Higgs Physics - Theory
Lecture I

The Higgs boson as predicted by the Standard Model of electroweak interactions

Laura Reina

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LHC Higgs-boson physics is as important as ever!

**Discovery**
(2012)

**After Run 2**
(2019)

Much improved statistics: main production and decay modes observed.

→ **Access to Higgs couplings:** Higgs portal to new physics!
Outline of these lectures

• **Lecture 1: the Standard-Model Higgs boson.**
  → EW gauge symmetry, Higgs mechanism.
  → Higgs-boson interactions.
  → Quantum constraints.

• **Lecture 2: Higgs-boson physics at the LHC.**
  → Production and decay modes, what do they probe.
  → Theoretical predictions and their accuracy.

• **Lecture 3: from Higgs-boson properties to new physics.**
  → Probing specific extensions of the SM.
  → Probing classes of interactions within SM boundaries.
The Standard Model of particle physics

“The Standard Model is a gauge invariant quantum field theory based on the local symmetry group \( SU(3) \times SU(2) \times U(1) \).”

\[
SU(3) \rightarrow \text{strong force } (g) \\
SU(2)_L \times U(1)_Y \rightarrow \text{electroweak force } (W_{1,2,3}, B_Y) \\
(Y = T^3 - Q)
\]

particle multiplets:

\[
\begin{pmatrix}
    \nu_e \\
    e_L
\end{pmatrix}, \begin{pmatrix}
    u \\
    d_L
\end{pmatrix} \leftrightarrow \begin{pmatrix}
    u \\
    u \\
    u \\
    d \\
    d \\
    d
\end{pmatrix}
\]

\[
SU(2) \leftarrow SU(3)
\]

with some caveats:

\( \leftrightarrow \) Masses of \( Z \) and \( W \) bosons breaks gauge invariance \( \leftrightarrow \) EWSB
\( \leftrightarrow \) Fermion masses breaks gauge invariance as well.
The Higgs discovery has constrained the mechanism of EWSB.

The **EW symmetry** is spontaneously broken (SSB) to $U(1)_Q$

$$SU(2)_L \times U(1)_Y \xrightarrow{\text{SSB}} U(1)_Q \quad \begin{cases} W^\pm, Z^0 & M_W, M_Z \neq 0 \\ \gamma & m_\gamma = 0 \end{cases}$$

After which, fermions get mass through Yukawa-type interactions.
The SM Lagrangian on a mug . . .

\[ \mathcal{L}_{SM} = \mathcal{L}_{QCD} + \mathcal{L}_{EW} \]

\[ \mathcal{L}_{QCD} \rightarrow \text{M. Grazzini’s lectures} \]

We will focus on:

\[ \mathcal{L}_{EW} = \mathcal{L}^{\text{gauge}}_{EW} + \mathcal{L}^{\text{ferm}}_{EW} + \mathcal{L}^{\text{Yukawa}}_{EW} + \mathcal{L}^{\text{scalar}}_{EW} \]

\[ \mathcal{L}^{\text{gauge}}_{EW} \rightarrow 1^{\text{st}} \text{ line} \]

\[ \mathcal{L}^{\text{ferm}}_{EW} \rightarrow 2^{\text{nd}} \text{ line} \]

and in particular:

\[ \mathcal{L}^{\text{Yukawa}}_{EW} \rightarrow 3^{\text{rd}} \text{ line} \]

\[ \mathcal{L}^{\text{scalar}}_{EW} \rightarrow 4^{\text{th}} \text{ line} \]

Very simple and very \textit{complete} \rightarrow contains all kinds of \( d = 4 \) renormalizable interactions between scalar, fermion, and vector fields.
From Global to Local: gauging a symmetry

Abelian case (→ QED)

A theory of free Fermi fields described by the Lagrangian density

$$\mathcal{L} = \bar{\psi}(x)(i\partial \psi - m)\psi(x)$$

is invariant under a global $U(1)$ transformation ($\alpha=$constant phase)

$$\psi(x) \rightarrow e^{i\alpha} \psi(x) \text{ such that } \partial_\mu \psi(x) \rightarrow e^{i\alpha} \partial_\mu \psi(x)$$

The same is not true for a local $U(1)$ transformation ($\alpha = \alpha(x)$) since

$$\psi(x) \rightarrow e^{i\alpha(x)} \psi(x) \text{ but } \partial_\mu \psi(x) \rightarrow e^{i\alpha(x)} \partial_\mu \psi(x) + ige^{i\alpha(x)} \partial_\mu \alpha(x) \psi(x)$$

Need to introduce a covariant derivative $D_\mu$ such that

$$D_\mu \psi(x) \rightarrow e^{i\alpha(x)} D_\mu \psi(x)$$
Only possibility: introduce a vector field $A_\mu(x)$ transforming as

$$A_\mu(x) \rightarrow A_\mu(x) - \frac{1}{g} \partial_\mu \alpha(x)$$

and define a covariant derivative $D_\mu$ according to

$$D_\mu = \partial_\mu + igA_\mu(x)$$

modifying $\mathcal{L}$ to accommodate $D_\mu$ and the gauge field $A_\mu(x)$ as

$$\mathcal{L} = \bar{\psi}(x)(i\slashed{D} - m)\psi(x) - \frac{1}{4} F^{\mu\nu}(x)F_{\mu\nu}(x)$$

where the last term is the Maxwell Lagrangian for a vector field $A^\mu$, i.e.

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) .$$

Requiring invariance under a local $U(1)$ symmetry has:

$\rightarrow$ promoted a free theory of fermions to an interacting one;

$\rightarrow$ fixed the form of the interaction in terms of a new vector field $A^\mu(x)$:

$$\mathcal{L}_{int} = -g \bar{\psi}(x)\gamma_\mu \psi(x)A^\mu(x)$$

$\rightarrow$ no mass term $A^\mu A_\mu$ allowed by the symmetry $\rightarrow$ this is QED.
Non-abelian case: Yang-Mills theories

Consider the same Lagrangian density

\[ \mathcal{L} = \bar{\psi}(x)(i\dot{\psi} - m)\psi(x) \]

where \( \psi(x) \rightarrow \psi_i(x) \) \((i = 1, \ldots, n)\) is a \(n\)-dimensional representation of a non-abelian compact Lie group (e.g. \(SU(N)\)).

\( \mathcal{L} \) is invariant under the global transformation \( U(\alpha) \)

\[ \psi(x) \rightarrow \psi'(x) = U(\alpha)\psi(x) , \quad U(\alpha) = e^{i\alpha^a T^a} = 1 + i\alpha^a T^a + O(\alpha^2) \]

where \( T^a \) \((a = 1, \ldots, d_{adj})\) are the generators of the group infinitesimal transformations with algebra,

\[ [T^a, T^b] = i f^{abc} T^c \]

and the corresponding Noether’s current are conserved. However, requiring \( \mathcal{L} \) to be invariant under the corresponding local transformation \( U(x) \)

\[ U(x) = 1 + i\alpha^a(x)T^a + O(\alpha^2) \]

brings us to replace \( \partial_\mu \) by a covariant derivative

\[ D_\mu = \partial_\mu - igA_\mu^a(x)T^a \]
in terms of vector fields $A^a_\mu(x)$ that transform as

\[ A^a_\mu(x) \rightarrow A^a_\mu(x) + \frac{1}{g} \partial_\mu \alpha^a(x) + f^{abc} A^b_\mu(x) \alpha^c(x) \]

such that

\[ D_\mu \rightarrow U(x) D_\mu U^{-1}(x) \]

\[ D_\mu \psi(x) \rightarrow U(x) D_\mu U^{-1}(x) U(x) \psi = U(x) D_\mu \psi(x) \]

\[ F_{\mu\nu} \equiv \frac{i}{g} [D_\mu, D_\nu] \rightarrow U(x) F_{\mu\nu} U^{-1}(x) \]

The invariant form of $\mathcal{L}$ or Yang Mills Lagrangian will then be

\[ \mathcal{L}_{YM} = \mathcal{L}(\psi, D_\mu \psi) - \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} = \bar{\psi} (i \not{D} - m) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \]

where $F_{\mu\nu} = F_{\mu\nu}^a T^a$ and

\[ F_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \]

Notice: boxed part is lines 1+2 of the mug Lagrangian!
Also notice that:

- **as in the abelian case:**
  - mass terms $A^{a,\mu} A^a_{\mu}$ are forbidden by symmetry: gauge bosons are massless.
  - the form of the interaction between fermions and gauge bosons is fixed by symmetry to be
    \[
    \mathcal{L}_{int} = -g \bar{\psi}(x) \gamma^\mu T^a \psi(x) A^{a,\mu}(x)
    \]

- **at difference from the abelian case:**
  - gauge bosons carry a group charge and therefore ...
  - gauge bosons have self-interaction.
  - the quantization procedure can be trickier (gauge fixing, ghosts).

Can we build a massive gauge theory?
Feynman rules, Yang-Mills theory:

\[ \frac{p}{a} \rightarrow \frac{i \delta^{ab}}{\not{p} - m} \]

\[ i \gamma^\mu (T^{c})_{ij} \]

\[ \frac{k}{\mu,a} \rightarrow \frac{-i}{k^2} \left[ g_{\mu \nu} - (1 - \xi) \frac{k_{\mu} k_{\nu}}{k^2} \right] \delta^{ab} \]

\[ g f^{abc} \left( g^{\beta \gamma} (q - r)^\alpha + g^{\gamma \alpha} (r - p)^\beta + g^{\alpha \beta} (p - q)^\gamma \right) \]

\[ -i g^2 \left[ f^{abe} f^{cde} (g^{\alpha \gamma} g^{\beta \delta} - g^{\alpha \delta} g^{\beta \gamma}) + f^{ace} f^{bde} (\cdots) + f^{ade} f^{bce} (\cdots) \right] \]
Spontaneous Breaking of a Gauge Symmetry

**Higgs mechanism, abelian case:** abelian gauge theory (one vector field $A^\mu(x)$) coupled to one complex scalar field $\phi(x)$:

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi$$

where

$$\mathcal{L}_A = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} = -\frac{1}{4} (\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

and ($D^\mu = \partial^\mu + ig A^\mu$)

$$\mathcal{L}_\phi = (D^\mu \phi)^* D_\mu \phi - V(\phi) = (D^\mu \phi)^* D_\mu \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

$\mathcal{L}$ invariant under local $U(1)$ symmetry:

$$\phi(x) \rightarrow e^{i\alpha(x)} \phi(x)$$

$$A^\mu(x) \rightarrow A^\mu(x) + \frac{1}{g} \partial^\mu \alpha(x)$$

Mass term for $A^\mu$ breaks the $U(1)$ gauge invariance (same as before).
Can we build a gauge invariant massive theory? Yes.

Consider the potential of the scalar field:

\[ V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2 \]

where \( \lambda > 0 \) (to be bounded from below), and observe that:

\[ \mu^2 > 0 \rightarrow \text{unique minimum: } \phi^* \phi = 0 \]

\[ \mu^2 < 0 \rightarrow \text{degeneracy of minima: } \phi^* \phi = \frac{-\mu^2}{2\lambda} \]
\( \mu^2 > 0 \rightarrow \) electrodynamics of a massless photon and a massive scalar field of mass \( \mu \) \((g = -e)\).

\( \mu^2 < 0 \rightarrow \) when we **choose a minimum**, the original \( U(1) \) symmetry is spontaneously broken or hidden.

\[
\phi_0 = \left( -\frac{\mu^2}{2\lambda} \right)^{1/2} = \frac{v}{\sqrt{2}} \quad \rightarrow \quad \phi(x) = \phi_0 + \frac{1}{\sqrt{2}} (\phi_1(x) + i\phi_2(x))
\]

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} g^2 v^2 A^\mu A_\mu + \frac{1}{2} (\partial^\mu \phi_1)^2 + \mu^2 \phi_1^2 + \frac{1}{2} (\partial^\mu \phi_2)^2 + gv A_\mu \partial^\mu \phi_2 + \ldots
\]

**Side remark:** The \( \phi_2 \) field actually generates the correct transverse structure for the mass term of the (now massive) \( A^\mu \) field propagator:

\[
\langle A^\mu(k) A^\nu(-k) \rangle = \frac{-i}{k^2 - m_A^2} \left( g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) + \ldots
\]
More convenient parameterization (unitary gauge):

\[
\phi(x) = \frac{e^{i\chi(x)/v}}{\sqrt{2}} (v + H(x)) \quad \overset{U(1)}{\longrightarrow} \quad \frac{1}{\sqrt{2}} (v + H(x))
\]

The \( \chi(x) \) degree of freedom (“would-be” Goldstone boson) is rotated away using gauge invariance, while the original Lagrangian becomes:

\[
\mathcal{L} = \mathcal{L}_A + \frac{g^2 v^2}{2} A^\mu A_\mu + \frac{1}{2} \left( \partial^\mu H \partial_\mu H + 2\mu^2 H^2 \right) + \ldots
\]

which describes now the dynamics of a system made of:

- a massive vector field \( A^\mu \) with \( m_A^2 = g^2 v^2 \);
- a real scalar field \( H \) of mass \( m_H^2 = -2\mu^2 = 2\lambda v^2 \): the Higgs field.

\[\downarrow\]

Total number of degrees of freedom is balanced
(2 vector + 2 scalar d.o.f) \( \rightarrow \) (3 vector+1 scalar d.o.f.)
Higgs mechanism, non-abelian case: several vector fields $A^a_\mu(x)$ and several (real) scalar field $\phi_i(x)$:

\[
\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi \quad , \quad \mathcal{L}_\phi = \frac{1}{2}(D^\mu \phi)^2 - V(\phi) \quad , \quad V(\phi) = \mu^2 \phi^2 + \frac{\lambda}{2} \phi^4
\]

($\mu^2 < 0$, $\lambda > 0$) invariant under a non-Abelian symmetry group $G$:

\[
\phi_i \longrightarrow (1 + i\alpha^a t^a)_{ij} \phi_j \quad t^a \overset{t^a = iT^a}{\longrightarrow} (1 - \alpha^a T^a)_{ij} \phi_j
\]

(s.t. $D_\mu = \partial_\mu + gA^a_\mu T^a$). In analogy to the Abelian case:

\[
\frac{1}{2}(D_\mu \phi)^2 \longrightarrow \ldots + \frac{1}{2} g^2 (T^a \phi)_i (T^b \phi)_i A^a_\mu A^{b\mu} + \ldots
\]

\[
\phi_{\text{min}} \overset{\phi \rightarrow \phi_0}{\longrightarrow} \ldots + \frac{1}{2} g^2 (T^a \phi_0)_i (T^b \phi_0)_i A^a_\mu A^{b\mu} + \ldots = m^2_{\mu
u}
\]

\[
\begin{align*}
T^a \phi_0 &\neq 0 \quad \longrightarrow \quad \text{massive vector boson + (Goldstone boson)} \\
T^a \phi_0 &= 0 \quad \longrightarrow \quad \text{massless vector boson + massive scalar field}
\end{align*}
\]
Classical $\rightarrow$ Quantum: 

$V(\phi) \rightarrow V_{eff}(\varphi_{cl})$

The stable vacuum configurations of the theory are now determined by the extrema of the Effective Potential:

$$V_{eff}(\varphi_{cl}) = -\frac{1}{VT} \Gamma_{eff}[\phi_{cl}] \ , \ \phi_{cl} = \text{constant} = \varphi_{cl}$$

where

$$\Gamma_{eff}[\phi_{cl}] = W[J] - \int d^4y J(y) \phi_{cl}(y) \ , \ \phi_{cl}(x) = \frac{\delta W[J]}{\delta J(x)} = \langle 0|\phi(x)|0 \rangle_J$$

$W[J] \rightarrow$ generating functional of connected correlation functions
$\Gamma_{eff}[\phi_{cl}] \rightarrow$ generating functional of 1PI connected correlation functions

$V_{eff}(\varphi_{cl})$ can be organized as a loop expansion (expansion in $\hbar$), s.t.:

$$V_{eff}(\varphi_{cl}) = V(\varphi_{cl}) + \text{loop effects}$$

SSB $\rightarrow$ non trivial vacuum configurations
The $R_\xi$ gauges: nature of would-be Goldstone bosons made explicit.

Consider the abelian case:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + (D^\mu \phi)^* D_\mu \phi - V(\phi)$$

upon SSB:

$$\phi(x) = \frac{1}{\sqrt{2}} ((v + \phi_1(x)) + i\phi_2(x))$$

$$\uparrow$$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} (\partial^\mu \phi_1 + gA^\mu \phi_2)^2 + \frac{1}{2} (\partial^\mu \phi_2 - gA^\mu (v + \phi_1))^2 - V(\phi)$$

Quantizing using the gauge fixing condition:

$$G = \frac{1}{\sqrt{\xi}} (\partial_\mu A^\mu + \xi g v \phi_2)$$

in the generating functional

$$Z = C \int \mathcal{D}A \mathcal{D}\phi_1 \mathcal{D}\phi_2 \exp \left[ \int d^4x \left( \mathcal{L} - \frac{1}{2} G^2 \right) \right] \det \left( \frac{\delta G}{\delta \alpha} \right)$$

($\alpha \rightarrow$ gauge transformation parameter)
\[
\mathcal{L} - \frac{1}{2} G^2 = -\frac{1}{2} A_\mu \left( -g^{\mu\nu} \partial^2 + \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu - (gv)^2 g^{\mu\nu} \right) A_\nu
\]

\[
\frac{1}{2} (\partial_\mu \phi_1)^2 - \frac{1}{2} m_{\phi_1}^2 \phi_1^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 - \frac{\xi}{2} (gv)^2 \phi_2^2 + \cdots
\]

\[+ \]

\[
\mathcal{L}_{ghost} = \bar{c} \left[ -\partial^2 - \xi (gv)^2 \left( 1 + \frac{\phi_1}{\nu} \right) \right] c
\]

such that:

\[
\langle A^\mu(k)A^\nu(-k) \rangle = \frac{-i}{k^2 - m_A^2} \left( g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) + \frac{-i\xi}{k^2 - \xi m_A^2} \left( \frac{k^\mu k^\nu}{k^2} \right)
\]

\[
\langle \phi_1(k)\phi_1(-k) \rangle = \frac{-i}{k^2 - m_{\phi_1}^2}
\]

\[
\langle \phi_2(k)\phi_2(-k) \rangle = \langle c(k)\bar{c}(-k) \rangle = \frac{-i}{k^2 - \xi m_A^2}
\]

Goldtone boson \( \phi_2 \), \( \Longleftrightarrow \) longitudinal gauge bosons
Glashow-Weinberg-Salam Model, i.e. the SM:
Spontaneously broken Yang-Mills theory based on $SU(2)_L \times U(1)_Y$.

- $SU(2)_L \rightarrow$ weak isospin group, gauge coupling $g$:
  - three generators: $T^i = \sigma^i / 2$ ($\sigma^i =$ Pauli matrices, $i = 1, 2, 3$)
  - three gauge bosons: $W^\mu_1$, $W^\mu_2$, and $W^\mu_3$
  - $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$ fields are doublets of $SU(2)$
  - $\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$ fields are singlets of $SU(2)$
  - mass terms not allowed by gauge symmetry

- $U(1)_Y \rightarrow$ weak hypercharge group ($Q = T_3 + Y$), gauge coupling $g'$:
  - one generator $\rightarrow$ each field has a $Y$ charge
  - one gauge boson: $B^\mu$

Example: first generation

$$L_L = \begin{pmatrix}
\nu_{eL} \\
e_L \\
\nu_{eR} \\
e_R
\end{pmatrix}_Y = -1/2 \quad \begin{array}{c}
(\nu_{eR})_Y = 0 \\
(e_R)_Y = -1
\end{array}$$

$$Q_L = \begin{pmatrix}
u_L \\
u_R \\
\frac{2}{3} \\
\frac{1}{3}
\end{pmatrix}_Y = -1/3 \quad \begin{array}{c}
u_R)_Y = 2/3 \\
(d_R)_Y = -1/3
\end{array}$$

\[
L_L = \begin{pmatrix}
u_{eL} \\
e_L \\
\nu_{eR} \\
e_R
\end{pmatrix}_Y = -1/2 \quad \begin{array}{c}
(\nu_{eR})_Y = 0 \\
(e_R)_Y = -1
\end{array}$$

\[
Q_L = \begin{pmatrix}
u_L \\
u_R \\
\frac{2}{3} \\
\frac{1}{3}
\end{pmatrix}_Y = -1/3 \quad \begin{array}{c}
u_R)_Y = 2/3 \\
(d_R)_Y = -1/3
\end{array}$$
Three fermionic generations, summary of gauge quantum numbers:

\[
\begin{align*}
Q^i_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix} & \quad SU(3)_C & SU(2)_L & U(1)_Y & U(1)_Q \\
u^i_R &= u_R \begin{pmatrix} c_R \\ t_R \end{pmatrix} & 3 & 2 & \frac{1}{6} & \frac{2}{3} \\
d^i_R &= d_R \begin{pmatrix} s_R \\ b_R \end{pmatrix} & 3 & 1 & -\frac{1}{3} & -\frac{1}{3}
\end{align*}
\]

\[
\begin{align*}
L^i_L &= \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} & 1 & 2 & -\frac{1}{2} & 0 \\
\nu^i_R &= \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} & 1 & 1 & -1 & -1
\end{align*}
\]

where a minimal extension to include \(\nu^i_R\) has been allowed (notice however that it has zero charge under the entire SM gauge group!)
Lagrangian of fermion fields

For each generation (here specialized to the first generation):

\[
\mathcal{L}_{\text{EW}}^{\text{ferm}} = \bar{L}_L (i \not\! \! \! \partial) L_L + \bar{e}_R (i \not\! \! \! \partial) e_R + \bar{\nu}_e R (i \not\! \! \! \partial) \nu_e R + \bar{Q}_L (i \not\! \! \! \partial) Q_L + \bar{u}_R (i \not\! \! \! \partial) u_R + \bar{d}_R (i \not\! \! \! \partial) d_R
\]

where in each term the covariant derivative is given by

\[
D_\mu = \partial_\mu - ig W_\mu^i T^i - ig' \frac{1}{2} Y B_\mu
\]

and \(T^i = \sigma^i / 2\) for L-fields, while \(T^i = 0\) for R-fields \((i = 1, 2, 3)\), i.e.

\[
D_{\mu,L} = \partial_\mu - \frac{ig}{\sqrt{2}} \begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} - \frac{i}{2} \begin{pmatrix} g W^3_\mu - g' Y B_\mu & 0 \\ 0 & g W^3_\mu - g' Y B_\mu \end{pmatrix}
\]

\[
D_{\mu,R} = \partial_\mu + ig' \frac{1}{2} Y B_\mu
\]

with

\[
W^\pm = \frac{1}{\sqrt{2}} (W^1_\mu \pm iW^2_\mu)
\]
$\mathcal{L}_{\text{EW}}^\text{ferm}$ can then be written as

$$\mathcal{L}_{\text{EW}}^\text{ferm} = \mathcal{L}_{\text{kin}}^\text{ferm} + \mathcal{L}_{\text{CC}} + \mathcal{L}_{\text{NC}}$$

where

$$\mathcal{L}_{\text{kin}}^\text{ferm} = \bar{L}_L(i\slashed{\partial})L_L + \bar{e}_R(i\slashed{\partial})e_R + \ldots$$

$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} W^+_{\mu} \bar{\nu}_e L \gamma^\mu e_L + W^-_{\mu} \bar{e}_L \gamma^\mu \nu_e L + \ldots$$

$$\mathcal{L}_{\text{NC}} = \frac{g}{2} W^3_{\mu} [\bar{\nu}_e L \gamma^\mu \nu_e L - \bar{e}_L \gamma^\mu e_L] + \frac{g'}{2} B_{\mu} [Y(L)(\bar{\nu}_e L \gamma^\mu \nu_e L + \bar{e}_L \gamma^\mu e_L)$$

$$+ Y(e_R)\bar{\nu}_e R \gamma^\mu \nu_e R + Y(e_R)\bar{e}_R \gamma^\mu e_R] + \ldots$$

where

$$W^\pm = \frac{1}{\sqrt{2}} (W^1_{\mu} \mp iW^2_{\mu}) \rightarrow \text{mediators of Charged Currents}$$

$W^3_{\mu}$ and $B_{\mu} \rightarrow \text{mediators of Neutral Currents}.$

$$\Downarrow$$

However neither $W^3_{\mu}$ nor $B_{\mu}$ can be identified with the photon field $A_{\mu},$ because they couple to neutral fields.
Rotate $W_\mu^3$ and $B_\mu$ introducing a weak mixing angle ($\theta_W$)

\[ W_\mu^3 = \sin \theta_W A_\mu + \cos \theta_W Z_\mu \]
\[ B_\mu = \cos \theta_W A_\mu - \sin \theta_W Z_\mu \]

such that the kinetic terms are still diagonal and the neutral current Lagrangian becomes

\[ \mathcal{L}_{NC} = \bar{\psi} \gamma^\mu \left( g \sin \theta_W T^3 + g' \cos \theta_W \frac{Y}{2} \right) \psi A_\mu + \bar{\psi} \gamma^\mu \left( g \cos \theta_W T^3 - g' \sin \theta_W \frac{Y}{2} \right) \psi Z_\mu \]

for $\psi^T = (\nu_{eL}, e_L, \nu_{eR}, e_R, \ldots)$. One can then identify ($Q \rightarrow$ e.m. charge)

\[ eQ = g \sin \theta_W T^3 + g' \cos \theta_W \frac{Y}{2} \]

and, e.g., from the leptonic doublet $L_L$ derive that

\[ \begin{cases} 
\frac{g}{2} \sin \theta_W - \frac{g'}{2} \cos \theta_W = 0 \\
-\frac{g}{2} \sin \theta_W - \frac{g'}{2} \cos \theta_W = -e
\end{cases} \quad \rightarrow \quad g \sin \theta_W = g' \cos \theta_W = e \]
\[ A_\mu = -ieQ_f \gamma^\mu \]
\[ W_\mu = \frac{ie}{2\sqrt{2}s_w} \gamma^\mu (1 - \gamma_5) \]
\[ Z_\mu = ie\gamma^\mu (v_f - a_f \gamma_5) \]

where

\[ v_f = -\frac{s_w}{c_w} Q_f + \frac{T_f^3}{2s_W c_W} \]
\[ a_f = \frac{T_f^3}{2s_W c_W} \]
Lagrangian of gauge fields

\[ \mathcal{L}_{EW}^{\text{gauge}} = -\frac{1}{4} W_\mu^a W^{a,\mu\nu} - \frac{1}{4} B_\mu B^{\mu\nu} \]

where

\[ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \]
\[ W_\mu^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + \epsilon^{abc} W_\mu^b W_\nu^c \]

in terms of physical fields:

\[ \mathcal{L}_{EW}^{\text{gauge}} = \mathcal{L}_{kin}^{\text{gauge}} + \mathcal{L}_{EW}^{3V} + \mathcal{L}_{EW}^{4V} \]

where

\[ \mathcal{L}_{kin}^{\text{gauge}} = -\frac{1}{2} (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) \]
\[ -\frac{1}{4} (\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^{-\nu} - \partial^\nu Z^{-\mu}) - \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^{-\nu} - \partial^\nu A^{-\mu}) \]
\[ \mathcal{L}_{EW}^{3V} = (3\text{-gauge-boson vertices involving } ZW^+W^- \text{ and } AW^+W^-) \]
\[ \mathcal{L}_{EW}^{4V} = (4\text{-gauge-boson vertices involving } ZZW^+W^-, AAW^+W^-, AZW^+W^-, \text{ and } W^+W^-W^+W^-) \]
\[
\begin{align*}
\frac{k}{\mu} & = \frac{-i}{k^2 - M_V^2} \left( g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{M_V^2} \right) \\
W_\mu^+ & = i e C_V \left[ g_{\mu\nu}(k^+_\nu - k^-\mu) + g_{\nu\rho}(k^-\mu - k_V\rho) + g_{\rho\mu}(k_V - k^+_\nu) \right] \\
W^-_\mu & = i e^2 C_{VV'} \left( 2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho} \right)
\end{align*}
\]

where

\[
C_\gamma = 1 \quad , \quad C_Z = -\frac{c_W}{s_W}
\]

and

\[
C_{\gamma\gamma} = -1 \quad , \quad C_{ZZ} = -\frac{c_W^2}{s_W^2} \quad , \quad C_{\gamma Z} = \frac{c_W}{s_W} \quad , \quad C_{WW} = \frac{1}{s_W^2}
\]
The Higgs sector of the Standard Model: $SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_Q$

Introduce one complex scalar doublet of $SU(2)_L$ with $Y = 1/2$:

$$
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \leftrightarrow \mathcal{L}^{SSB}_{EW} = (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2
$$

where $D_\mu \phi = (\partial_\mu - igW^a_\mu T^a - ig'Y_\phi B_\mu), (T^a = \sigma^a/2, a = 1, 2, 3)$.

The SM symmetry is spontaneously broken when $\langle \phi \rangle$ is chosen to be (e.g.):

$$
\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \text{ with } v = \left( \frac{-\mu^2}{\lambda} \right)^{1/2} \quad (\mu^2 < 0, \lambda > 0)
$$

The gauge boson mass terms arise from:

$$
(D^\mu \phi)^\dagger D_\mu \phi \rightarrow \cdots + \frac{1}{8} (0 \ v) \left( gW^a_\mu \sigma^a + g' B_\mu \right) \left( gW^{b\mu} \sigma^b + g' B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} + \cdots
$$

$$
\rightarrow \cdots + \frac{1}{2} \frac{v^2}{4} \left[ g^2 (W^1_\mu)^2 + g^2 (W^2_\mu)^2 + (-gW^3_\mu + g' B_\mu)^2 \right] + \cdots
$$
And correspond to the weak gauge bosons:

\[
W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \mp iW^2_\mu) \quad \rightarrow \quad M_W = g\frac{v}{2}
\]

\[
Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (gW^3_\mu - g'B_\mu) \quad \rightarrow \quad M_Z = \sqrt{g^2 + g'^2} \frac{v}{2}
\]

while the linear combination orthogonal to \(Z_\mu\) remains massless and corresponds to the photon field:

\[
A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g'W^3_\mu + gB_\mu) \quad \rightarrow \quad M_A = 0
\]

**Notice:** using the definition of the weak mixing angle, \(\theta_w\):

\[
\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}} \quad , \quad \sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}
\]

the \(W\) and \(Z\) masses are related by:

\[
M_W = M_Z \cos \theta_w
\]
The scalar sector becomes more transparent in the unitary gauge:

\[
\phi(x) = e^{i \frac{x}{v} \vec{\chi}(x) \cdot \vec{\tau}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \xrightarrow{SU(2)} \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}
\]

after which the Lagrangian becomes

\[
\mathcal{L} = \mu^2 H^2 - \lambda v H^3 - \frac{1}{4} H^4 = -\frac{1}{2} M_H^2 H^2 - \sqrt{\lambda} M_H H^3 - \frac{1}{4} \lambda H^4
\]

Three degrees of freedom, the \( \chi^a(x) \) Goldstone bosons, have been reabsorbed into the longitudinal components of the \( W^\pm_\mu \) and \( Z_\mu \) weak gauge bosons. One real scalar field remains:

the Higgs boson, \( H \), with mass \( M_H^2 = -2\mu^2 = 2\lambda v^2 \)

and self-couplings:

\[
H \quad H = -3i \frac{M_H^2}{v}
\]

\[
H \quad H = -3i \frac{M_H^2}{v^2}
\]
From \((D^\mu \phi)^\dagger D_\mu \phi \rightarrow\) Higgs-Gauge boson couplings:

\[ H = 2i \frac{M^2_V}{v} g^{\mu\nu}, \]

\[ = 2i \frac{M^2_V}{v^2} g^{\mu\nu}. \]

**Notice**: The entire Higgs sector depends on only two parameters, e.g. \(M_H\) and \(v\).

\(v\) measured in \(\mu\)-decay:

\[ v = (\sqrt{2}G_F)^{-1/2} = 246 \text{ GeV} \rightarrow \text{SM Higgs Physics depends on } M_H \]

Run 1+2 (combined): \(M_H = 125.09 \pm 0.24 (\pm 0.21) \text{ GeV} \)
Also: remember Higgs-gauge boson loop-induced couplings:

Surprisingly important in Higgs-boson phenomenology!
Higgs boson couplings to quarks and leptons

The gauge symmetry of the SM also forbids fermion mass terms $m_{Q_i} Q^i_L u^i_R, \ldots$, but all fermions are massive.

Fermion masses are generated via gauge invariant Yukawa couplings:

$$\mathcal{L}_{Yukawa}^{EW} = -\Gamma_{u}^{ij} \bar{Q}^i_L \phi^c u^j_R - \Gamma_{d}^{ij} \bar{Q}^i_L \phi d^j_R - \Gamma_{e}^{ij} \bar{L}^i_L \phi l^j_R + h.c.$$ such that, upon spontaneous symmetry breaking:

$$\mathcal{L}_{Yukawa}^{EW} = -\Gamma_{u}^{ij} \bar{u}^i_L \frac{v + H}{\sqrt{2}} u^j_R - \Gamma_{d}^{ij} \bar{d}^i_L \frac{v + H}{\sqrt{2}} d^j_R - \Gamma_{e}^{ij} \bar{l}^i_L \frac{v + H}{\sqrt{2}} l^j_R + h.c.$$

$$= - \sum_{f,i,j} \bar{f}^i_L M_f^{ij} f^j_R \left(1 + \frac{H}{v}\right) + h.c.$$

where

$$M_f^{ij} = \Gamma_f^{ij} \frac{v}{\sqrt{2}}$$

is a non-diagonal mass matrix.
Upon diagonalization (by unitary transformation $U_L$ and $U_R$)

$$M_D = (U_L^f)^\dagger M_f U_R^f$$

and defining mass eigenstates:

$$f'_L = (U_L^f)_{ij} f_L^j \quad \text{and} \quad f'_R = (U_R^f)_{ij} f_R^j$$

the fermion masses are extracted as

$$\mathcal{L}_{EW}^{\text{Yukawa}} = \sum_{f,i,j} \bar{f}'_L [(U_L^f)^\dagger M_f U_R^f] f'_R \left( 1 + \frac{H}{v} \right) + \text{h.c.}$$

$$= \sum_{f,i} m_f \left( \bar{f}'_L f'_R + \bar{f}'_R f'_L \right) \left( 1 + \frac{H}{v} \right)$$

$$\bar{f} \quad \quad \quad \quad H \quad = -i \frac{m_f}{v} = -i y_f$$

\[ \]
In terms of the new mass eigenstates the quark part of $\mathcal{L}_{CC}$ now reads

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \bar{u}'_L [ (U^u_L)^\dagger U^d_R ] \gamma^\mu d^j_L + \text{h.c.}$$

where

$$V_{CKM} = (U^u_L)^\dagger U^d_R$$

is the Cabibbo-Kobayashi-Maskawa matrix, origin of flavour mixing in the SM → G.Wilkinson’s lectures
LHC Run 1+Run 2: first measurements of Higgs couplings

Higgs couplings to gauge bosons measured to 10-15% level.
Higgs couplings to $3^{rd}$-generation fermions measured at 20-30% level.
First bound on Higgs self-coupling ($\kappa_\lambda = \lambda_3^{} / \lambda_3^{SM}$)

$-11.8 \leq \kappa_\lambda \leq 18.8$ (95% CL) [CMS, PRL 122, 121803]

$-5.0 \leq \kappa_\lambda \leq 12.0$ (95% CL) [ATLAS, arXiv:1906.02025]
SM Higgs-boson decay branching ratios and width

These curves include: **tree level** + **QCD and EW loop corrections**.

- Can you make sense of these plots?
- You have all the building blocks to calculate them! How do your results compare with the plots above?
- You can also use automated tools (see e.g. HDECAY, and its extensions).
- Observe difference between light and heavy Higgs.