Useful resources & acknowledgments

- Heavy Flavour Averaging Group (HFLAV)  [https://hflav.web.cern.ch]
- CKMfitter  [ckmfitter.in2p3.fr]  Utfit  [www.utfit.org/UTfit/]
- Particle Data Group reviews  [pdg.lbl.gov]
- Books:  
  - CP violation, I.I. Bigi and A.I. Sanda (CUP, 2000)
  - CP violation, G.C. Branco, L. Lavoura & J.P. Silva (OUP, 1999)
- Reviews & lectures:  
  - M. Blanke, [arXiv:1704.03753]
  - J.F. Kamenik, [arXiv:1708.00771]
  - Z. Ligeti, [arXiv:1502.01372]

Thanks to flavour lecturers at this school in previous years, who provided inspiration for some of the material shown (esp. T. Gerson, J. Zupan & M-H. Schune).
What is flavour physics and why should we care?
What is flavour physics?

The concept of ‘flavour’ in particle physics relates to the existence of different families of quarks, and how they couple to each other.

\[ \text{i.e. 6 known flavours of quark, grouped into 3 generations} \]

Open questions:

- why 3 generations?
- why do the quarks exhibit this striking hierarchy in mass?

No answer yet!

These mysteries make the ‘flavour sector’ of the Standard Model of great interest.
In the Standard Model quarks can only change flavour through emission of a W boson (i.e. weak force). For example a t quark can decay into a b, s or d quark:

$$t \xrightarrow{V_{tb}} b$$  
$$t \xrightarrow{V_{ts}} s$$  
$$t \xrightarrow{V_{td}} d$$

But these decays are not equally likely. At the amplitude level they are weighted by factors that are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and these factors vary dramatically – here is another hierarchy we don’t understand!

$$\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
0.9705 - 0.9770 & 0.21 - 0.24 & 0 - 0.014 \\
0.21 - 0.24 & 0.971 - 0.973 & 0.036 - 0.070 \\
0 - 0.014 & 0.036 - 0.070 & 0.997 - 0.999
\end{pmatrix}$$

These elements of the CKM matrix are also fundamental parameters of the Standard Model. Why they have these values is another great mystery.
Parameters of the Standard Model

- 3 gauge couplings
- 2 Higgs parameters
- strong CP parameter $\theta$
- 6 quark masses
- 3 quark mixing angles + 1 phase \textit{[i.e. CKM matrix]}
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase \textit{[i.e. PMNS matrix]})

() = with Dirac neutrino masses
Parameters of the Standard Model

- 3 gauge couplings
- 2 Higgs parameters
- strong CP parameter $\theta$
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\(\Rightarrow\) with Dirac neutrino masses

These are all flavour parameters!
Parameters of the Standard Model

- 3 gauge couplings
- 2 Higgs parameters
- strong CP parameter $\theta$
- 6 quark masses
- 3 quark mixing angles + 1 phase \([i.e. \text{CKM matrix}]\)
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase \([i.e. \text{PMNS matrix}]\))

() = with Dirac neutrino masses

This is of particular relevance…
CP violation

CP violation (CPV) → difference in behaviour between matter and anti-matter.

First discovered in the kaon system in 1964, opportunities of study were limited until colliders arrived that could make lots & lots of b-quark hadrons, e.g. the LHC

A recent example from LHCb - look at B meson decaying into a pion & two kaons…

…the decay probabilities are manifestly different for $B^- \text{ & } B^+$. In the Standard Model CPV is accommodated, but not explained, by an imaginary phase in the CKM matrix.
As first pointed out by Andrei Sakharov, CP-violation is one requirement for explaining *baryogenesis* – the process that took us from the equal amounts of matter and anti-matter produced in the Big Bang, to the matter dominated universe of today.

The problem is that the CP-violation that appears in the Standard Model, is woefully inadequate to explain the matter-antimatter asymmetry we have today.

This is a big problem with the Standard Model!

More & better measurements may point a way forward.
Problems with the Standard Model

The Standard Model (SM) cannot be a final theory

We have already encountered the following shortcomings:

- No explanation for baryogenesis
- No explanation for the quark or CKM hierarchy
- No real explanation for CP violation, and why it is only found in the weak interaction.

And there are plenty of others, for example:

- No explanation for dark matter or dark energy
- No explanation for neutrino masses
- Gravity not included
- No explanation for why the Higgs boson has the mass it does (left to itself the theory would make it much, much heavier)

More ambitious theories (e.g. supersymmetry or SUSY) can solve at least some of these problems. They generally predict new particles or effects outside the SM. Finding these effects is the goal of the LHC & many other present/planned facilities!
Breaching the walls of the Standard Model

The HEP community is searching for ‘New Physics’ - to find this we need to penetrate the walls of the Standard Model fortress. There are two strategies used in this search.

Direct

Use the high energy of, e.g. the LHC to produce the New Physics particles, which we then detect.

Indirect

Make precise measurements of processes in which New Physics particles enter through ‘virtual loops’

Both methods are powerful. Flavour physics follows the ‘indirect’ approach.
Indirect measurements – an established tradition in science

Eratosthenes was able to determine the circumference of the earth using indirect means...
Indirect measurements – an established tradition in science

Eratosthenes was able to determine the circumference of the earth using indirect means…

…around 2.2 thousand years prior to the direct observation.
Indirect measurements – an established tradition in science

In flavour physics the guiding principle is to probe processes where loop diagrams are important, as here non-SM particles may contribute.

(but as we will see, tree-mediated decays also have their role to play)

Indirect search principle

Precise measurements of low energy phenomena tells us about unknown physics at energies far beyond direct searches (\(~10^4\) TeV in some cases)
Indirect measurements – an established tradition in science

In flavour physics the guiding principle is to probe processes where loop diagrams are important, as here non-SM particles may contribute (but as we will see, tree-mediated decays also have their role to play).

Indirect search principle  Precise measurements of low energy phenomena tells us about unknown physics at energies far beyond direct searches (~$10^4$ TeV in some cases).
Indirect measurements – an established tradition in science

Indirect search principle $\rightarrow$ Precise measurements of low energy phenomena tell us about unknown physics at energies far beyond direct searches ($\sim 10^4 \text{ TeV}$ in some cases)

For this reason its rather surprising that (spoiler alert !) most flavour measurements so far agree with the SM, as naturalness told us New Physics is expected at TeV scale $\rightarrow$ the New Physics Flavour Puzzle.

Either, there is something specific about the flavour-structure of the New Physics that is masking the effect…

…or we have put too much trust in naturalness.

Either way, flavour is central to the story !
Outline of the lecture contents and schedule
Flavour topics that we won’t be covering

Flavour encompasses a huge range of areas of study & corresponding experimental activity. The following are genuine topics of flavour, but ones we will not cover.

- Kaon physics  (OK, we will say a little, but not really do it justice)
- Suppressed top decays
- Flavour and CPV violation in the Higgs sector
- Charged lepton-flavour violation, e.g. $\mu \rightarrow e\gamma$
- (g-2) muon anomaly
- All neutrino physics

Instead we will focus on beauty physics, with some discussion on charm.
Lecture outline

• Introduction ✓
• Birth of flavour physics & the kaon sector
• The beautiful millennium
• Flavour structure of the SM
• The Unitarity Triangle and CPV measurements
• Spectroscopy (a brief digression)
• FCNCs or ‘rare decays’
• Charm physics
• Future of flavour

Note the approach will (necessarily) be from an experimentalist’s perspective.
The birth of experimental flavour physics and the (continuing) importance of kaon studies
Events of 1964

Nelson Mandela sentenced to life imprisonment

Cassius Clay becomes heavyweight champion of the world

Martin Luther King Jnr. wins Nobel Peace Prize

Change of face in the Kremlin

Nelson Mandela sentenced to life imprisonment
EVIDENCE FOR THE 2π DECAY OF THE $K^0_2$ MESON*†
J. H. Christenson, J. W. Cronin, ‡ V. L. Fitch, ‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

This Letter reports the results of experimental studies designed to search for the $2\pi$ decay of the $K^0_2$ meson. Several previous experiments have served to set an upper limit of 1/300 for the fraction of $K^0_2$'s which decay into two charged pions. The present experiment, using spark chamber techniques, proposed to extend this limit.

In this measurement, $K^0_2$ mesons were produced at the Brookhaven AGS in an internal Be target bombarded by 30-BeV protons. A neutral beam was defined at 30 degrees relative to the circulating protons by a 1 1/2-in. × 1 1/2-in. × 48-in. collimator at an average distance of 14.5 ft. from the internal target. This collimator was followed by a sweeping and a 6-in. × 48-in. 1 1/2-in. thick first collimator which led to the beam.

The experimental layout is shown in relation to the beam in Fig. 1. The detector for the decay of the $K^0_2$ mesons produced by coherent regeneration in 43 gm/cm² of tungsten. Since the $K^0_2$ mesons produced by coherent regeneration

Discovery of CP violation (in kaon decays)
Nobel Prize for physics in 1980

Events of 1964
Discovery of CP violation

Observation of $45 \pm 10 \pi^+\pi^-$ decays in a $K^0_L$ beam [Christenson et al., PRL 13 (1964) 138].

Interpretation: $K^0_L$ not a pure CP-odd eigenstate. Level of CP-even ‘contamination’ given by $\varepsilon$, which is now measured to be $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$ [PDG].
The heroic quest for $\varepsilon'$

In the CKM paradigm $K^0_L \to \pi\pi$ is readily explained as *indirect* CPV. CKM also allows for the possibility of *direct* CPV, which can be revealed by measuring the relative rates of $K^0_S$ and $K^0_L$ into $\pi^+\pi^-$ and $\pi^0\pi^0$, which give the parameter $\text{Re}(\varepsilon'/\varepsilon)$.

A non-zero $\text{Re}(\varepsilon'/\varepsilon)$ implies direct CPV, & is consistent with CKM picture. Historically, other models (*e.g.* superweak [*Wolfenstein, PRL 13 (1964) 562*]) predicted zero direct CPV.

Effect is very small, experiment is hard, and first measurements were ambiguous.
The heroic quest for $\epsilon'$

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Effect is very small, experiment is hard, and first measurements were ambiguous. A second round of experiments (NA48, KTeV) was required to show $\text{Re}(\epsilon'/\epsilon) \neq 0$.

Lattice QCD prediction not as precise as experiment, but progress being made. Heroic work! Kaon physics is very difficult - small effects & theoretically challenging.
In search of the ultra-rare

CP violation is not the whole story. Kaons system is also well suited for searches for forbidden or ultra-suppressed decays, the most topical of which is $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. In SM $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (8.39 \pm 0.30) \times 10^{-11}$ [Buras et al., JHEP 1511 (2015) 033], but New Physics enhancements possible with sensitivity to mass scales under 100 TeV.

BNL experiments E949 & E787 saw 3 events [PRD 77 (2008) 052003], consistent with SM. NA62, here at CERN, aims to observe ~100 and make precise measurement of BR.

Pilot measurement released [PLB 791 (2019) 156], based on a few weeks of data taking.
In search of the ultra-rare

CP violation is not the whole story. Kaons system is also well suited for searches for forbidden or ultra-suppressed decays, the most topical of which is $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

New Physics enhancements possible with sensitivity to mass scales under 100 TeV.

In search of the ultra-rare

Kaon studies have played a critical role in the development of flavour physics. They will continue to do so in future.

However, in the past 2-3 decades the focus has been on beauty:

• A huge number of decays and processes to explore
• Sizable CPV effects expected (& observed)
• In many cases theoretically clean predictions are available

Pilot measurement released [PLB 791 (2019) 156], based on a few weeks of data taking.
We live in a golden age of flavour!

An introduction to the experiments of B physics
We can date the start of modern flavour physics to the 2001 measurements of the CP-violating asymmetry in $B^0 \rightarrow J/\psi K^0$ decays that give unitarity triangle angle $\beta$.

These studies, when improved with larger samples, confirmed the CKM paradigm as the dominant mechanism of CP violation in nature (→ 2008 Nobel Prize), and also opened up a rich and wide spectrum of complementary measurements.
2001 – opening of the age of flavour

We can date the start of modern flavour physics to the 2001 measurements of the CP-violating asymmetry in $B^0 \to J/\psi K^0$ decays that give unitarity triangle angle $\beta$.


These studies, when improved with larger samples, confirmed the CKM paradigm as the dominant mechanism of CP violation in nature (→ 2008 Nobel Prize), and also opened up a rich and wide spectrum of complementary measurements.

No other area of particle physics has delivered such a rich and extensive set of results during this period!
Very important flavour-physics measurements were performed prior to 2001 (e.g. at ARGUS, CLEO, the SPS and LEP), but since then there has been an avalanche of results. What has enabled this explosion of progress?

- High-luminosity accelerators with large $b\bar{b}$ production cross-sections;
  - Number of $b$-hadrons produced at LEP $\sim 10^7$
  - Number of $b$-hadrons produced (so far) at LHCb $\sim 10^{12}$
- Improved and dedicated instrumentation, e.g. vertex detectors and RICHes;
- Improved triggering, essential for hadron collider experiments;
- And not forgetting progress in theory, in particular lattice QCD.
Heroes of the age of flavour

b-factories

BaBar (SLAC) & Belle (KEK)

Operated in the 2000’s e⁺e⁻ machines with asymmetric beams for time-dep studies, mainly at Y(4S), hence B⁰ and B⁺ samples. Considered ‘clean’ environments.

Tevatron experiments

CDF & D0


LHC high-pₜ experiments

ATLAS & CMS

Their excellent instrumentation gives them great capabilities in certain b-physics topics, especially those with dilepton final states.

Important contributions also from BESIII, an e⁺e⁻ experiment in Beijing. Operates below the Y(4S), but provides critical measurements of open charm & spectroscopy (at did CLEO-c).
Heroes of the age of flavour - LHCb

Designed to be a *dedicated* experiment for b- and c-physics at the LHC.
Heroes of the age of flavour - LHCb

Designed to be a dedicated experiment for b- and c-physics at the LHC.

Dedicated in the sense of the following attributes:

• Acceptance

Spectrometer geometry is optimised to capture forward-peaked $b\bar{b}$ production.
Heroes of the age of flavour - LHCb

Designed to be a *dedicated* experiment for b- and c-physics at the LHC.

Dedicated in the sense of the following attributes:

- Acceptance
- Instrumentation

Vertex locator (VELO) and RICH system give unique capabilities for b-physics.

- One-half of the VELO under construction
- A b-hadron decay vertex
- Assembling RICH 2
- Array of RICH photodetectors
Heroes of the age of flavour - LHCb

Designed to be a *dedicated* experiment for b- and c-physics at the LHC.

Dedicated in the sense of the following attributes:

- Acceptance
- Instrumentation
- Trigger

Trigger fully optimised for b-physics. Allows lower $p_T$ thresholds than at ATLAS and CMS and ability to select hadronic final states.
Heroes of the age of flavour - LHCb

Designed to be a dedicated experiment for b- and c-physics at the LHC.

Dedicated in the sense of the following attributes:

- Acceptance
- Instrumentation
- Trigger
- Operating luminosity

In run 1 & 2 luminosity deliberately set to be lower than at ATLAS & CMS, in order to provide best environment for b-physics measurements.

Total data sample from run 1 & run 2 around 9 fb$^{-1}$.
Heroes of the age of flavour - LHCb

Designed to be a *dedicated* experiment for b- and c-physics at the LHC.

Dedicated in the sense of the following attributes:

- Acceptance
- Instrumentation
- Trigger
- Operating luminosity

(But these attributes allow for important & unique studies beyond flavour, *e.g.* spectroscopy, electroweak, fixed-target proton-gas collisions…).

LHCb data-taking is now complete, and an upgraded detector is being installed.
Flavour structure of the Standard Model
Neutral currents are flavour conserving at tree level

- Photon, gluon, Z have flavour (generation) – universal interactions

- Higgs has flavour-diagonal interactions proportional to quark mass

Whereas only the charged-current $W$ couplings are flavour changing, with a very non-trivial structure $\to V_{\text{CKM}}$. 
Cabibbo-Kobayashi-Maskawa matrix

The CKM matrix appears in the SM Lagrangian as a consequence of diagonalising the mass matrices. Therefore connected to quark masses (& Higgs mechanism).

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

It must be unitarity, i.e. $V_{\text{CKM}}^\dagger V_{\text{CKM}} = V_{\text{CKM}}V_{\text{CKM}}^\dagger = 1$, and can be parameterised with three angles and one imaginary phase, which is the origin of SM CPV.

This tight system of four parameters means that CKM physics is highly predictive!

One representation [Chau & Keung, PRL 53 (1984) 1802]:

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Measurements indicate a striking hierarchy: $s_{12} \sim 0.2$, $s_{23} \sim 0.04$, $s_{12} \sim 0.004$. 
A fit to data, imposing unitarity constraint [PDG review], and showing magnitudes:

\[ V_{\text{CKM}} = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359 \pm 0.00010 & 0.04214 \pm 0.00076 \\ 0.00896 \pm 0.00024 & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix} \]

This is presumably telling us something, but what? (very different picture to one seen in neutrino sector)

Hierarchy motivates an alternative representation based on expansion in \( \lambda = \sin \theta_c \).
In the Wolfenstein parameterisation the matrix is expanded in orders of $\lambda \sim 0.23$.

$$V_{\text{CKM}} = \begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix}$$

This is expanded to $\lambda^3$, which will be adequate for most of our subsequent discussion, but not all…

$$V_{\text{CKM}} = \begin{pmatrix}
    1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\
    -\lambda & 1 - \frac{1}{2} \lambda^2 & A\lambda^2 \\
    A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4)$$
In the Wolfenstein parameterisation the matrix is expanded in orders of $\lambda \sim 0.23$.

In the Wolfenstein parameterisation the matrix is expanded in orders of $\lambda \sim 0.23$.

Note that at order $\lambda^3$ only two elements are complex: $V_{ub}$ and $V_{td}$. Thus transitions involving these vertices will be of great interest in CPV studies (but please don’t forget that it is only phase differences between transitions that are physical).
Back to FCNCs – although forbidden at tree level, they still occur, albeit suppressed

FCNCs do occur, but through higher-order diagrams

<table>
<thead>
<tr>
<th>Charged currents</th>
<th>Neutral currents</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR($K^+ \to \mu^+ \nu$) = 64 %</td>
<td>BR($K_L \to \mu^+ \mu^-$) = 7 $\times$ 10$^{-9}$</td>
</tr>
<tr>
<td>BR($D^+ \to K^0 \mu^+ \nu$) = 9 %</td>
<td>BR($D^0 \to \pi^0 \mu^+ \mu^-$) &lt; 1.8 $\times$ 10$^{-4}$</td>
</tr>
<tr>
<td>BR($B^- \to D^0 l\bar{\nu}$) = 2.3 %</td>
<td>BR($B^- \to K^{*-} l^+ l^-$) = 5 $\times$ 10$^{-7}$</td>
</tr>
</tbody>
</table>

The decay rates of FCNCs tend to be highly suppressed w.r.t. tree-level processes.
Back to FCNCs – although forbidden at tree level, they still occur, albeit suppressed

Suppression of FCNCs is explained by the GIM mechanism:

- Cancellation of diagrams relies on unitarity of $V_{\text{CKM}}$
- Suppression set by the mass-squared difference of the virtual quarks, & would be perfect in the degenerate limit
- GIM, and the smallness of $\text{BR}(K^0_L \rightarrow \mu^+\mu^-)$ led to the prediction of the charm quark

The Unitarity Triangle and CPV measurements
The CKM matrix must be unitarity:
\[ V_{CKM}^{\dagger} V_{CKM} = V_{CKM} V_{CKM}^{\dagger} = 1 \]

This imposes various constraints, including
\[ \sum_k V_{ik} V_{jk}^* = 0 \quad \text{where} \ i \neq j \ . \]

The are 6 such independent relations, which can be represented as unitarity triangles in the complex plane. Experimentally, the most interesting is:
\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \]

As the sides are of similar length, & its parameters can be studied in $B^0$, $B^+$ decays. Another, relevant for $B^0_s$ physics is:
\[ V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0 \]

Note that the area of all triangles is the same = \( 1/2 \ J \), the Jarlskog invariant.
\[ J = c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} \sin \delta \approx 3 \times 10^{-5} \]

[Jarlskog, PRL 55 (1985) 1039]
‘The’ Unitarity Triangle

Three complex vectors sum to zero

→ triangle in Argand plane

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

\[ \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0 \]

Expressions for angles:

\[ \alpha = \text{arg} \left[ -\frac{V_{td}V_{tb}^*}{V_{ub}V_{cb}^*} \right] \]

\[ \beta = \text{arg} \left[ -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right] \]

\[ \gamma = \text{arg} \left[ -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right] \]

Upper vertex:

\[ \bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*) \]

\[ \bar{\rho} = \rho(1 - \lambda^2/2 + \cdots) \]

\[ \bar{\eta} = \eta(1 - \lambda^2/2 + \cdots) \]

(\(\varphi_2\), \(\varphi_1\) & \(\varphi_3\) alternative notation)
‘The’ Unitarity Triangle

Three complex vectors sum to zero

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

\[ \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0 \]

\[ \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right| \]

Goal of Unitarity Triangle tests

Over-constrain triangle by making measurements of all parameters, in particular, comparing those made in tree-level processes (pure SM) and those made with loops (New Physics sensitive).

We hope to find inconsistencies!

Upper vertex:

\[ \bar{\rho} + i \bar{\eta} = -\frac{(V_{ud}V_{ub}^*)}{(V_{cd}V_{cb}^*)} \]

\[ \bar{\rho} = \rho(1 - \lambda^2/2 + \cdots) \quad \bar{\eta} = \eta(1 - \lambda^2/2 + \cdots) \]

(\(\phi_2, \phi_1 \& \phi_3\) alternative notation)
The $B^0_s$ Unitarity Triangle

\[ V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \]

The $B^0_s$ triangle is very squashed, & contains a small angle $\beta_s$ ($= -\phi_s/2$ – see later).
The Unitarity Triangle – how do we know what we know?

[CKMfitter, 2018]
The Unitarity Triangle – how do we know what we know?

Length of side opposite $\gamma$ is given by ratio of $B^0$ & $B^0_s$ mixing freq.s & lattice QCD.

\[
\frac{|V_{td}V^*_{tb}|}{|V_{cd}V^*_{cb}|} = \frac{1}{\lambda} \frac{|V_{td}|}{V_{ts}}
\]
Digression on neutral-meson mixing

Mixing is critical for much of what follows, so warrants a recap of essentials. Phenomenon occurs for $K^0$, $D^0$, $B^0$ and $B^0_s$ systems. Physically caused by either

Virtual, Short-range (box diagrams) and/or

On-shell, long-range (common intermediate states)

Physical states are superposition of flavour eigenstates

Subscripts indicate Short or Long lived (see $K^0$ system); sometimes Heavy or Light used, or 1, 2.

$$B_{S,L}^0 = pB^0 \pm q\overline{B^0}$$

$p$ & $q$ are complex and

$$|p|^2 + |q|^2 = 1$$

If CP is conserved the physical states = CP eigenstates, which means $\left|\frac{q}{p}\right| = 1$.

Known not to be the case in the $K^0$ system, where $\varepsilon = \frac{p-q}{q-p} \approx 2 \times 10^{-3}$, and the SM calculations indicate small, but finite, breaking in other systems too.

Mass and width splittings between physical states:

$$\Delta m = m_L - m_S \quad \text{set by short-range effects}$$

$$\Delta \Gamma = \Gamma_S - \Gamma_L \quad \text{set by long-range effects}$$
Digression on neutral-meson mixing

There is a wide range in the sizes of the mixing parameters across the four systems, which has significant practical consequences for measurements.

<table>
<thead>
<tr>
<th></th>
<th>( \Delta m / \Gamma )</th>
<th>( \Delta \Gamma/2\Gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^0 )</td>
<td>Large (~500)</td>
<td>Maximal (~1)</td>
</tr>
<tr>
<td>( D^0 )</td>
<td>Small (0.39 \pm 0.11%)</td>
<td>Small (0.65 \pm 0.06%)</td>
</tr>
<tr>
<td>( B^0 )</td>
<td>Medium (0.769 \pm 0.004)</td>
<td>Small ((20 \pm 5) \times 10^{-4})</td>
</tr>
<tr>
<td>( B_{s}^0 )</td>
<td>Large (26.81 \pm 0.08)</td>
<td>Medium (0.0675 \pm 0.004)</td>
</tr>
</tbody>
</table>

Aside: the New Physics flavour puzzle

Remark – mixing parameters are what they are because of SM (CKM, GIM & quark masses) and could easily be perturbed by New Physics, so bounds can be set.

Add to SM Lagrangian higher order terms that would contribute to neutral meson mixing and CPV

\[ \Delta L_{\text{NP}} = \sum_{i \neq j} \frac{c_{ij}^{\text{NP}}}{\Lambda_{\text{NP}}^2} (Q_{Li} \gamma^\mu Q_{Lj})^2, \]

where \( c_{ij}^{\text{NP}} \) is the coupling, and \( \Lambda_{\text{NP}} \) the mass scale of the New Physics.

If we assuming the coupling is \(~1, (i.e. generic) obtain the following \( \rightarrow \) bounds on \( \Lambda_{\text{NP}} \) [Nir, arXiv:1605.00433].

<table>
<thead>
<tr>
<th>System</th>
<th>CP-conserving observables</th>
<th>CP-violating Observables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^0 )</td>
<td>( 1 \times 10^3 \text{ TeV} )</td>
<td>( 2 \times 10^4 \text{ TeV} )</td>
</tr>
<tr>
<td>( D^0 )</td>
<td>( 1 \times 10^3 \text{ TeV} )</td>
<td>( 3 \times 10^3 \text{ TeV} )</td>
</tr>
<tr>
<td>( B^0 )</td>
<td>( 4 \times 10^2 \text{ TeV} )</td>
<td>( 8 \times 10^2 \text{ TeV} )</td>
</tr>
<tr>
<td>( B^0_s )</td>
<td>( 7 \times 10^1 \text{ TeV} )</td>
<td>( 2 \times 10^2 \text{ TeV} )</td>
</tr>
</tbody>
</table>

These are enormous! And naturalness told us to expect New Physics at the TeV scale. Something is wrong…

Get out clause: couplings are not \(~1. One possibility, structure is more specific e.g. same as in the SM (‘minimal flavour violation’ [Ambrosio et al., NPB 645 (2002) 155]).
Digression on neutral-meson mixing

Mixing leads to an oscillation of probability to observe meson in either flavour eigenstate with proper time, e.g. if at $t=0$ we have a $B^0$, then at later time $t$:

$$\text{Prob. to decay as } \frac{B^0}{B^0} \propto e^{-\Gamma_d t} (1 \mp \cos \Delta m_d t)$$

Time-integrated B-oscillations were first observed by UA1 \[PLB 186 (1987) 247\] & ARGUS \[PLB 192 (1987) 245\]. $B^0$ ($B^0_s$) oscillations first resolved by ALEPH (CDF).

$B^0$ discovery $\rightarrow$ state-of-the-art

$B^0_s$ discovery $\rightarrow$ state-of-the-art

\( \mathbf{B}^0(s) - \overline{\mathbf{B}}^0(s) \) mixing — accessing CKM elements

In \( \mathbf{B}^0 \) and \( \mathbf{B}^0_s \) systems, mixing driven by \( \Delta m_{d(s)} \) and is calculable in SM.

Depends on CKM elements in box & factors that can be calculated in lattice QCD.

For \( \mathbf{B}^0_s \) case →

\[
\Delta m_s = \frac{G_F^2}{6 \pi^2} m_{B_s} m_W^2 \eta_B S_0(x_t) f_{B_s}^2 B_s \left| V_{ts} V_{tb}^* \right|^2
\]

Equivalent expression for \( \mathbf{B}^0 \) mixing, involving \( V_{td} \). Ratio of frequencies is then

\[
\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d}}{m_{B_s}} \xi_{\Delta m}^{-2} \left| \frac{V_{td}}{V_{ts}} \right|^2
\]

\( \xi_{\Delta m} \), being a ratio of QCD factors of value close to 1 can be calculated to a few % in lattice QCD, hence giving access to \( |V_{td}|/|V_{ts}| \).
The Unitarity Triangle – how do we know what we know?

\[
\frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left|\frac{V_{ub}}{V_{cb}}\right|
\]

Length of side opposite $\beta$ is given by measuring $|V_{ub}|/|V_{cb}|$ from ratio $b\to u / b\to c$. 

CKMfitter, 2018
Measuring $|V_{ub}| / |V_{cb}|$

We can measure the ratio of $b \rightarrow ul \nu$ to $b \rightarrow cl \nu$ processes at hadron level, but then must use theory or lattice QCD to correct back to quark level.

Two broad strategies followed:

- **Inclusive** $b \rightarrow X_u l \nu$, using *e.g.* endpoint of $p_l$ spectrum to isolate signal from $b \rightarrow X_c l \nu$
  \[
  |V_{ub}| = (4.49 \pm 0.28) \times 10^{-3} \quad \text{[2018 PDG review]}
  \]

- **Exclusive**, *e.g.* $B \rightarrow \pi l \nu$. But then need calculation of hadronic form factor.
  \[
  |V_{ub}| = (3.70 \pm 0.16) \times 10^{-3} \quad \text{[2018 PDG review]}
  \]

There is tension between these two numbers at the $\sim 2.5\sigma$ level, which means that a conservative approach is advisable when using the results to set UT constraints. Much activity underway to understand this issue, & we can be hopeful of progress!