Low x physics and saturation in terms of TMD distributions

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Low x 2019

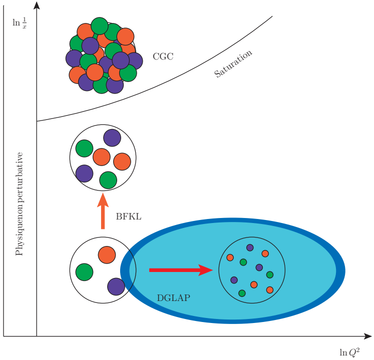
[Altinoluk, RB, Kotko], JHEP 1905 (2019) 156

[Altinoluk, RB], 1902.07930[hep-ph]

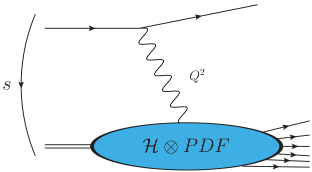
[RB, Mehtar-Tani]

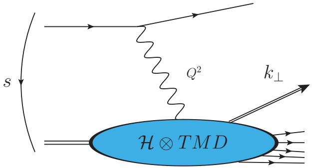
QCD at moderate x

$$Q^2 \sim s$$



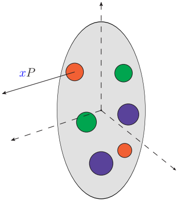
Parton Distributions

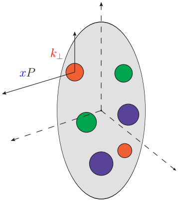




Parton Distribution Fonction (PDF)

Transverse Momentum Dependent distributions (TMD)





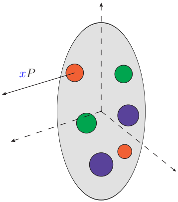
Operator definition for parton distributions

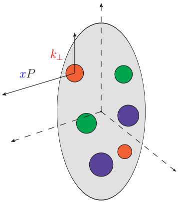
Parton distribution function

$$\mathcal{F}(x) \propto \int dz^+ e^{ixP^- z^+} \left\langle P \middle| F^{-i}(z^+) \left[z^+, 0^+\right] F^{-i}(0) \left[0^+, z^+\right] \middle| P \right\rangle$$

Transverse Momentum Dependent distribution

$$\mathcal{F}(x, k_\perp) \propto \int d^4 z \delta(z^-) e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P \middle| F^{-i}(z) \mathcal{U}_{z,0} F^{-i}(0) \mathcal{U}_{0,z} \middle| P \right\rangle$$





(So-called) non-universality of TMD distributions:

The importance of gauge links

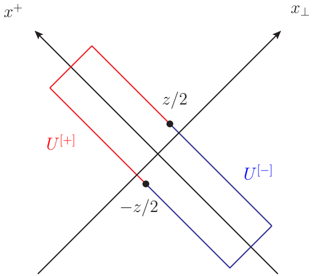
[Collins, Soper, Sterman], [Brodsky, Hwang, Schmidt], [Belitsky, Ji, Yuan], [Bomhof, Mulders, Pijlman], [Boer, Mulders, Pijlman]

[Kharzeev, Kovchegov, Tuchin]

TMD gauge links

"Non-universality" of quark TMD distributions

Gauge links can be future-pointing or past-pointing



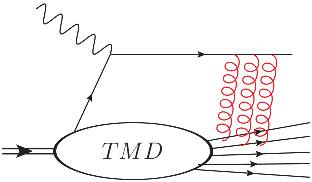
$$q^{[+]}(x,k_\perp) \propto \left\langle P,S \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[+]}_{\frac{z}{2},-\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P,S \right\rangle$$

$$q^{[-]}(x,k_\perp) \propto \left\langle P,S \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[-]}_{\frac{z}{2},-\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P,S \right\rangle$$

For naive T-odd distributions, $q^{[+]} = -q^{[-]}$: Sivers effect

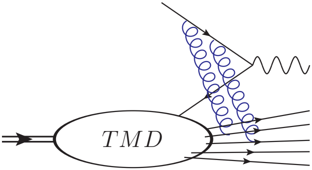
The Sivers effect

SIDIS



Final state interactions: $q^{[+]}$

Drell-Yan

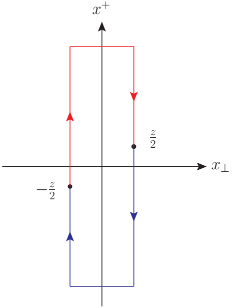


Initial state interactions: $q^{[-]}$

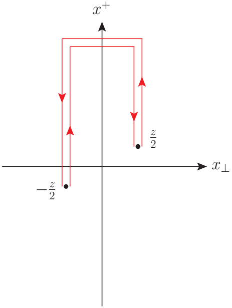
The Sivers distribution comes with a relative $-$ sign between SIDIS and DY: different gauge links for a naive T-odd quantity!

TMD gauge links

"Non-universality" of gluon TMD distributions



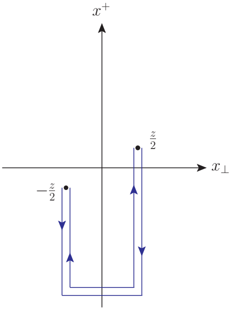
$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[-]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$



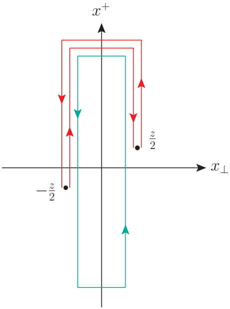
$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[+]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$

TMD gauge links

"Non-universality" of gluon TMD distributions



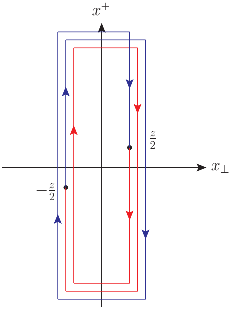
$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[-]\dagger}F^{i-}\mathcal{U}^{[-]}\right]$



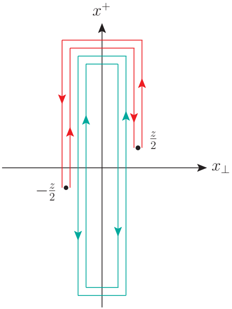
$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]$

TMD gauge links

"Non-universality" of gluon TMD distributions



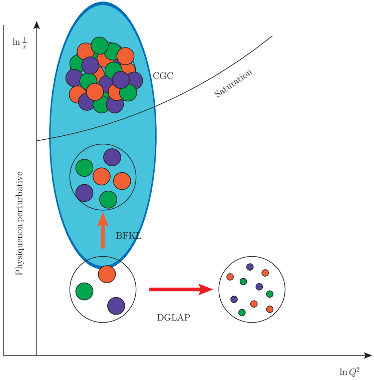
$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]$$



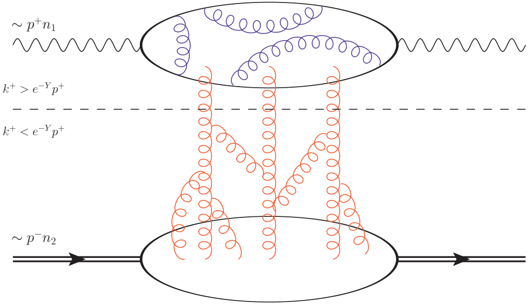
$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]$$

QCD at small x

$$Q^2 \ll s$$



Rapidity separation

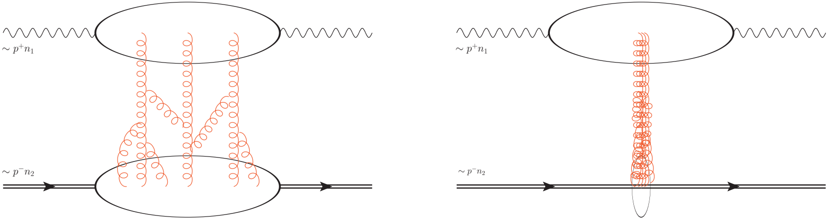


Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A^{\mu a}_{Y_c}(|k^+| > e^{-Y_c} p^+, k^-, \vec{k}) + b^{\mu a}_{Y_c}(|k^+| < e^{-Y_c} p^+, k^-, \vec{k})$$

$e^{-Y_c} \ll 1$

Large longitudinal boost to the projectile frame



$$b^+(x^+, x^-, \vec{x}) \qquad\qquad \frac{1}{\Lambda} b^+(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$b^-(x^+, x^-, \vec{x}) \quad \longrightarrow \quad \Lambda b^-(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

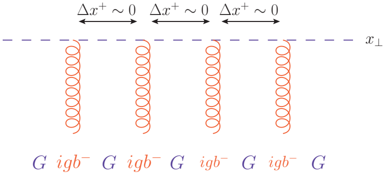
$$b^k(x^+, x^-, \vec{x}) \qquad \Lambda \sim \sqrt{\frac{s}{m_t^2}} \qquad b^k(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

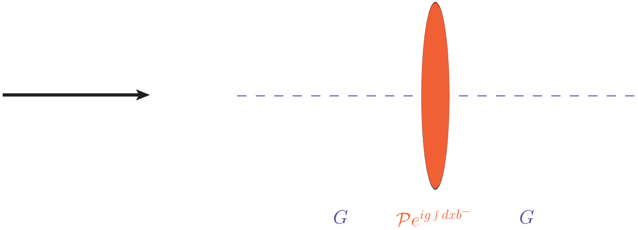
$$b^\mu(x) \to b^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + O(\sqrt{\frac{m_t^2}{s}})$$

Shockwave approximation

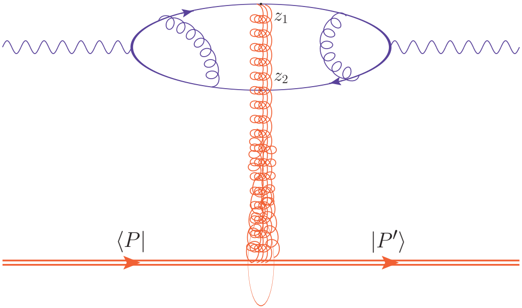
Effective Feynman rules in the slow background field

The interactions with the background field can be exponentiated





Factorized picture



Factorized amplitude

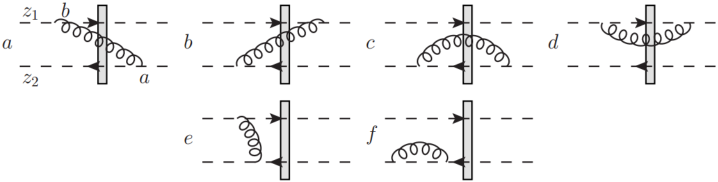
$$\mathcal{A}^{Y_c} = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^{Y_c}(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U_{\vec{z}_1}^{Y_c} U_{\vec{z}_2}^{Y_c\dagger}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}_{ij}^{Y_c} = \frac{1}{N_c}\mathrm{Tr}(U_{\vec{z}_i}^{Y_c} U_{\vec{z}_j}^{Y_c\dagger}) - 1$

Written similarly for any number of Wilson lines in any color representation!

Evolution for the dipole operator

$$\mathcal{U}_{12}^{Y_c+\delta Y} - \mathcal{U}_{12}^{Y_c}$$



B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

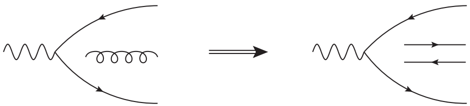
$$\frac{\partial \mathcal{U}_{12}^{Y_c}}{\partial Y_c} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{Y_c} + \mathcal{U}_{32}^{Y_c} - \mathcal{U}_{12}^{Y_c} + \mathcal{U}_{13}^{Y_c}\mathcal{U}_{32}^{Y_c} \right]$$

$$\frac{\partial \mathcal{U}_{13}^{Y_c}\mathcal{U}_{32}^{Y_c}}{\partial Y_c} = \ldots$$

Evolves a dipole into a double dipole

The BK equation

Mean field approximation, or 't Hooft planar limit $N_c \to \infty$ in the dipole B-JIMWLK equation



[Mueller]

⇒ BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \left\langle \mathcal{U}_{12}^{Y_c} \right\rangle}{\partial Y_c} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\left\langle \mathcal{U}_{13}^{Y_c} \right\rangle + \left\langle \mathcal{U}_{32}^{Y_c} \right\rangle - \left\langle \mathcal{U}_{12}^{Y_c} \right\rangle + \left\langle \mathcal{U}_{13}^{Y_c} \right\rangle \left\langle \mathcal{U}_{32}^{Y_c} \right\rangle \right]$$

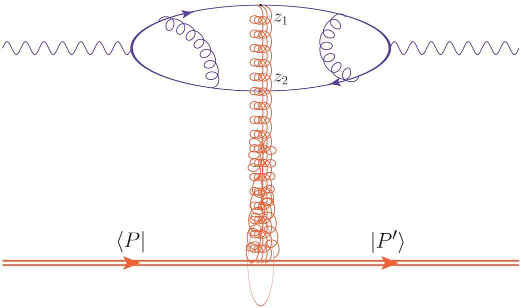
BFKL/BKP part Triple pomeron vertex

Non-linear term : one type of saturation

Non-perturbative elements are compatible with CGC-type models

Factorized picture

Semi-classical approach to small x physics
[McLerran, Venugopalan], [Balitsky]



$$\mathcal{S} = \int d^2\vec{z}_1 d^2\vec{z}_2\, \Phi^{Y_c}(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U_{\vec{z}_1}^{Y_c} U_{\vec{z}_2}^{Y_c\dagger}) - N_c]|P\rangle$$

Written similarly for any number of Wilson lines in any color representation!

Y_c independence: B-JIMWLK hierarchy of equations
[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

The seemingly incompatible nature of the distributions

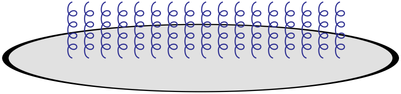
Two different kinds of gluon distributions

Moderate x distributions	Low x distributions
GTMD, GPD, TMD, PDF...	Dipole scattering amplitude
$\langle P^{(\prime)} \| F^{-i} W F^{-j} W \| P \rangle$	$\langle P^{(\prime)} \| \mathrm{tr}(U_1 U_2^\dagger) \| P \rangle$

TMD distributions from QCD shockwaves

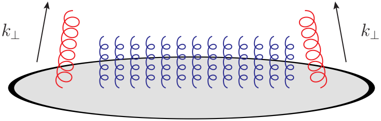
From the CGC to a TMD

From Wilson lines...



$$\left\langle P \left| \mathrm{Tr}\left(U_{\frac{r}{2}} U^{\dagger}_{-\frac{r}{2}} \right) \right| P \right\rangle$$

To a parton distribution



$$\left\langle P \left| \mathrm{Tr}\left(\partial^i U_{\frac{r}{2}} \partial^i U^{\dagger}_{-\frac{r}{2}} \right) \right| P \right\rangle$$

From the CGC to a TMD

Staple gauge links from a Wilson line operator

[Dominguez, Marquet, Xiao, Yuan]

Consider the derivative of a path-ordered Wilson line, denoting

$$[x_1^+, x_2^+]_{\vec{x}} \equiv \mathcal{P} \exp \left[ig \int_{x_1^+}^{x_2^+} dx^+ b^-(x^+, \vec{x}) \right]$$

For a given shockwave operator $U_{\vec{x}} = [-\infty, +\infty]_{\vec{x}}$

$$\partial^i U_{\vec{x}} = ig \int dx^+ [-\infty, x^+]_{\vec{x}} F^{-i}(x^+, \vec{x}) [x^+, +\infty]_{\vec{x}}$$

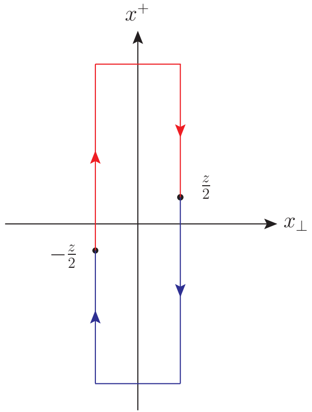
$$\partial^j U^\dagger_{\vec{x}} = -ig \int dx^+ [+\infty, x^+]_{\vec{x}} F^{-j}(x^+, \vec{x}) [x^+, -\infty]_{\vec{x}}$$

$$(\partial^i U^\dagger_{\vec{x}}) U_{\vec{x}} = -ig \int dx^+ [+\infty, x^+]_{\vec{x}} F^{-i}(x^+, \vec{x}) [x^+, +\infty]_{\vec{x}}$$

Taking the derivative of a shockwave operator allows to extract a physical gluon

From the CGC to a TMD

The dipole TMD

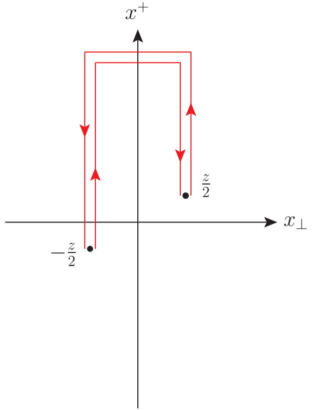


$$\mathcal{F}_{qg}^{(1)}(x, k_\perp) \propto \int d^4 z \delta\left(z^+\right) e^{ix(P\cdot z)+i(k_\perp \cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right) \mathcal{U}^{[-]\dagger} F^{i-}\left(-\frac{z}{2}\right) \mathcal{U}^{[+]} \right] \right| P \right\rangle$$

$$\to \int d^2 z_\perp e^{i(k_\perp \cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[\left(\partial^i U^\dagger_{\frac{z}{2}}\right) \left(\partial^i U_{-\frac{z}{2}}\right) \right] \right| P \right\rangle$$

From the CGC to a TMD

The Weizsäcker-Williams TMD



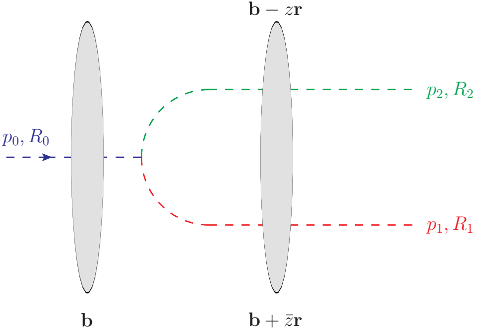
$$\mathcal{F}_{gg}^{(3)}(x,k_\perp) \propto \int d^4z\,\delta\left(z^+\right) e^{ix(P\cdot z)+i(k_\perp\cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[+]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]\right| P\right\rangle$$

$$\to \int dz_\perp e^{i(k_\perp\cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[\left(\partial^i U_{\frac{z}{2}}\right) U^\dagger_{-\frac{z}{2}} \left(\partial^i U_{-\frac{z}{2}}\right) U^\dagger_{\frac{z}{2}}\right]\right| P\right\rangle$$

General $1 \to 2$ process in the shockwave framework

Splitting of a particle into two particles in the external shockwave field

$$\mathcal{A} = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right) \int d^2\boldsymbol{b}\, d^2\boldsymbol{r}\, e^{-i(\boldsymbol{q}\cdot\boldsymbol{r}) - i(\boldsymbol{k}\cdot\boldsymbol{b})} \mathcal{H}(\boldsymbol{r})$$
$$\times \left[\left(U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}}\, T^{R_0}\, U^{R_2}_{\boldsymbol{b}-z\boldsymbol{r}}\right) - \left(U^{R_1}_{\boldsymbol{b}}\, T^{R_0}\, U^{R_2}_{\boldsymbol{b}}\right)\right],$$



Matching shockwave amplitudes and TMD amplitudes

[Altinoluk, RB]

We can cast the shockwave amplitude into a 1-body amplitude

$$\mathcal{A}_1 = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right)\int d^2\boldsymbol{b}\, e^{-i(\boldsymbol{k}\cdot\boldsymbol{b})}(-i)\int d^2\boldsymbol{r}\, e^{-i(\boldsymbol{q}\cdot\boldsymbol{r})} r_\perp^\alpha \mathcal{H}(\boldsymbol{r}) \times\left[\left(\frac{e^{i\bar{z}(\boldsymbol{k}\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}\cdot\boldsymbol{r})}\right)\left(\partial_\alpha U_{\boldsymbol{b}}^{R_1}\right) T^{R_0} U_{\boldsymbol{b}}^{R_2} + \left(\frac{e^{-iz(\boldsymbol{k}\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}\cdot\boldsymbol{r})}\right) U_{\boldsymbol{b}}^{R_1} T^{R_0}\left(\partial_\alpha U_{\boldsymbol{b}}^{R_2}\right)\right]$$

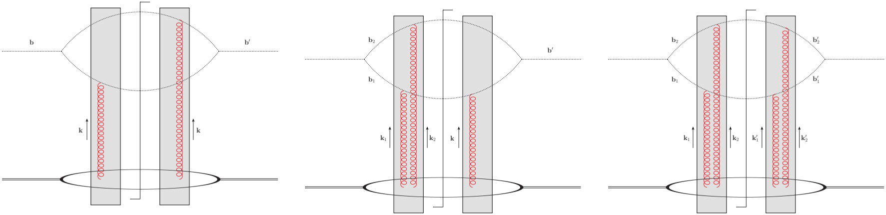
and a 2-body amplitude

$$\mathcal{A}_2 = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right)\int \frac{d^2\boldsymbol{k}_1}{(2\pi)^2}\frac{d^2\boldsymbol{k}_2}{(2\pi)^2}(2\pi)^2\delta^2(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}) \times\int d^2\boldsymbol{b}_1 d^2\boldsymbol{b}_2 e^{-i(\boldsymbol{k}_1\cdot\boldsymbol{b}_1)-i(\boldsymbol{k}_2\cdot\boldsymbol{b}_2)}\left(\partial^i U_{\boldsymbol{b}_1}^{R_1}\right) T^{R_0}\left(\partial^j U_{\boldsymbol{b}_2}^{R_2}\right) \times\left[-\int d^2\boldsymbol{r} e^{-i(\boldsymbol{q}\cdot\boldsymbol{r})}\boldsymbol{r}^i\boldsymbol{r}^j\mathcal{H}(\boldsymbol{r})\left(\frac{e^{-iz(\boldsymbol{k}\cdot\boldsymbol{r})}}{(\boldsymbol{k}\cdot\boldsymbol{r})}\frac{e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}_1\cdot\boldsymbol{r})} + \frac{e^{i\bar{z}(\boldsymbol{k}\cdot\boldsymbol{r})}}{(\boldsymbol{k}\cdot\boldsymbol{r})}\frac{e^{-i(\boldsymbol{k}_2\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}_2\cdot\boldsymbol{r})}\right)\right]$$

Inclusive low x cross section

Inclusive low x cross section = TMD cross section

[Altinoluk, RB, Kotko], [Altinoluk, RB]

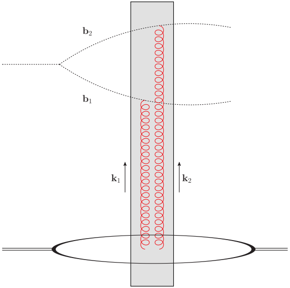


$$\sigma = \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \left| F^{-i} W F^{-j} W \right| P \right\rangle$$
$$+ \mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W F^{-k} W \right| P \right\rangle$$
$$+ \mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \right| P \right\rangle$$

Exclusive low x cross section

Exclusive low x amplitude = GTMD amplitude

[Altinoluk, RB]



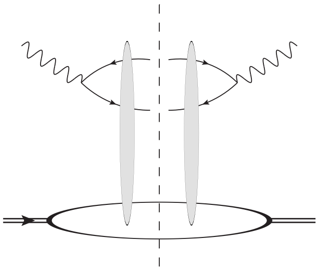
$$\mathcal{H}^{ij}\left(k_{1\perp},\, k_{2\perp}\right) \otimes \left\langle P' \left| F^{-i} W F^{-j} W \right| P \right\rangle$$

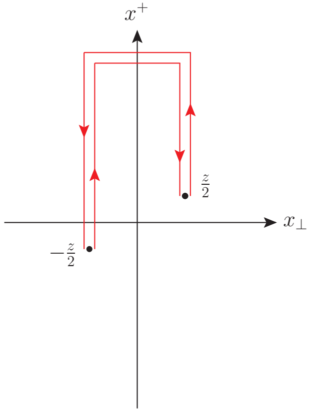
Every exclusive low x process probes a Wigner distribution!

Dijet electro- or photoproduction

Weizsäcker-Williams TMD

$T^{R_0} = 1, U^{R_1} = U, U^{R_2} = U^\dagger$



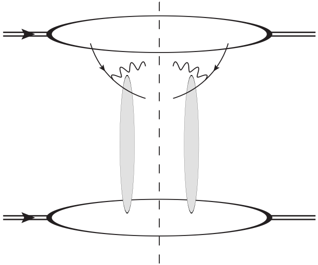


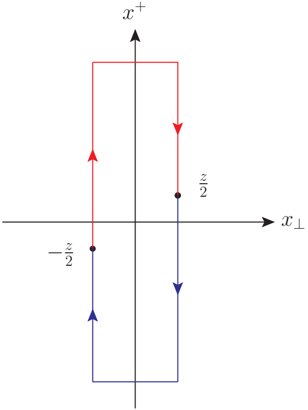
$$\mathcal{F}_{gg}^{(3)}(x \sim 0, k_\perp) \propto \int d^2 z_\perp e^{-i(k_\perp \cdot z_\perp)} \langle P | \mathrm{Tr}(\partial^i U^\dagger_{\frac{z}{2}}) U_{\frac{z}{2}} (\partial^i U^\dagger_{-\frac{z}{2}}) U_{-\frac{z}{2}} | P \rangle$$

Jet+photon production in pA collisions

Dipole TMD

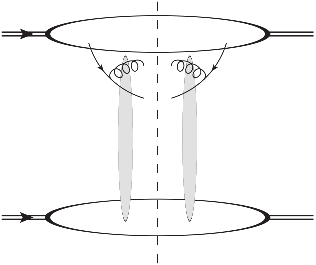
$$T^{R_0} = 1, U^{R_1} = U, U^{R_2} = 1$$

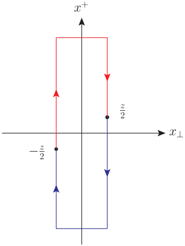




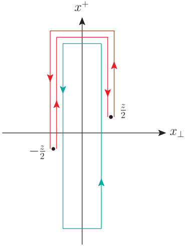
$$\mathcal{F}_{qg}^{(1)}(x \sim 0, k_\perp) \propto \int d^2 z_\perp e^{-i(k_\perp \cdot z_\perp)} \langle P| \mathrm{tr}(\partial^i U_{\frac{z}{2}})(\partial^i U^\dagger_{-\frac{z}{2}})|P\rangle$$

Forward dijet production in pA collisions



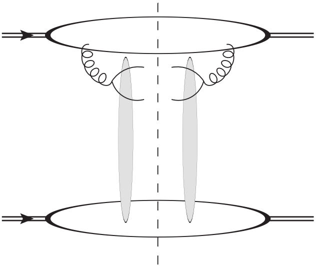


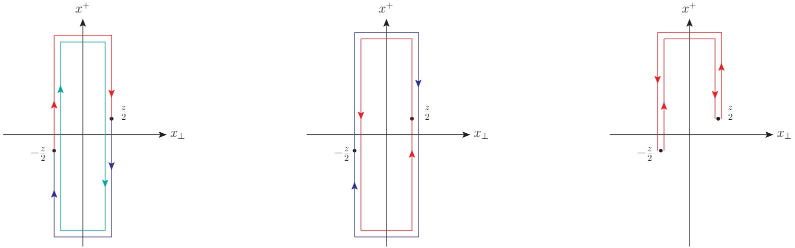
$\mathcal{F}_{qg}^{(1)}$



$\mathcal{F}_{qg}^{(2)}$

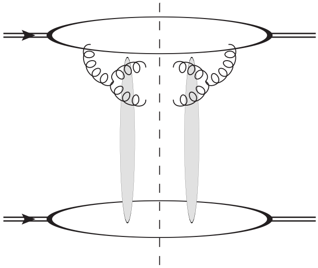
Forward dijet production in pA collisions

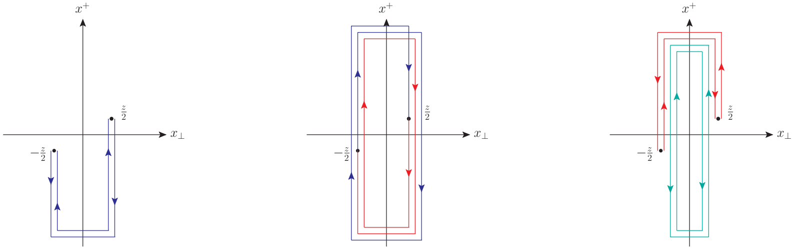




$\mathcal{F}_{gg}^{(1)}$ $\mathcal{F}_{gg}^{(2)}$ $\mathcal{F}_{gg}^{(3)}$

Forward dijet production in pA collisions





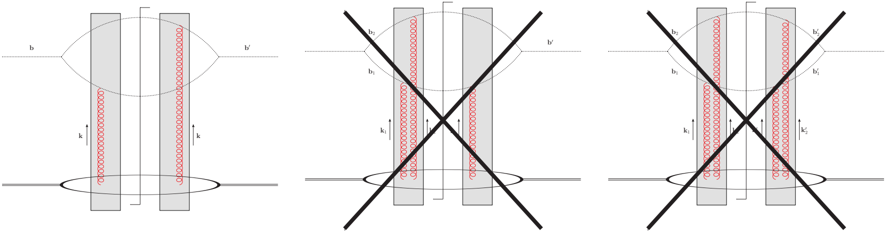
$\mathcal{F}_{gg}^{(4)}$ $\mathcal{F}_{gg}^{(5)}$ $\mathcal{F}_{gg}^{(6)}$

The so-called dilute limit in terms of TMD distributions

Inclusive low x cross section

First, take the Wandzura-Wilczek approximation

[Altinoluk, RB, Kotko]: matches iTMD cross sections



$$\sigma = \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \left| F^{-i} W F^{-j} W \right| P \right\rangle$$

$$+\mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W F^{-k} W \right| P \right\rangle$$

$$+\mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \right| P \right\rangle$$

WW approximation at large k_t: the BFKL limit

- At large transverse momentum transfer, no multiple scattering from the gauge links

TMD with staple gauge links

$$\int \frac{d^2\boldsymbol{k}}{(2\pi)^2} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \int dx^+ \left\langle P \left| F^{i-}(x) \left[x^+, \pm\infty\right]_{\boldsymbol{x}} \left[\pm\infty, 0^+\right]_{\boldsymbol{0}} F^{j-}(0) \left[0^+, \pm\infty\right]_{\boldsymbol{0}} \left[\pm\infty, x^+\right]_{\boldsymbol{x}} \right| P \right\rangle$$

Large $k_\perp \sim Q \Rightarrow$ small transverse distance $x_\perp$

$$\left[x^+, \pm\infty\right]_{\boldsymbol{x}} \left[\pm\infty, y^+\right]_{\boldsymbol{0}} \sim \left[x^+, y^+\right]_{\boldsymbol{x}\sim\boldsymbol{0}} .$$

All TMD distributions shrink into the unintegrated PDF

$$\int \frac{d^2\boldsymbol{k}}{(2\pi)^2} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \int dx^+ \left\langle P \left| F^{i-}(x) \left[x^+, 0^+\right]_{\boldsymbol{0}} F^{j-}(0) \left[0^+, x^+\right]_{\boldsymbol{0}} \right| P \right\rangle \Big|_{x^-=0}$$

and one recovers a BFKL cross section.

BFKL distributions and genuine twist corrections

Unintegrated PDF = 2-Reggeon matrix element

$$\int d^2\boldsymbol{x} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \int dx^+ \left\langle P \left| F^{i-}(x) \left[x^+, 0^+\right]_{\boldsymbol{0}} F^{j-}(0) \left[0^+, x^+\right]_{\boldsymbol{0}} \right| P \right\rangle \Big|_{x^-=0}$$

Integration by parts

$$\int dx^+ \int d^2\boldsymbol{x} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \boldsymbol{k}^i \boldsymbol{k}^j \langle P \left| \left[-\infty, x^+\right]_{\boldsymbol{0}} A^-(x) \left[x^+, +\infty\right]_{\boldsymbol{0}} \left[+\infty, 0^+\right]_{\boldsymbol{0}} A^-(0) \left[0^+, -\infty\right]_{\boldsymbol{0}} \right| P \rangle$$

We recognize the so-called nonsense polarizations in axial gauge. We could define a Reggeon operator:

$$R(x) = \int dx^+ \left[-\infty, x^+\right]_{\boldsymbol{0}} A^-(x) \left[x^+, +\infty\right]_{\boldsymbol{0}}$$

and rewrite the unintegrated PDF as

$$\int \frac{d^2\boldsymbol{k}}{(2\pi)^2} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \frac{\boldsymbol{k}^i \boldsymbol{k}^j}{\boldsymbol{k}^2} \boldsymbol{k}^2 \left\langle P \left| R(\boldsymbol{x}) R^\dagger(0) \right| P \right\rangle$$

BFKL distributions and genuine twist corrections

What is neglected in BFKL: 3- and 4-Reggeon matrix elements.

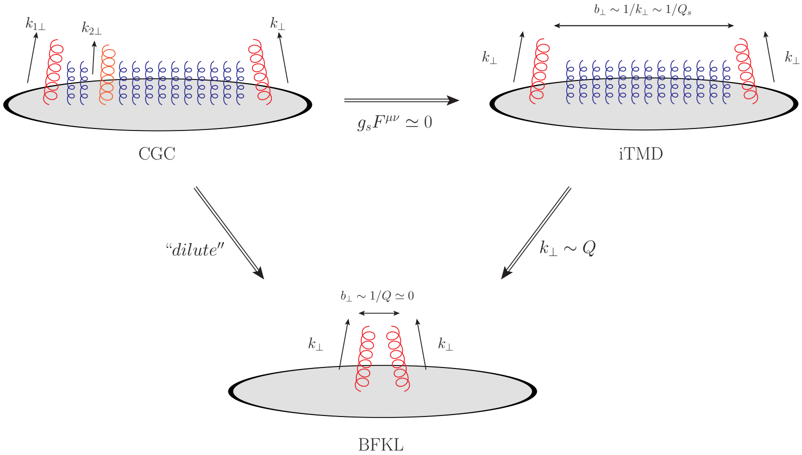
$$\langle P\,|RR|\,P\rangle\,,\quad \langle P\,|R(g_s R)R|\,P\rangle\,,\quad \langle P\,|R(g_s R)(g_s R)R|\,P\rangle$$

They are not perturbatively suppressed.

Suppression = WW approximation (unquantifiable)

The dilute limit

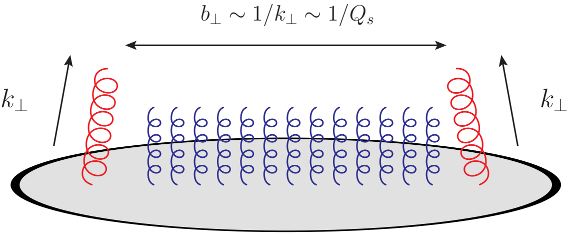
The dilute limit in terms of TMD distributions



Two kinds of multiple scattering effects: higher genuine twists and higher kinematic twists

Kinematic saturation

"Saturation" from a TMD gauge link



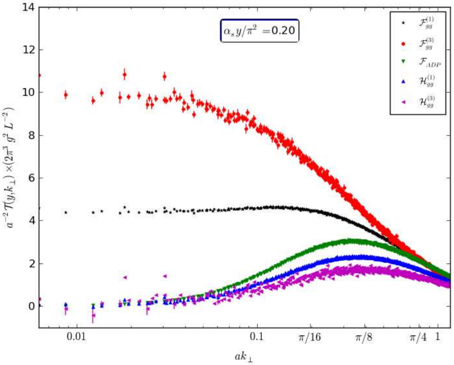
$$g_s^2 \int d^4 b\, \delta\left(b^-\right) e^{i(k\cdot b)} \left\langle P \left| F^{i-}(b)\, \mathcal{U}_{b,0}^{[\pm]} F^{j-}(0)\, \mathcal{U}_{0,b}^{[\pm]} \right| P \right\rangle$$

Expected at small $k_\perp/Q$

Kinematic saturation

"Saturation" from a TMD gauge link

Link length $\sim 1/|k_\perp|$, hence effect suppressed at large $k_\perp$

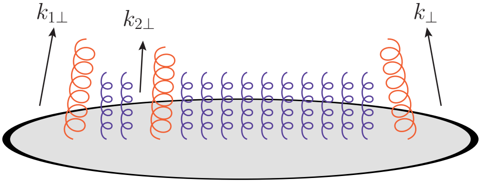


[Marquet, Petreska, Roiesnel ; Marquet, Roiesnel, Taels]

Genuine saturation

"Saturation" as an enhancement of genuine twists

Large gluon occupancy $\Rightarrow g_s F \sim 1$



$$g_s^2 \int d^4 b_1 d^4 b_2 d^4 b' \delta\left(b_1^-\right) \delta\left(b_2^-\right) \delta\left(b'^-\right) e^{i(k_1 \cdot b_1)+i(k_2 \cdot b_2)-i\left(k \cdot b'\right)}$$

$$\times \frac{\left\langle P \left| F^{i-}(b_1) \mathcal{U}^{[\pm]}_{b_1,b_2} g_s F^{j-}(b_2) \mathcal{U}^{[\pm]}_{b_2,b'} F^{k-}(b') \mathcal{U}^{[\pm]}_{b',b_1} \right| P \right\rangle}{\langle P|P\rangle}$$

$k_\perp/Q$-suppressed: expected at large $k_\perp$?

Linearly polarized gluons at small x

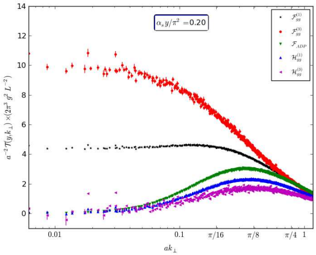
Polarized TMD in the CGC

Wilson line operators also contain linearly polarized gluon TMDs

$$\left\langle P \left| \partial^i U \partial^j U \right| P \right\rangle \to \frac{\delta^{ij}}{2} \mathcal{F}\left(k_\perp\right) + \left(\frac{k_\perp^i k_\perp^j}{k_\perp^2} - \frac{\delta^{ij}}{2} \right) \mathcal{H}\left(k_\perp\right)$$

- $\mathcal{F}$: unpolarized TMD, $\mathcal{H}$: linearly polarized (Boer Mulders) TMD
- $\mathcal{H}$ can be observed in processes with massive quarks [Marquet, Roiesnel, Taels]
- Or in processes with 3 body final states (requires an extension of the notion of the correlation limit) [Altinoluk, RB, Marquet, Taels]
- Can also be seen from loop corrections to 2-body observables, for example prompt photon+jet production in pA collisions [Benić, Dumitru], based on a computation by [Benić, Fukushima, Garcia-Montero, Venugopalan]

Polarized TMD in the CGC



In the large $k_\perp \sim Q$ limit (BFKL limit), all TMDs are equal:

$$\mathcal{F}(k_\perp) = \mathcal{H}(k_\perp), \text{ then } \left\langle P \left| \partial^i U \partial^j U \right| P \right\rangle \to \frac{k_\perp^i k_\perp^j}{k_\perp^2} \mathcal{F}(k_\perp)$$

We can recognize the so-called *non-sense polarization* in lightcone gauge: $\frac{k_\perp^i}{|k_\perp|}$.
BFKL contains as many linearly polarized gluon pairs as unpolarized ones.

At large $k_\perp$, gluon distributions are very polarized

Conclusions

- TMD distributions are what allows to match standard parton distributions and semi-classical descriptions of small x physics
- Dipole and Color Glass Condensate models can give insights on TMDs at small x
- The reformulation of shockwaves in terms of TMD distributions allows to understand polarized gluon distributions at small x
- Two distinct kinds of multiple scattering effects must be distinguished to understand gluonic saturation models

Backup

Operator product expansion (OPE)

- Moderate x OPE: factorization

$$\mathcal{O}(z) \to \sum_n C_n(z,\mu)\mathcal{O}_n(\mu)$$

  - Operators are ordered in twists (dimension − spin)
  - Divergences in C_n are canceled via renormalization of $\mathcal{O}_n$
  - Easy task: resumming powers of s and logarithms of Q^2. Difficulty: including twist corrections and logarithms of s

- Low x OPE:

$$\mathcal{O}(z) \to C_0(z,Y)\mathcal{O}_0(Y) + \alpha_s C_1(z,Y)\mathcal{O}_1(Y) + \ldots$$

  - Operators are sorted by representations of $SU(N_c)$, order by order in α_s
  - Built order by order in α_s. The spurious pole in $C_n(z,Y)$ is canceled via the B/JIMWLK RGE of $\mathcal{O}_{n-1}(Y)$
  - Easy task: resumming twists and logarithms of s. Difficulty: including subeikonal corrections and logarithms of Q^2

Small dipole "correlation" expansion

Taylor expansion of the Wilson line operators

$$U^{R_1}_{\boldsymbol{b}+\frac{\boldsymbol{r}}{2}} T^{R_0} U^{R_2}_{\boldsymbol{b}-\frac{\boldsymbol{r}}{2}} - U^{R_1}_{\boldsymbol{b}} T^{R_0} U^{R_2}_{\boldsymbol{b}} = \frac{\boldsymbol{r}^i}{2}\left[\left(\partial^i U^{R_1}_{\boldsymbol{b}}\right) T^{R_0} U^{R_2}_{\boldsymbol{b}} - U^{R_1}_{\boldsymbol{b}} T^{R_0} \left(\partial^i U^{R_2}_{\boldsymbol{b}}\right)\right] + O\left(\boldsymbol{r}^2\right)$$

allows for a match at leading twist

$$d\sigma = \mathcal{H}(b, r) \otimes \left[U^{R_1}_{\boldsymbol{b}+\frac{\boldsymbol{r}}{2}} T^{R_0} U^{R_2}_{\boldsymbol{b}-\frac{\boldsymbol{r}}{2}} - U^{R_1}_{\boldsymbol{b}} T^{R_0} U^{R_2}_{\boldsymbol{b}}\right]$$

$$\times \mathcal{H}^*(b', r') \otimes \left[U^{R_2 \dagger}_{\boldsymbol{b}'-\frac{\boldsymbol{r}'}{2}} T^{R_0 \dagger} U^{R_1 \dagger}_{\boldsymbol{b}'+\frac{\boldsymbol{r}'}{2}} - U^{R_2 \dagger}_{\boldsymbol{b}'} T^{R_0 \dagger} U^{R_1 \dagger}_{\boldsymbol{b}'}\right]$$

$$\to d\sigma^{(i)}_{\boldsymbol{k}=\boldsymbol{0}} \otimes \Phi^{(i)}(x, \boldsymbol{k}) + O\left(\boldsymbol{r}^2\right)$$

How to extend this to higher twist corrections?

Power expansion for TMD observables:
dealing with powers of $k_\perp/Q$

Consider (hypothetical) hard subamplitudes with non-zero transverse momenta in the t channel. The amplitude would read:

$$\mathcal{H}_1^i(\boldsymbol{k}) \otimes \int d^2\boldsymbol{x}_1 e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}_1)} [\pm\infty, x_1] F^{i-}(x_1) [x_1, \pm\infty]$$

$$+ \mathcal{H}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2) \otimes \int d^2\boldsymbol{x}_1 d^2\boldsymbol{x}_2 e^{-i(\boldsymbol{k}_1\cdot\boldsymbol{x}_1)-i(\boldsymbol{k}_2\cdot\boldsymbol{x}_2)} [\pm\infty, x_1] F^{i-}(x_1) [x_1, x_2] F^{j-}(x_2) [x_2, \pm\infty]$$

$$+ \ldots$$

$$= \mathcal{H}_1^i(\boldsymbol{k}) \otimes \mathcal{O}_1^i(\boldsymbol{k}) + \mathcal{H}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2) \otimes \mathcal{O}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2) + \ldots$$

Power expansion for TMD amplitudes:
dealing with powers of $k_\perp/Q$

Leading twist amplitude

$$\mathcal{A}_{LT} = \mathcal{H}_1^i\,(\mathbf{0}) \otimes \mathcal{O}_1^i\,(\boldsymbol{k})$$

Next-to-leading twist amplitude

$$\mathcal{A}_{NLT} = \boldsymbol{k} \cdot \left(\partial_{\boldsymbol{k}} \mathcal{H}_1^i\right)(\mathbf{0}) \otimes \mathcal{O}_1^i\,(\boldsymbol{k}) + \mathcal{H}_2^{ij}\,(\mathbf{0},\mathbf{0}) \otimes \mathcal{O}_2^{ij}\,(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

First term: kinematic twist correction, second term: genuine twist corrections

Match without an expansion

Trick: rewrite operators in terms of their derivatives

$$U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}} - U^{R_1}_{\boldsymbol{b}} = -ir^{\mu}_{\perp} \int \frac{d^2\boldsymbol{k}_1}{(2\pi)^2} \int d^2\boldsymbol{b}_1 e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{b}_1-\boldsymbol{b})} \frac{e^{i\bar{z}(\boldsymbol{k}_1\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}_1\cdot\boldsymbol{r})} \left(\partial_{\mu} U^{R_1}_{\boldsymbol{b}}\right)$$

Rewrite the amplitude

$$\mathcal{A} = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right) \int d^2\boldsymbol{b}\, d^2\boldsymbol{r}\, e^{-i(\boldsymbol{q}\cdot\boldsymbol{r})-i(\boldsymbol{k}\cdot\boldsymbol{b})}\mathcal{H}(\boldsymbol{r})$$
$$\times \left[\left(U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}} - U^{R_1}_{\boldsymbol{b}}\right) T^{R_0} \left(U^{R_2}_{\boldsymbol{b}-z\boldsymbol{r}} - U^{R_2}_{\boldsymbol{b}}\right) + \left(U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}} - U^{R_1}_{\boldsymbol{b}}\right) T^{R_0} U^{R_2}_{\boldsymbol{b}} + U^{R_1}_{\boldsymbol{b}} T^{R_0} \left(U^{R_2}_{\boldsymbol{b}-z\boldsymbol{r}} - U^{R_2}_{\boldsymbol{b}}\right)\right]$$

genuine twist — kinematic + genuine twists

Extracting genuine twists: Taylor, IbP, resummation.

Saturation effects: Shockwaves vs iTMD vs BFKL