Flavour Physics lecture 1

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CERN-Fermilab HCPSS
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Useful resources & acknowledgments

- Heavy Flavour Averaging Group (HFLAV) https://hflav.web.cern.ch
- CKMfitter ckmfitter.in2p3.fr Utfit www.utfit.org/UTfit/
- Particle Data Group reviews pdg.lbl.gov
- Books:
  - CP violation, I.I. Bigi and A.I. Sanda (CUP, 2000)
  - CP violation, G.C. Branco, L. Lavoura & J.P.Silva (OUP, 1999)
- Reviews & lectures:
  - M. Blanke, arXiv:1704.03753
  - O. Gedalia & G. Perez, arXiv:1005.3106
  - Y. Grossman & P. Tanedo, arXiv:1711.03624
  - J.F. Kamenik, arXiv:1708.00771
  - Z. Ligeti, arXiv:1502.01372
  - Y. Nir, arXiv:0708.1872, arXiv:1605.00433

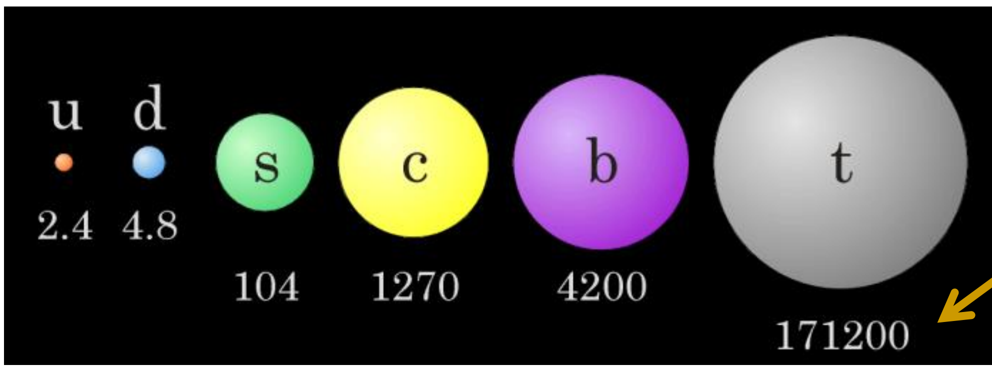
Thanks to flavour lecturers at this school in previous years, who provided inspiration for some of the material shown (esp. T. Gerson, J. Zupan & M-H. Schune).

What is flavour physics and why should we care ?

What is flavour physics?

The concept of 'flavour' in particle physics relates to the existence of different families of quarks*, and how they couple to each other

i.e. 6 known flavours of quark, grouped into 3 generations



Not to linear scale !

mass in MeV/c^2

Open questions:

- why 3 generations ?
- why do the quarks exhibit this striking hierarchy in mass ?

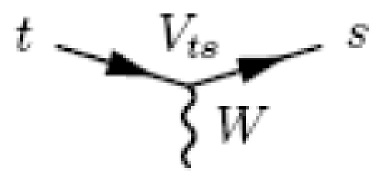
No answer yet !
These values (i.e. '3' & the masses) are free parameters of the SM

These mysteries make the 'flavour sector' of the Standard Model of great interest.

* the concept of flavour extends to the lepton sector too

Flavour and the CKM matrix

In the Standard Model quarks can only change flavour through emission of a W boson (*i.e.* weak force). For example a t quark can decay into a b, s or d quark:



But these decays are not equally likely. At the amplitude level they are weighted by factors that are elements of the Cabibbo-Kobyashi-Maskawa (CKM) matrix, and these factors vary dramatically – here is another hierarchy we don't understand !

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9705-0.9770 & 0.21-0.24 & 0-0.014 \\ 0.21-0.24 & 0.971-0.973 & 0.036-0.070 \\ 0-0.014 & 0.036-0.070 & 0.997-0.999 \end{pmatrix}$$

These elements of the CKM matrix are also fundamental parameters of the Standard Model. Why they have these values is another great mystery.

Parameters of the Standard Model

- 3 gauge couplings
- 2 Higgs parameters
- strong CP parameter θ
- 6 quark masses
- 3 quark mixing angles + 1 phase [*i.e.* CKM matrix]
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase [*i.e.* PMNS matrix])

() = with Dirac neutrino masses

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This is of particular relevance…

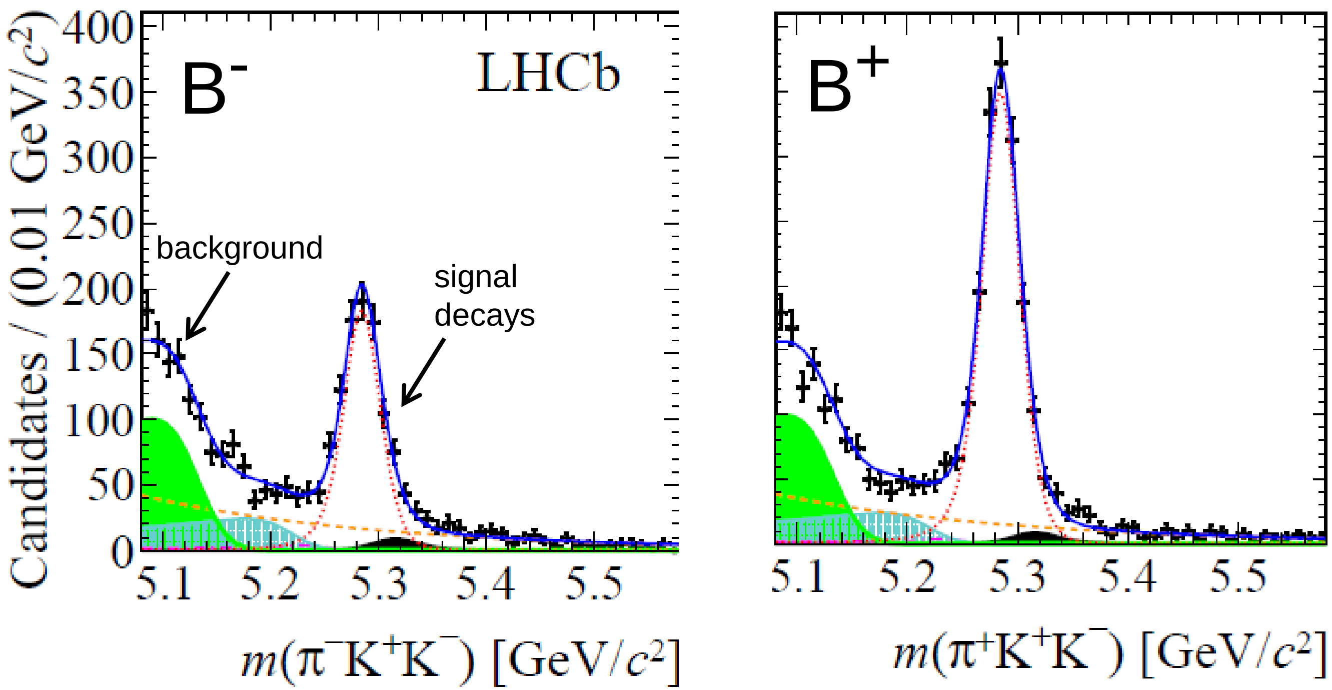
() = with Dirac neutrino masses

CP violation

CP violation (CPV) → difference in behaviour between matter and anti-matter.

First discovered in the kaon system in 1964, opportunities of study were limited until colliders arrived that could make lots & lots of b-quark hadrons, *e.g.* the LHC

A recent example from LHCb - look at B meson decaying into a pion & two kaons…

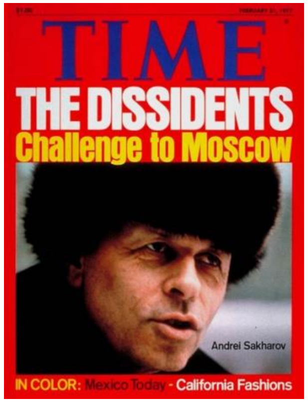


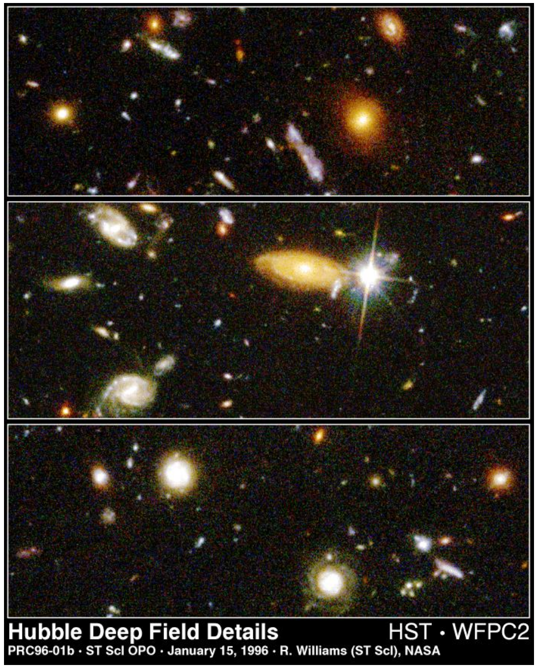
[LHCb, PRD 90 (2014) 112004]

…the decay probabilities are manifestly different for B$^-$ & B$^+$! In the Standard Model CPV is accommodated, *but not explained*, by an imaginary phase in the CKM matrix

Cosmological connections ?

As first pointed out by Andrei Sakharov, CP-violation is one requirement for explaining *baryogenesis* – the process that took us from the equal amounts of matter and anti-matter produced in the Big Bang, to the matter dominated universe of today





The problem is that the CP-violation that appears in the Standard Model, is woefully inadequate to explain the matter-antimatter asymmetry we have today.

This is a big problem with the Standard Model !

More & better measurements may point a way forward.

Problems with the Standard Model

The Standard Model (SM) cannot be a final theory

We have already encountered the following shortcomings:

- No explanation for baryogenesis
- No explanation for the quark or CKM hierarchy
- No real explanation for CP violation, and why it is only found in the weak interaction.

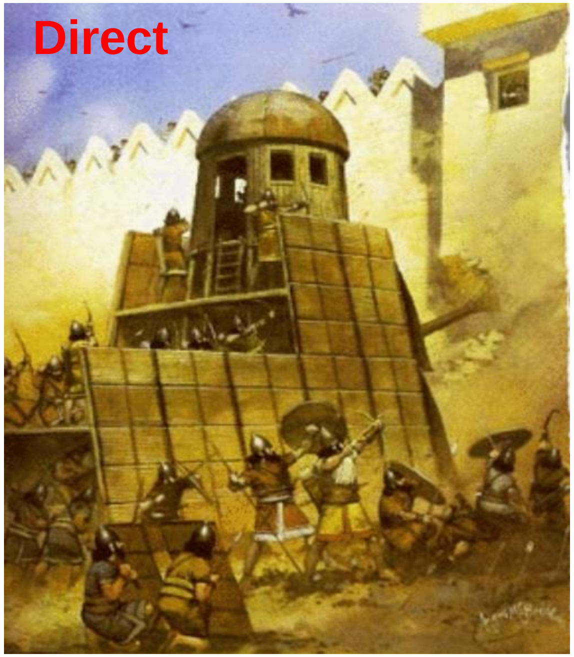
And there are plenty of others, for example:

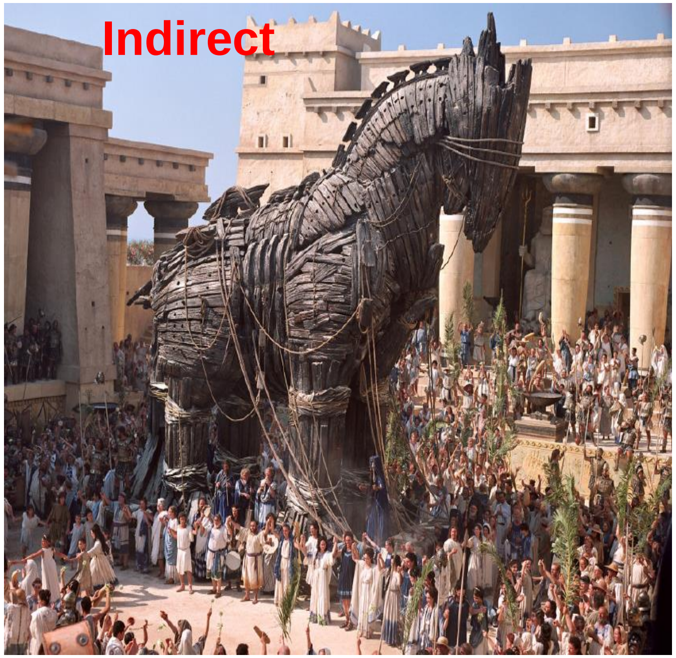
- No explanation for dark matter or dark energy
- No explanation for neutrino masses
- Gravity not included
- No explanation for why the Higgs boson has the mass it does (left to itself the theory would make it much, much heavier)

More ambitious theories (*e.g.* supersymmetry or SUSY) can solve at least some of these problems. They generally predict new particles or effects outside the SM. Finding these effects is the goal of the LHC & many other present/planned facilities !

Breaching the walls of the Standard Model

The HEP community is searching for ‘New Physics’ - to find this we need to penetrate the walls of the Standard Model fortress. There are two strategies used in this search.





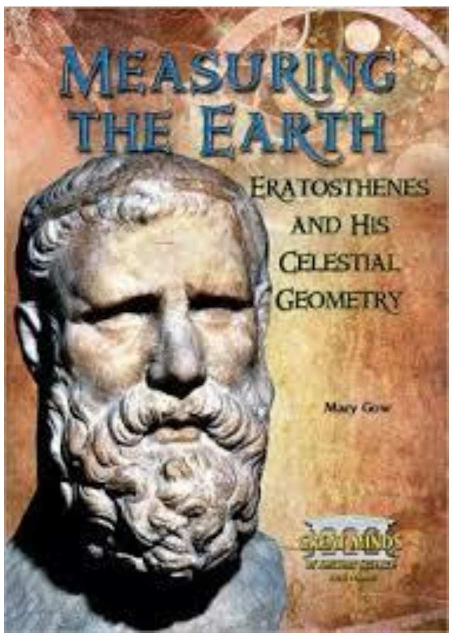
Use the high energy of, *e.g.* the LHC to produce the New Physics particles, which we then detect

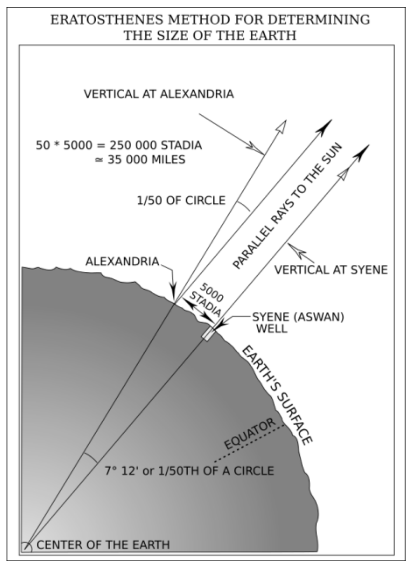
Make precise measurements of processes in which New Physics particles enter through ‘virtual loops’

Both methods are powerful. Flavour physics follows the ‘indirect’ approach.

Indirect measurements – an established tradition in science

Eratosthenes was able to determine the circumference of the earth using indirect means…





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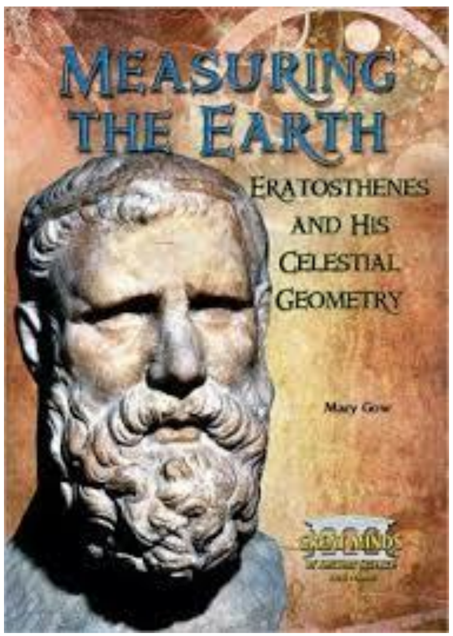

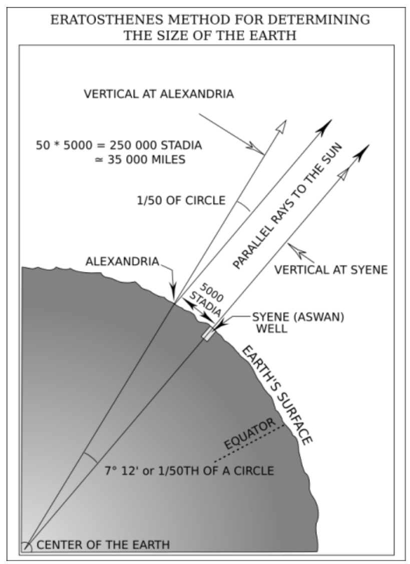

…around 2.2 thousand years prior to the direct observation.

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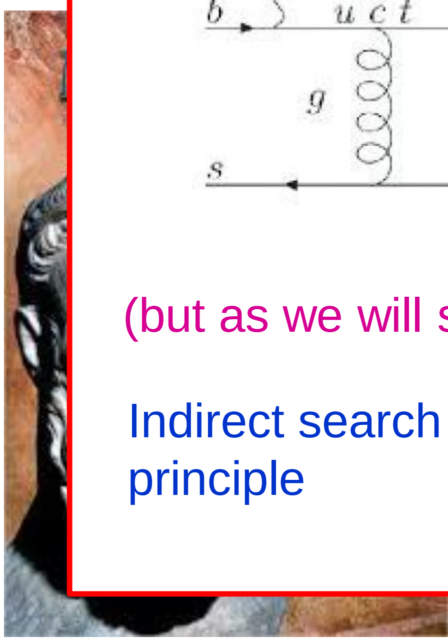
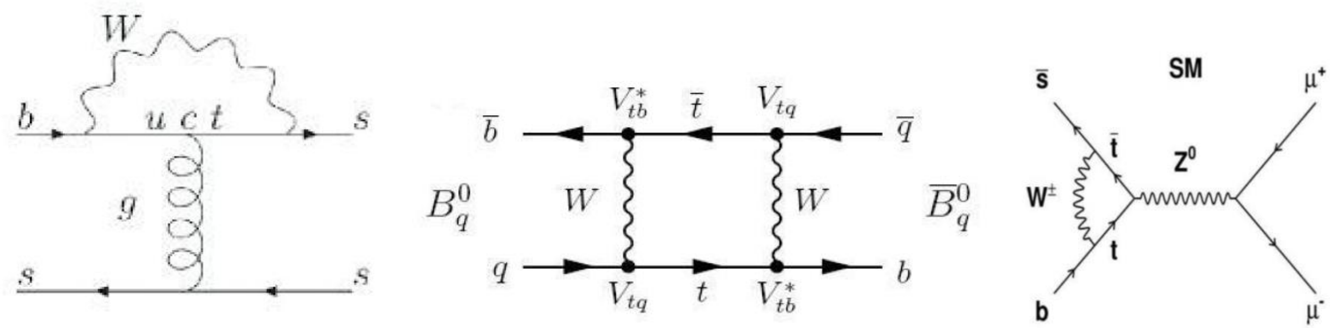


In flavour physics the guiding principle is to probe processes where loop diagrams are important, as here non-SM particles may contribute



(but as we will see, tree-mediated decays also have their role to play)

Indirect search principle ➡ Precise measurements of low energy phenomena tells us about unknown physics at energies *far* beyond direct searches (~10^4 TeV in some cases)

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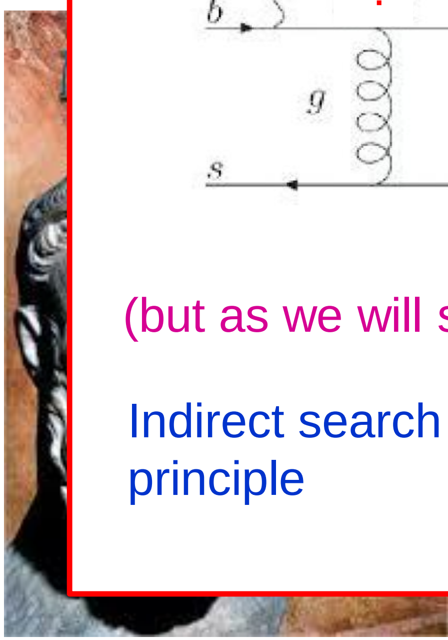

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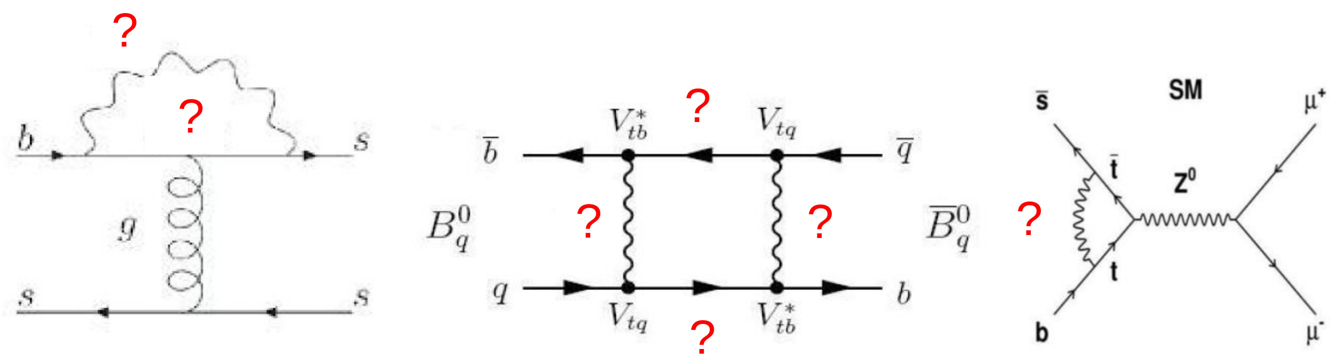

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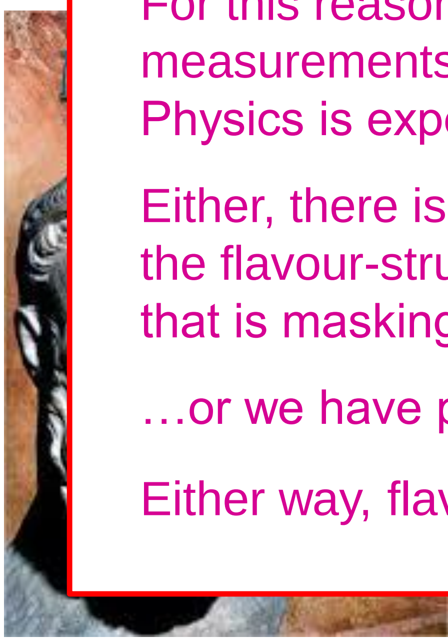

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For this reason its rather surprising that (spoiler alert !) most flavour measurements so far agree with the SM, as naturalness told us New Physics is expected at TeV scale → the **New Physics Flavour Puzzle**.

Either, there is something specific about the flavour-structure of the New Physics that is masking the effect…

…or we have put too much trust in naturalness.

Either way, flavour is central to the story !

Outline of the lecture contents and schedule

Flavour topics that we won't be covering

Flavour encompasses a huge range of areas of study & corresponding experimental activity. The following are genuine topics of flavour, but ones we will not cover.

- Kaon physics (OK, we will say a little, but not really do it justice)
- Suppressed top decays
- Flavour and CPV violation in the Higgs sector
- Charged lepton-flavour violation, *e.g.* $\mu \to e\gamma$
- (g-2) muon anomaly
- All neutrino physics

Instead we will focus on beauty physics, with some discussion on charm.

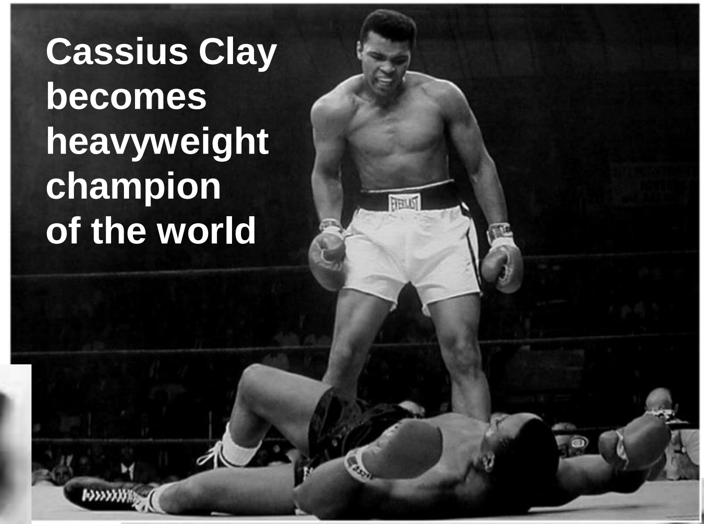
Lecture outline

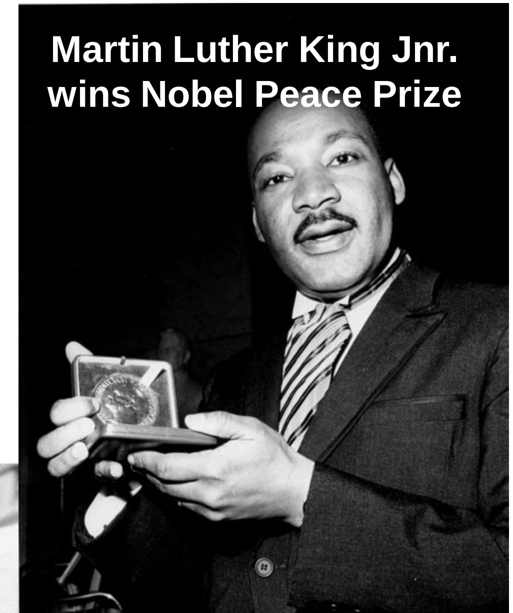
- Introduction ✓
- Birth of flavour physics & the kaon sector
- The beautiful millennium
- Flavour structure of the SM
- The Unitarity Triangle and CPV measurements
- Spectroscopy (a brief digression)
- FCNCs or ‘rare decays’
- Charm physics
- Future of flavour

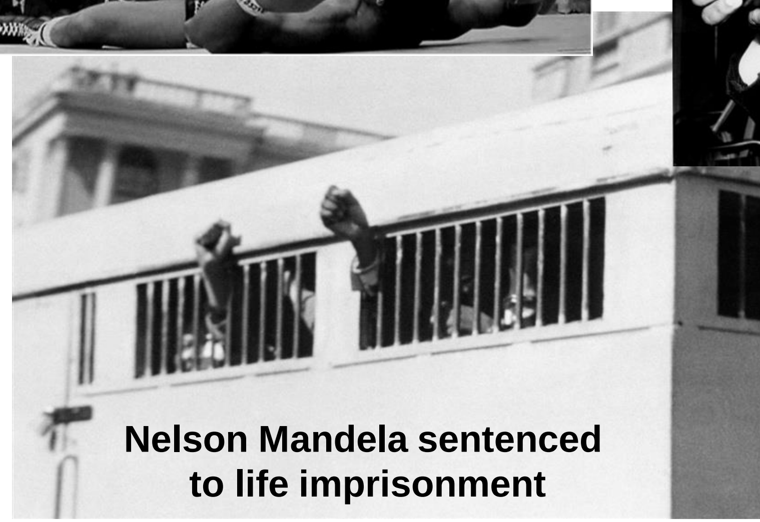
Note the approach will (necessarily) be from an experimentalist’s perspective.

The birth of experimental flavour physics and the (continuing) importance of kaon studies

Events of 1964







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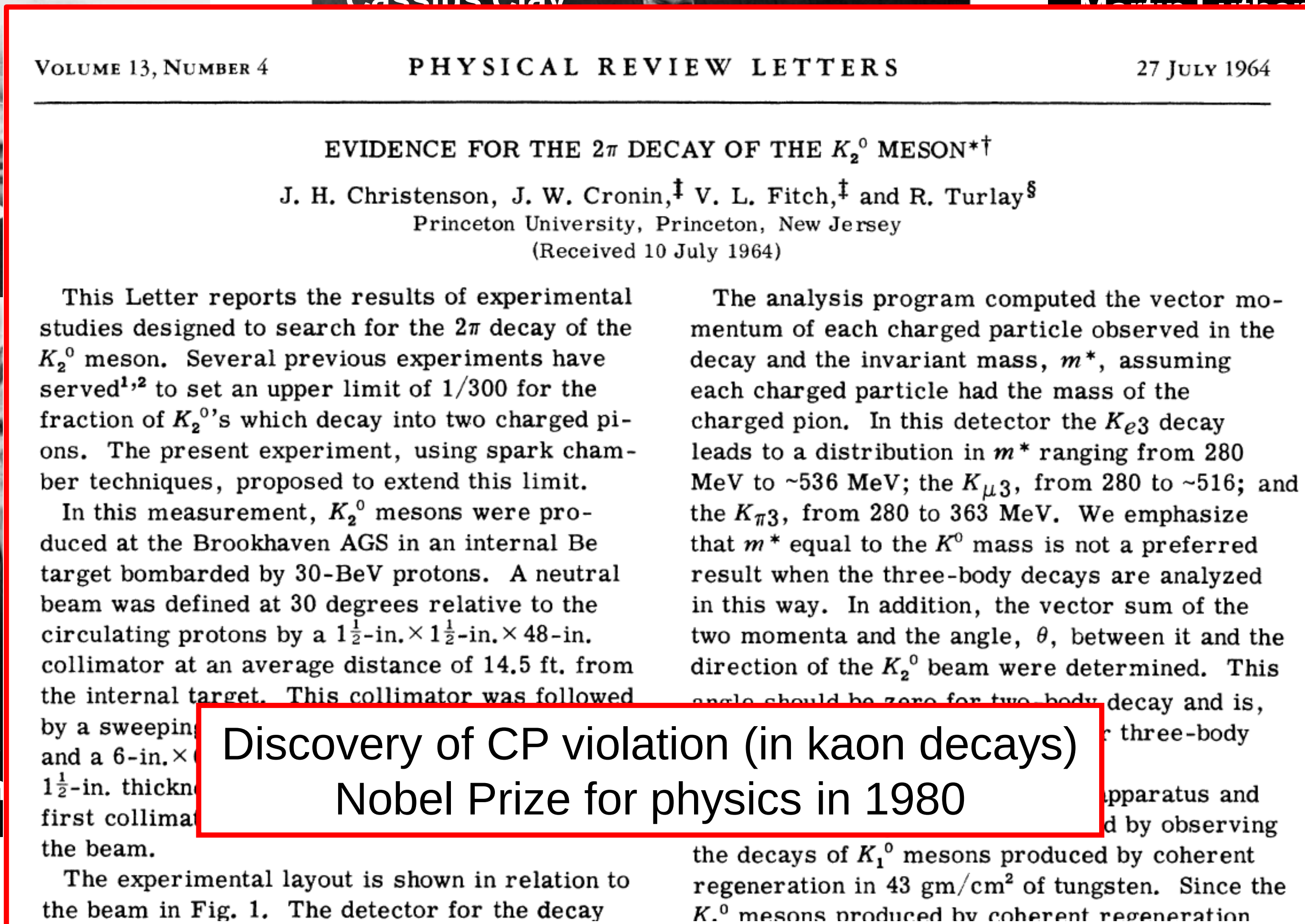

VOLUME 13, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1964

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

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(Received 10 July 1964)

This Letter reports the results of experimental studies designed to search for the 2π decay of the K_2^0 meson. Several previous experiments have served[1,2] to set an upper limit of 1/300 for the fraction of K_2^0's which decay into two charged pions. The present experiment, using spark chamber techniques, proposed to extend this limit.

In this measurement, K_2^0 mesons were produced at the Brookhaven AGS in an internal Be target bombarded by 30-BeV protons. A neutral beam was defined at 30 degrees relative to the circulating protons by a $1\frac{1}{2}$-in.$\times 1\frac{1}{2}$-in.$\times$48-in. collimator at an average distance of 14.5 ft. from the internal target. This collimator was followed by a sweeping
and a 6-in.×
$1\frac{1}{2}$-in. thickn
first collimat
the beam.

The experimental layout is shown in relation to the beam in Fig. 1. The detector for the decay

The analysis program computed the vector momentum of each charged particle observed in the decay and the invariant mass, m^*, assuming each charged particle had the mass of the charged pion. In this detector the K_{e3} decay leads to a distribution in m^* ranging from 280 MeV to ~536 MeV; the $K_{\mu3}$, from 280 to ~516; and the $K_{\pi3}$, from 280 to 363 MeV. We emphasize that m^* equal to the K^0 mass is not a preferred result when the three-body decays are analyzed in this way. In addition, the vector sum of the two momenta and the angle, θ, between it and the direction of the K_2^0 beam were determined. This angle should be zero for two-body decay and is,
three-body

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Discovery of CP violation (in kaon decays)
Nobel Prize for physics in 1980

Discovery of CP violation

Observation of 45 ± 10 $\pi^+\pi^-$ decays in a K^0_L beam [Christenson *et al.*, PRL 13 (1964) 138].

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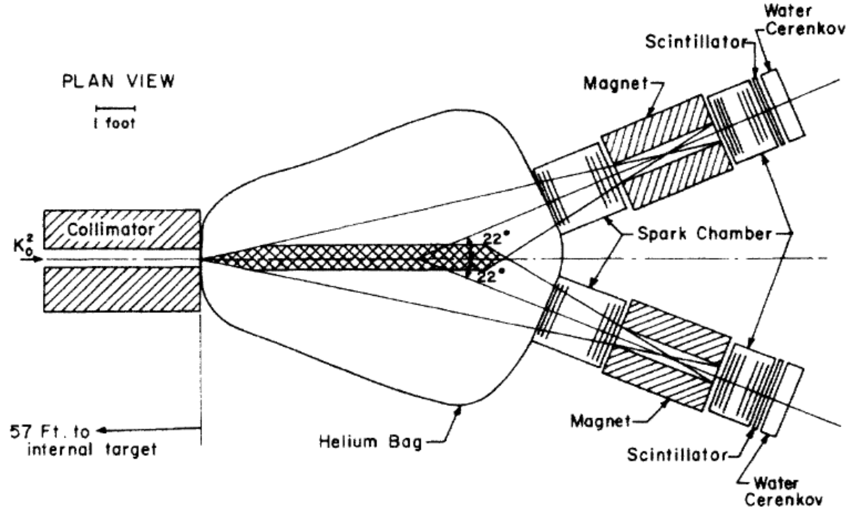
An important calibration of the apparatus and data reduction system was afforded by observing the decays of K_1^0 mesons produced by coherent regeneration in 43 gm/cm² of tungsten. Since the K_1^0 mesons produced by coherent regeneration

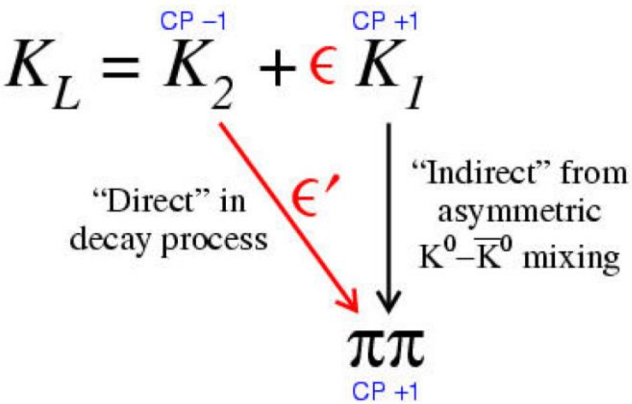


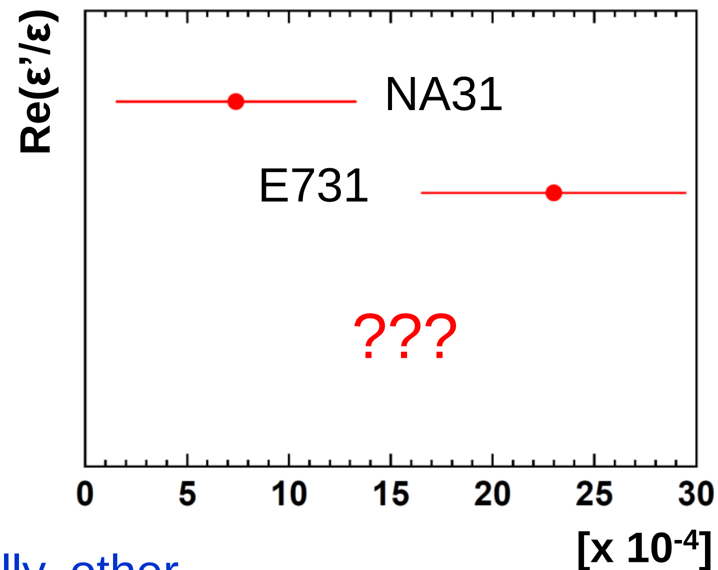
Interpretation: K^0_L not a pure CP-odd eigenstate. Level of CP-even 'contamination' given by ε, which is now measured to be $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$ [PDG].

The heroic quest for ε′

(more thorough discussion on direct & indirect CPV will come later…)

In the CKM paradigm $K^0_L \to \pi\pi$ is readily explained as *indirect* CPV. CKM also allows for the possibility of *direct* CPV, which can be revealed by measuring the relative rates of K^0_S and K^0_L into $\pi^+\pi^-$ and $\pi^0\pi^0$, which give the parameter Re(ε'/ε).





[E731, PRL 70 (1993) 1203]
[NA31, PRL 317 (1993) 233]

A non-zero Re(ε'/ε) implies direct CPV, & is consistent with CKM picture. Historically, other models (*e.g.* superweak [Wolfenstein, PRL 13 (1964) 562]) predicted zero direct CPV.

Effect is very small, experiment is hard, and first measurements were ambiguous.

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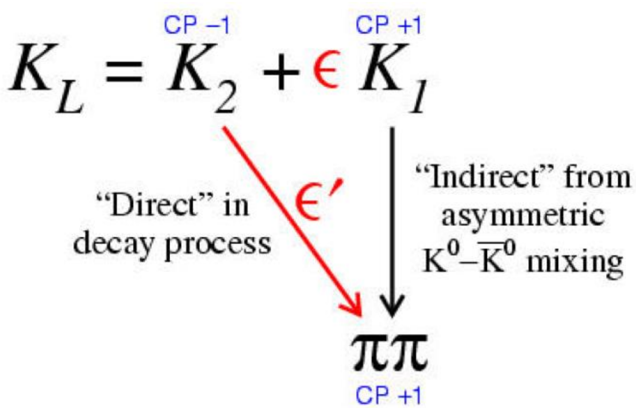

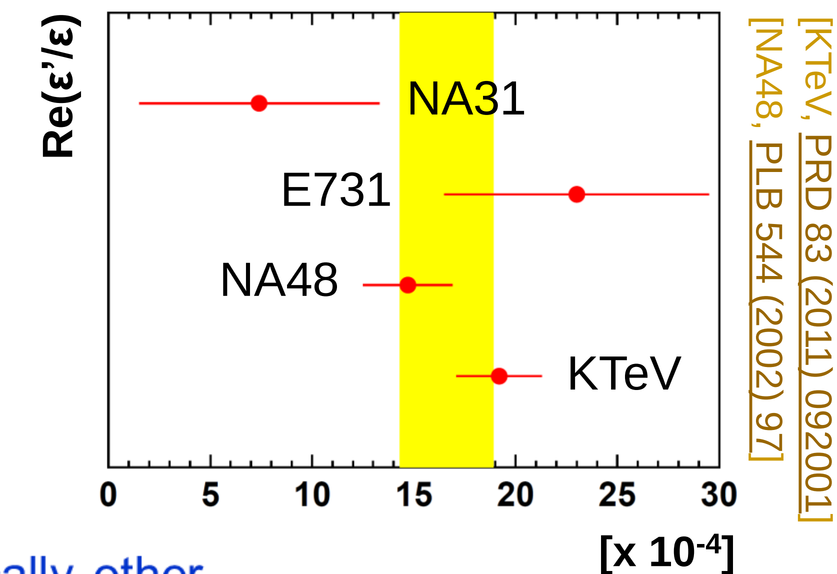

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Effect is very small, experiment is hard, and first measurements were ambiguous. A second round of experiments (NA48, KTeV) was required to show Re(ε’/ε)$\neq 0$.

Lattice QCD prediction not as precise as experiment, but progress being made.

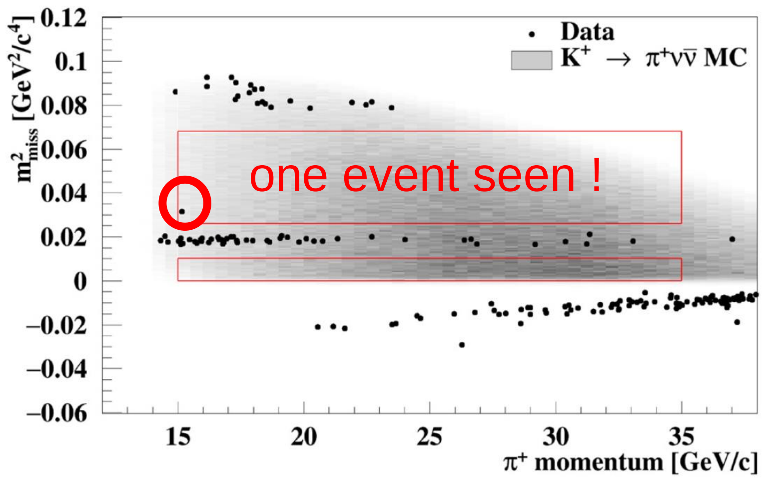
Heroic work! Kaon physics is very difficult - small effects & theoretically challenging.

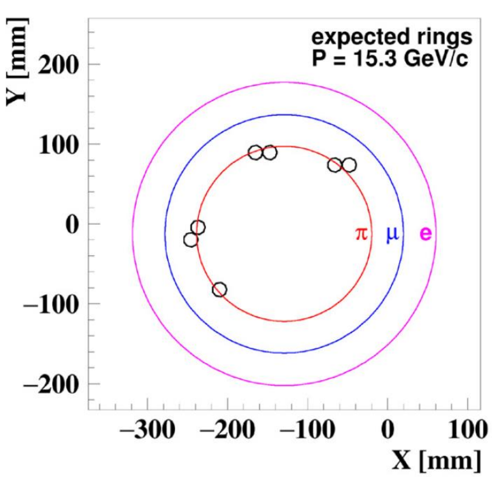
In search of the ultra-rare

CP violation is not the whole story. Kaons system is also well suited for searches for forbidden or ultra-suppressed decays, the most topical of which is $K^+ \to \pi^+ \nu\bar{\nu}$.

In SM $BR(K^+ \to \pi^+ \nu\bar{\nu}) = (8.39 \pm 0.30) \times 10^{-11}$ [Buras *et al.*, JHEP 1511 (2015) 033], but New Physics enhancements possible with sensitivity to mass scales under 100 TeV.

BNL experiments E949 & E787 saw 3 events [PRD 77 (2008) 052003], consistent with SM. NA62, here at CERN, aims to observe ~100 and make precise measurement of BR.





Pilot measurement released [PLB 791 (2019) 156], based on a few weeks of data taking.

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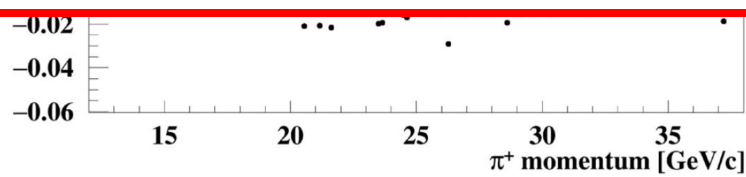
In SI … , but
New … 0 TeV.

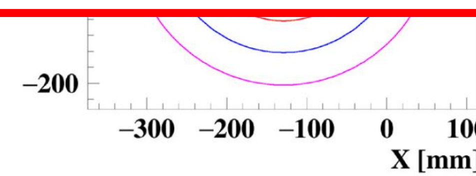
BNL … vith SM.
NA6 … of BR.

Kaon studies have played a critical role in the development of flavour physics. They will continue to do so in future.

However, in the past 2-3 decades the focus has been on beauty:

- A huge number of decays and processes to explore
- Sizable CPV effects expected (& observed)
- In many cases theoretically clean predictions are available





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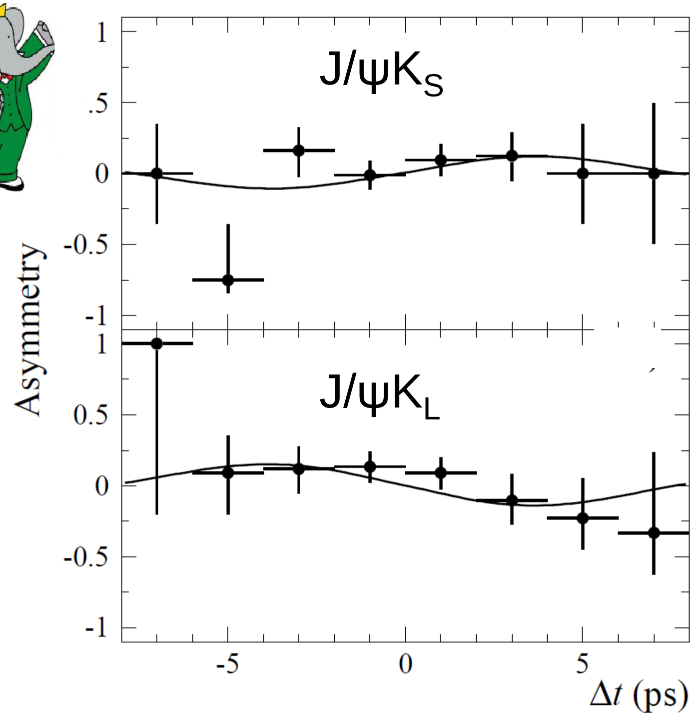
We live in a golden age of flavour !

An introduction to the experiments of B physics

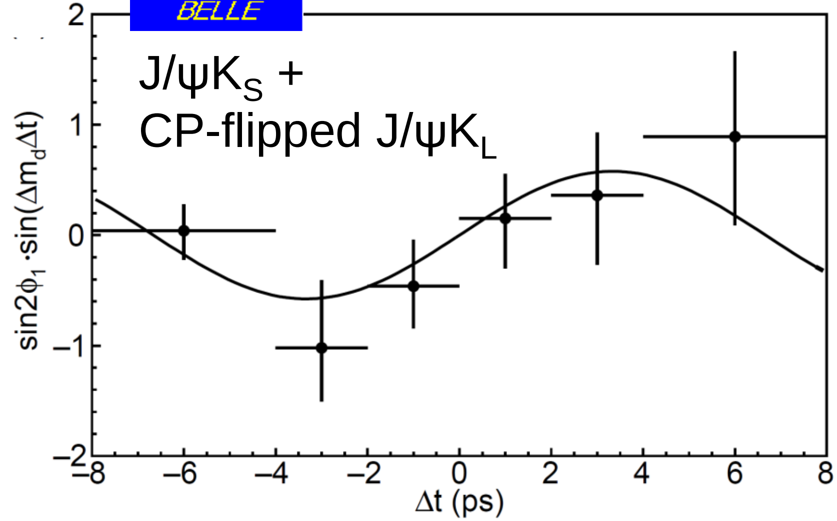
2001 – opening of the age of flavour

2008 Nobel Prize

We can date the start of modern flavour physics to the 2001 measurements of the CP-violating asymmetry in $B^0 \to J/\psi K^0$ decays that give unitarity triangle angle β.



[BaBar, PRL 86 (2001) 2515]



[Belle, PRL 86 (2001) 2509]

These studies, when improved with larger samples, confirmed the CKM paradigm as the dominant mechanism of CP violation in nature (→ 2008 Nobel Prize), and also opened up a rich and wide spectrum of complementary measurements.

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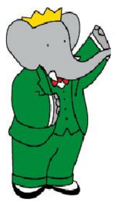
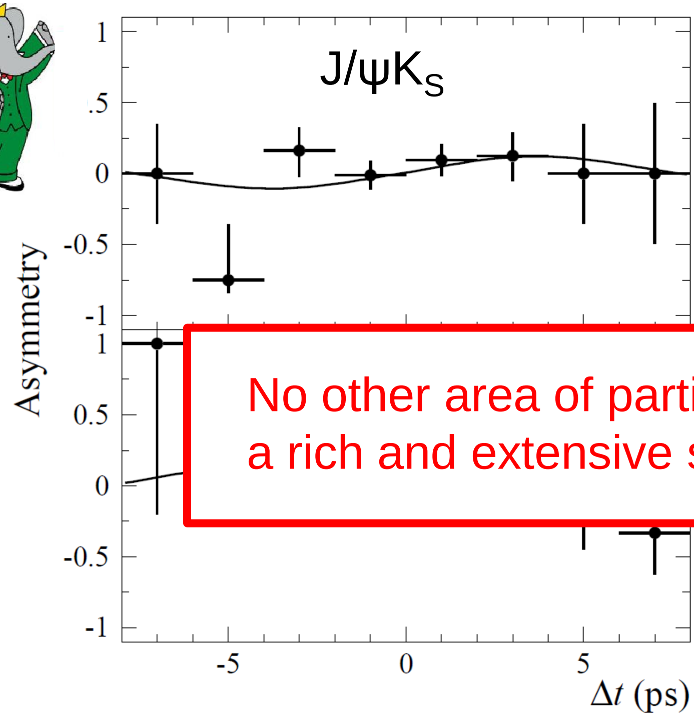
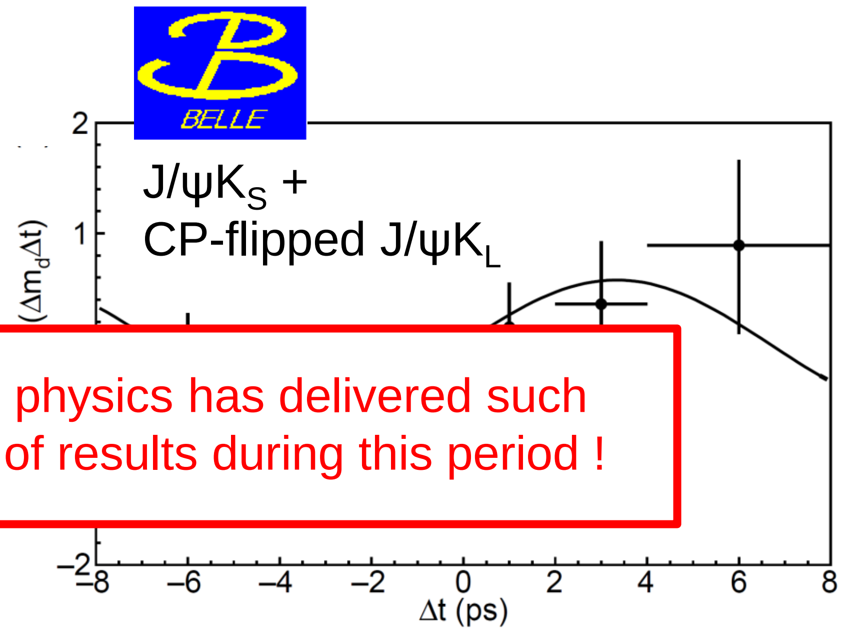






No other area of particle physics has delivered such a rich and extensive set of results during this period !

[BaBar, PRL 86 (2001) 2515]

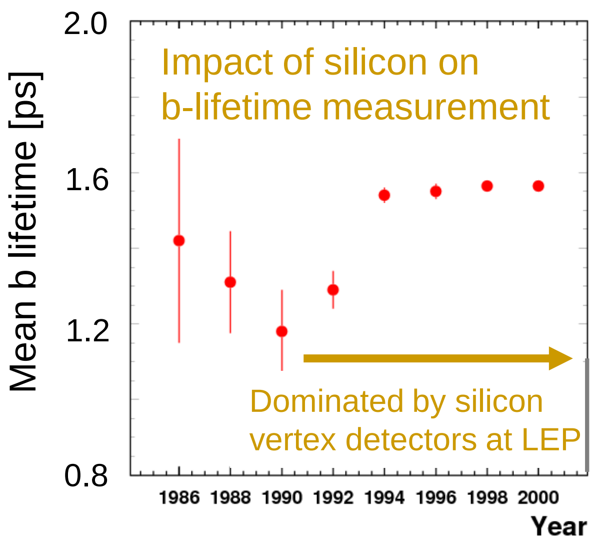
[Belle, PRL 86 (2001) 2509]

These studies, when improved with larger samples, confirmed the CKM paradigm as the dominant mechanism of CP violation in nature (→ 2008 Nobel Prize), and also opened up a rich and wide spectrum of complementary measurements.

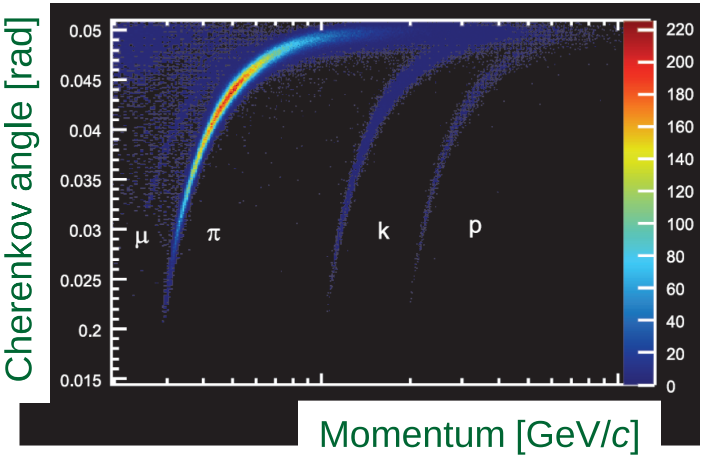
Why have we made progress?

Very important flavour-physics measurements were performed prior to 2001 (*e.g.* at ARGUS, CLEO, the SPS and LEP), but since then there has been an avalanche of results. What has enabled this explosion of progress?

- High-luminosity accelerators with large bbbar production cross-sections;
  - Number of b-hadrons produced at LEP ~ 10^7
  - Number of b-hadrons produced (so far) at LHCb ~ 10^{12}
- Improved and dedicated instrumentation, *e.g.* vertex detectors and RICHes;



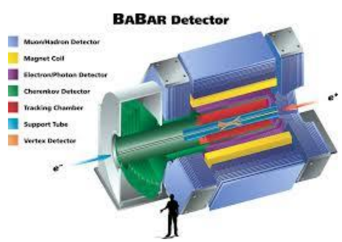
Cherenkov angle vs momentum in LHCb RICH



- Improved triggering, essential for hadron collider experiments;
- And not forgetting progress in theory, in particular lattice QCD.

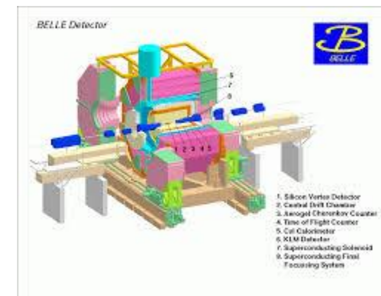
Heroes of the age of flavour

b-factories

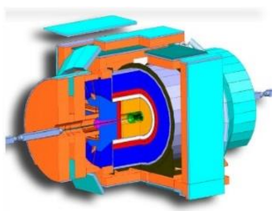


BaBar (SLAC) & Belle (KEK)

Operated in the 2000's
e^+e^- machines with asymmetric beams for time-dep studies, mainly at Y(4S), hence B^0 and B^+ samples.
Considered 'clean' environments.

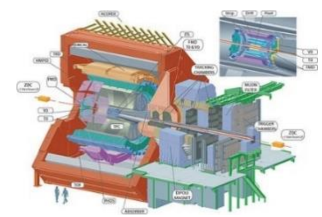


Tevatron experiments



CDF & D0

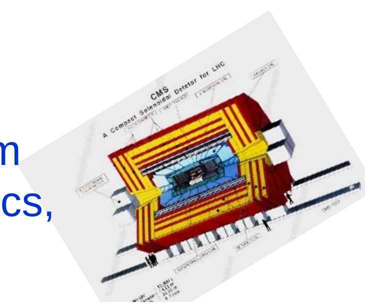
Tevatrons 'general purpose detectors'.
Pioneered *b*-physics in hadronic collisions.
Important early B_s and b-baryon studies.



LHC high-p_T experiments

ATLAS & CMS

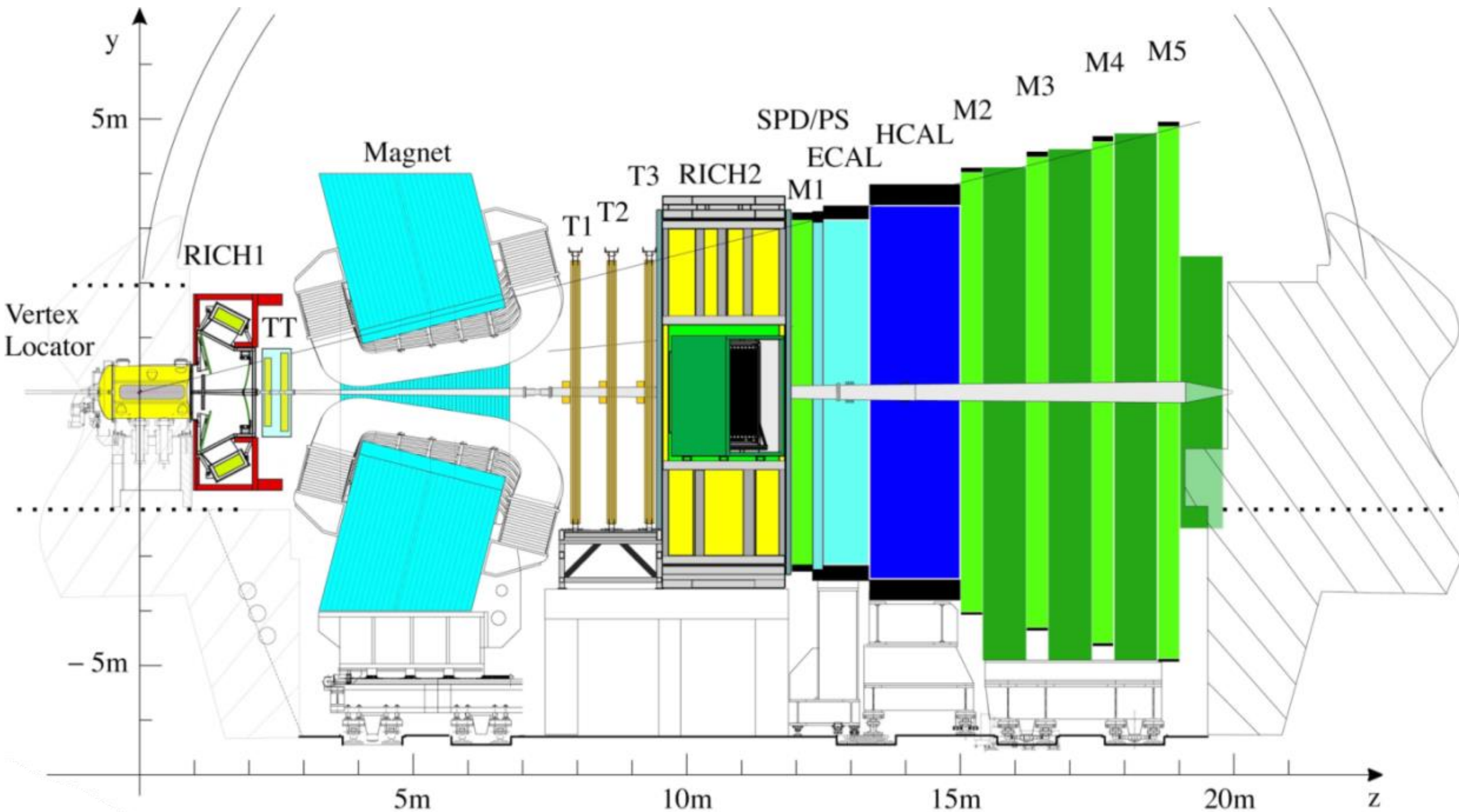
Their excellent instrumentation gives them great capabilities in certain b-physics topics, especially those with dilepton final states.



Important contributions also from BESIII, an e^+e^- experiment in Beijing. Operates below the Y(4S), but provides critical measurements of open charm & spectroscopy (at did CLEO-c).

Heroes of the age of flavour - LHCb

Designed to be a *dedicated* experiment for b- and c-physics at the LHC.



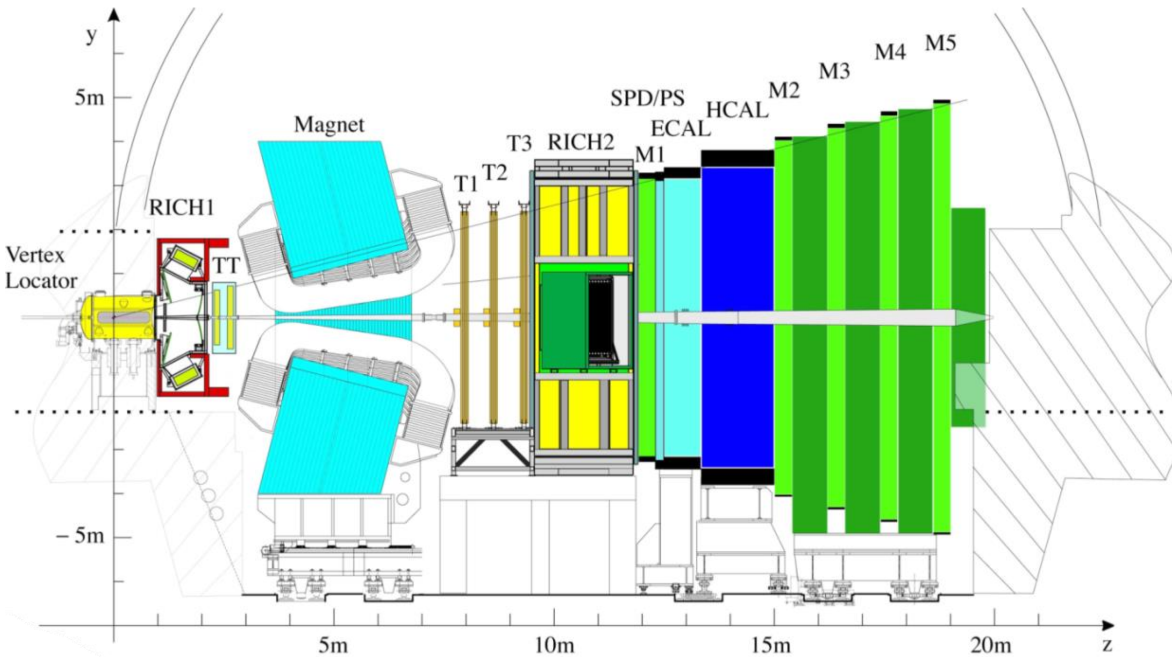
Heroes of the age of flavour - LHCb

Designed to be a *dedicated* experiment for b- and c-physics at the LHC.

Dedicated in the sense of the following attributes:

- Acceptance

  Spectrometer geometry is optimised to capture forward-peaked bbbar production.



Heroes of the age of flavour - LHCb

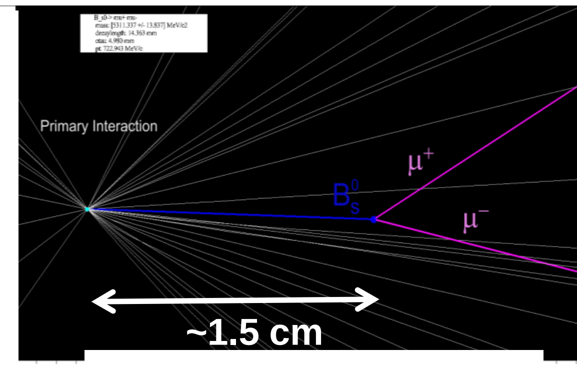
Designed to be a *dedicated* experiment for b- and c-physics at the LHC.

Dedicated in the sense of the following attributes:

- Acceptance
- Instrumentation

Vertex locator (VELO) and RICH system give unique capabilities for b-physics.

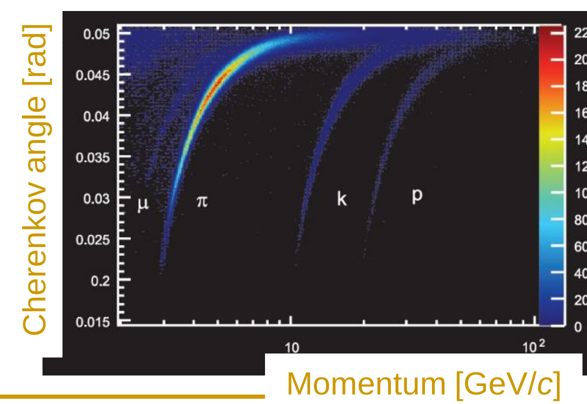
One-half of the VELO under construction



A b-hadron decay vertex

Assembling RICH 2

Array of RICH photodetectors



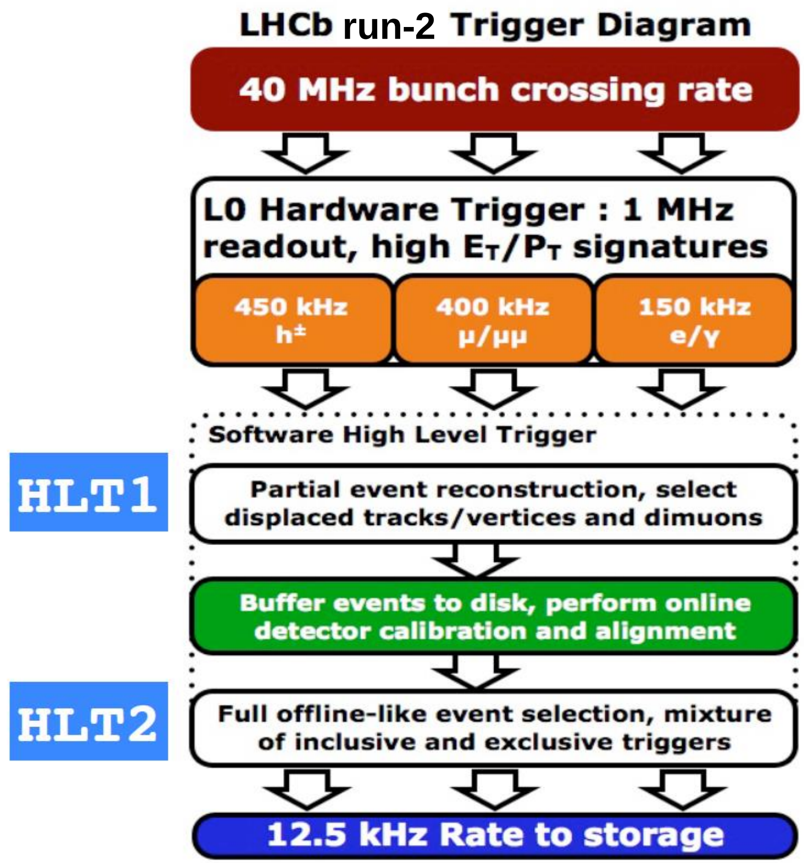
Heroes of the age of flavour - LHCb

Designed to be a *dedicated* experiment for b- and c-physics at the LHC.

Dedicated in the sense of the following attributes:

- Acceptance
- Instrumentation
- Trigger

  Trigger fully optimised for b-physics. Allows lower p_T thresholds than at ATLAS and CMS and ability to select hadronic final states.

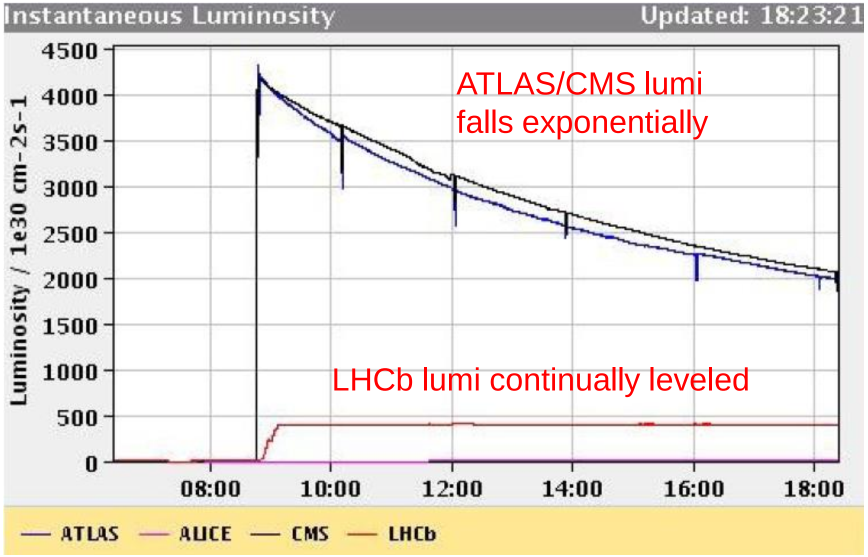


Heroes of the age of flavour - LHCb

Designed to be a *dedicated* experiment for b- and c-physics at the LHC.

Dedicated in the sense of the following attributes:

- Acceptance
- Instrumentation
- Trigger
- Operating luminosity



~4 x 10^{32} $cm^{-2}s^{-1}$

In run 1 & 2 luminosity deliberately set to be lower than at ATLAS & CMS, in order to provide best environment for b-physics measurements.

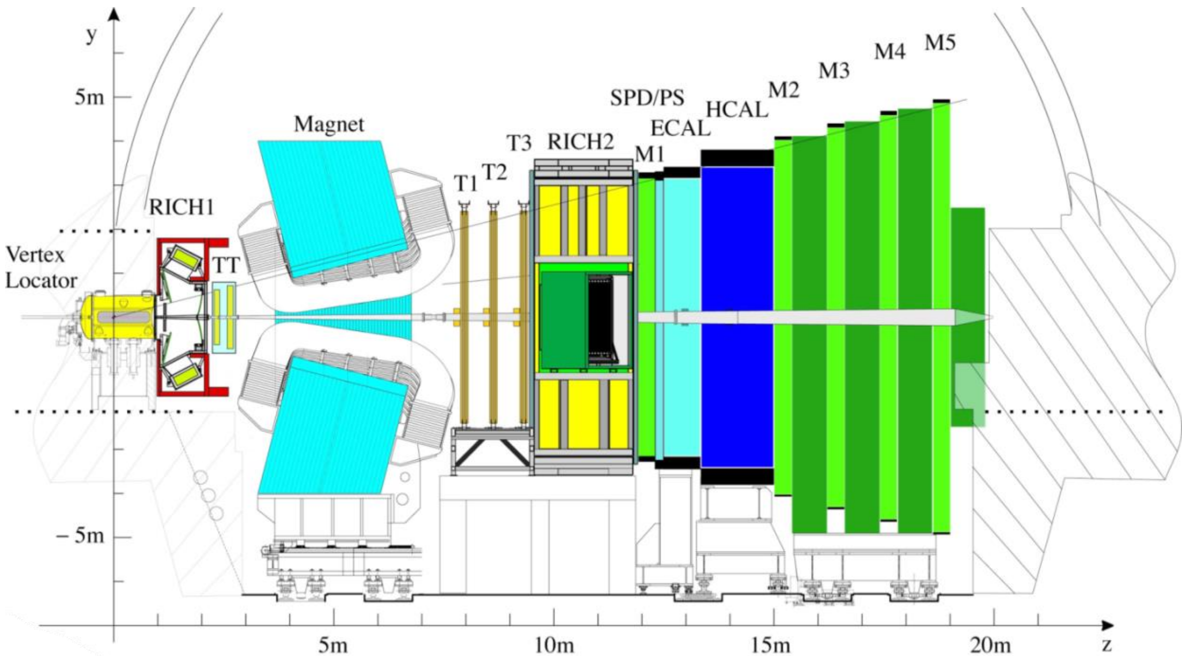
Total data sample from run 1 & run 2 around 9 fb^{-1}.

Heroes of the age of flavour - LHCb

Designed to be a *dedicated* experiment for b- and c-physics at the LHC.

Dedicated in the sense of the following attributes:

- Acceptance
- Instrumentation
- Trigger
- Operating luminosity



(But these attributes allow for important & unique studies beyond flavour, *e.g.* spectroscopy, electroweak, fixed-target proton-gas collisions…).

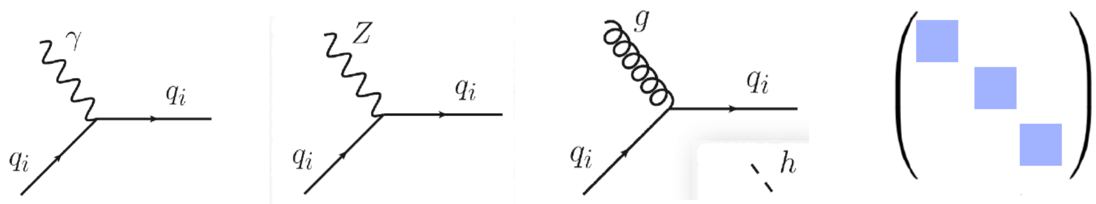
LHCb data-taking is now complete, and an upgraded detector is being installed.

Flavour structure of the Standard Model

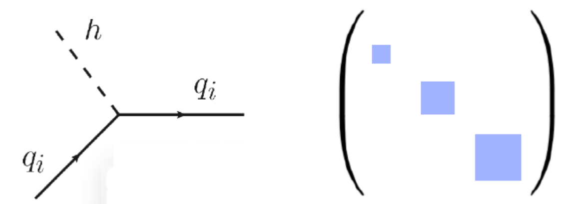
No Flavour-Changing Neutral Currents (FCNCs) at tree level

Neutral currents are flavour conserving at tree level

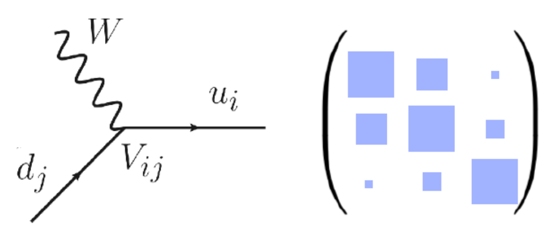
- Photon, gluon, Z have flavour (generation) –universal interactions



- Higgs has flavour-diagonal interactions proportional to quark mass



Whereas only the charged-current W couplings are flavour changing, with a very non-trivial structure → V_{CKM}



Cabibbo-Kobayashi-Maskawa matrix

The CKM matrix appears in the SM Lagrangian as a consequence of diagonalising the mass matrices. Therefore connected to quark masses (& Higgs mechanism).

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

It must be unitarity, *i.e.* $V_{\rm CKM}^{\dagger} V_{\rm CKM} = V_{\rm CKM} V_{\rm CKM}^{\dagger} = 1$, and can be parameterised with three angles and one imaginary phase, which is the origin of SM CPV.

This tight system of four parameters means that CKM physics is highly predictive !

One representation [Chau & Keung, PRL 53 (1984) 1802]:

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

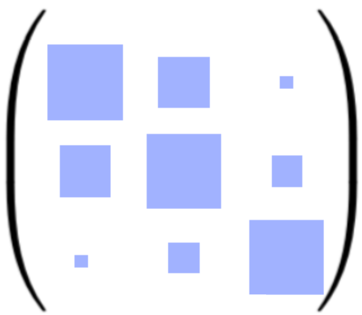
Measurements indicate a striking hierarchy: s_{12}~0.2, s_{23}~0.04, s_{12}~0.004.

Observed hierarchy of CKM matrix

A fit to data, imposing unitarity constraint [PDG review], and showing magnitudes:

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix}$$

or represented graphically:



This is presumably telling us something, but what? (very different picture to one seen in neutrino sector)

Hierarchy motivates an alternative representation based on expansion in $\lambda = \sin\theta_c$.

CKM matrix expressed in Wolfenstein parametrisation

[Wolfenstein, PRL 51 (1983) 1945]

In the Wolfenstein parameterisation the matrix is expanded in orders of $\lambda \sim 0.23$.

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

This is expanded to λ^3, which will be adequate for most of our subsequent discussion, but not all…

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

CKM matrix expressed in Wolfenstein parametrisation

[Wolfenstein, PRL 51 (1983) 1945]

In the Wolfenstein parameterisation the matrix is expanded in orders of λ ~ 0.23.

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & \boxed{V_{ub}} \\ V_{cd} & V_{cs} & V_{cb} \\ \boxed{V_{td}} & V_{ts} & V_{tb} \end{pmatrix}$$

This is expanded to λ^3, which will be adequate for most of our subsequent discussion, but not all…

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \boxed{A\lambda^3(\rho - i\eta)} \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ \boxed{A\lambda^3(1-\rho-i\eta)} & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Note that at order λ^3 only two elements are complex: V_{ub} and V_{td}. Thus transitions involving these vertices will be of great interest in CPV studies (but please don't forget that it is only phase *differences* between transitions that are physical).

Back to FCNCs – although forbidden at tree level, they still occur, albeit suppressed

FCNCs do occur, but through higher-order diagrams

Charged currents

$$BR(K^+ \to \mu^+ \upsilon) = 64\ \%$$

$$BR(D^+ \to K^0 \mu^+ \nu) = 9\ \%$$

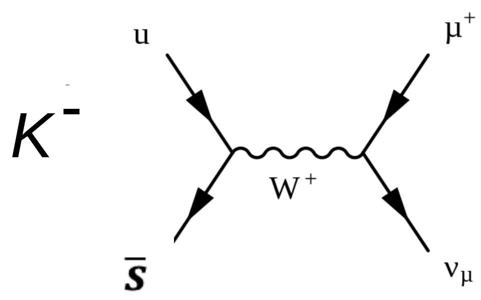
$$BR(B^- \to D^0 l \bar{\upsilon}) = 2.3\ \%$$

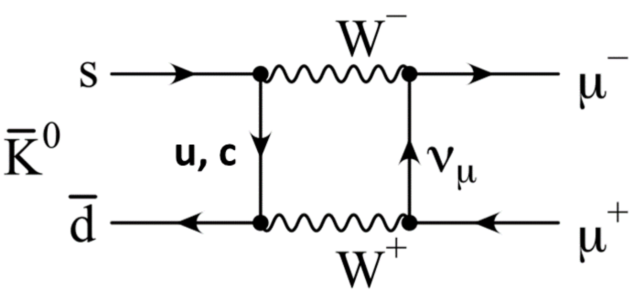
Neutral currents

$$BR(K_L \to \mu^+ \mu^-) = 7\ \times\ 10^{-9}$$

$$BR(D^0 \to \pi^0 \mu^+ \mu^-) < 1.8\ \times 10^{-4}$$

$$BR(B^- \to K^{*-} l^+ l^-) = 5\ \times 10^{-7}$$



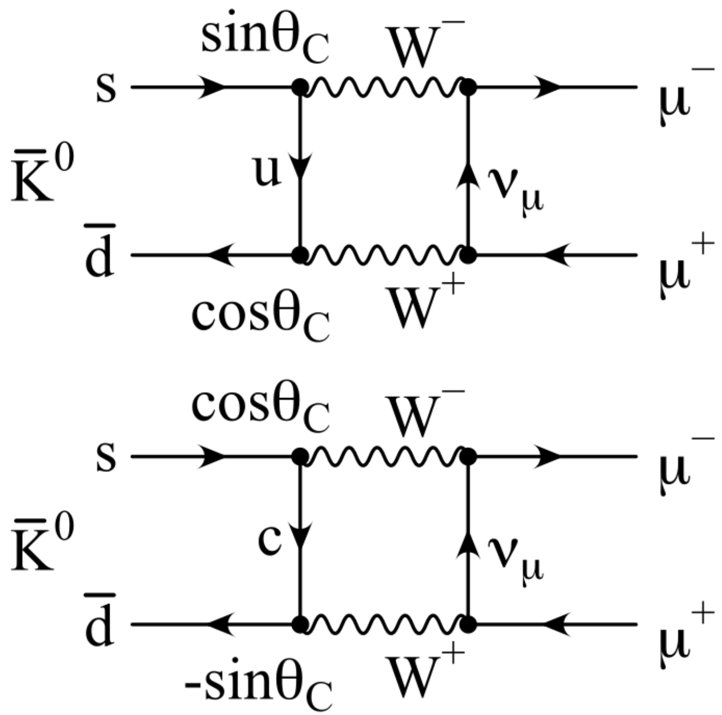


The decay rates of FCNCs tend to be highly suppressed w.r.t. tree-level processes.

Back to FCNCs – although forbidden at tree level, they still occur, albeit suppressed

Suppression of FCNCs is explained by the GIM mechanism:

- Cancellation of diagrams relies on unitarity of V_{CKM}
- Suppression set by the mass-squared difference of the virtual quarks, & would be perfect in the degenerate limit
- GIM, and the smallness of BR($K^0_L \rightarrow \mu^+\mu^-$) led to the prediction of the charm quark



[Glashow, Iliopoulos & Maiani, PRD 2 (1970)1285]

The Unitarity Triangle and CPV measurements

Unitarity Triangles(s)

The CKM matrix must be unitarity: $V_{\mathrm{CKM}}^{\dagger} V_{\mathrm{CKM}} = V_{\mathrm{CKM}} V_{\mathrm{CKM}}^{\dagger} = 1$

This imposes various constraints, including $\sum_k V_{ik} V_{jk}^* = 0$ where $i \neq j$.

The are 6 such independent relations, which can be represented as **unitarity triangles** in the complex plane. Experimentally, the most interesting is:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

As the sides are of similar length, & its parameters can be studied in B^0, B^+ decays.

Another, relevant for B^0_s physics is:

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0$$

Note that the area of all triangles is the same = ½ J, the Jarlskog invariant.

$$J = c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} \sin\delta \approx 3 \times 10^{-5}$$

[Jarlskog, PRL 55 (1985) 1039]

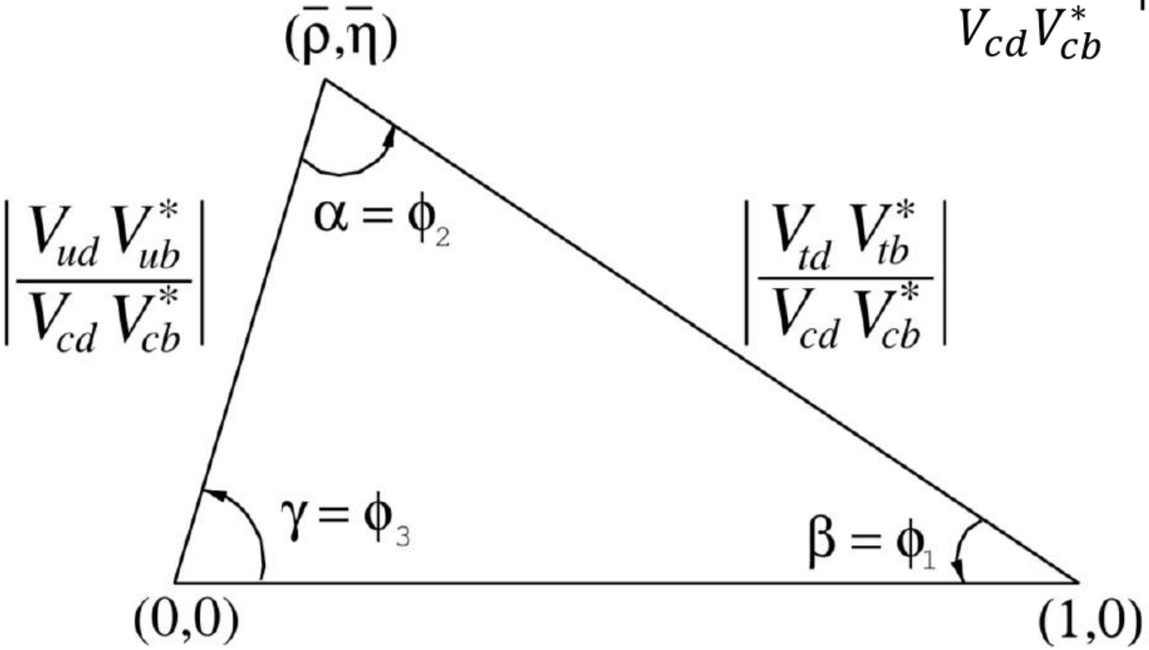
‘The’ Unitarity Triangle

Three complex vectors sum to zero

→ triangle in Argand plane

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$



Expressions for angles:

$$\alpha = \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ub}V_{cb}^*}\right]$$

$$\beta = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

$$\gamma = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

Upper vertex: $\bar\rho + i\bar\eta = -(V_{ud}V_{ub}^*)/(V_{cd}\,V_{cb}^*)$

$\bar\rho = \rho(1 - \lambda^2/2 + \cdots)$ $\bar\eta = \eta(1 - \lambda^2/2 + \cdots)$

(φ_2, φ_1 & φ_3 alternative notation)

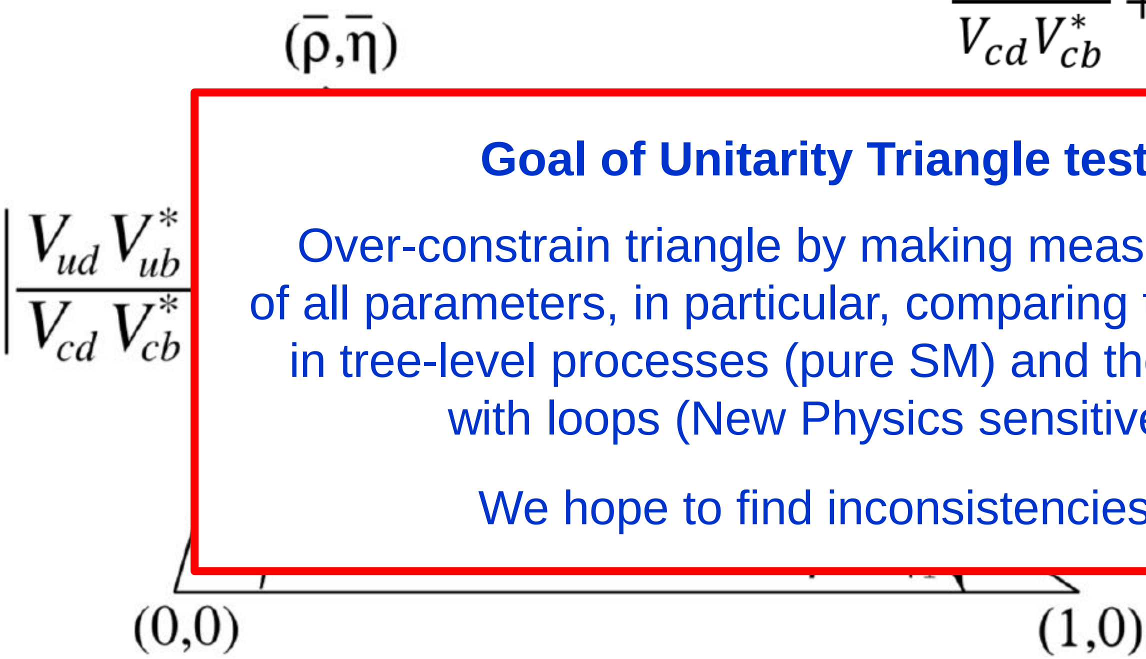
‘The’ Unitarity Triangle

Three complex vectors sum to zero

→ triangle in Argand plane

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$



for angles:

$$\left[-\frac{V_{td}V_{tb}^*}{V_{ub}V_{cb}^*}\right]$$

$$\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

$$\gamma = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

Goal of Unitarity Triangle tests

Over-constrain triangle by making measurements of all parameters, in particular, comparing those made in tree-level processes (pure SM) and those made with loops (New Physics sensitive).

We hope to find inconsistencies !

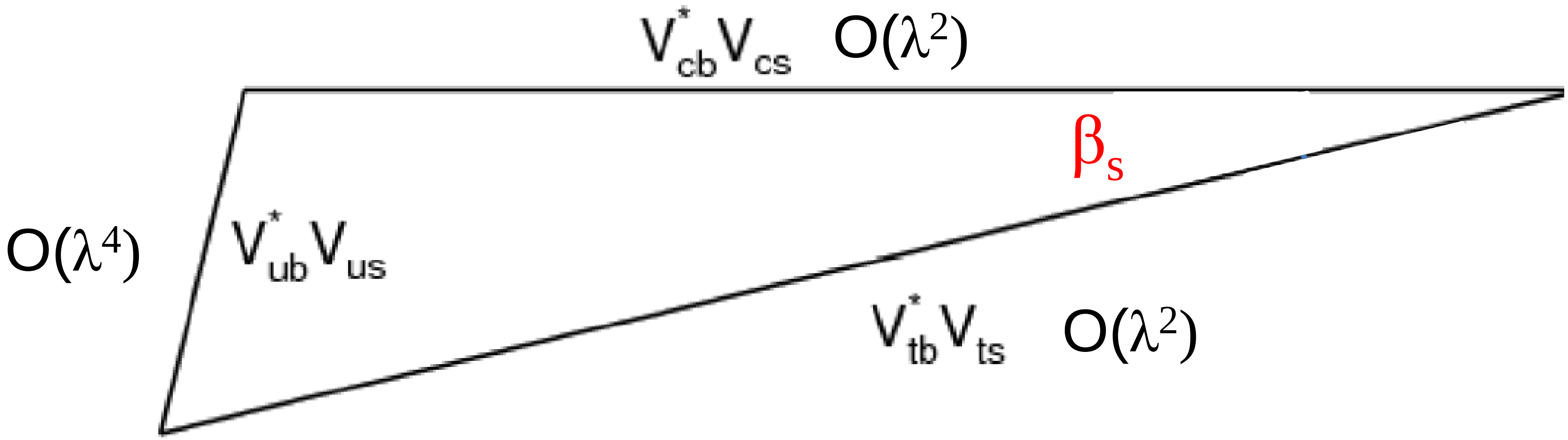
Upper vertex: $\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}\,V_{cb}^*)$

$\bar{\rho} = \rho(1-\lambda^2/2+\cdots)$ $\bar{\eta} = \eta(1-\lambda^2/2+\cdots)$

(φ_2, φ_1 & φ_3 alternative notation)

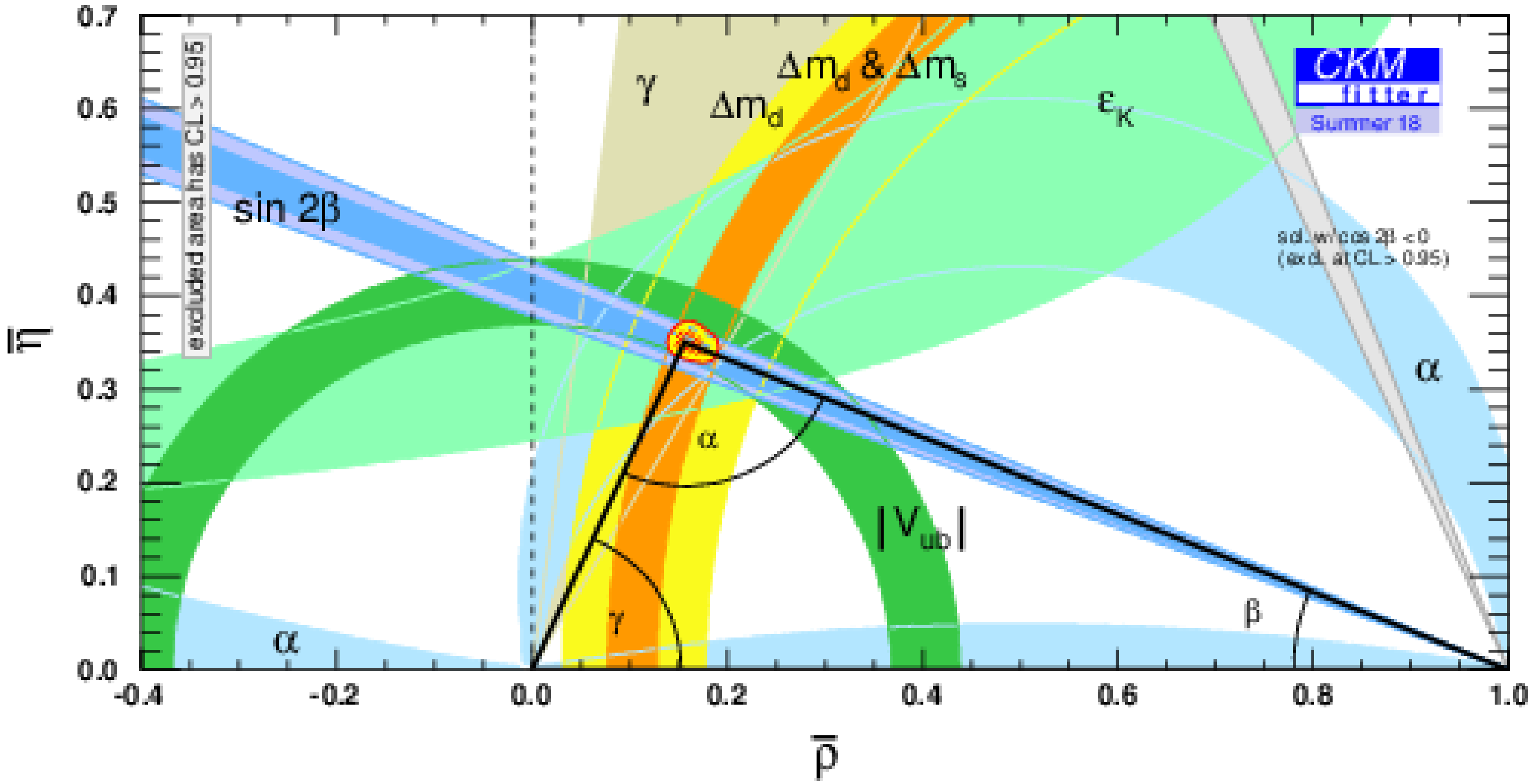
The B^0_s Unitarity Triangle

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$



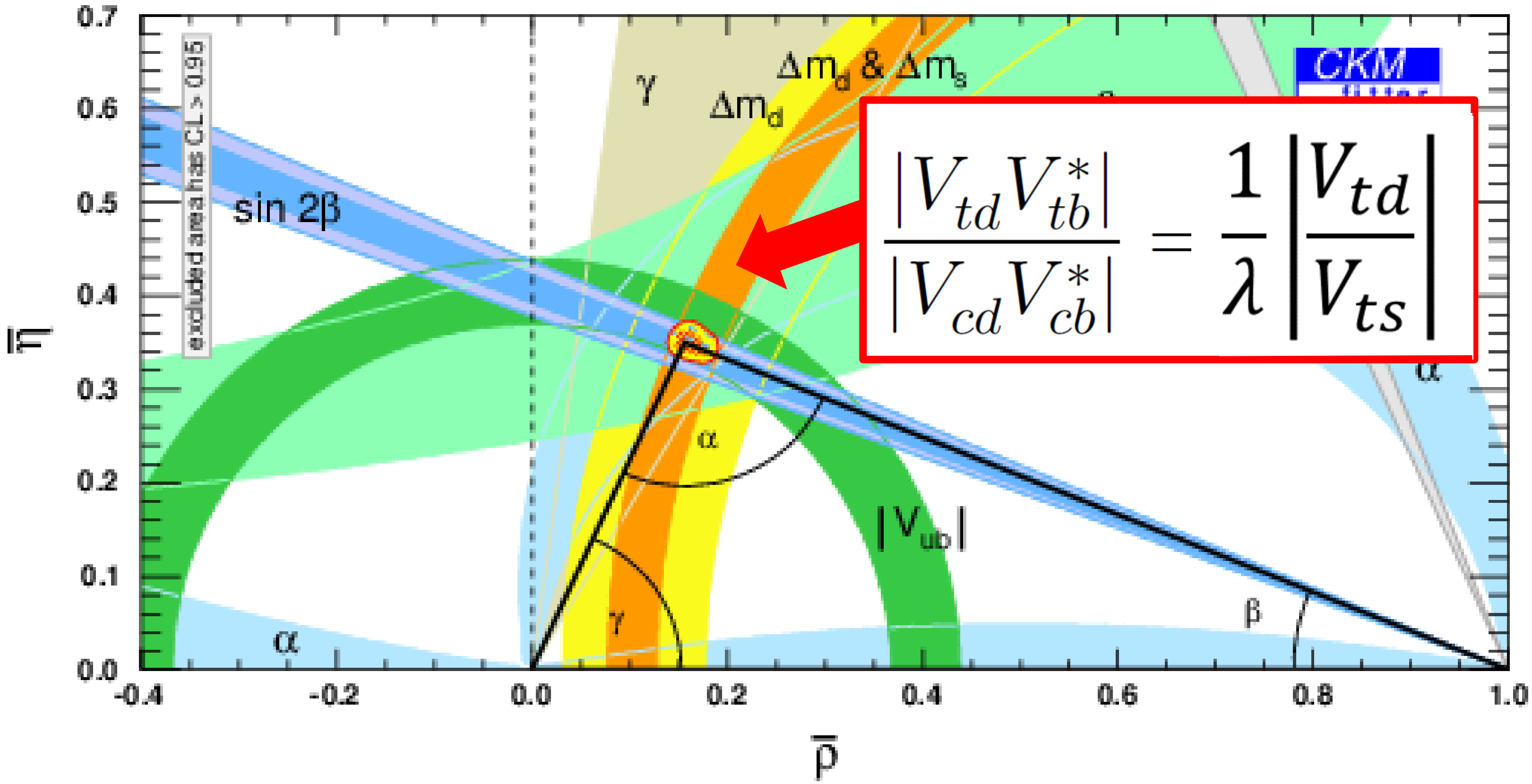
The B^0_s triangle is very squashed, & contains a small angle β_s (= $-\varphi_s/2$ – see later).

The Unitarity Triangle – how do we know what we know ?



[CKMfitter, 2018]

The Unitarity Triangle – how do we know what we know ?



$$\frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{ts}}\right|$$

[CKMfitter, 2018]

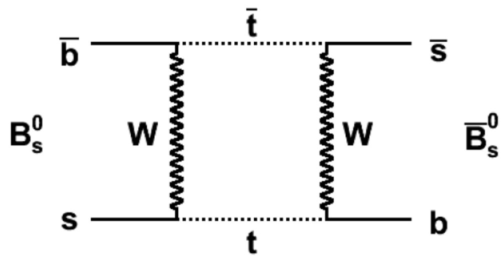
Length of side opposite γ is given by ratio of B^0 & B^0_s mixing freq.s & lattice QCD.

Digression on neutral-meson mixing

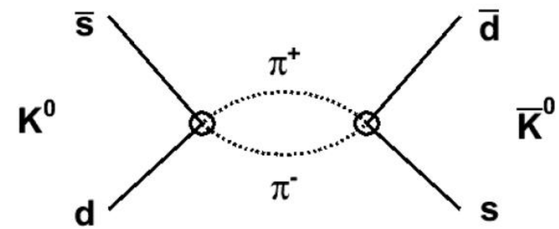
Mixing is critical for much of what follows, so warrants a recap of essentials.

Phenomenon occurs for K^0, D^0, B^0 and B^0_s systems. Physically caused by either

Virtual,
Short-range
(box diagrams)



and/or



On-shell,
long-range
(common
intermediate
states)

Physical states are superposition of flavour eigenstates

Subscripts indicate Short or Long lived (see K^0 system); sometimes Heavy or Light used, or 1, 2.

$$B^0_{S,L} = pB^0 \pm q\overline{B}^0$$

p & q are complex and $|p|^2 + |q|^2 = 1$

If CP is conserved the physical states = CP eigenstates, which means $\left|\frac{q}{p}\right| = 1$.

Known not to be the case in the K^0 system, where $\varepsilon = \frac{p-q}{q-p} \approx 2 \times 10^{-3}$, and the SM calculations indicate small, but finite, breaking in other systems too.

Mass and width splittings between physical states:

$$\Delta m = m_L - m_S$$ set by short-range effects

$$\Delta\Gamma = \Gamma_S - \Gamma_L$$ set by long-range effects

Digression on neutral-meson mixing

There is a wide range in the sizes of the mixing parameters across the four systems, which has significant practical consequences for measurements.

	Δm / Γ		$\Delta\Gamma/2\Gamma$	
K^0	Large	~500	Maximal	~1
D^0	Small	0.39 ± 0.11 %	Small	$0.65 \pm 0.06\%$
B^0	Medium	0.769 ± 0.004	Small	$(20 \pm 5) \times 10^{-4}$
B^0_s	Large	26.81 ± 0.08	Medium	0.0675 ± 0.004

Refs: PDG, HFLAV and [Lenz & Nierste, JHEP 0706 (2007) 072]

Aside: the New Physics flavour puzzle

Remark – mixing parameters are what they are because of SM (CKM, GIM & quark masses) and could easily be perturbed by New Physics, so bounds can be set.

Add to SM Lagrangian higher order terms that would contribute to neutral meson mixing and CPV

$$\Delta\mathcal{L}_{\mathrm{NP}} = \sum_{i\neq j} \frac{c_{ij}^{\mathrm{NP}}}{\Lambda_{\mathrm{NP}}^2} (\overline{Q}_{Li}\gamma^\mu Q_{Lj})^2 ,$$

where c_{ij}^{NP} is the coupling, and Λ_{NP} the mass scale of the New Physics.

If we assuming the coupling is ~1, (*i.e.* generic) obtain the following → bounds on Λ_{NP} [Nir, arXiv:1605.00433].

System	CP-conserving observables	CP-violating Observables
K^0	1×10^3 TeV	2×10^4 TeV
D^0	1×10^3 TeV	3×10^3 TeV
B^0	4×10^2 TeV	8×10^2 TeV
B^0_s	7×10^1 TeV	2×10^2 TeV

These are enormous ! And naturalness told us to expect New Physics at the TeV scale. Something is wrong…

Get out clause: couplings are not ~1. One possibility, structure is more specific *e.g.* same as in the SM ('minimal flavour violation' [Ambrosio *et al.*, NPB 645 (2002) 155]).

Digression on neutral-meson mixing

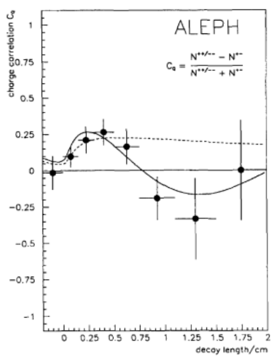
Mixing leads to an oscillation of probability to observe meson in either flavour eigenstate with proper time, *e.g.* if at *t*=0 we have a B^0, then at later time *t*:

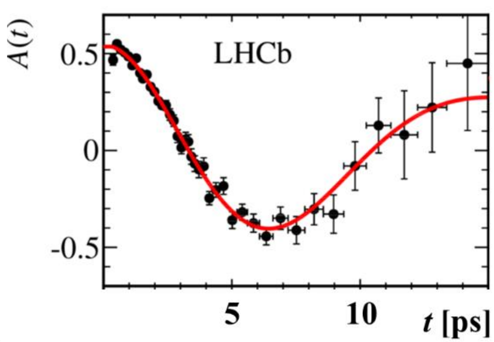
Prob. to decay as $\begin{matrix}\overline{B}^0 \\ B^0\end{matrix} \propto e^{-\Gamma_d t}(1 \mp \cos \Delta m_d\, t)$

Time-integrated B-oscillations were first observed by UA1 [PLB 186 (1987) 247] & ARGUS [PLB 192 (1987) 245]. B^0 (B^0_s) oscillations first resolved by ALEPH (CDF).

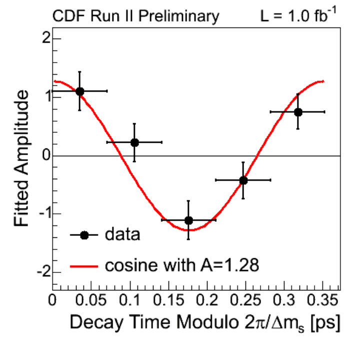
B^0 discovery → state-of-the-art B^0_s discovery → state-of-the-art

[PLB 313 (1993) 498]

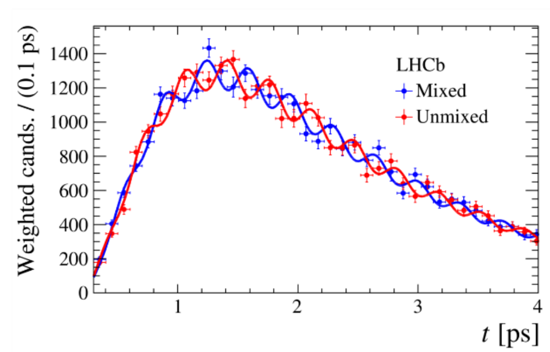




[EPJC 76 (2016) 412]



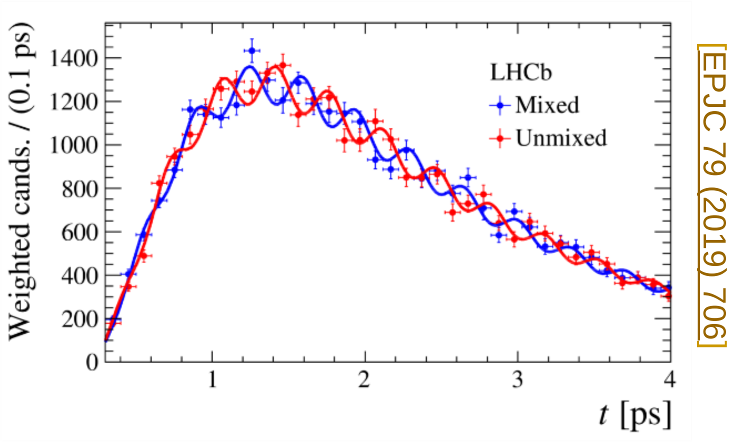
[PRL 97 (2006) 242003]

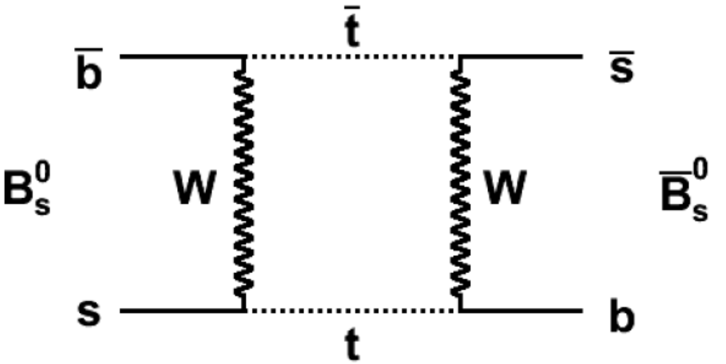


[EPJC 79 (2019) 706]

$B^0_{(s)}$-$\bar{B}^0_{(s)}$ mixing – accessing CKM elements

In B^0 and B^0_s systems, mixing driven by $\Delta m_{d(s)}$ and is calculable in SM.





Depends on CKM elements in box & factors that can be calculated in lattice QCD.

For B^0_s case ⟶

$$\Delta m_s = \frac{G_F^2}{6\pi^2} m_{B_s} m_W^2 \eta_B S_0(x_t) f_{B_s}^2 B_s \left|V_{ts} V_{tb}^*\right|^2$$

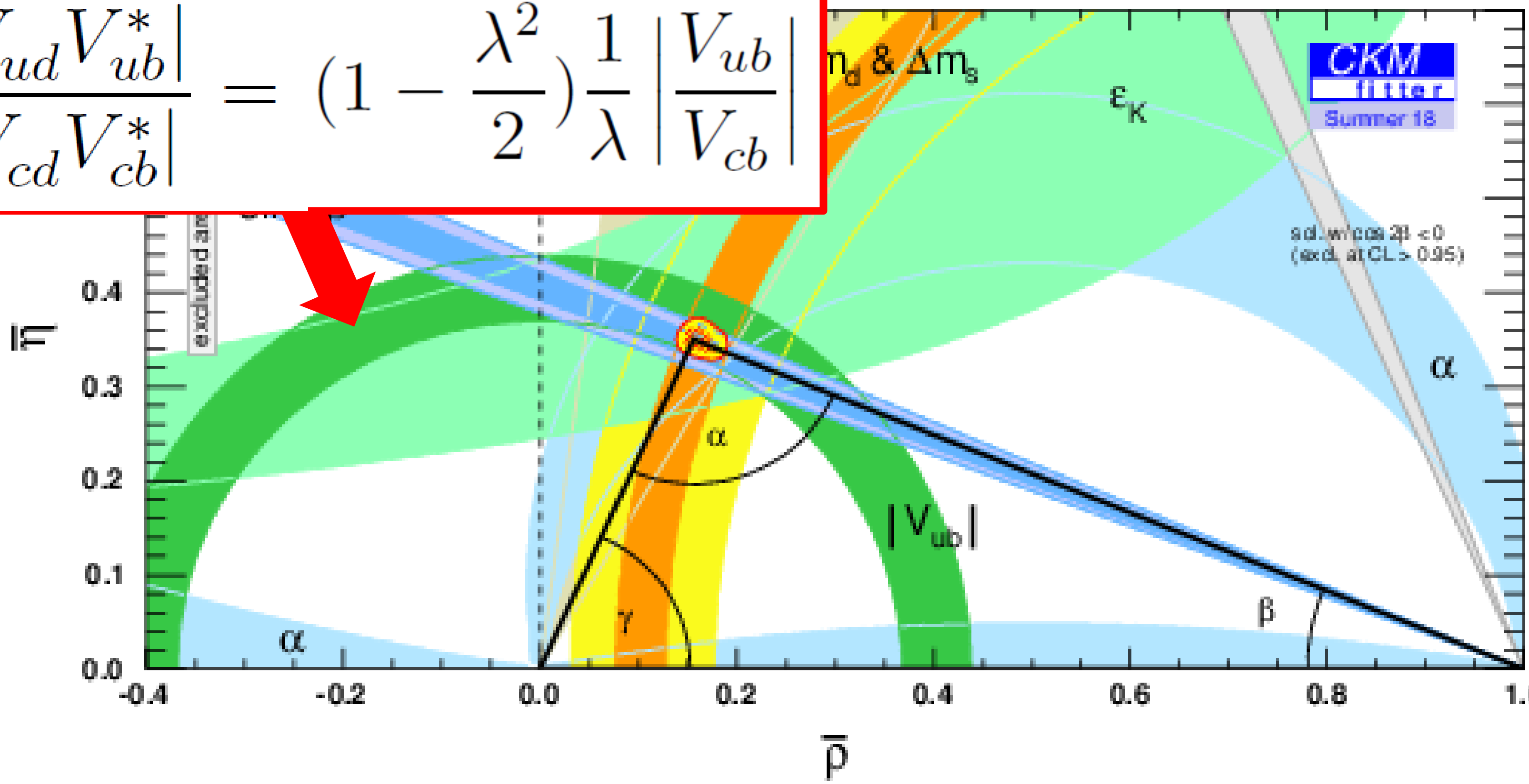
Equivalent expression for B^0 mixing, involving V_{td}. Ratio of frequencies is then

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{Bd}}{m_{Bs}} \xi_{\Delta m}^{-2} \frac{|V_{td}|^2}{|V_{ts}|^2}$$

$\zeta_{\Delta m}$, being a ratio of QCD factors of value close to 1 can be calculated to a few % in lattice QCD, hence giving access to $|V_{td}|/|V_{ts}|$.

The Unitarity Triangle – how do we know what we know ?

$$\frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$



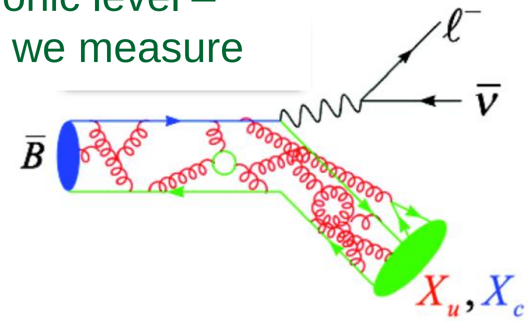
[CKMfitter, 2018]

Length of side opposite β is given by measuring $|V_{ub}|/|V_{cb}|$ from ratio b→u / b→c.

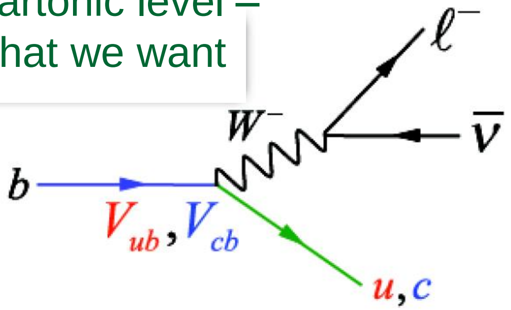
Measuring $|V_{ub}|/|V_{cb}|$

We can measure the ratio of b→ulν to b→clν processes at hadron level, but then must use theory or lattice QCD to correct back to quark level.

Hadronic level –
what we measure



Partonic level –
what we want



Two broad strategies followed:

- Inclusive $b \to X_u l \nu$, using *e.g.* endpoint of p_l spectrum to isolate signal from $b \to X_c l \nu$

$$|V_{ub}| = (4.49 \pm 0.28) \times 10^{-3}$$ [2018 PDG review]

- Exclusive, *e.g.* $B \to \pi l \nu$. But then need calculation of hadronic form factor.

$$|V_{ub}| = (3.70 \pm 0.16) \times 10^{-3}$$ [2018 PDG review]

There is tension between these two numbers at the ~2.5σ level, which means that a conservative approach is advisable when using the results to set UT constraints.

Much activity underway to understand this issue, & we can be hopeful of progress !