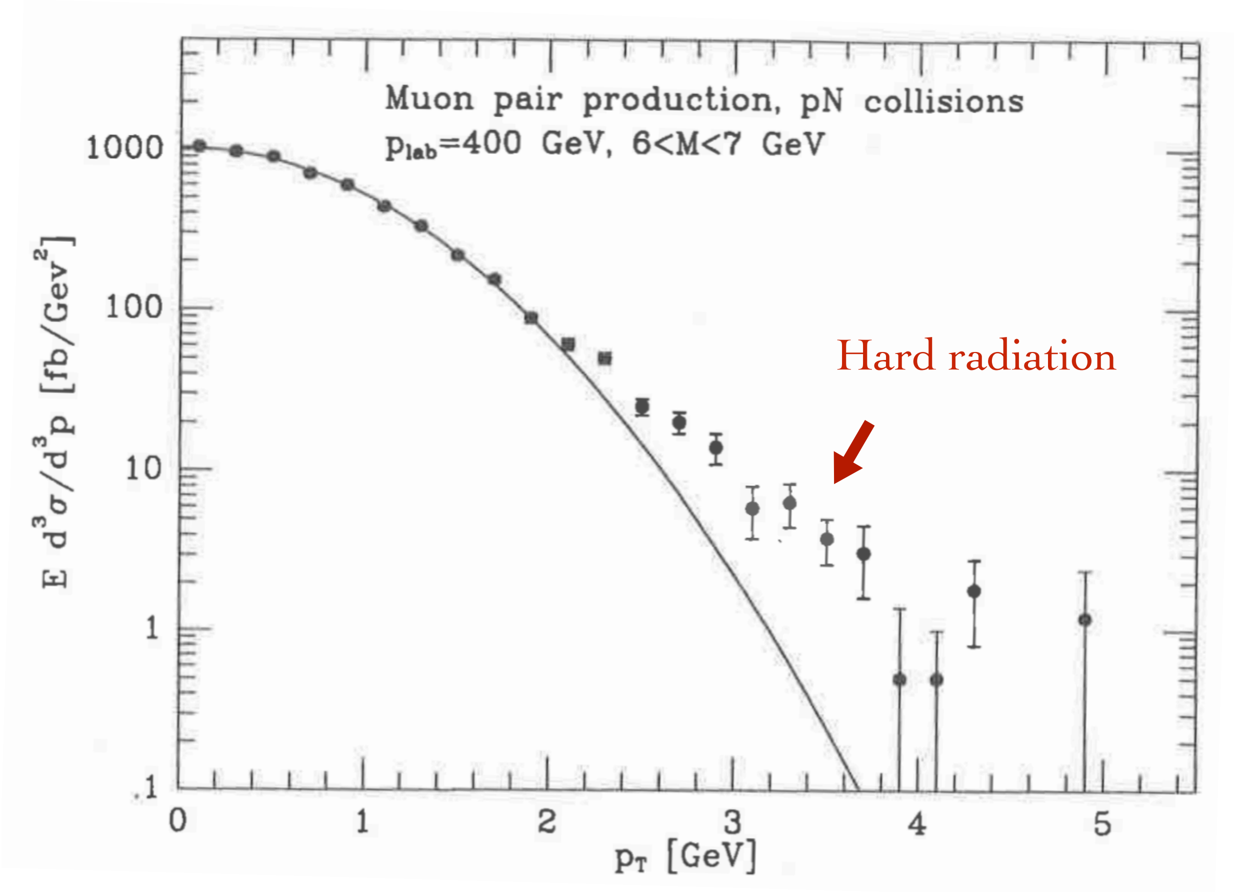
Dilepton spectrum from the CFS collaboration (1981)

the data correspond to

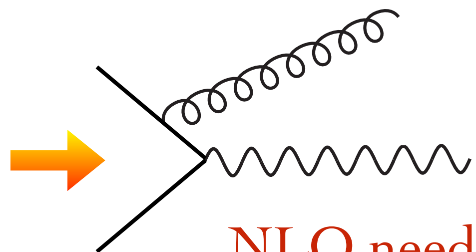
$$\langle k_T \rangle = \sqrt{\pi/4b} \sim 760\,\mathrm{MeV}$$

indeed of the order of the typical hadronic mass scale !

Historically the relative abundance of Drell-Yan lepton pairs with large transverse momenta provided one of the evidences that the parton model was incomplete



Transverse momentum is not generated only by "intrinsic" motion of the quarks in the hadrons but also by hard gluon radiation

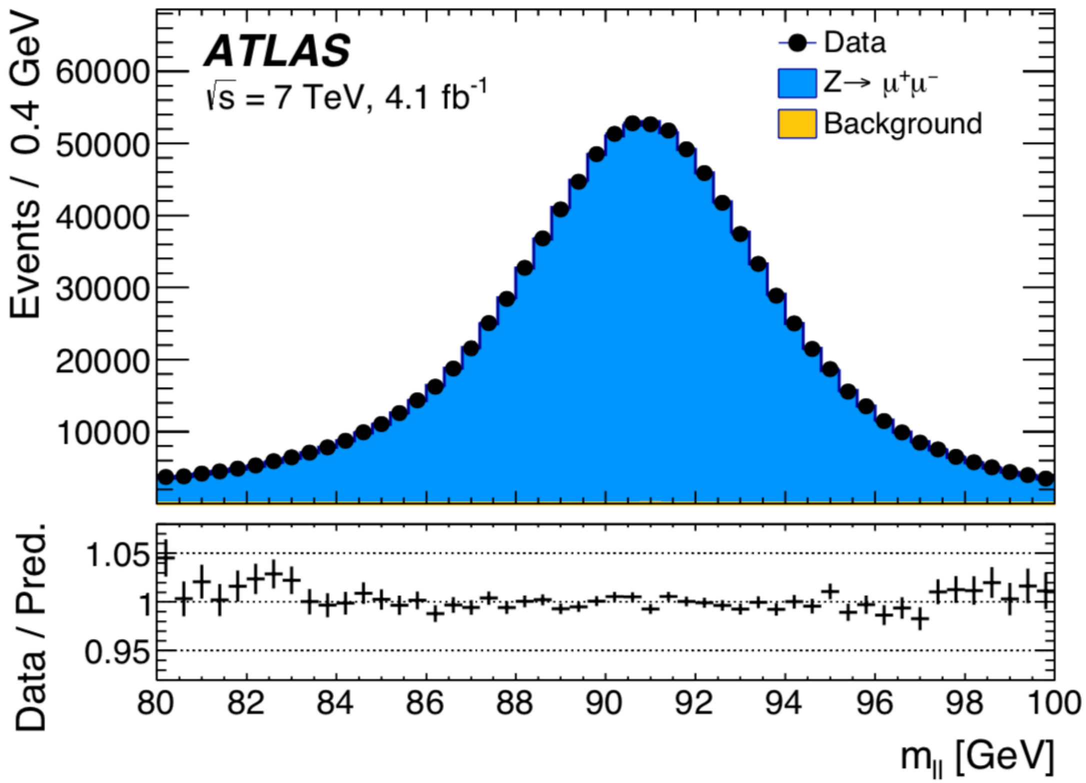


NLO needed

Z production

At higher energies the photon contribution must be supplemented with Z exchange

In practice lepton pair production around m_Z
is often analyzed using the narrow width approximation



$$\frac{1}{(\hat{s}-m_Z)^2+m_Z^2\Gamma_Z^2} \sim \frac{\pi}{m_Z\Gamma_Z}\delta(\hat{s}-m_Z^2)$$

The normalization is fixed by the condition that the two distributions have the same integral

W production

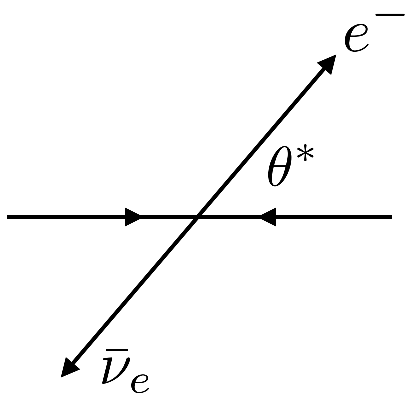
Since in the W→ lν decay the neutrino momentum is not reconstructed the W invariant mass cannot be measured → m_W measurement more difficult

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta^*} = \frac{3}{8}(1+\cos^2\theta^*)$$

angular distribution of the charged lepton in the W rest frame

The transverse momentum of the charged lepton carries information on m_W

At LO, however the W has zero transverse momentum



$$\cos\theta^* = \left(1 - \frac{4p_{Te}^2}{m_W^2}\right)^{1/2}$$

→ $$\frac{1}{\sigma}\frac{d\sigma}{dp_{Te}^2} = \frac{3}{m_W^2}\left(1 - \frac{4p_{Te}^2}{m_W^2}\right)^{-1/2}\left(1 - \frac{2p_{Te}^2}{m_W^2}\right)$$

strong peak at $p_{Te} = m_W/2$ (Jacobian peak)

In practice the peak is smeared by finite-width effects and QCD radiation

W production: transverse mass

Define now $m_T = \sqrt{2p_T^l p_T^{\text{miss}}(1-\cos\phi)}$

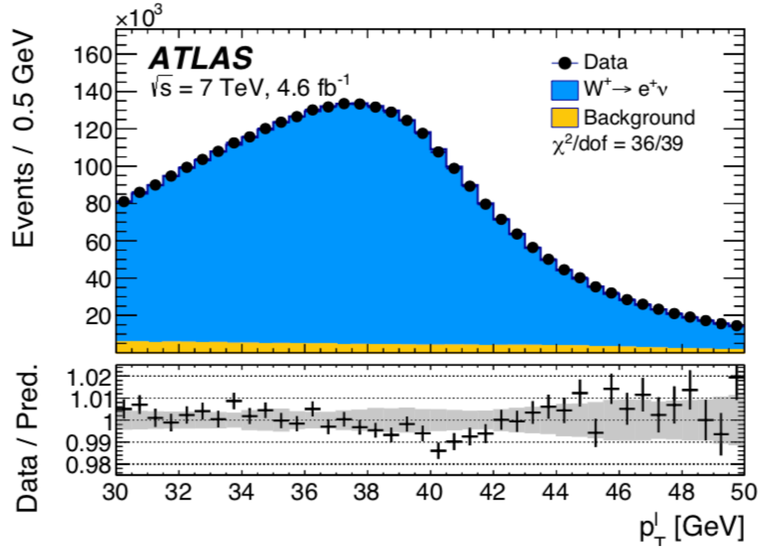
azimuthal angle between electron and neutrino momenta

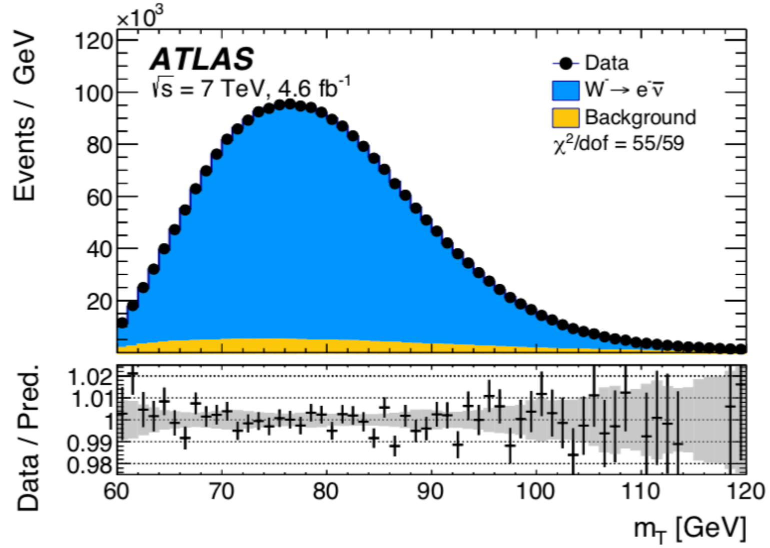
At LO $\phi = \pi$ and $p_{Te} = p_T^{\text{miss}}$

imply $m_T = 2p_{Te}$

➡ The transverse mass distribution has also a jacobian peak at $m_T = m_W$

The advantage of the transverse mass is that it is less sensitive to the W transverse momentum with respect to the electron p_T

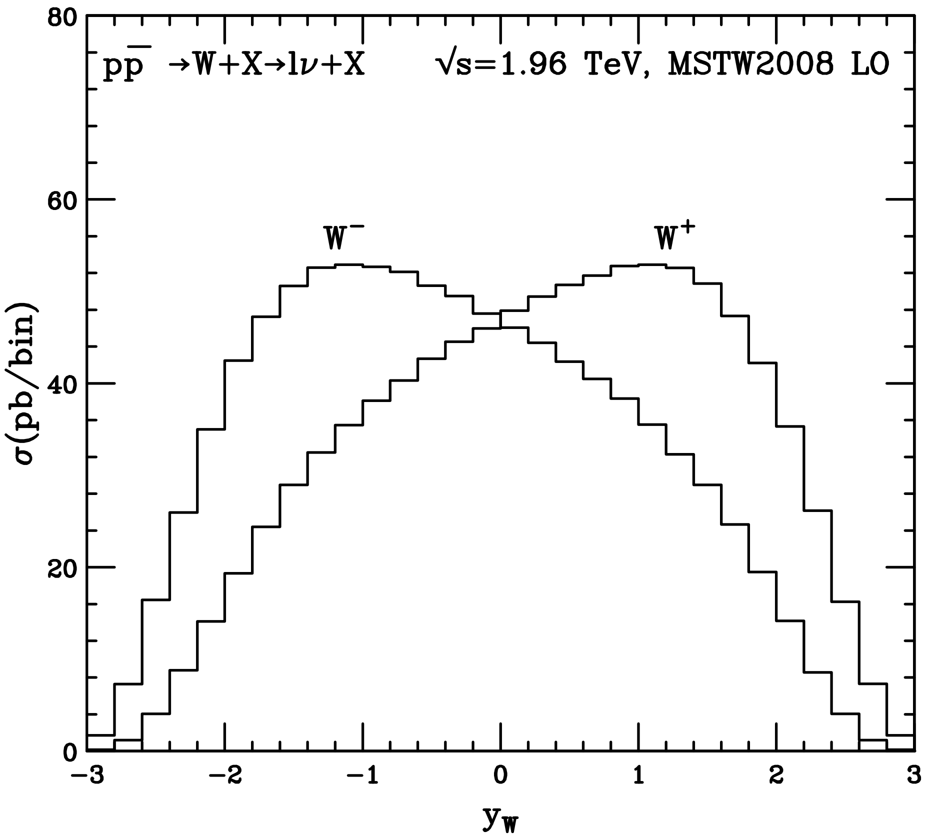




NB: If p_T^W is small $p_{Te,\nu} = \pm p + p_T^W/2$ leave the transverse mass invariant to first order

W charge asymmetry

An important observable in W hadroproduction is the asymmetry in the rapidity distributions of the W bosons



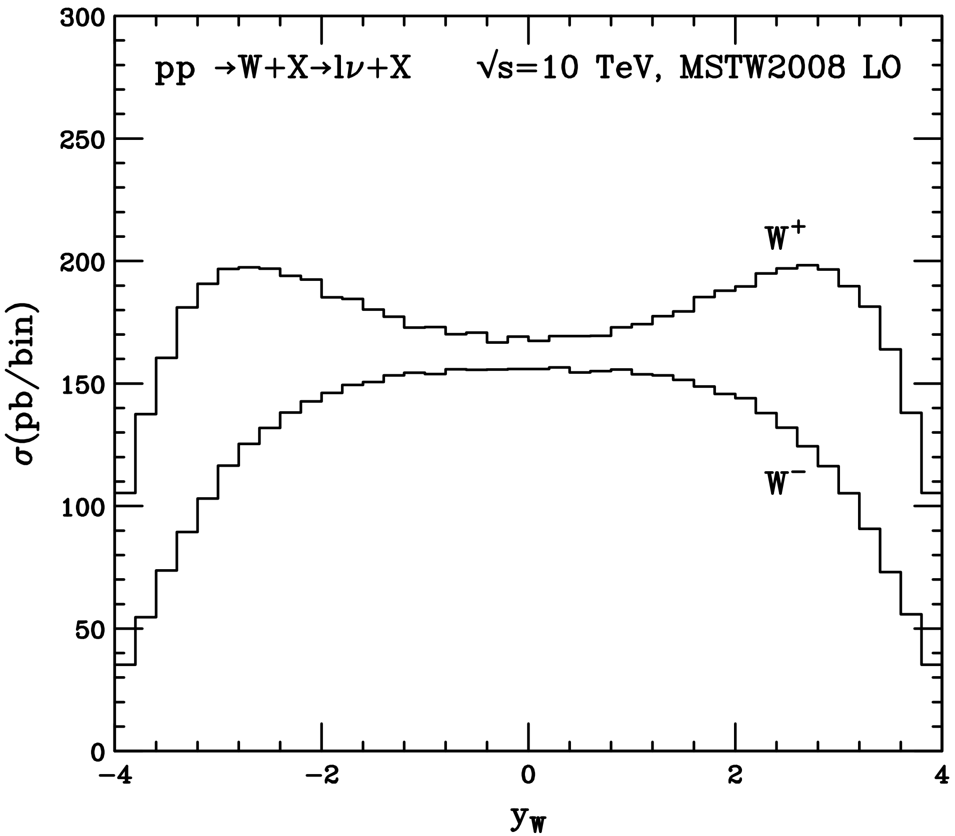
$$A(y_W) = \frac{\frac{d\sigma(W^+)}{dy_W} - \frac{d\sigma(W^-)}{dy_W}}{\frac{d\sigma(W^+)}{dy_W} + \frac{d\sigma(W^-)}{dy_W}}$$

In $p\bar{p}$ collisions the W^+ and W^- are produced with equal rates but W^+ (W^-) is produced mainly in the proton (antiproton) direction

These asymmetries are mainly due to the fact that, on average, the u quark carries more proton momentum fraction than the d quark

W charge asymmetry

An important observable in W hadroproduction is the asymmetry in the rapidity distributions of the W bosons



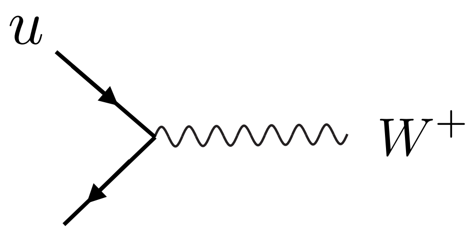
$$A(y_W) = \frac{\frac{d\sigma(W^+)}{dy_W} - \frac{d\sigma(W^-)}{dy_W}}{\frac{d\sigma(W^+)}{dy_W} + \frac{d\sigma(W^-)}{dy_W}}$$

In pp collisions the W^+ and W^- are produced with different rates but W^+ and W^- rapidity distributions are forward-backward symmetric W^- distribution is central, whereas W^+ is produced at larger rapidities

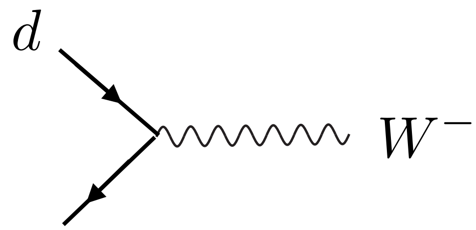
These asymmetries are mainly due to the fact that, on average, the u quark carries more proton momentum fraction than the d quark

W charge asymmetry

In $p\bar{p}$ collisions:



$\bar{d} = d$ in the proton



$\bar{u} = u$ in the proton

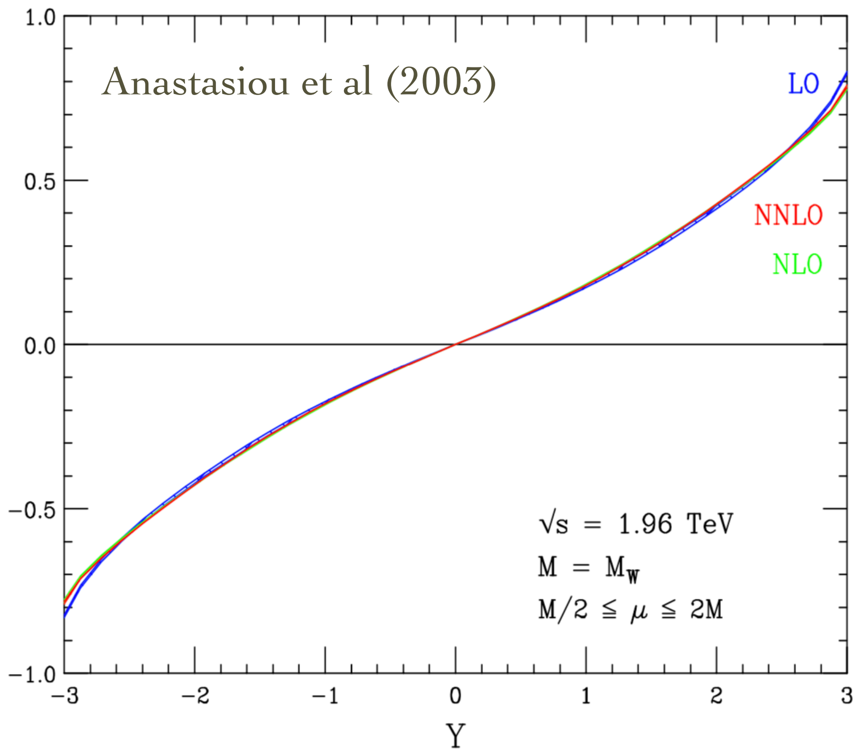
If u in the proton is faster than d

➡ W^+ (W^-) produced mainly in p ($\bar{p}$) direction

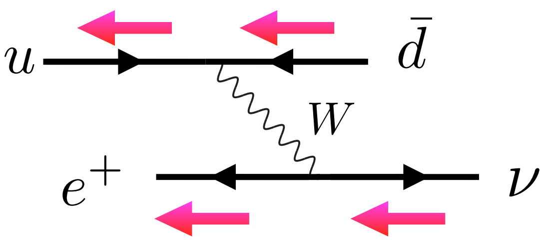
The W asymmetry $$A(y) = \frac{\frac{d\sigma(W^+)}{dy} - \frac{d\sigma(W^-)}{dy}}{\frac{d\sigma(W^+)}{dy} + \frac{d\sigma(W^-)}{dy}}$$

is a measure of $$\frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)}$$

➡ probes the relative shape of u and d quarks



In practice W→lν → measure the charged lepton asymmetry



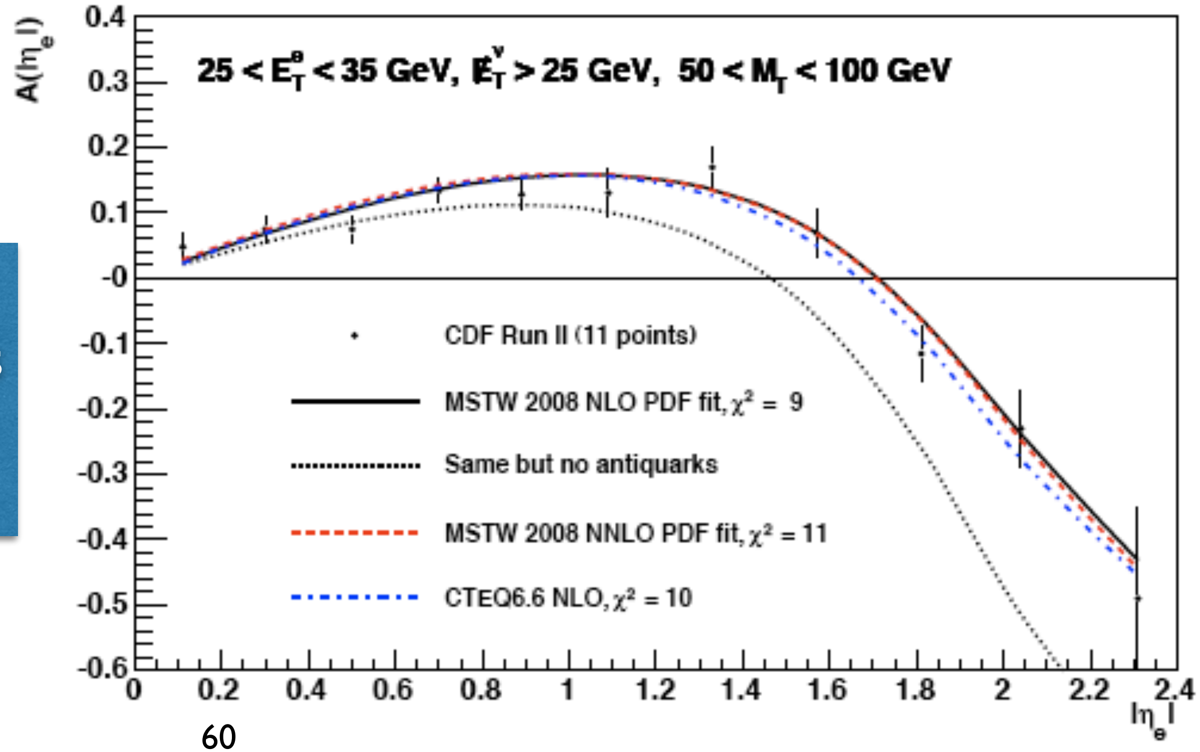
Angular momentum conservation: the e⁺ is mainly produced in the direction of the antiquark

Scattering angle in the W rest frame

$$\frac{1}{\hat{\sigma}^{(0)}_{U\overline{D}}}\frac{d\hat{\sigma}^{(0)}_{U\overline{D}}}{d\cos\theta^*_{lD}} = \frac{1}{\hat{\sigma}^{(0)}_{D\overline{U}}}\frac{d\hat{\sigma}^{(0)}_{D\overline{U}}}{d\cos\theta^*_{lD}} = \frac{3}{8}(1+\cos\theta^*_{lD})^2$$

In the case of pp̄ collisions the W boson tends to follow the colliding up quark

The V-A decay acts in the opposite direction and tends to dilute the effect in the lepton asymmetry



Jets

It is common to discuss QCD at high-energy in terms of partons

But quarks and gluons are never really visible since, immediately after being produced they fragment and hadronize

A jet is a collimated spray of energetic hadrons and is one of the most typical manifestation of QCD at high energy

By measuring its energy and direction one can get a handle on the the original parton

How to define a jet ? A proper jet definition requires:

- a jet algorithm
- a recombination scheme

Jet algorithm: a set of rules for grouping particles into jets usually involves a set of parameters that specify how close two particles must be to belong to the same jet

Recombination scheme: indicates what momentum must be assigned to the combination of two particles (the simplest is the sum of the 4-momenta)

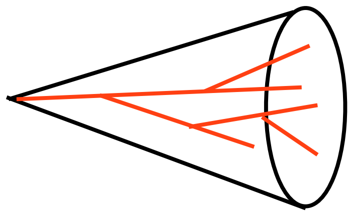
There are two broad categories of jet algorithms:

1) cone algorithms

2) sequential recombination algorithms

1) cone algorithms

they are based on a “top-bottom” approach: rely on the idea that QCD branching and hadronization do not change the energy flow



G.Sterman, S.Weinberg (1977)

2) sequential recombination algorithms

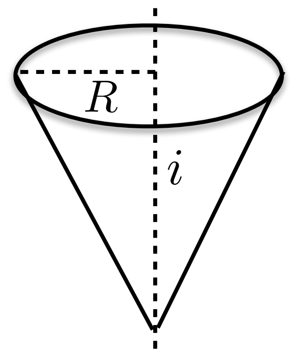
they are based on a “bottom-up” approach: repeatedly recombine the closest pair of particles according to some distance measure

Cone algorithms

The cone algorithms used in practice are "iterative cones" (IC) and were mostly used at the Tevatron

A seed particle i sets some initial direction, then one draws a circle around the seed of radius R in rapidity (or pseudorapidity) and azimuth, taking all j such that

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 < R^2$$



The direction of the resulting sum is then taken as a new seed and the procedure is iterated until a stable cone is found

Questions:

- How to choose the seeds ?
- What should be done when cones obtained by iterating two different seeds share some particles ?

Overlapping cones:

- First solution: *progressive removal approach*

 (often referred to as UA1-type cone algorithms)

 - Start from the particle with the largest transverse momentum
 - Once a stable cone is found, call it a jet
 - Remove all the particles contained in the cone
 - Iterate

The use of the hardest particle as seed make these algorithms *collinear* unsafe

- Second solution: *split-merge approach*

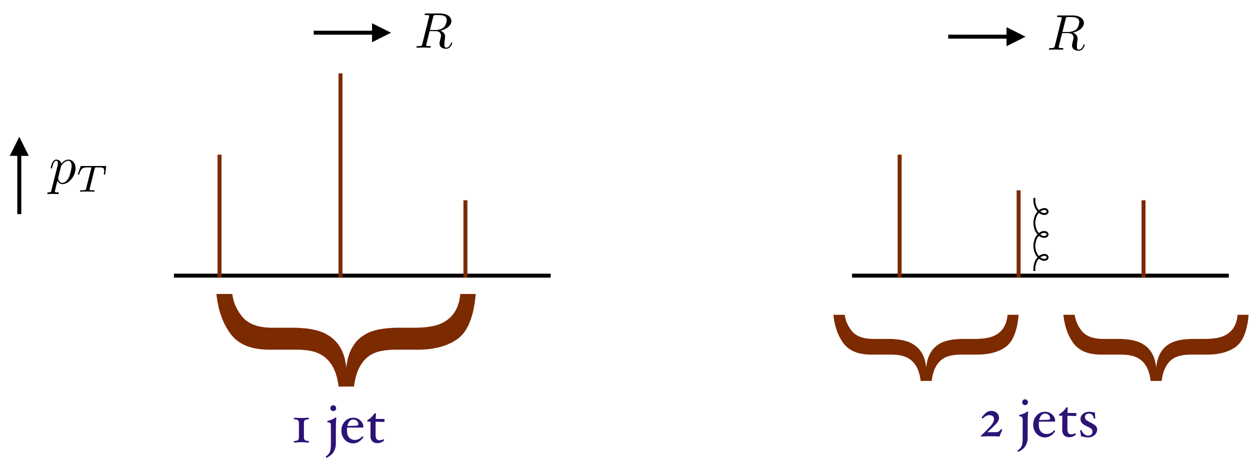
 - Find all the stable cones (protojets) starting from ALL the particles as seeds (often a threshold in p_T is assumed)

 - Run a split-merge procedure to merge a pair of cones if more than a fraction f of the softer cone's transverse momentum is shared by the harder cone

The use of seeds make these algorithms *infrared* unsafe

Infrared and collinear safety

Iterative cone algorithms with progressive removal are collinear unsafe



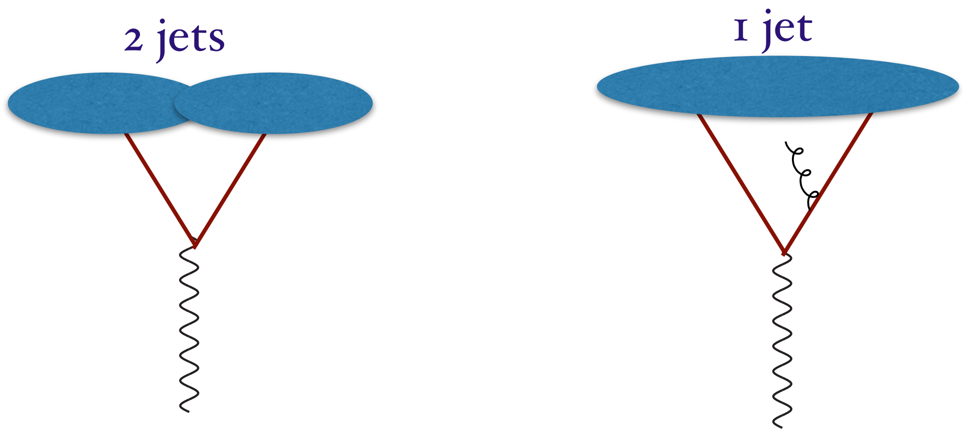
In the first configuration the hardest parton is the central one and if the cone is large enough we get one jet

In the second configuration the central quark has split in a collinear qg pair

➡ The number of jets should be insensitive to such a collinear splitting but now the hardest parton is the left one and we get two jets

In practice detectors provide a regularization to the collinear unsafety, but how this happens depends on the detector details and a jet cross section should be independent on them

Iterative cone algorithms with split-merge are infrared unsafe



a) In an event with 2 hard partons both acts as seeds and give a two jet configuration

b) A soft gluon acts as a seed and may give a new stable cone: a one jet configuration is found after the split-merge procedure

→ **The algorithm is infrared unsafe and the jet cross section is divergent !**

Midcone fix: search for additional stable cones by iterating from midpoints

Presented as IR safe and widely used in Run II at the Tevatron but still unsafe for three hard patron configurations

Solution: find all stable cones through some exact procedure → SIScone

Slow when number of particles to be clustered is large G.Salam, G.Soyez (2007)

Sequential algorithms

Sequential recombination algorithms find their roots in e^+e^- experiments

- Much simpler to state than cone algorithms
- Go beyond just finding the jets: they assign a sequence to the clustering procedure that is somewhat connected to the branching at parton level

Examples:

- Jade algorithm
- k_T algorithm
- Cambridge-Achen algorithm
- anti-k_T

Jade

The first sequential recombination algorithm was introduced by the JADE collaboration in the 80's

1. For each pair ij compute the distance:

$$y_{ij} = \frac{2E_iE_j(1-\cos\theta_{ij})}{Q^2}$$

Q total energy

2. Find the minimum y_{min} of all y_{ij}

3. If y_{min} is below a threshold y_{cut} recombine i and j in a single particle (pseudojet) and go back to 1.

4. If not declare all remaining particles as jets and terminate

Jade

It depends on a single parameter y_{cut}: reducing y_{cut} resolves more jets

We may define the variable $y_{n(n+1)}$ as the value of y_{cut} at which a n jet event becomes n+1-jet like

The JADE algorithm is infrared and collinear safe: soft and collinear splitting give very small y_{ij} and thus are recombined first

However the presence of E_iE_j in the distance let two soft particles moving in opposite directions to be recombined in the same jet

→ This is against physical intuition !
We expect a jet to be limited in angular reach

Another consequence is a complication in higher order logarithmic contributions to y_{23} that cannot be resummed to all orders

The k_T algorithm in e^+e^- collisions

S.Catani et al. (1991)

The k_T algorithm in e^+e^+ collisions is identical to the JADE algorithm except for the distance measure, which is

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{Q^2}$$

In the collinear limit $\theta_{ij} \ll 1$ and the numerator becomes $(\min(E_i, E_j)\theta_{ij})^2$

It's nothing but the squared transverse momentum of i relative to j (i being the softer particle) → that's why it is called k_T algorithm

In this way the distance between two soft and back to back particles is larger than that between a soft particle and a hard one close in angle

The clustering sequence retains useful approximate information of the QCD branching process

The k_T algorithm in hadron collisions

In hadronic collisions there are two difficulties to face:

S.Catani et al. (1993)
S.D.Ellis and D.Soper (1993)

- The total energy Q is not defined
- besides the divergences involving outgoing particles, there are divergences between final state and *incoming* particles

Inclusive k_T algorithm:

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$$d_{ij} = \min(p_{Ti}^2, p_{Tj}^2) \frac{\Delta R_{ij}^2}{D^2} \qquad d_{iB} = p_{Ti}^2$$

1. Compute all the distances d_{ij} and d_{iB}
2. Find the minimum.
3. If it is a d_{ij} recombine i and j and return to 1.
4. If it is a d_{iB} declare i to be a final state jet, remove it and return to 1.

The parameter D determines what it is called a jet:
Suppose i has no particles at a distance smaller than D:

→ d_{ij} will be larger than d_{iB} for any j harder than i

Arbitrarily soft particles can become jets in their own

→ A minimum transverse momentum for jets should be specified

The k_T algorithm has been advocated by theorists because of its good properties

Experimentalists have questioned the use of the algorithm because of its speed limit (the clustering time for N particles naively increases as N^3) and because it tends to produce rather irregular jets

The issue of speed is crucial in high-multiplicity environments like LHC or heavy-ion collisions

→ The algorithm has been reformulated by using techniques borrowed from computational geometry: in this way it scales as N lnN

M.Cacciari, G.Salam (2006)

The Cambridge/Achen algorithm

It works like the inclusive k_T algorithm but using ΔR_{ij} as distance measure

It works by recombining the pair of particles with smallest ΔR_{ij} and repeating the procedure until all the clusters are separated by $\Delta R_{ij} > R$

The final objects are called jets

The clustering hierarchy is in angle rather than in transverse momentum

➡ makes possible to look at the jet at different angular resolutions

G.Salam et al. (2008)

Important for “filtering”, “trimming” and “pruning” techniques

Like the k_T algorithm it tends to produce rather irregular jets

The anti-k_T algorithm

M.Cacciari, G.Salam, G.Soyez (2008)

Define a family of algorithms each characterized by an integer p

$$d_{ij} = \min(p_{Ti}^{2p}, p_{Tj}^{2p}) \frac{\Delta R_{ij}^2}{D^2} \qquad d_{iB} = p_{Ti}^{2p}$$

- p=1 k_T algorithm
- p=0 Cambridge-Aachen

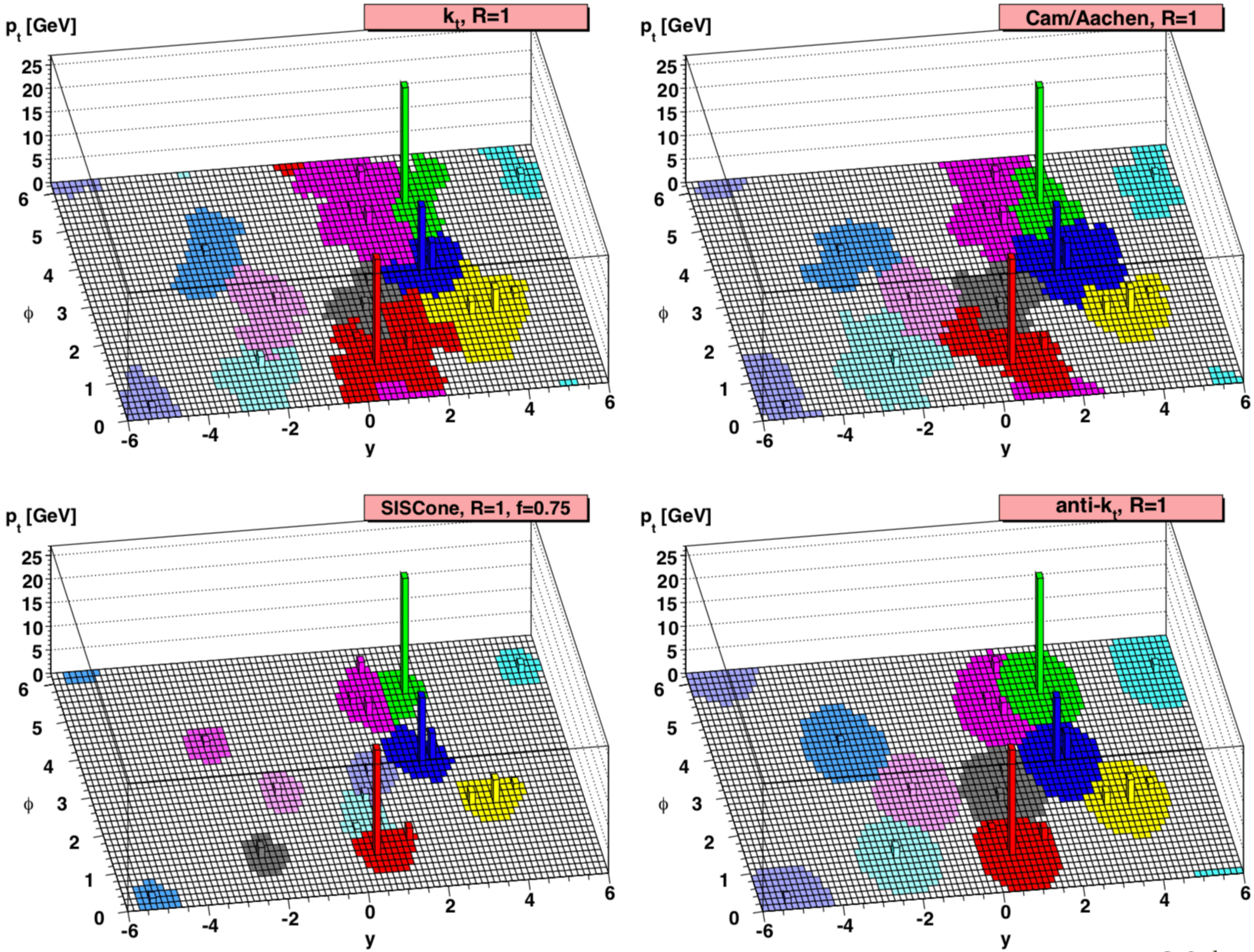
What about p=-1 ? It seems a rather odd choice but...

Soft particles tend to cluster with hard ones long before they cluster among themselves

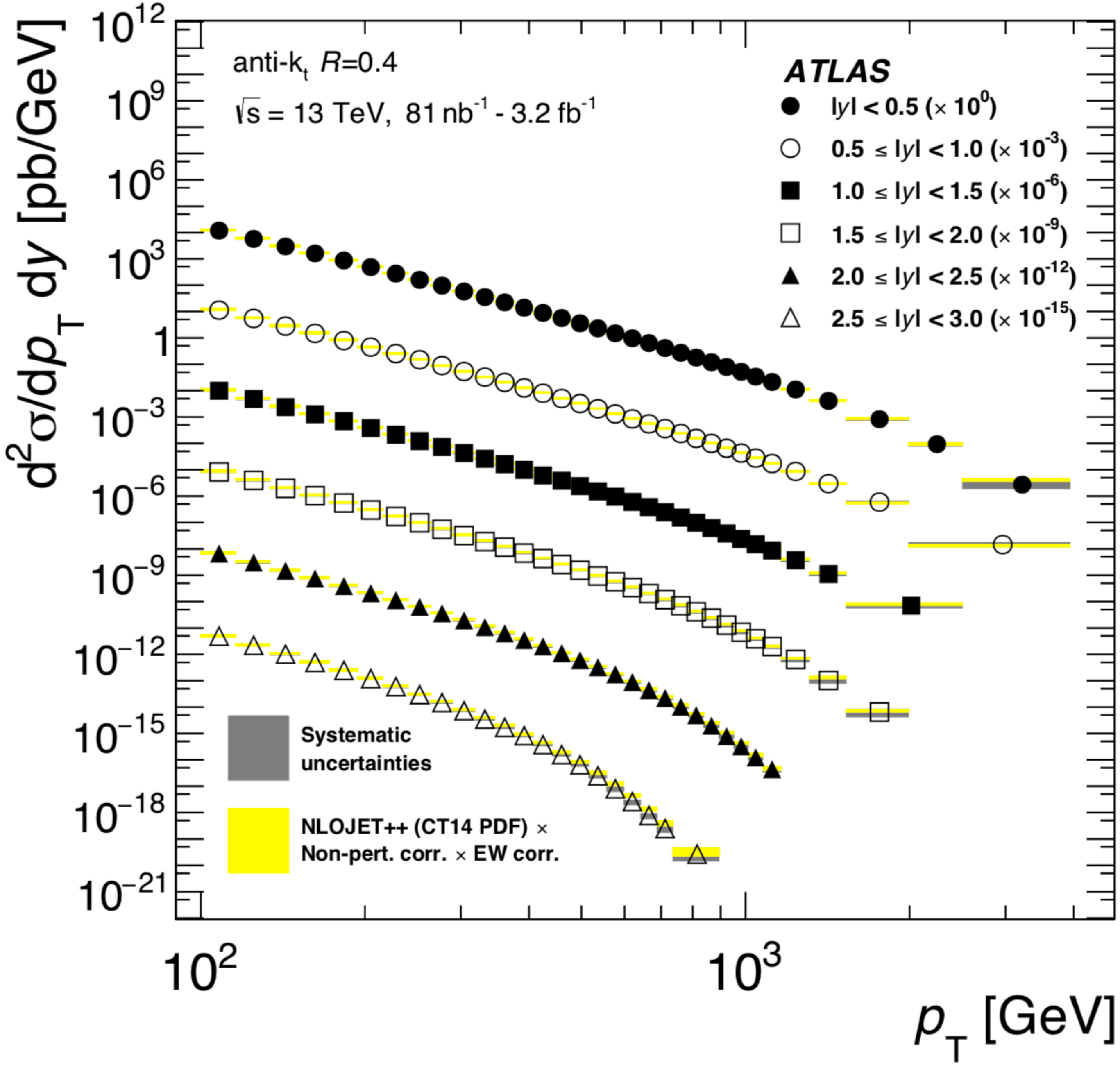
It produces regular (circular) jets

A sequential recombination algorithm is the perfect cone algorithm !

→ Now the default for ATLAS and CMS experiments



G.Salam (2009)



Inclusive jet cross section