Forward jet production in proton-nucleus collisions at high energy: from trijet to NLO dijet

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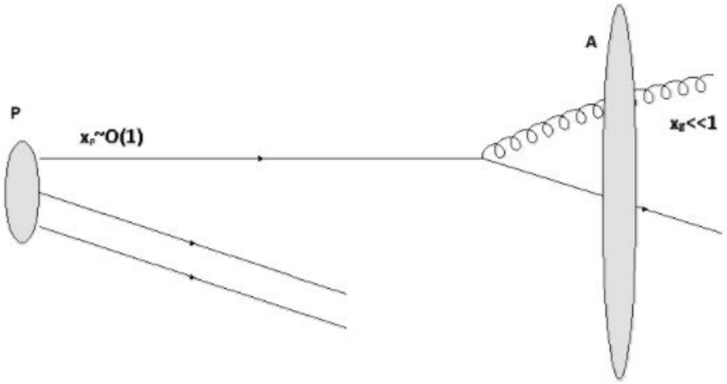
Forward Jet Production

By using the formalism of the light-cone wave function in perturbative QCD, together with the hybrid factorization, the derivation of the forward LO dijet cross-section was done in hep-ph/0708.0231 (C. Marquet).

The basic setup: a large-x parton from the proton scatters off the small-x gluon distribution in the target nucleus. Large-x parton is most likely a quark.



Quark fragmentation in the presence of a shockwave.

The time evolution of the initial (bare) quark state is given by:

$$\left|q_{\lambda}^{\alpha}(q^{+},\boldsymbol{q})\right\rangle_{\text{in}} \equiv U(0,-\infty)\left|q_{\lambda}^{\alpha}(q^{+},\boldsymbol{q})\right\rangle$$

Where U denotes a unitary operator:

$$U(t,t_0) = \mathrm{T}\exp\left\{-i\int_{t_0}^{t} dt_1\, H_I(t_1)\right\}$$

The information both on the time evolution and interaction of the bare quark with the target nucleus is given by the “outgoing state”:

$$\left|q_{\lambda}^{\alpha}(q^{+},\boldsymbol{w})\right\rangle_{out} \equiv U(\infty,0)\,\hat{S}\,U(0,-\infty)\left|q_{\lambda}^{\alpha}(q^{+},\boldsymbol{w})\right\rangle$$

This state will be shown to generate all the possible insertions of the shockwave. More importantly, the outgoing state is directly related to expectation values:

$$\left\langle \hat{\mathcal{O}}\right\rangle = \left\langle \langle q|\, U^{\dagger}\,\hat{S}\,U\,\hat{\mathcal{O}}\,U^{\dagger}\,\hat{S}\,U\,|q\rangle\right\rangle_{cgc}$$

The LO Outgoing State

The production state at leading order is given by

$$\left|q^{\alpha}_{\lambda}(q^{+},\, \boldsymbol{w})\right\rangle^{(g)}_{out} \equiv U(\infty,\, 0)\, \hat{S}\, U(0,\, -\infty)\, \left|q^{\alpha}_{\lambda}(q^{+},\, \boldsymbol{w})\right\rangle \;=\; \left|\psi^{\alpha}_{\lambda}(q^{+},\, \boldsymbol{w})\right\rangle_{qg} \;+\; \left|q^{\alpha}_{\lambda}(q^{+},\, \boldsymbol{w})\right\rangle$$

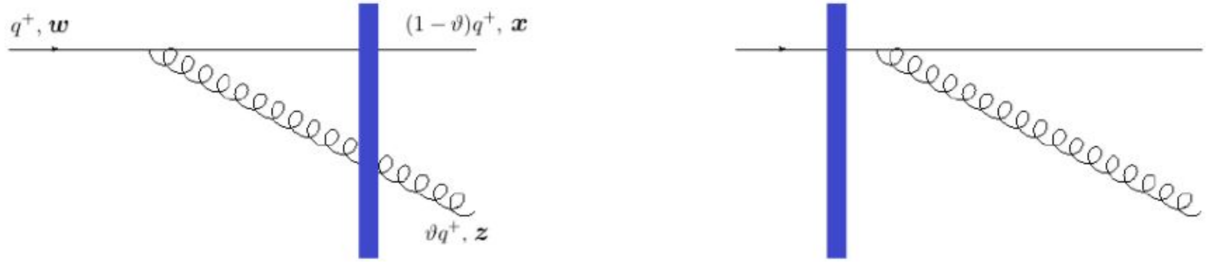
$$\left|\psi^{\alpha}_{\lambda}\right\rangle_{qg} = \left|q^{\gamma} g^{b}\right\rangle \left(-\left\langle q^{\gamma} g^{b}\right| \hat{S} \left|q^{\beta} g^{a}\right\rangle \frac{\left\langle q^{\beta} g^{a}\right| H_{q\to qg} \left|q^{\alpha}\right\rangle}{E_{qg} - E_{q}} + \frac{\left\langle q^{\gamma} g^{b}\right| H_{q\to qg} \left|q^{\beta}\right\rangle}{E_{qg} - E_{q}} \left\langle q^{\beta}\right| \hat{S} \left|q^{\alpha}\right\rangle \right)$$

Where only terms of order g were kept. The following result is obtained for the |qg> contribution:

$$\left|\psi^{\alpha}_{\lambda}(q^{+},\, \boldsymbol{w})\right\rangle_{qg} = \int_{\boldsymbol{x},\, \boldsymbol{z}} \int_{0}^{1} d\vartheta\, \frac{ig\phi^{ij}_{\lambda_1\lambda}(\vartheta)\sqrt{q^{+}}\, \boldsymbol{X}^{j}}{4\pi^{3/2}\sqrt{\vartheta}\, \boldsymbol{X}^{2}}\, \delta^{(2)}(\boldsymbol{w} - (1-\vartheta)\boldsymbol{x} - \vartheta \boldsymbol{z})$$
$$\times \left[V^{\gamma\beta}(\boldsymbol{x})\, U^{ba}(\boldsymbol{z})\, t^{a}_{\beta\alpha} - t^{b}_{\gamma\beta}\, V^{\beta\alpha}(\boldsymbol{w}) \right] \left| q^{\gamma}_{\lambda_1}((1-\vartheta)q^{+},\, \boldsymbol{x})\, g^{b}_{i}(\vartheta q^{+},\, \boldsymbol{z}) \right\rangle$$

$$\boldsymbol{X} \equiv \boldsymbol{x} - \boldsymbol{z}$$

Diagrammatically (blue bar denotes a shockwave = interaction with the target):



One gluon production at leading order with shockwave before and after the emission.

The LO forward dijet cross-section

From the production state we can pass easily to the quark-gluon dijet cross section:

$$\frac{d\sigma_{\rm LO}^{qA\to qg+X}}{d^3k\,d^3p} \equiv \frac{1}{2N_c\,L}\,{}^{(g)}_{out}\langle q_\lambda^\alpha(q^+,\boldsymbol{q})|\,\hat{\mathcal{N}}_q(p)\,\hat{\mathcal{N}}_g(k)\,|q_\lambda^\alpha(q^+,\boldsymbol{q})\rangle_{out}^{(g)}$$

$$= \frac{1}{2N_c\,L}\int_{\boldsymbol{w},\,\overline{\boldsymbol{w}}} e^{i(\boldsymbol{w}-\overline{\boldsymbol{w}})\cdot\boldsymbol{q}}\,{}_{qg}\langle\psi_\lambda^\alpha(q^+,\overline{\boldsymbol{w}})|\,\hat{\mathcal{N}}_q(p)\,\hat{\mathcal{N}}_g(k)\,|\psi_\lambda^\alpha(q^+,\boldsymbol{w})\rangle_{qg}$$

The following number density operators were introduced:

$$\hat{\mathcal{N}}_q(p) \equiv \frac{1}{(2\pi)^3}\,b_\lambda^{\alpha\dagger}(p)\,b_\lambda^\alpha(p) \qquad\qquad \hat{\mathcal{N}}_g(k) \equiv \frac{1}{(2\pi)^3}\,a_i^{a\dagger}(k)\,a_i^a(k)$$

Then the result for the leading-order dijet cross section is given by:

$$\frac{d\sigma_{\rm LO}^{qA\to qg+X}}{dk^+\,d^2\boldsymbol{k}\,dp^+\,d^2\boldsymbol{p}} = \frac{2\alpha_s C_F\left(1+(1-\vartheta)^2\right)}{(2\pi)^6\vartheta q^+}\,\delta(q^+-k^+-p^+)$$

$$\times\int_{\boldsymbol{x},\overline{\boldsymbol{x}},\boldsymbol{z},\overline{\boldsymbol{z}}}\frac{\boldsymbol{X}\cdot\overline{\boldsymbol{X}}}{\boldsymbol{X}^2\,\overline{\boldsymbol{X}}^2}\,\mathrm{e}^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{k}\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})}\,\mathbb{S}_{\rm LO}\left(\overline{\boldsymbol{w}},\,\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\boldsymbol{w},\,\boldsymbol{x},\,\boldsymbol{z}\right)$$

with $\boldsymbol{X}\equiv\boldsymbol{x}-\boldsymbol{z}$, $\overline{\boldsymbol{X}}\equiv\overline{\boldsymbol{x}}-\overline{\boldsymbol{z}}$, $\boldsymbol{w}=(1-\vartheta)\boldsymbol{x}+\vartheta\boldsymbol{z}$ and $\overline{\boldsymbol{w}}=(1-\vartheta)\overline{\boldsymbol{x}}+\vartheta\overline{\boldsymbol{z}}$.

$$\mathbb{S}_{\rm LO}\left(\overline{\boldsymbol{w}},\,\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\boldsymbol{w},\,\boldsymbol{x},\,\boldsymbol{z}\right) \equiv S_{qgqg}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\boldsymbol{x},\,\boldsymbol{z}\right) - S_{qqg}\left(\overline{\boldsymbol{w}},\,\boldsymbol{x},\,\boldsymbol{z}\right) - S_{qqg}\left(\overline{\boldsymbol{x}},\,\boldsymbol{w},\,\overline{\boldsymbol{z}}\right) + \mathcal{S}\left(\overline{\boldsymbol{w}},\,\boldsymbol{w}\right)$$

Where the following combinations of Wilson lines were introduced (in the large Nc limit these combinations represent the quadropole-dipole and dipole-dipole interactions):

$$S^{(1)}_{q\bar{q}gg}(\overline{x}, \overline{z}, x, z) \equiv \frac{1}{C_F N_c} \operatorname{tr}\left(V^\dagger(\overline{x}) V(x) t^a t^c\right) \left[U^\dagger(\overline{z}) U(z)\right]^{ca}$$
$$= \frac{1}{2C_F N_c}\left(N_c^2\, \mathcal{Q}(\overline{x}, x, z, \overline{z})\, \mathcal{S}(\overline{z}, z) - \mathcal{S}(\overline{x}, x)\right) \simeq \mathcal{Q}(\overline{x}, x, z, \overline{z})\, \mathcal{S}(\overline{z}, z)$$

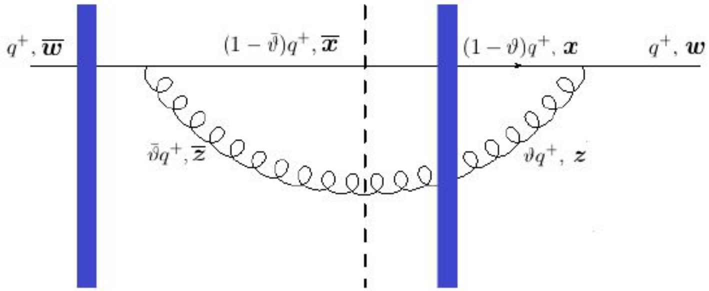
$$S_{q\bar{q}g}(\overline{w}, x, z) \equiv \frac{1}{C_F N_c} \operatorname{tr}\left(V^\dagger(\overline{w}) t^b V(x) t^a\right) U^{ba}(z)$$
$$= \frac{1}{2C_F N_c}\left(N_c^2\, \mathcal{S}(\overline{w}, z)\, \mathcal{S}(z, x) - \mathcal{S}(\overline{w}, x)\right) \simeq \mathcal{S}(\overline{w}, z)\, \mathcal{S}(z, x)$$

The dipole and quadropole are defined by:

$$\mathcal{S}(\overline{\boldsymbol{w}}, \boldsymbol{w}) \equiv \frac{1}{N_c} \operatorname{tr}\left[V^\dagger(\overline{\boldsymbol{w}})\, V(\boldsymbol{w})\right] \qquad \mathcal{Q}(\overline{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{z}, \overline{\boldsymbol{z}}) \equiv \frac{1}{N_c} \operatorname{tr}\left[V^\dagger(\overline{\boldsymbol{x}})\, V(\boldsymbol{x})\, V^\dagger(\boldsymbol{z})\, V(\overline{\boldsymbol{z}})\right]$$

$$U(\boldsymbol{x}) = \mathrm{T}\exp\left\{ig\int dx^+\, T^a A^-_a(x^+, \boldsymbol{x})\right\}, \qquad V(\boldsymbol{x}) = \mathrm{T}\exp\left\{ig\int dx^+\, t^a A^-_a(x^+, \boldsymbol{x})\right\}$$

In total there are four different insertions of Wilson lines (each diagram corresponds to a different term in the rectangular brackets). For example, below is the relevant diagram which corresponds to $S_{q\bar{q}g}(\overline{\boldsymbol{w}}, \boldsymbol{x}, \boldsymbol{z})$ (the location of the measurement is denoted by a dashed line).

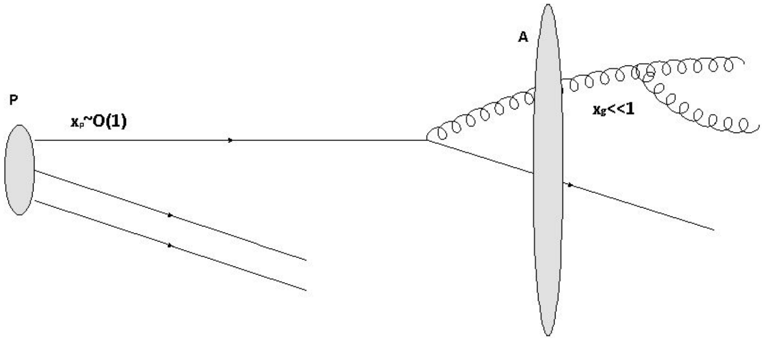


The Trijet Setup

In the new setup, we have to produce three particles in the final state. There are two configurations of particles:

a) Quark, quark and anti-quark
b) Quark together with two gluons.

Due to the fact that we are using the light-cone gauge, the production of these configurations can happen both instantaneously (via one emission), or in the regular way, via two successive emissions or one emission followed by splitting process.



An example for a contribution with three particles in the final state

The Trijet Outgoing State

The perturbative expression for the outgoing state is:

$$|out\rangle = |in\rangle + |out\rangle^{(1)} + |out\rangle^{(2)} + \cdots$$

with:

$$|out\rangle^{(1)} = -\sum_{f,j} |f\rangle\langle f|S|j\rangle \frac{\langle j|H_{\rm int}|in\rangle}{E_j - E_{in}} + \sum_{f,j} |f\rangle \frac{\langle f|H_{\rm int}|j\rangle}{E_f - E_j} \langle j|S|in\rangle$$

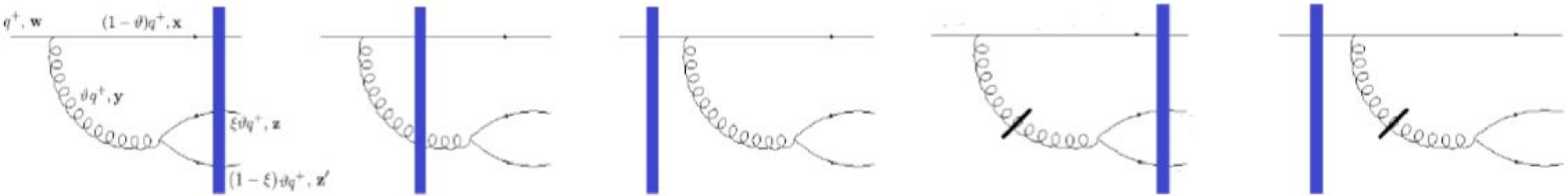
$$|out\rangle^{(2)} = \sum_{f,j,i} |f\rangle\langle f|S|j\rangle \frac{\langle j|H_{\rm int}|i\rangle \, \langle i|H_{\rm int}|in\rangle}{(E_j - E_{in})(E_i - E_{in})} + \sum_{f,j,i} |f\rangle \frac{\langle f|H_{\rm int}|j\rangle \, \langle j|H_{\rm int}|i\rangle}{(E_f - E_j)(E_f - E_i)} \langle i|S|in\rangle$$
$$- \sum_{f,j,i} |f\rangle \frac{\langle f|H_{\rm int}|j\rangle}{E_f - E_j} \langle j|S|i\rangle \frac{\langle i|H_{\rm int}|in\rangle}{E_i - E_{in}}$$

Where i, j and k runs over the relevant bare states, and H_{int} represent the interaction part of the QCD Hamiltonian.In the following we will focus only on the contribution to the outgoing states from the quark, quark, anti-quark configuration:

$$|\psi^\alpha\rangle_{qq\bar{q}}^{inst} \equiv |\bar{q}^\rho q^\varrho q^\sigma\rangle \left(\frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \,\mathrm{H}_{q\to qq\bar{q}}\, |q^\beta\rangle \, \langle q^\beta | \,\hat{S}\, |q^\alpha\rangle}{E_{qq\bar{q}} - E_q} - \frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \,\hat{S}\, |\bar{q}^\epsilon q^\delta q^\beta\rangle \, \langle \bar{q}^\epsilon q^\delta q^\beta | \,\mathrm{H}_{q\to qq\bar{q}}\, |q^\alpha\rangle}{E_{qq\bar{q}} - E_q} \right)$$

$$|\psi^\alpha\rangle_{qq\bar{q}}^{reg} \equiv |\bar{q}^\rho q^\varrho q^\sigma\rangle \left(\frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \,\hat{S}\, |\bar{q}^\delta q^\epsilon q^\kappa\rangle \, \langle \bar{q}^\delta q^\epsilon q^\kappa | \,\mathrm{H}_{g\to q\bar{q}}\, |q^\beta g^i\rangle \, \langle q^\beta g^i | \,\mathrm{H}_{q\to qg}\, |q^\alpha\rangle}{(E_{qq\bar{q}} - E_q)(E_{qg} - E_q)} \right.$$
$$\left. + \frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \,\mathrm{H}_{g\to q\bar{q}}\, |q^\gamma g^i\rangle \, \langle q^\gamma g^i | \,\mathrm{H}_{q\to qg}\, |q^\beta\rangle \, \langle q^\beta | \,\hat{S}\, |q^\alpha\rangle}{(E_{qq\bar{q}} - E_{qg})(E_{qq\bar{q}} - E_q)} - \frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \,\mathrm{H}_{g\to q\bar{q}}\, |q^\gamma g^j\rangle \, \langle q^\gamma g^j | \,\hat{S}\, |q^\beta g^i\rangle \, \langle q^\beta g^i | \,\mathrm{H}_{q\to qg}\, |q^\alpha\rangle}{(E_{qq\bar{q}} - E_{qg})(E_{qg} - E_q)} \right)$$

The Quark Quark Anti-quark Outgoing State



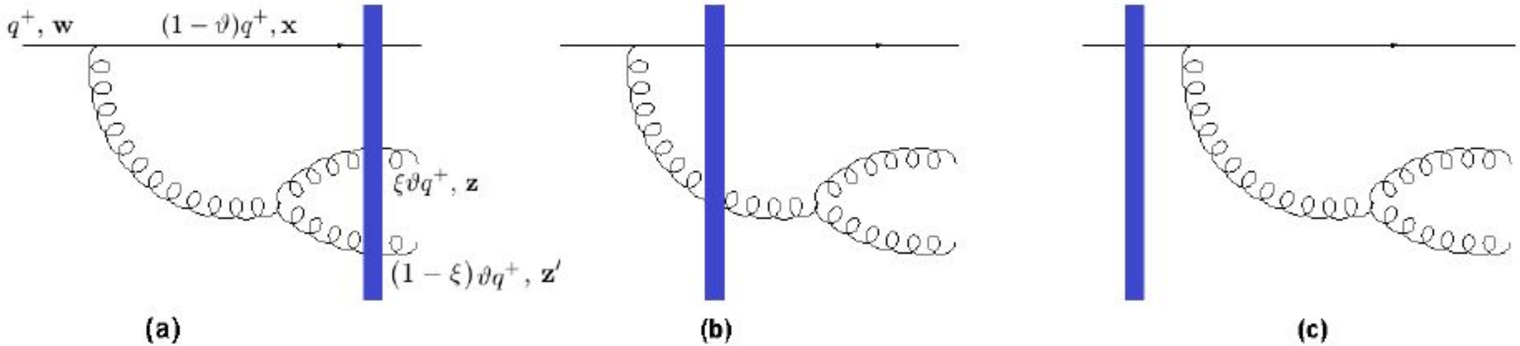
$$
\begin{aligned}
\left|q^{\alpha}_{\lambda}(q^+,\, \boldsymbol{w})\right\rangle_{qq\bar{q}} = & -\int_{\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'} \int_0^1 d\vartheta\, d\xi\, \frac{g^2\, q^+}{(2\pi)^4\, \left((1-\vartheta)\,(\boldsymbol{X}+\xi\boldsymbol{Z})^2 + \xi(1-\xi)\boldsymbol{Z}^2\right)} \\
& \times \Big\{ F^{\lambda_3\lambda_2\lambda_1}_{qq\bar{q}1}(\vartheta,\, \xi,\, \mathbf{X},\, \mathbf{Z}) \left[V^{\varrho\delta}(\boldsymbol{z}')\, t^a_{\delta\epsilon}\, V^{\dagger\epsilon\rho}(\boldsymbol{z})\, V^{\sigma\beta}(\boldsymbol{x})\, t^a_{\beta\alpha} - t^b_{\varrho\rho}\, V^{\sigma\beta}(\boldsymbol{x})\, U^{ba}(\boldsymbol{y})\, t^a_{\beta\alpha} \right] \\
& + F^{\lambda_3\lambda_2\lambda_1}_{qq\bar{q}2}(\vartheta,\, \xi,\, \mathbf{X},\, \mathbf{Z}) \left[t^b_{\varrho\rho}\, V^{\sigma\beta}(\boldsymbol{x})\, U^{ba}(\boldsymbol{y})\, t^a_{\beta\alpha} - t^a_{\varrho\rho}\, t^a_{\sigma\beta}\, V^{\beta\alpha}(\boldsymbol{w}) \right] \Big\} \\
& \times \delta^{(2)}\left(\boldsymbol{w} - \boldsymbol{C}\right) \left| \bar{q}^{\rho}_{\lambda_3}((1-\xi)\vartheta q^+,\, \boldsymbol{z})\, q^{\varrho}_{\lambda_2}(\xi\vartheta q^+,\, \boldsymbol{z}')\, q^{\sigma}_{\lambda_1}((1-\vartheta)q^+, \boldsymbol{x}) \right\rangle
\end{aligned}
$$

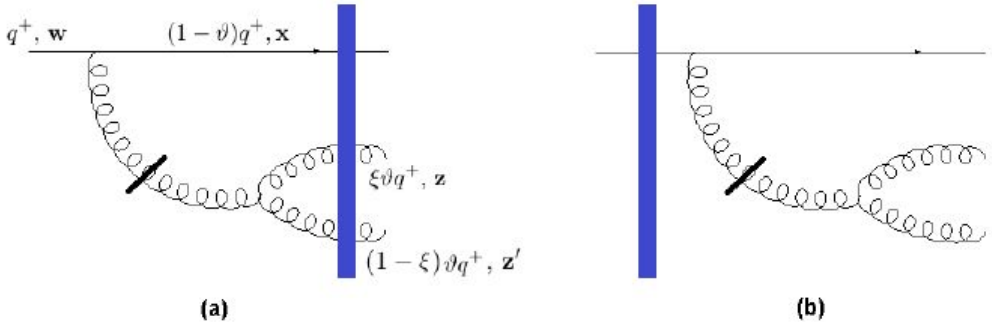
$$
F^{\lambda_3\lambda_2\lambda_1}_{qq\bar{q}1}(\vartheta,\, \xi,\, \mathbf{X},\, \mathbf{Z}) \equiv (1-\vartheta) \left(\frac{\varphi^{il}_{\lambda_2\lambda_3}(\xi)\, \phi^{ij}_{\lambda_1\lambda}(\vartheta)\, Z^l\, \left(X^j + \xi Z^j\right)}{2Z^2} + \xi(1-\xi)\delta_{\lambda_3\lambda_2}\delta_{\lambda_1\lambda} \right)
$$

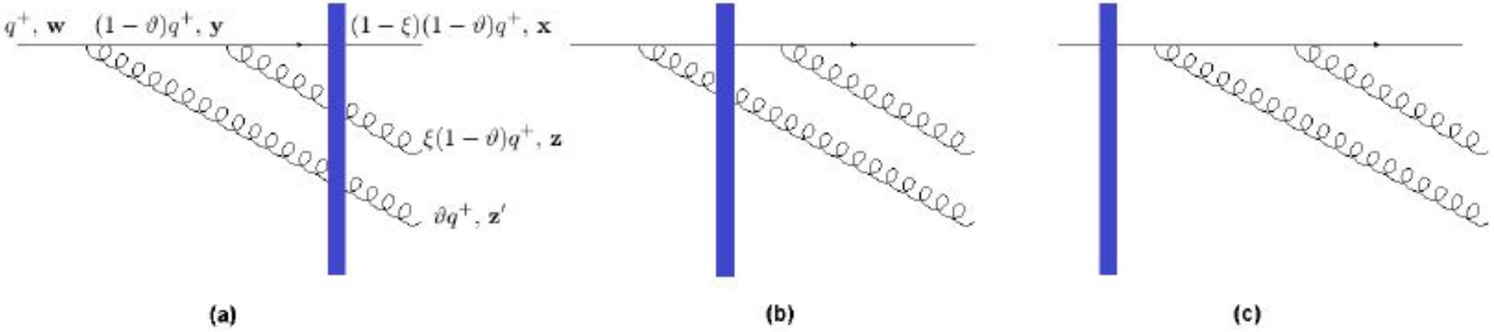
$$
F^{\lambda_3\lambda_2\lambda_1}_{qq\bar{q}2}(\vartheta,\, \xi,\, \mathbf{X},\, \mathbf{Z}) \equiv \xi(1-\xi) \left(-\frac{\varphi^{il}_{\lambda_2\lambda_3}(\xi)\, \phi^{ij}_{\lambda_1\lambda}(\vartheta)\, Z^l\, \left(X^j + \xi Z^j\right)}{2\,(X+\xi Z)^2} + (1-\vartheta)\delta_{\lambda_3\lambda_2}\delta_{\lambda_1\lambda} \right)
$$

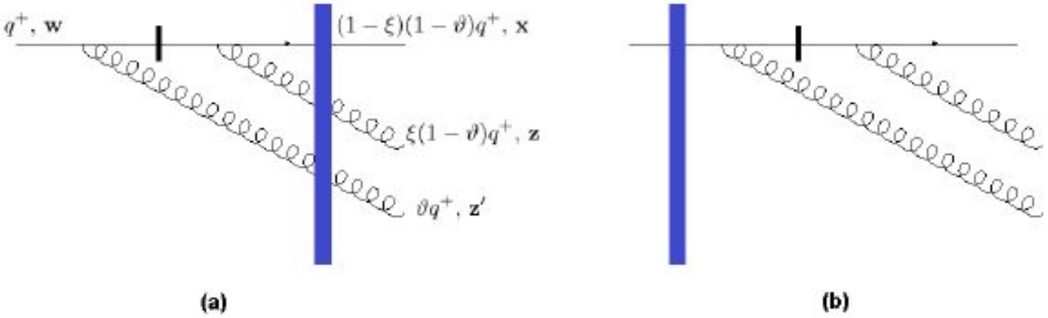
C denotes the c.o.m for of the three produced particles: $\boldsymbol{C} \equiv (1-\vartheta)\boldsymbol{x} + \xi\vartheta\boldsymbol{z} + (1-\xi)\vartheta\boldsymbol{z}'$.
Note that the result above and in the previous slide vanishes under the limit S->1. This property of the results has to be expected since the new particles are produced by the shockwave.

The Diagrams for the Quark and Two Gluons Outgoing States






q^+, $\mathbf{w}$
$(1-\vartheta)q^+$, $\mathbf{y}$
$(1-\xi)(1-\vartheta)q^+$, $\mathbf{x}$
$\xi(1-\vartheta)q^+$, $\mathbf{z}$
ϑq^+, $\mathbf{z}'$
(a)
(b)
(c)


q^+, $\mathbf{w}$
$(1-\xi)(1-\vartheta)q^+$, $\mathbf{x}$
$\xi(1-\vartheta)q^+$, $\mathbf{z}$
ϑq^+, $\mathbf{z}'$
(a)
(b)

The results for the forward trijet cross section

The expression for the forward trijet cross section is composed by two contributions:

$$\frac{d\sigma^{pA\to 3jet+X}}{d^3q_1\,d^3q_2\,d^3q_3} = \int dx_p\, q(x_p,\mu^2)\left(\frac{d\sigma^{qA\to qgg+X}}{d^3q_1\,d^3q_2\,d^3q_3} + \frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\,d^3q_2\,d^3q_3}\right)$$

The two contributions to the two final partonic state:

$$\frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\,d^3q_2\,d^3q_3} \equiv \frac{1}{2N_c\,L}\,{}_{out}^{(g^2)}\langle q_\lambda^\alpha(q^+,\,\boldsymbol{q}=0_\perp)|\,\hat{\mathcal{N}}_q(q_1)\,\hat{\mathcal{N}}_q(q_2)\,\hat{\mathcal{N}}_{\bar{q}}(q_3)\,|q_\lambda^\alpha(q^+,\,\boldsymbol{q}=0_\perp)\rangle_{out}^{(g^2)}$$

$$\frac{d\sigma^{qA\to qgg+X}}{d^3q_1\,d^3q_2\,d^3q_3} \equiv \frac{1}{2N_c\,L}\,{}_{out}^{(g^2)}\langle q_\lambda^\alpha(q^+,\,\boldsymbol{q}=0_\perp)|\,\hat{\mathcal{N}}_q(q_1)\,\hat{\mathcal{N}}_g(q_2)\,\hat{\mathcal{N}}_g(q_3)\,|q_\lambda^\alpha(q^+,\,\boldsymbol{q}=0_\perp)\rangle_{out}^{(g^2)}$$

In hep-ph/1809.05526, the result is given by:

$$\frac{d\sigma^{qA\to qq\overline{q}+X}}{d^3q_1\,d^3q_2\,d^3q_3} \equiv \frac{\alpha_s^2\,C_F\,N_f}{2(2\pi)^{10}(q^+)^2}\,\delta(q^+-q_1^+-q_2^+-q_3^+)\int_{\overline{x},\overline{z},\overline{z}',x,z,z'} \mathrm{e}^{-iq_1\cdot(x-\overline{x})-iq_2\cdot(z-\overline{z})-iq_3\cdot(z'-\overline{z}')}$$
$$\times\Big\{K^1_{qq\overline{q}}(\overline{x},\overline{z},\overline{z}',x,z,z')\big[\overline{\Theta}_1\Theta_1 S_{q\overline{q}qq\overline{q}q}(\overline{x},\overline{z},\overline{z}',x,z,z') - \overline{\Theta}_1 S_{q\overline{q}qqg}(\overline{x},\overline{z},\overline{z}',x,y)$$
$$-\Theta_1 S_{qgq\overline{q}q}(\overline{x},\overline{y},x,z,z') + \overline{\Theta}_2\Theta_1 S_{qq\overline{q}q}(\overline{w},x,z,z') + \overline{\Theta}_1\Theta_2 S_{qq\overline{q}q}(\overline{x},w,\overline{z}',\overline{z})$$
$$+S_{qgqg}(\overline{x},\overline{y},x,y) - \overline{\Theta}_2 S_{qqg}(\overline{w},y,x) - \Theta_2 S_{qqg}(\overline{x},w,\overline{y}) + \overline{\Theta}_2\Theta_2\mathcal{S}(\overline{w},w)\big]$$
$$+K^2_{qq\overline{q}}(\overline{x},\overline{z},\overline{z}',x,z,z')\big[\Theta_1 S_{q\overline{q}qq\overline{q}q}(\overline{x},\overline{z},\overline{z}',x,z,z') - S_{q\overline{q}qqg}(\overline{x},\overline{z},\overline{z}',x,y)$$
$$-\Theta_1 S_{qq\overline{q}q}(\overline{w},x,z,z') + \Theta_2 S_{qq\overline{q}q}(\overline{x},w,\overline{z}',\overline{z}) + S_{qqg}(\overline{w},y,x) - \Theta_2\mathcal{S}(\overline{w},w)\big]$$
$$+K^2_{qq\overline{q}}(x,z,z',\overline{x},\overline{z},\overline{z}')\big[\overline{\Theta}_1 S_{q\overline{q}qq\overline{q}q}(\overline{x},\overline{z},\overline{z}',x,z,z') - S_{qgq\overline{q}q}(\overline{x},\overline{y},x,z,z')$$
$$+\overline{\Theta}_2 S_{qq\overline{q}q}(\overline{w},x,z,z') - \overline{\Theta}_1 S_{qq\overline{q}q}(\overline{x},w,\overline{z}',\overline{z}) + S_{qqg}(\overline{x},w,\overline{y}) - \overline{\Theta}_2\mathcal{S}(\overline{w},w)\big]$$
$$+K^3_{qq\overline{q}}(\overline{x},\overline{z},\overline{z}',x,z,z')\big[S_{q\overline{q}qq\overline{q}q}(\overline{x},\overline{z},\overline{z}',x,z,z') - S_{qq\overline{q}q}(\overline{w},x,z,z')$$
$$-S_{qq\overline{q}q}(\overline{x},w,\overline{z}',\overline{z}) + \mathcal{S}(\overline{w},w)\big]\Big\} + (q_1^+\leftrightarrow q_2^+,\,q_1\leftrightarrow q_2)$$

Or equivalently:

$$\frac{d\sigma^{qA\to qq\overline{q}+X}}{d^3q_1\,d^3q_2\,d^3q_3} \equiv \frac{\alpha_s^2\,C_F\,N_f}{2(2\pi)^{10}(q^+)^2}\,\delta(q^+-q_1^+-q_2^+-q_3^+)\int_{\overline{\boldsymbol{x}},\overline{\boldsymbol{z}},\overline{\boldsymbol{z}}',\boldsymbol{x},\boldsymbol{z},\boldsymbol{z}'} \mathrm{e}^{-i\boldsymbol{q}_1\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{q}_2\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})-i\boldsymbol{q}_3\cdot(\boldsymbol{z}'-\overline{\boldsymbol{z}}')}$$
$$\times\big[K^1_{qq\overline{q}}(\vartheta,\xi,\overline{\mathbf{X}},\overline{\mathbf{Z}},\mathbf{X},\mathbf{Z})\,\mathbb{S}^1_{qq\overline{q}}(\overline{\mathbf{x}},\overline{\mathbf{z}},\overline{\mathbf{z}}',\mathbf{x},\mathbf{z},\mathbf{z}') + K^2_{qq\overline{q}}(\vartheta,\xi,\overline{\mathbf{X}},\overline{\mathbf{Z}},\mathbf{X},\mathbf{Z})$$
$$\times\mathbb{S}^2_{qq\overline{q}}(\overline{\mathbf{x}},\overline{\mathbf{z}},\overline{\mathbf{z}}',\mathbf{x},\mathbf{y}) + h.c. + K^3_{qq\overline{q}}(\vartheta,\xi,\overline{\mathbf{X}},\overline{\mathbf{Z}},\mathbf{X},\mathbf{Z})\,\mathbb{S}_{\rm LO}(\overline{\boldsymbol{w}},\overline{\boldsymbol{x}},\overline{\boldsymbol{z}},\boldsymbol{w},\boldsymbol{x},\boldsymbol{z})\big]$$
$$+(q_1^+\leftrightarrow q_2^+,\,q_1\leftrightarrow q_2)$$

$$\mathbb{S}^1_{qq\overline{q}}(\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\overline{\boldsymbol{z}},\overline{\boldsymbol{z}}',\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{z}') \equiv S_{q\overline{q}qq\overline{q}q}(\overline{\boldsymbol{x}},\overline{\boldsymbol{z}},\overline{\boldsymbol{z}}',\boldsymbol{x},\boldsymbol{z},\boldsymbol{z}') - S_{q\overline{q}qqg}(\overline{\boldsymbol{x}},\overline{\boldsymbol{z}},\overline{\boldsymbol{z}}',\boldsymbol{x},\boldsymbol{y}) - S_{qgq\overline{q}q}(\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\boldsymbol{x},\boldsymbol{z},\boldsymbol{z}') + S_{qgqg}(\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\boldsymbol{x},\boldsymbol{y})$$
$$\mathbb{S}^2_{qq\overline{q}}(\overline{\boldsymbol{w}},\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{z}') \equiv S_{qgq\overline{q}q}(\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\boldsymbol{x},\boldsymbol{z},\boldsymbol{z}') - S_{qq\overline{q}q}(\overline{\boldsymbol{w}},\boldsymbol{x},\boldsymbol{z},\boldsymbol{z}') - S_{qgqg}(\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\boldsymbol{x},\boldsymbol{y}) + S_{qqg}(\overline{\boldsymbol{w}},\boldsymbol{y},\boldsymbol{x})$$

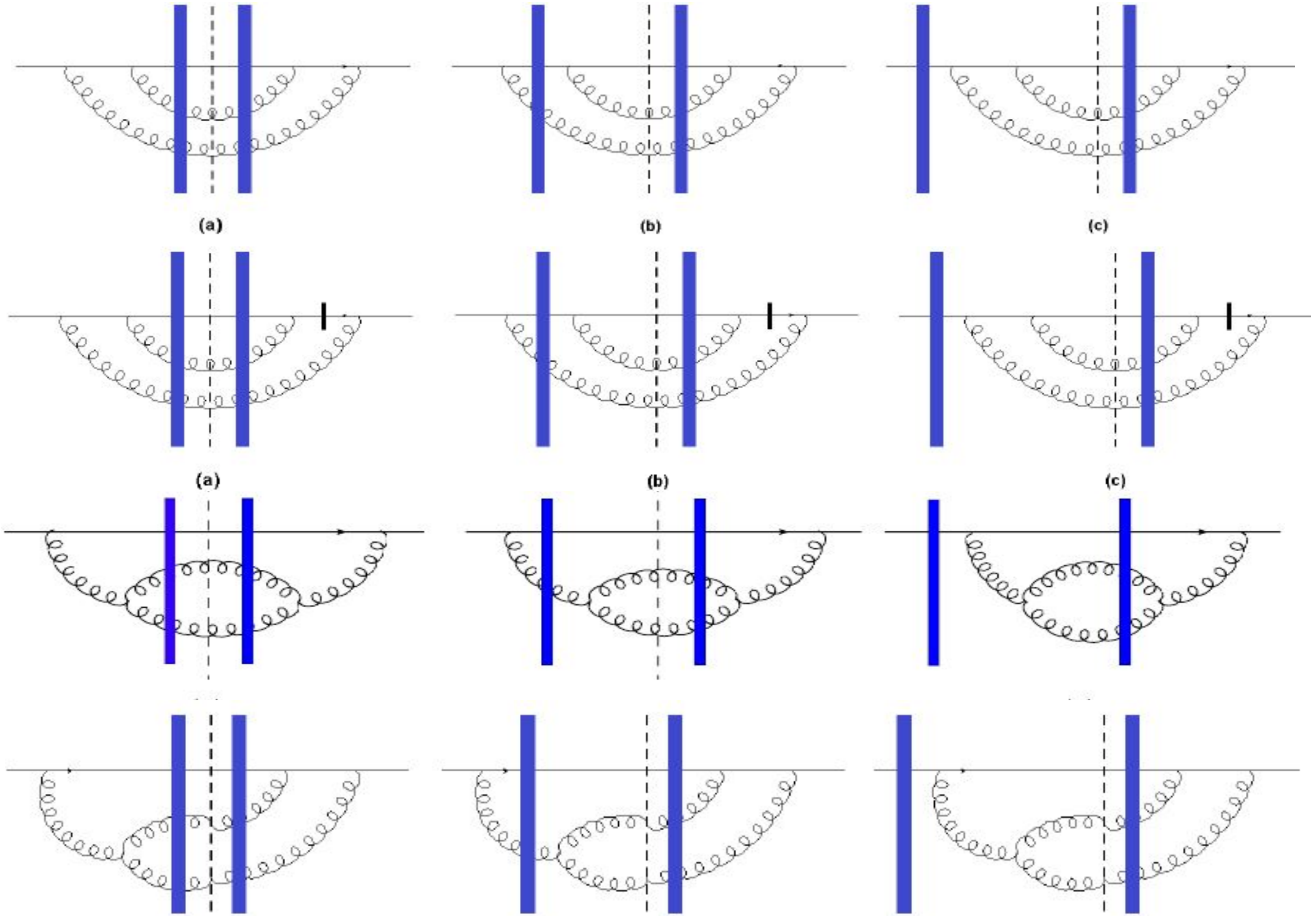
$$S_{q\bar{q}q\bar{q}q\bar{q}}\left(\overline{x},\,\overline{z},\,\overline{z}',\,x,\,z,\,z'\right) \equiv \frac{2}{C_F\,N_c}\,\mathrm{tr}\left(V^{\dagger}(\bar{x})\,V(x)\,t^a\,t^b\right)\,\mathrm{tr}\left(V(\bar{z}')\,t^b\,V^{\dagger}(\bar{z})\,V(z)\,t^a\,V^{\dagger}(z')\right)$$
$$= \frac{1}{2C_F N_c}\,\left(N_c^2\,\mathcal{Q}(\overline{x},\,x,\,z',\,\overline{z}')\,\mathcal{S}(\overline{z},\,z)\;-\;\mathcal{H}(\overline{x},\,x,\,z',\,\overline{z}',\,\overline{z},\,z) - \mathcal{H}(\overline{x},\,x,\,\overline{z},\,z,\,z',\,\overline{z}')\right.$$
$$\left.+\,\mathcal{S}(\overline{x},\,x)\,\mathcal{Q}(\overline{z},\,z,\,z',\,\overline{z}')\right)\;\simeq\;\mathcal{Q}(\overline{x},\,x,\,z',\,\overline{z}')\,\mathcal{S}(\overline{z},\,z),$$

$$S^{(1)}_{q\bar{q}q\bar{q}g}\left(\overline{x},\,\overline{z},\,\overline{z}'\,,x,\,y\right)\;\equiv\;\frac{2}{C_F\,N_c}\,\mathrm{tr}\left[t^a\,V^{\dagger}(\overline{x})\,V(x)\,t^d\right]\;\mathrm{tr}\left[t^a\,V^{\dagger}(\overline{z})\,t^c\,V(\overline{z}')\right]\;U^{cd}(y)\;=\;\frac{1}{2C_F\,N_c}$$
$$\times\left(N_c^2\,\mathcal{Q}(\overline{x},\,x,\,y,\,\overline{z}')\,\mathcal{S}(\overline{z},\,y) - \mathcal{H}(\overline{x},\,x,\,y,\,\overline{z}',\,\overline{z},\,y) - \mathcal{Q}(\overline{x},\,x,\,\overline{z},\,\overline{z}') + \mathcal{S}(\overline{x},\,x)\,\mathcal{S}(\overline{z},\,\overline{z}')\right)$$
$$\simeq\;\mathcal{Q}(\overline{x},\,x,\,y,\,\overline{z}')\,\mathcal{S}(\overline{z},\,y).$$

$$S^{(2)}_{q\bar{q}q\bar{q}g}\left(\bar{x},\,\bar{y},\,x,\,z,\,z'\right)\;\equiv\;\frac{2}{C_F\,N_c}\,\mathrm{tr}\left[t^d V^{\dagger}(\bar{x})\,V(x)\,t^a\right]\;\mathrm{tr}\left[t^c\,V(z)\,t^a\,V^{\dagger}(z')\right]\;U^{cd}(\bar{y})\;=\;\frac{1}{2C_F\,N_c}$$
$$\times\left(N_c^2\,\mathcal{Q}(\overline{x},\,x,\,z',\,\overline{y})\,\mathcal{S}(\overline{y},\,z) - \mathcal{H}(\overline{x},\,x,\,\overline{y},\,z,\,z',\,\overline{y}) - \mathcal{Q}(\overline{x},\,x,\,z',\,z) + \mathcal{S}(\overline{x},\,x)\,\mathcal{S}(z',\,z)\right)$$
$$\simeq\;\mathcal{Q}(\overline{x},\,x,\,z',\,\overline{y})\,\mathcal{S}(\overline{y},\,z),$$

$$S_{q\bar{q}q\bar{q}}\left(\bar{w},\,x,\,z,\,z'\right)\;=\;\frac{2}{C_F\,N_c}\,\mathrm{tr}\left[V^{\dagger}(\overline{w})\,t^b\,V(x)\,t^a\right]\;\mathrm{tr}\left[V(z)\,t^a\,V^{\dagger}(z')\,t^b\right]\;=\;\frac{1}{2C_F\,N_c}$$
$$\times\left(N_c^2\,\mathcal{S}(\overline{w},\,z)\,\mathcal{S}(z',\,x)\;-\;\mathcal{Q}(\overline{w},\,x,\,z',\,z)\;-\;\mathcal{Q}(\overline{w},\,z,\,z',\,x)\;+\;\mathcal{S}(\overline{w},\,x)\,\mathcal{S}(z',\,z)\right)\;\simeq\;\mathcal{S}(\overline{w},\,z)\,\mathcal{S}(z',\,x)$$

Contribution from the Gluons



The forward dijet cross section

In order to allow phenomenology to be reliable, higher order corrections as dictated by pQCD must be included in the result of hep-ph/0708.0231.

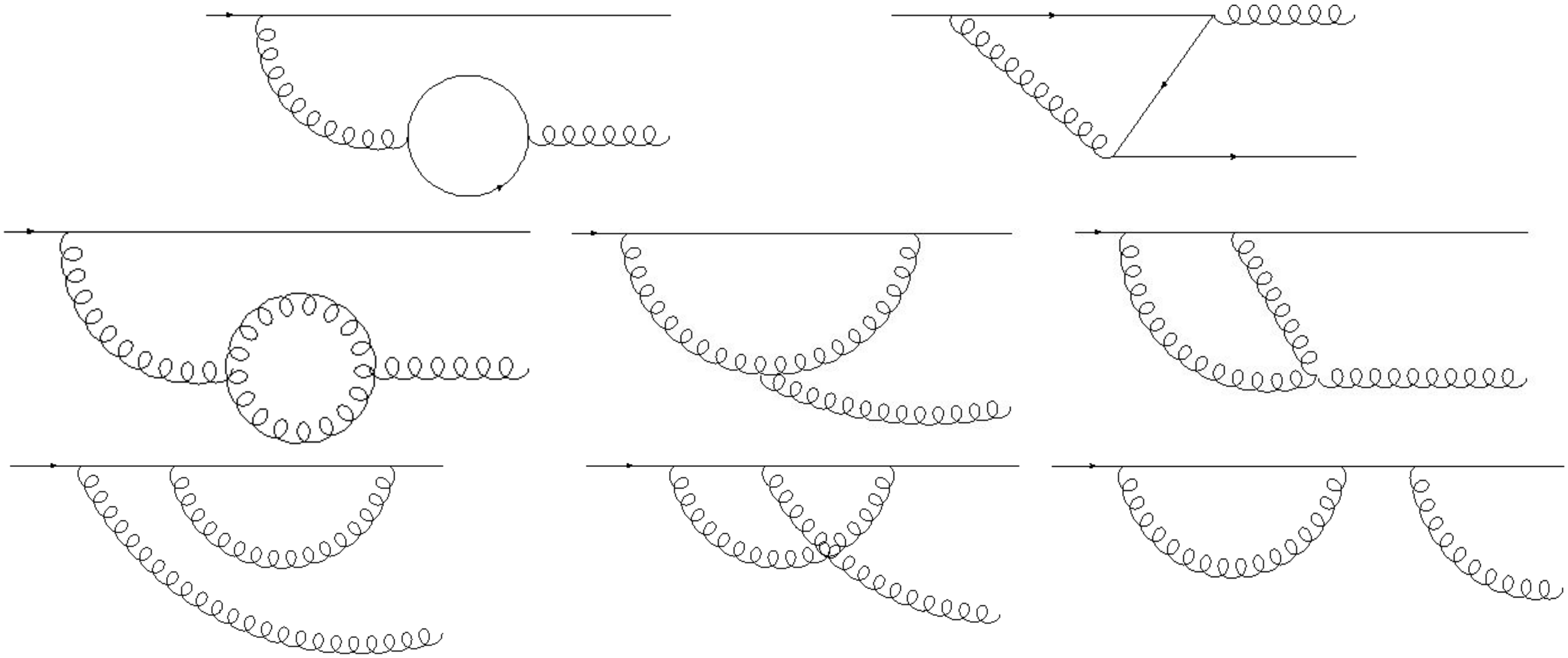
The missing part of the new outgoing state is the part which involves the production of a quark and a gluon together with a loop (virtual) correction.

Each of these diagrams has a dependence on an IR longitudinal momentum cutoff. This dependence must not be a part of the final result for the cross section.

The NLO outgoing quark state has the following structure:

$$\left|q_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{NLO} = \hat{\mathcal{Z}}_{NLO}\left|q_\lambda^\alpha(q^+,\mathrm{w})\right\rangle + \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{LO} + \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{qg} + \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{qq\bar{q}} + \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{qgg}.$$

The Diagrams



First calculated also in hep-ph/1611.00497 (T. Lappi and R. Paatelainen)

Cancellation of the IR Divergence

Example of results for the diagrams:

$$|\psi_\lambda^\alpha\rangle_{qg}^2 = \int_0^1 d\vartheta \int d^2\tilde{\mathbf{k}} \frac{g^3 N_c t^a_{\beta\alpha} \phi^{ij}_{\lambda_1\lambda}(\vartheta) \tilde{\mathbf{k}}^j \sqrt{q^+}}{4(2\pi)^5\sqrt{2\vartheta}\,\tilde{\mathbf{k}}^2} \left(\left[\frac{11}{3} + 4\ln\left(\frac{\Lambda}{\vartheta q^+}\right)\right] \left[-\frac{2}{\epsilon} + \ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)\right] \right.$$
$$\left. +2\ln^2\left(\frac{\Lambda}{\vartheta(1-\vartheta)q^+}\right) - \frac{67}{9} + \frac{2\pi^2}{3} - \frac{11}{3}\ln(1-\vartheta) - 2\ln^2(1-\vartheta) \right) \left| q^\beta_{\lambda_1}((1-\vartheta)q^+, (1-\vartheta)\mathbf{q} - \tilde{\mathbf{k}})\, g^a_i(\vartheta q^+, \vartheta\mathbf{q} + \tilde{\mathbf{k}}) \right\rangle$$

$$|\psi_\lambda^\alpha\rangle_{qg}^{5a} \equiv -\int_0^1 d\vartheta \int d^2\tilde{\mathbf{k}} \frac{g^3 t^a_{\beta\alpha} \phi^{ij}_{\lambda_1\lambda}(\vartheta) \tilde{\mathbf{k}}^j \sqrt{q^+}}{4(2\pi)^5\sqrt{2\vartheta}\,\tilde{\mathbf{k}}^2} \left(\left[\frac{3}{2} + 2\ln\left(\frac{\Lambda}{(1-\vartheta)q^+}\right)\right] \left[-\frac{2}{\epsilon} + \ln\left(\frac{\tilde{\mathbf{k}}^2}{\vartheta\mu^2_{\overline{MS}}}\right)\right] \right.$$
$$\left. +\ln^2\left(\frac{\Lambda}{(1-\vartheta)q^+}\right) + \frac{\pi^2}{3} - \frac{7}{2} \right) \left| q^\beta_{\lambda_1}((1-\vartheta)q^+, (1-\vartheta)\mathbf{q} - \tilde{\mathbf{k}})\, g^a_i(\vartheta q^+, \vartheta\mathbf{q} + \tilde{\mathbf{k}}) \right\rangle .$$

hep-ph/1606.00777

The IR logs cancellation pattern is:

	$\ln\left(\frac{\Lambda}{q^+}\right)\ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)$	$\ln^2\left(\frac{\Lambda}{\vartheta(1-\vartheta)q^+}\right)$
$\|\psi_\lambda^\alpha\rangle_{qg}^{2a}$	4	2
$\|\psi_\lambda^\alpha\rangle_{qg}^{3}$	-3	-2
$\|\psi_\lambda^\alpha\rangle_{qg}^{4}$	1	1
$\|\psi_\lambda^\alpha\rangle_{qg}^{5a}$	-2	-1

Results for the NLO WF

By combining all the loop contribution:

$$\left|\psi_{\lambda}^{\alpha}(q^+,\boldsymbol{w})\right\rangle_{qg} = -\int_0^1 d\vartheta \int_{\mathbf{x},\mathbf{z}} \frac{ig^3 N_c t^a_{\beta\alpha}\sqrt{q^+}\,\mathbf{X}^j}{4(2\pi)^4\sqrt{2\vartheta}\,\mathbf{X}^2}\left\{\phi^{ij}_{\lambda_1\lambda}(\vartheta)\left(-\beta(\vartheta)\left[\frac{2}{\epsilon}+\ln\left(\frac{\mathbf{X}^2\mu^2_{\overline{MS}}}{4e^{-2\gamma}}\right)\right]+\gamma(\vartheta)+\mathcal{I}(\vartheta)\right)+\kappa^{ij}_{\lambda_1\lambda}(\vartheta)\right\}$$
$$\times\,\delta(\boldsymbol{w}-(1-\vartheta)\boldsymbol{x}-\vartheta\boldsymbol{z})\left|q^{\beta}_{\lambda_1}((1-\vartheta)q^+,\boldsymbol{x})\,g^a_i(\vartheta q^+,\boldsymbol{z})\right\rangle$$

with:

$$\beta(\vartheta)\equiv\left(\frac{11}{3}+\ln(1-\vartheta)\right)N_c-\frac{2}{3}N_f$$

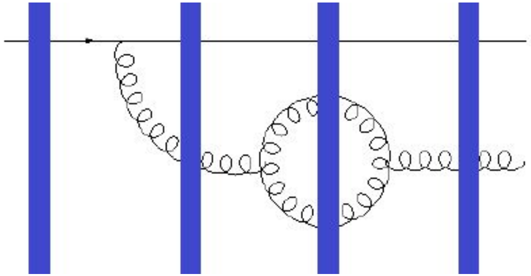
$$\gamma(\vartheta)\equiv\left(\frac{67}{9}-\frac{\pi^2}{3}+3Li_2(\vartheta)+\frac{1}{2}\ln\left(e^{\frac{13}{3}}\vartheta^2(1-\vartheta)\right)\ln(1-\vartheta)\right)N_c-\frac{10}{9}N_f$$

$$\mathcal{I}(\vartheta)\equiv\left(3+4\ln\left(\frac{\Lambda}{\vartheta q^+}\right)\right)N_c\int\frac{d^2\tilde{\boldsymbol{p}}}{\tilde{\boldsymbol{p}}^2}$$

$$\kappa^{ij}_{\lambda_1\lambda}(\vartheta)\equiv\chi^{\dagger}_{\lambda_1}\left\{\frac{\vartheta(2-\vartheta)}{2}\delta^{ij}+\left[4\vartheta(2-\vartheta)+(2-\vartheta)\ln(1-\vartheta)+\frac{3\vartheta^2}{1-\vartheta}\ln(\vartheta)\right]i\varepsilon^{ij}\sigma^3\right.$$
$$\left.+\vartheta(\vartheta-4)\left(-\frac{2}{\epsilon}+\ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)\right)\left(\delta^{ij}-i\varepsilon^{ij}\sigma^3\right)\right\}\chi_{\lambda}.$$

The cubic term of the outgoing qg state (the construction involves four shockwave insertions):

$$|out\rangle^{(3)} = -\left[\frac{\langle q\, g_2|\, H\, |q\, g_3\, g_4\rangle\, \langle q\, g_3\, g_4|\, H\, |q\, g_1\rangle\, \langle q\, g_1|\, H\, |q\rangle}{(E_{qgg}-E_{qg_2})\,(E_{qgg}-E_{qg_1})\,(E_{qg_1}-E_q)}\,(S_F\, S_{A_1}-S_F)\right.$$
$$+\frac{\langle q\, g_2|\, H\, |q\, g_3\, g_4\rangle\, \langle q\, g_3\, g_4|\, H\, |q\, g_1\rangle\, \langle q\, g_1|\, H\, |q\rangle}{(E_{qgg}-E_{qg_2})\,(E_{qgg}-E_q)\,(E_{qg_1}-E_q)}\,(S_F\, S_{A_3}\, S_{A_4}-S_F)$$
$$\left.+\frac{\langle qg_2|\, H\, |q\, g_3\, g_4\rangle\, \langle q\, g_3\, g_4|\, H\, |q\, g_1\rangle\, \langle q\, g_1|\, H\, |q\rangle}{(E_{qg_2}-E_q)\,(E_{qgg}-E_q)\,(E_{qg_1}-E_q)}\,(S_F\, S_{A_2}-S_F)\right]|qg_2\rangle\,.$$



Which can be written as a sum of five different contributions:

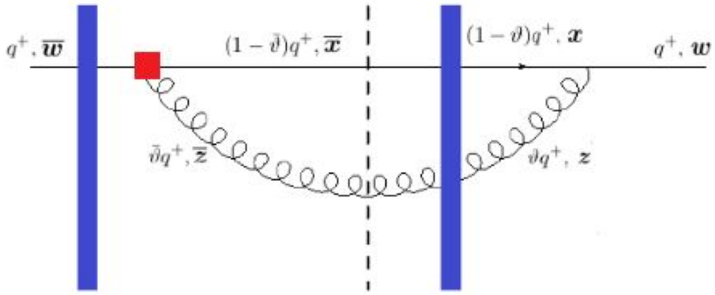
$$\frac{d\sigma^{dijet}}{d^3k\, d^3p} \equiv \frac{d\sigma_R^{q\to qqX}}{d^3k\, d^3p}+\frac{d\sigma_R^{q\to q\bar{q}X}}{d^3k\, d^3p}+\frac{d\sigma_R^{q\to ggX}}{d^3k\, d^3p}+\frac{d\sigma_R^{q\to qgX}}{d^3k\, d^3p}+\frac{d\sigma_V^{q\to qgX}}{d^3k\, d^3p}$$

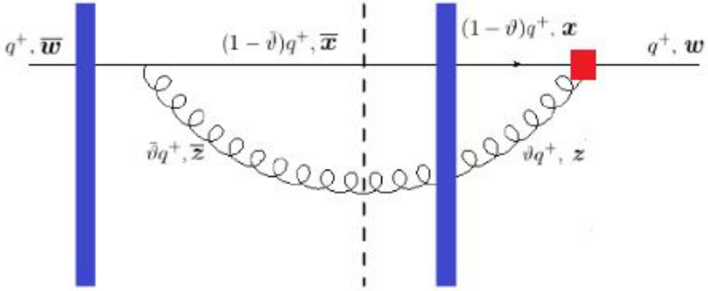
$$\frac{d\sigma_V^{q\to qgX}}{d^3k\, d^3p} = \frac{d\sigma_I^{q\to qgX}}{d^3k\, d^3p}+\frac{d\sigma_{II}^{q\to qgX}}{d^3k\, d^3p}+\frac{d\sigma_{III}^{q\to qgX}}{d^3k\, d^3p}$$

The real contributions are directly related to the results in hep-ph/1809.05526.
Analogous calculation by Chirilli, Xiao, Yuan hep-ph/1112.1061,hep-ph/1203.6139.

Partial Results for the NLO dijet

The corresponding diagrams for NLO dijet cross section:





Contributions to the dijet cross section when the shockwave is inserted at final - initial state:

$$\frac{d\sigma_I^{qA\to qg+X}}{dk^+\, d^2\boldsymbol{k}\, dp^+\, d^2\boldsymbol{p}} = \frac{\alpha_s^2}{(2\pi)^6 q^+}\,\delta(q^+ - k^+ - p^+)$$

$$\times \int_{\boldsymbol{x},\overline{\boldsymbol{x}},\boldsymbol{z},\overline{\boldsymbol{z}}} \frac{\mathbf{X}\cdot\overline{\mathbf{X}}}{\mathbf{X}^2\,\overline{\mathbf{X}}^2}\, K\left(\vartheta, \mathbf{X}^2, \overline{\mathbf{X}}^2\right)\, \mathrm{e}^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{k}\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})}\, \mathbb{S}_{\mathrm{LO}}\left(\overline{\boldsymbol{w}}, \overline{\boldsymbol{x}}, \overline{\boldsymbol{z}}, \boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z}\right)$$

$$K\left(\vartheta, \mathbf{X}^2, \overline{\mathbf{X}}^2\right) = \left\{\frac{1+(1-\vartheta)^2}{\vartheta}\left(\beta(\vartheta)\left[-\frac{2}{\epsilon} + \ln\left(\frac{\mathbf{X}^2\mu^2_{\overline{MS}}}{4e^{-2\gamma}}\right)\right] + \gamma(\vartheta) + \mathcal{I}(\vartheta)\right) + F(\vartheta) + \vartheta\beta(\vartheta)\right\} + (\mathbf{X} \longleftrightarrow \overline{\mathbf{X}})$$

$$F(\vartheta) \equiv \frac{1}{2}\left[(\vartheta - 4)\left(-\frac{2}{\epsilon} + \ln\left(\frac{\mathbf{X}^2\mu^2_{\overline{MS}}}{4e^{-2\gamma}}\right)\right) + \frac{1}{2}(13\vartheta^2 - 36\vartheta + 4) - (1-\vartheta)\ln(1-\vartheta) - \frac{3\vartheta^2}{2(1-\vartheta)}\ln(\vartheta)\right] N_c$$

Summary

1) Generalization of the method by C. Marquet (2007) to all orders, for the calculation of the forward particle production in proton-nucleus collisions at high energy, was shown to be possible by adopting the outgoing state approach.

2) We computed the full light-cone wave function of the incoming quark, and partially its corresponding outgoing state.

3) IR divergences has been shown to cancel after combining all the loop contributions (except normalization contribution).

4) Partial results for the inclusive forward NLO dijet cross section are available.