Vertices of three reggeized gluons and the unitarity corrections to the propagator of reggeized gluons

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Plan

1. The Lipatov's effective action
2. How correlators and evolution equations are derived from it?
3. One loop QCD Regge Field Theory
4. Vertices of three reggeized gluons

Effective action

The Lipatov's effective action is gauge invariant and written in the covariant form in terms of gluon field v as

$$S_{eff} \; = \; - \int d^4 x \left(\frac{1}{4} \, G^a_{\mu\nu} \, G_a^{\mu\nu} \right. \\ \left. + \, \frac{1}{N} tr \, \left[\, (\, A_+(v_+) \, - \, A_+ \,) \, \partial_i^2 \, A_a^+ \, + \, (\, A_-(v_-) \, - \, A_- \,) \, \partial_i^2 \, A_a^- \, \right] \right),$$

where

$$A_\pm(v_\pm) \; = \; \frac{1}{g} \, \partial_\pm \, O(x^\pm, v_\pm); \;\; O(x^\pm, v_\pm) \; = \; P \, e^{g \int_{-\infty}^{x^\pm} \, dx'^\pm \, v_\pm} \, .$$

There are additional kinematical constraints for the reggeon fields

$$\partial_- \, A_+ \; = \; \partial_+ \, A_- \; = \; 0 \, ,$$

[1] L. N. Lipatov, Nucl. Phys. B **452**, 369 (1995); Phys. Rept. **286**, (1997) 131;

In the framework with an external source of the color charge introduced, keeping only gluon field depending terms, we rewrite action as

$$S_{eff} \,=\, -\int d^4 x \left(\frac{1}{4}\, G^a_{\mu\nu}\, G_a^{\mu\nu} \,+\, v_-\, J^-(v_-) \,+\, v_+\, J^+(v_+) \right)\,,$$

Under variation on the gluon fields these currents reproduce the Lipatov's induced currents

$$\delta\,\left(\, v_\pm\, J^\pm(v_\pm)\,\right) \,=\, (\delta\, v_\pm)\, j^{ind}_\mp(v_\pm) \,=\, (\delta\, v_\pm)\, j^\pm(v_\pm)\,,$$

The classical equations of motion for the gluon field v_μ which arose from the action are the following:

$$(\, D_\mu\, G^{\mu\nu}\,)_a \,=\, \partial_\mu\, G_a^{\mu\nu} \,+\, g\, f_{abc} v_\mu^b\, G^{c\,\mu\nu} \,=\, j_a^+\,\delta^{\nu +} \,+\, j_a^-\,\delta^{\nu -}$$

One-loop correction

We have found a solution to these equations of motion $v^a_{i\,cl}$, $v^a_{+\,cl}$ with NNLO precision and determined in the original Lagrangian:

$$v^a_i \to v^a_{i\,cl} + \varepsilon^a_i\,, \quad v^a_+ \to v^a_{+\,cl} + \varepsilon^a_+\,, \quad v^a_- = 0\,,$$

$$v^{ia}_{cl} = \sum_{k=0}^{+\infty} g^k v^{ia}_k(A_+, A_-), \quad v^a_{+cl} = \sum_{k=0}^{+\infty} g^k v^a_{+k}(A_+, A_-)$$

and after integration over fluctuation $\varepsilon_{i,+}$ we have an expression for an effective one-loop action:

$$\begin{aligned}\Gamma \;&=\; \int d^4x\, \left(L_{YM}(v^{cl}_i, v^{cl}_+) - v^a_{+\,cl}\, J^+_a(v^{cl}_+) - A^a_+\, \left(\partial^2_i\, A^a_- \right) \right) \\ &+\; \frac{\imath}{2}\, \ln \left(1 + G(v^{cl})\, M(v^{cl}) \right)\,,\end{aligned}$$

which is the functional of only reggeized gluon fields.

The Lipatov's effective action formulated as RFT

In general case, consider the Lipatov's effective action for reggeized gluons $A_{\pm}$, formulated as RFT (Regge Field Theory) which can be obtained by an integration out the gluon fields v in the generating functional for the $S_{eff}[v, A]$:

$$e^{\imath\,\Gamma[A]} = \int Dv\, e^{\imath\, S_{eff}[v,\, A]}$$

The Lipatov's effective action formulated as RFT

The result of this action is that the effective action Γ can be expanded in terms of reggeon fields A_- and A_+ as

$$
\begin{aligned}
\Gamma &= \sum_{n,m=1} \left(\mathcal{A}_{+}^{a_1} \cdots \mathcal{A}_{+}^{a_n} \left(K_{-\,\cdots\,-}^{+\,\cdots\,+}\right)_{b_1 \cdots\, b_m}^{a_1 \cdots\, a_n} \mathcal{A}_{-}^{b_1} \cdots \mathcal{A}_{-}^{b_m} \right) \\
&= -\mathcal{A}_{+\,x}^{a}\, \partial_i^2\, \mathcal{A}_{-\,x}^{a} + \mathcal{A}_{+\,x}^{a} \left(K_{xy}^{a\,b}\right)_{-}^{+} \mathcal{A}_{-\,y}^{b} + \cdots ,
\end{aligned}
$$

that determines this expression as functional of reggeon fields and provides effective vertices of the interactions of the reggeized gluons in the RFT calculus.

[2] S. Bondarenko, L. Lipatov, S. Pozdnyakov, A. Prygarin, Eur. Phys. J. C **77** (2017) no.9, 630.

A hierarchy of correlators in the formalism of Lipatov's effective action

Generating functional for calculation the correlators of the reggeon fields

$$Z[J] = \int D\exp\Big(\imath\,\Gamma[A] - \imath \int d^4x\, J^a_-(x^-, x_\perp)\, A^a_+(x^+, x_\perp) - \imath \int d^4x\, J^a_+(x^+, x_\perp)\, A^a_-(x^-, x_\perp)\Big)$$

$$< A^a_\pm(x, \eta) > = \sum_{n=1} \Big(\tilde{K}(\eta; x, x_1 \cdots, x_n) \Big)^{a\, a_1 \cdots a_n} < A^{a_1}_\pm(x_1) \cdots A^{a_n}_\pm(x_n) >$$

Taking the derivative of this equation with respect to η, we can also obtain a BFKL-like evolution equation for reggeized gluon fields:

$$\frac{\partial}{\partial \eta} < A^a_\pm > = \frac{\partial}{\partial \eta} \left(\sum_{n=1} \Big(\tilde{K}(\eta) \Big)^{a\, a_1 \cdots a_a} < A^{a_1}_\pm \cdots A^{a_n}_\pm > \right)$$

Results of variation on currents J

$$\partial^2_{\perp\,1} < A^{a_1}_{\pm}\, A^{a_2}_{\mp} >= -\imath\, \delta^{\,a_1\,a_2}\, \delta_{\pm 1\;\pm 2}\, \delta(x_{\perp\,1} - x_{\perp\,2}) + \left(K^{\,a_1}_{\,b_1} \right)^{\pm}_{\mp} < A^{b_1}_{\pm}\, A^{a_2}_{\mp} > + \cdots .$$

$$< A^{a}_{\pm}\, A^{a_1}_{\pm} \cdots A^{a_m}_{\pm} >= \sum_{n=1} \left(\hat{K}(\eta) \right)^{a\, b_1 \cdots b_n} < A^{b_1}_{\pm} \cdots A^{b_n}_{\pm}\, A^{a_1}_{\pm} \cdots A^{a_m}_{\pm} > .$$

Equations analogous to the hierarchy of Wilson line correlators in the Balitsky-JIMWLK formalism

$$\frac{\partial}{\partial\,\eta} < A^{a}_{\pm}\, A^{a_1}_{\pm} \cdots A^{a_m}_{\pm} >= \frac{\delta}{\delta\,\eta} \sum_{n=1} \left(\hat{K}(\eta) \right)^{a\, b_1 \cdots b_n} < A^{b_1}_{\pm} \cdots A^{b_n}_{\pm}\, A^{a_1}_{\pm} \cdots A^{a_m}_{\pm} > .$$

Propagator of reggeized gluons

1) Effective vertex of the interaction with NLO

$$\left(K^{a\,b}_{x\,y}\right)^{+\,-} = K^{a\,b}_{x\,y} = \left(\frac{\delta^2\,\Gamma}{\delta A^a_{+\,x}\,\delta A^b_{-\,y}}\right)_{A_+,\,A_-,\,v_{f\,\perp}=0}.$$

2) Propagator of reggeized gluons in the form of the perturbative series

$$D^{ac}_{xy} = D^{ac}_{xy\,0} - \int d^4z \int d^4w\, D^{ab}_{xz\,0}\left(\sum_{k=1} K^{bd}_{zw\,k}\right) D^{dc}_{wy}\,.$$

3) Final expression for the propagator with one-loop precision

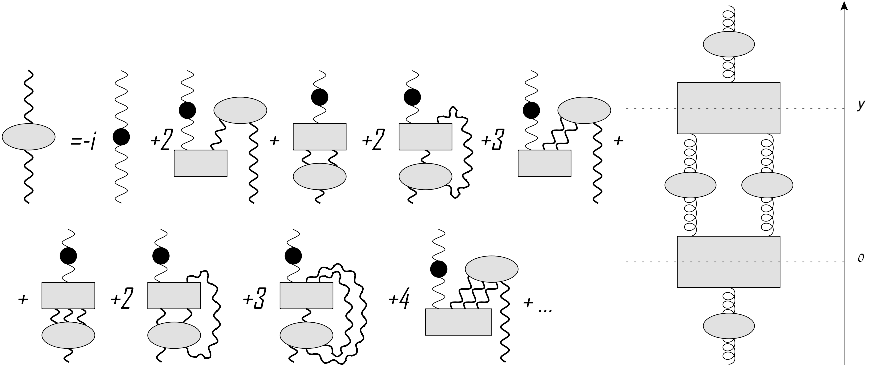
$$D^{ac}_{xy} = \delta^{ac}\delta(y^- - x^-)\,\delta(x^+ - y^+)\int \frac{d^2p}{(2\pi)^2}\,\tilde{D}(p_\perp,\,\eta)\,e^{-\imath\,p_i\,(x^i - y^i)}\,,$$

$$\tilde{D}^{ab}(p_\perp,\eta) = \frac{\delta^{ab}}{p^2_\perp}\,e^{\eta\,\epsilon(p^2_\perp)}\,,\quad \epsilon(p^2_\perp) = -\frac{\alpha_s\,N}{4\,\pi^2}\int d^2k_\perp\,\frac{p^2_\perp}{k^2_\perp\,(p_\perp - k_\perp)^2}$$

One-loop correction from a hierarchy of correlators

Here in a diagrammatic form: the ovals are the correlators, the blocks are vertices (interaction kernel), and the bold dot is the bare propagator of the Reggeon field D_0:

$$\left(\delta^{a\,b} \partial_\perp^2 - (K^{\,a}_{\,b})^{+}_{-} \right) D_{0\,-}^{bc\,+} = \delta^{a\,c}\,.$$



Violation of kinematical conditions

$$
\begin{aligned}
& \partial^2_{\perp\, x} < A^a_+(x^+, x_\perp) A^{a_1}_-(y^-, y_\perp) > = -\imath\, \delta^{a a_1}\, \delta(x^+)\, \delta(y^-)\, \delta^2(x_\perp - y_\perp) \, + \\
+ & \int d^2 z_\perp\, dz_1^+\, d^2 z_{1\perp}\, K^{b_1 b_2 a}_{++-}(x^+, z_\perp; z_1^+, z_{1\perp}; x_\perp) < A^{b_1}_+(x^+, z_\perp) A^{b_2}_+(z_1^+, z_{1\perp}) A^{a_1}_-(y^-, y_\perp) > + \\
+ & \int d^2 z_\perp\, dz_1^-\, d^2 z_{1\perp}\, K^{b_1 b_2 a}_{+--}(z_\perp; z_1^-, z_{1\perp}; x^-, x_\perp) < A^{b_1}_+(x^+, z_\perp) A^{b_2}_-(z_1^-, z_{1\perp}) A^{a_1}_-(y^-, y_\perp) > .
\end{aligned}
$$

The third term in the r.h.s. of the equation depends on x^- variable, whereas the correlator in the l.h.s. does not. This discrepancy means a violation of kinematical conditions, which in fact were known as not-absolute and therefore, generalizing the approach, we have to consider the reggeon fields as four-dimensional ones:

$$
A_+(x^+, x_\perp) \rightarrow \mathcal{B}_+(x^+, x^-, x_\perp) = A_+(x^+, x_\perp) + \mathcal{D}_+(x^+, x^-, x_\perp), \;\; \mathcal{D}_+(x^+, x^- = 0, x_\perp) = 0
$$

and

$$
A_-(x^-, x_\perp) \rightarrow \mathcal{B}_-(x^+, x^-, x_\perp) = A_-(x^-, x_\perp) + \mathcal{D}_-(x^+, x^-, x_\perp), \; \mathcal{D}_-(x^+ = 0, x^-, x_\perp) = 0 .
$$

Violation of kinematical conditions for correlator

$$
\begin{aligned}
& < A_+^a(x^+,x_\perp) A_-^{a_1}(y^-,y_\perp) > \to < \mathcal{B}_+^a(x^+,x^- \, x_\perp) A_-^{a_1}(y^-,y_\perp) > \\
= \; & < A_+^a(x^+,x_\perp) A_-^{a_1}(y^-,y_\perp) > + < \mathcal{D}_+^a(x^+,x^-,x_\perp) A_-^{a_1}(y^-,y_\perp) >
\end{aligned}
$$

Treating the expression perturbatively and writing the correlators as propagators

$$
< A_+^a(x^+,x_\perp) A_-^b(y^-,y_\perp) > = \imath\, \delta(x^+)\, \delta(y^-)\, \delta^{ab}\, D(x_\perp,y_\perp)
$$

and

$$
< \mathcal{D}_+^a(x^+,x^-,x_\perp) A_-^{a_1}(y^-,y_\perp) > = \imath\, \mathcal{G}_+^{aa_1}(x^+,x^-,x_\perp; y^-,y_\perp)\,,
$$

Violation of the reggeized form of the initial propagator

$$D^{ab}(x^{+},x^{-},x_{\perp};y^{+},y^{-},y_{\perp};Y) \;=\; D^{ab}(x_{\perp},y_{\perp};Y) + \mathcal{G}_{+}^{aa_1}(x^{+},x^{-},x_{\perp};y^{-},y_{\perp};Y)$$
$$+\; \mathcal{G}_{-}^{aa_1}(x^{+},x_{\perp};y^{+},y^{-},y_{\perp};Y)\,.$$

Fourier transform of $\mathcal{G}_{+}^{aa_1}$ with respect to x^{+} and $x_{\perp}-y_{\perp}$ is

$$\tilde{\mathcal{G}}_{+}^{aa_1}(p_{+},p_{\perp};y_{-},x^{-};Y) =$$
$$= \frac{2\,\delta^{aa_1}}{p_{\perp}^{2}}\left(e^{\,\epsilon(p_{\perp}^{2})\,Y} - e^{2\,\epsilon(p_{\perp}^{2})\,Y} - \int_{-\,\epsilon(p_{\perp}^{2})\,Y}^{-\,2\,\epsilon(p_{\perp}^{2})\,Y}\frac{dy}{y}\left(e^{-y}-1\right)\right)\left(\tilde{G}_{x-\,0}^{-\,0} - \tilde{G}_{0\,x-}^{-\,0}\right)$$

Correction $\mathcal{G}_{-}^{aa_1}$ can be directly obtained by change of the $+$ sign in the expression on the $-$ sign everywhere and corresponding change of the coordinates

This correction is large and does not suppressed perturbatively

$\tilde{\mathcal{G}}_{+}^{aa_1}(p_+, p_\perp; x^-, y^-; Y) =$

$$\frac{2\delta^{aa_1}}{p_\perp^2}\left(1 - e^{\epsilon(p_\perp^2)Y} - \int_0^{-\epsilon(p_\perp^2)Y}\frac{dy}{y}\left(e^{-y} - 1\right)\right)\left(\tilde{G}^{-0}_{x-0} - \tilde{G}^{-0}_{0x-}\right)\delta(p_+)\delta(y^-)\,.$$

The last step in the Fourier transform is the transforms of the theta functions x^- coordinat. We note that expression is zero at $x^- = 0$, as it must be. Therefore, in order to preserve the property of the expression, we have to regularize the Fourier transform of the difference of the theta functions in this limit, we do so with the help of Sokhotski expressions:

$$\frac{1}{p_- + \imath\varepsilon} + \frac{1}{p_- - \imath\varepsilon} = 2\,\mathscr{P}(\frac{1}{p_-}) \tag{1}$$

where

$$\left(\mathscr{P}(\frac{1}{p_-}), f(p_-)\right) = PV\int\frac{f(p_-)}{p_-}\,dp_- = \int\frac{f(p_-) - f(0)}{p_-}\,dp_-\,. \tag{2}$$

On reggeization of vertex of three reggeized gluons

Any vertex of the three Reggeon fields interactions is defined as following

$$\left(K^{a\,b\,c}_{x\,y\,z} \right)^{\mu\nu\rho} = \left(\frac{\delta^3\,\Gamma(v_{i\,cl}(A_+,A_-), v_{+\,cl}(A_+,A_-))}{\delta\,A^a_\mu\,\delta\,A^b_\nu\,\delta\,A^c_\rho} \right)_{A_+,\,A_-,\,v_{f\,\perp}=0} .$$

$$\begin{aligned} \left(K^{abc}_{xzy;\eta}\right)^{++} &= \left(K^{abc}_{xzy}\right)^{++}_0 - \frac{2\alpha_s N}{(2\pi)^4}\int_0^\eta d\,s \int d^2\omega_\perp \left(K^{abc}_{xzy;s}\right)^{++} \\ &* \int d^2k_\perp \int d^2k_{1\perp}\,\frac{k^2_{1\perp}}{k^2_\perp\,(k_\perp - k_{1\perp})^2}\,e^{-\imath(y^i-\omega^i)k_{1i}} . \end{aligned}$$

where

$$\left(K^{abc}_{xzy}\right)^{++}_0 = \frac{1}{2} g f^{abc}(\theta_{x^+-z^+} - \theta_{z^+-x^+})\delta^2_{y_\perp\,x_\perp}\,\delta^2_{z_\perp\,y_\perp}\,\partial^2_{iy} .$$

The solution of this equation after Fourier transform leads to the reggeization of the vertex with the trajectory twice larger than the trajectory of the propagator of reggeized gluons.

$$\frac{\partial \bar{K}^{abc}}{\partial \eta} = 2\varepsilon(p_{2\perp})\bar{K}^{abc}\,,\ \varepsilon(p_{2\perp}) = - - \frac{\alpha_s N}{(2\pi)^2}\int d^2 k_\perp \frac{k_{1\perp}^2}{p_\perp^2\left(k_\perp - p_\perp\right)^2}$$

$$\bar{K}^{abc}\left(x^+, z^+; p_\perp, p_{1\perp}, p_{2\perp}; \eta\right) = \bar{K}_0^{abc}\left(x^+, z^+; p_\perp, p_{1\perp}, p_{2\perp}; \eta\right) e^{2\eta\varepsilon(p_{2\perp})}\,.$$

[3] S. Bondarenko, M.A. Zubkov, Eur.Phys.J. C **78** (2018) no.8, 617

The relation of reggeon correlators to the correlators of Wilson lines operators built from the longitudinal gluons is also clarified and results of this relation, we hope, will help to understand general landscape of the high energy QCD approaches.

Correlators of Lipatov's operators

$$\hat{O}(v_{\pm}) = \frac{1}{2}\left(P\, e^{g\, \int_{-\infty}^{\infty} dx^{\pm}\, v_{\pm}(x^{\pm},\, x^{\mp},\, x_{\perp})} - \bar{P}\, e^{-\,g\, \int_{-\infty}^{\infty} dx^{\pm}\, v_{\pm}(x^{\pm},\, x^{\mp},\, x_{\perp})} \right)$$

$$< \left(T^{a_1}\, \hat{O}_1(v_{\pm}) \right) \otimes \left(T^{a_2}\, \hat{O}_2(v_{\pm}) \right) \otimes \cdots \otimes \left(T^{a_n}\, \hat{O}_n(v_{\pm}) \right) > =$$

$$= \quad C(R)^n\, (2\, g)^n \left(\frac{\delta^{\,n}}{\delta J_{\mp}^{a_1}\, \cdots\, \delta J_{\mp}^{a_n}} \log Z[J] \right)_{J=0} = C(R)^n\, (2g)^n$$

$$\int dx_1^{\pm} \cdots \int dx_n^{\pm} \left(\sum_{j=0}^{J} C_{b_1 \cdots\, b_{n-2j}}^{a_1 \cdots a_n} \left(\frac{\imath}{2}\, \partial_{\perp}^{-2} \right)^j (-\imath)^{n-2j} < A_{\pm}^{b_1} \cdots A_{\pm}^{b_{n-2j}} > \right)$$

Conclusion

1. This effective action formalism, based on the reggeized gluons as main degrees of freedom, can be considered as reformulation of the RFT calculus for the case of high energy QCD.
2. We calculated the one loop RFT contribution to the propagator of reggeized gluons in the framework of Lipatov's effective action and Dyson-Schwinger hierarchy of the equations for the correlators of reggeized gluon fields.
3. We find additional part of the corrections that breaks the propagator's reggeization.
4. The one loop QCD triple vertices will change the rapidity structure of the two reggeized gluons correlator as well and will allow understand better the quantum structure of the theory.
5. The continuation of high energy QCD Lipatov's effective action to Euclidean space is performed.

Solution to these equations of motion

[2] S. Bondarenko, L. Lipatov and A. Prygarin, Eur. Phys. J. C **77** (2017) no.8, 527.

$$v_{+0\;a} = A_{+\;a}, \tag{3}$$

$$v_{i0\;a} = \rho_i^b(x_\perp, x^-)U_{ab}, \tag{4}$$

$$v_{+1\;a} = -\frac{2}{g}\Box^{-1}\Big((\partial_+\partial^i U_{ab})\rho_i^b\Big), \tag{5}$$

$$v_{i1\;a} = -\Box^{-1}\Big[\partial^j F_{ji\;a} + \frac{1}{g}\partial_i\Big((\partial^j U_{ab})\rho_j^b\Big) - \partial_i^{-1} j_{a1}^{+}\Big]. \tag{6}$$

$$U^{ab}(v_+) = -tr\Big[T^a\Big(Pe^{g\int_{-\infty}^{x^+} dx'^+ v_{+c}(x'^+, x^-, x_\perp)T^c}\Big)T^b\Big(Pe^{-g\int_{x^+}^{+\infty} dx'^+ v_{+d}(x'^+, x^-, x_\perp)T^d}\Big)\Big]. \tag{7}$$

NNLO solution to these equations of motion

$$v_{+2\ a} = \Box^{-1}\partial_-^{-1}\Big[\frac{2}{g}\partial_+ j^+_{a1} + f_{abc}\Big(2\partial_- v^b_{i0}\partial_+ v_1^{ic} + (\partial_i v_0^{i\ c})\partial_- v^b_{1+} + 2\partial_i(A^b_+\partial_- v_1^{i\ c})\Big)\Big], \quad (8)$$

$$v^i_{2a} = \Box^{-1}\Big[\partial_-^{-1}\partial^i\Big(L^+_{a2} - (\partial^j\rho^b_j)(\frac{1}{g^2}\partial_- U_{ab})\Big) - f_{abc}\Big(\frac{1}{g}v^b_{j0}\Big(\partial^j v_0^{i\ c} - \partial^i v_0^{j\ c}\Big) + v^b_{0j}(\partial^j v_1^{i\ c} - \partial^i v_1^{j\ c}) - v_0^{i\ c}v^b_{1+} + 2A^b_+ v_1^{i\ c} + + \partial_j(v_0^{j\ b}v_1^{i\ c} - v_0^{i\ b}v_1^{j\ c}) + f^{cde}v^b_{j0}v^j_{0d}v^i_{0e}\Big)\Big]. \quad (9)$$

Extending by including the quarks

We have added a part of the Lagrangian L_{quark}. We find only one correction of the order g^2 and ε^2. This addition will allow us to take into account the contribution with a quark loop in the gluon. Integrating out the fluctuation we obtain the following expression for the contribution to the action:

$$-\frac{g^2}{4} tr\Big(G_q^0(y,x)(\gamma^\nu v_{a\nu}(x)) G_q^0(x,y)(\gamma^\mu v_\mu^a(y))\Big), \qquad (10)$$

where

$$G_q^0(x,y) = (i\gamma_\nu \partial_x^\nu + m)\Delta_{xy}, \quad \Delta_{xy} = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 - m^2 + i0}$$

Then after varying by $v_{a\rho}(z)$ we obtain the following contributions to the equation of motion

$$j^\rho_{quark\ a}(z) = -\frac{g^2}{2} tr\Big(\gamma^\rho G_q^0(z,y)\gamma^\mu v_{a\ \mu}(y) G_q^0(y,z)\Big) \qquad (11)$$

We verified, that condition $\partial_\rho j^\rho_{quark\ a} = 0$ is true. Then the classical solutions have additional contributions v^i_{q2} and v_{q+2}, respectively:

$$v^i_{q2a} = \Box^{-1}\left[j^i_{quark\ 0a} - \partial^i \partial_-^{-1} j^+_{quark\ 0a}\right], \tag{12}$$

$$v_{q+2a} = \Box^{-1}\left[j^-_{quark\ 0a} - \partial_+ \partial_-^{-1} j^+_{quark\ 0a}\right], \tag{13}$$

where

$$j^\rho_{quark\ a}(z) = -\frac{g^2}{2} tr\Big(\gamma^\rho G^0_q(z,y)\gamma^\mu v_{a\ \mu}(y) G^0_q(y,z))\Big) \tag{14}$$

We want to estimate the contribution of quarks to the reggeon propagator

Inserting obtained classical gluon fields solutions in the Eq. (**??**) action, we will obtain a action which will depend only on the reggeon fields. In order to calculate this action we need to know the components of the field strength tensor, with LO precision we have:

$$G^{a}_{+-\;0} = 0\,,\;\; G^{a}_{i+\;0} = \partial_i A^{a}_{+}\,,\;\; G^{a}_{i-\;0} = -\,\partial_- v^{a}_{i0}\,,\;\; G_{i\,j\;0} = 0\,, \tag{15}$$

and components of the field strength tensor with included quarks

$$G^{a}_{+-\;q2} = 0\,,\;\; G^{a}_{i+\;q2} = g^2\,\partial_i\, v^{a}_{q+2}\,,\;\; G^{a}_{i-\;q2} = -\,g^2\partial_-\, v^{a}_{qi2}\,,\;\; G_{i\,j\;q2} = 0\,, \tag{16}$$

that gives

$$\frac{1}{2}\,G_{\mu\,\nu\;0}\,G^{\mu\,\nu}_{q2} = g^2\,\left(\partial_-\, v^{a}_{i0}\right)\left(\partial_i\, v^{a}_{q+2}\right) + g^2\,\left(\partial_-\, v^{a}_{qi2}\right)\left(\partial_i\, A^{a}_{+}\right)\,. \tag{17}$$

Therefore, for the additional contributions to the effective action from the inclusion of quarks we obtain to LO:

$$S_{eff\ \ q2} = -\frac{g^2}{N}\int d^4x\Big[\partial_i\left(v^a_{q+2}\partial_i A^b_-\right) + N\left(\partial_-\left(v^b_{qi2}\partial_i A^a_+\right) - v^b_{qi2}\left(\partial_i\partial_- A^a_+\right)\right)\Big] = 0, \quad (18)$$

In addition, it is a direct contribution to the effective action:

$$-\frac{g^2}{4}\left(\frac{\delta^2\, tr\Big(G^0_q(y,x)(\gamma^\nu v_{a\nu}(x))G^0_q(x,y)(\gamma^\mu v^a_\mu(y))\Big)}{\delta A^a_{+\,x}\,\delta A^b_{-\,y}}\right)_{A_+,\,A_-=0}. \quad (19)$$

After the calculations, we showed that this contribution is also zero.