Correlators at Unequal Rapidity

in the dilute limit of JIMWLK

arXiv:1904.00782[hep-ph]

Andrecia Ramnath (with Tuomas Lappi)

University of Jyväskylä, Finland

Low-x 2019 Nicosia, Cyprus 27.08.2019

Motivation

Particle correlations appear in cross section expressions.

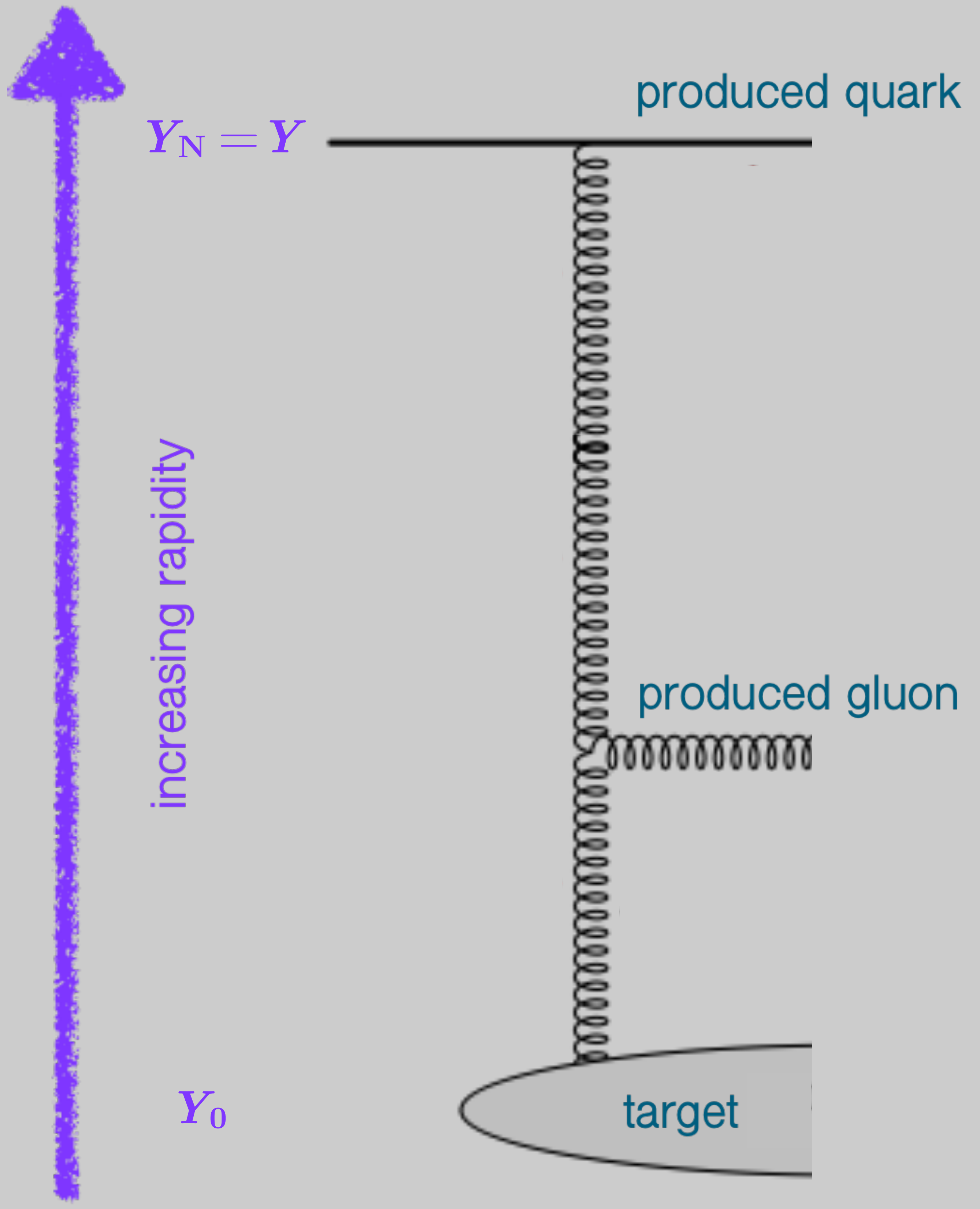
What are the QCD dynamics of correlations?

Consider produced particles separated in rapidity.

Required framework: Colour Glass Condensate.

Evolution in rapidity is governed by the JIMWLK equation.

Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner



$$Y_N - Y_0 \geq 1/\alpha_s$$

JIMWLK Evolution

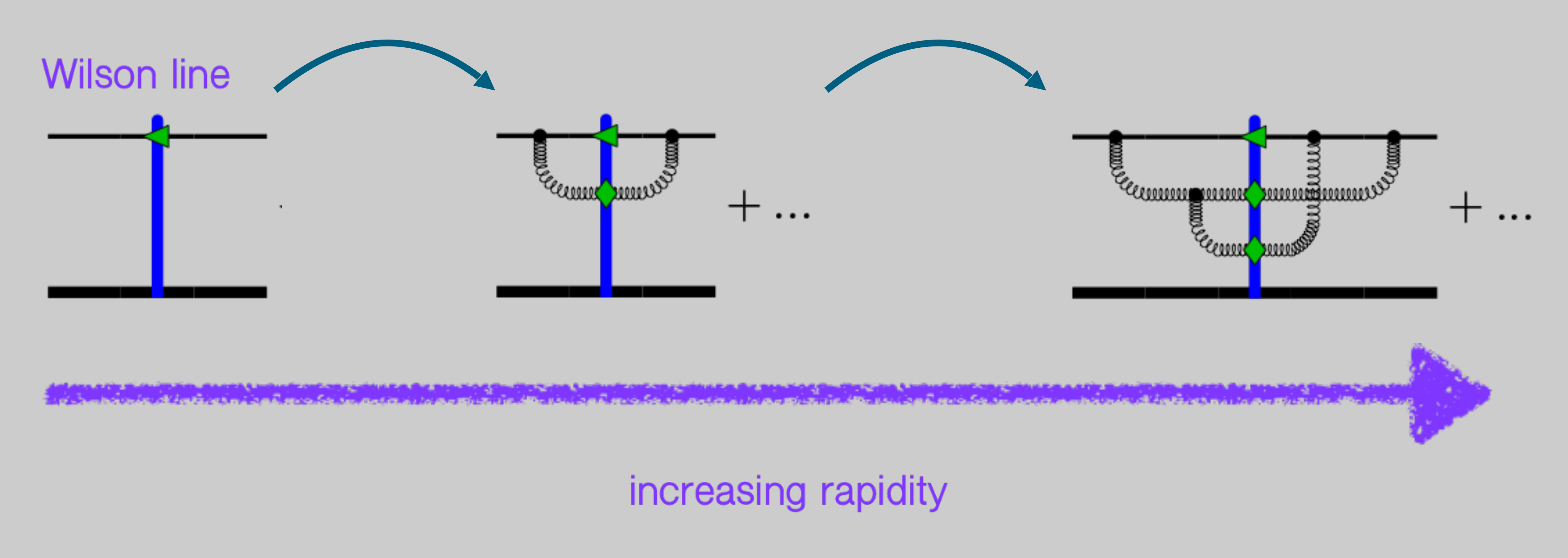
Fokker - Planck

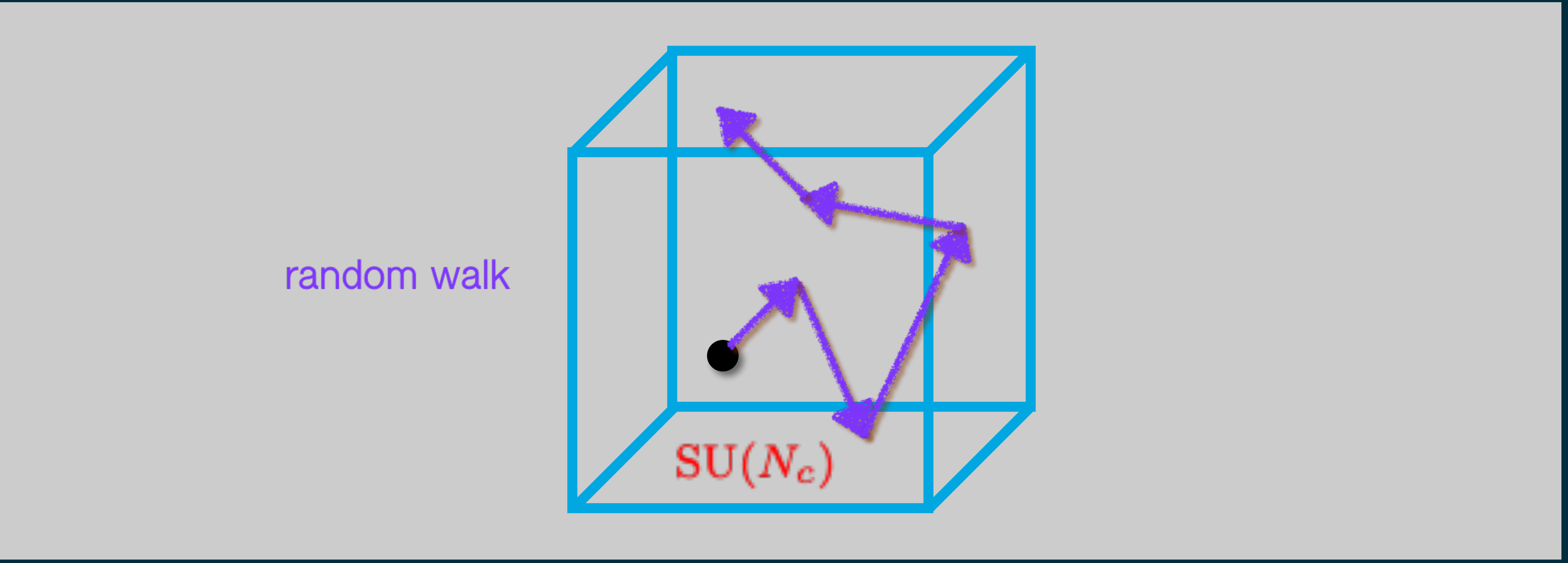
- better known

Langevin

- stochastic interpretation
- better for numerics

Blaizot, Iancu & Weigert [Nucl.Phys. A713 (2003) 441-469], Kovner & Lublinsky [JHEP 0611 (2006) 083], Iancu & Triantafyllopoulos [JHEP 1311 (2013) 067], Hatta & Iancu [JHEP 1608 (2016) 083]





Langevin Picture

Blaizot, Iancu & Weigert [Nucl.Phys. A713 (2003) 441-469]

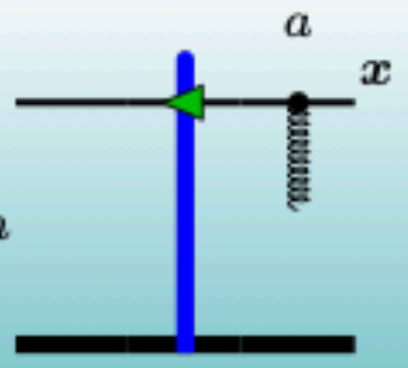
Discretise rapidity as $Y - Y_0 = N\epsilon, \quad \epsilon \to 0, \quad n = 0, \ldots, N$

The Langevin equation for a Wilson line is

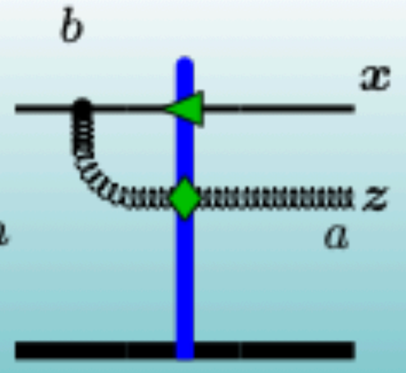
$$U^\dagger_{\boldsymbol{x},n+1} = \exp\left\{i\epsilon g \alpha^L_{\boldsymbol{x},n}\right\} U^\dagger_{\boldsymbol{x},n} \exp\left\{-i\epsilon g \alpha^R_{\boldsymbol{x},n}\right\}$$

The colour rotations are

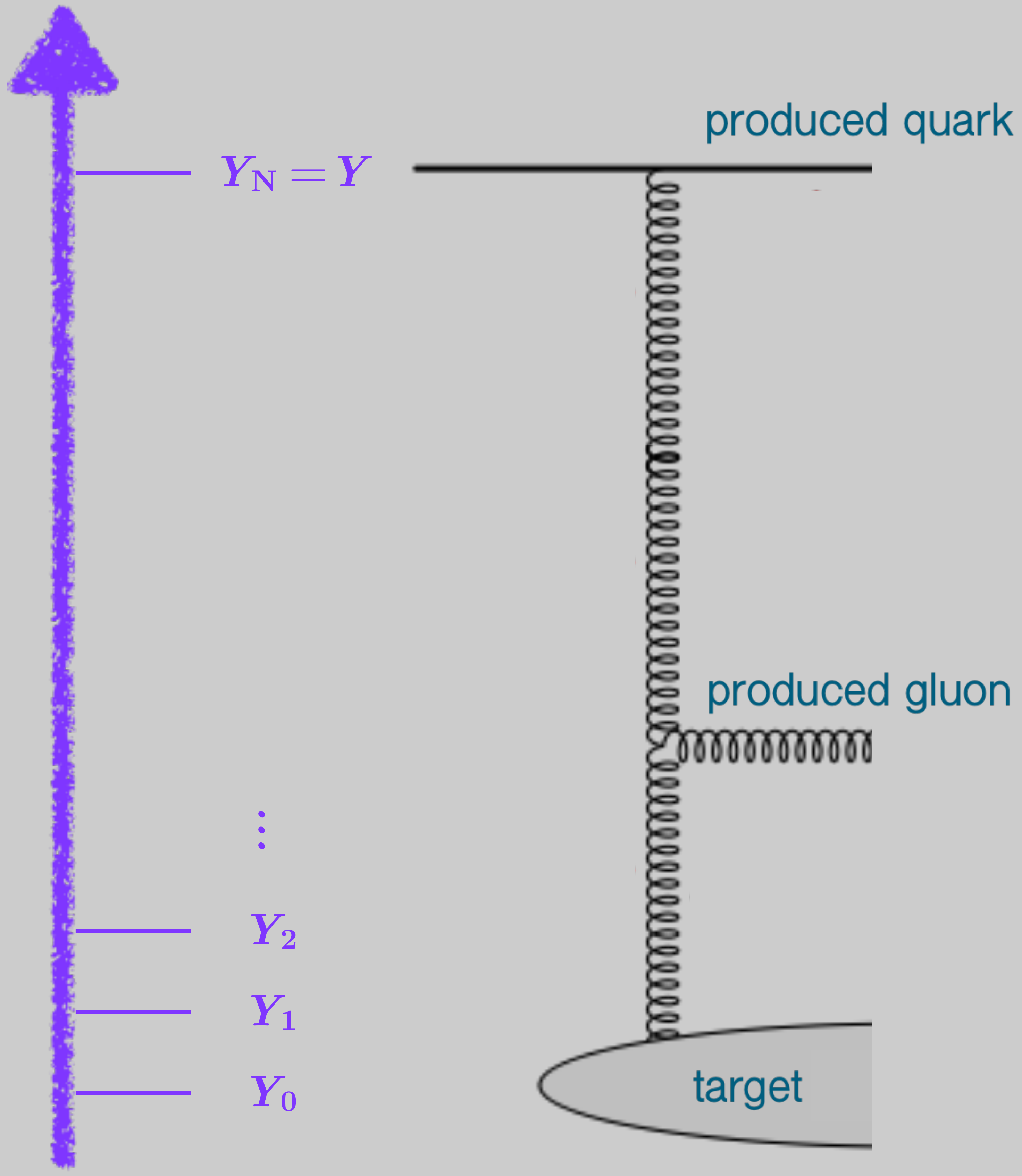
$$\alpha^L_{\boldsymbol{x},n} U^\dagger_{\boldsymbol{x},n} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}} \nu^{i,a}_{\boldsymbol{z},n}$$



$$\alpha^R_{\boldsymbol{x},n} U^\dagger_{\boldsymbol{x},n} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}} \nu^{i,a}_{\boldsymbol{z},n}$$



$$\langle \nu^{i,a}_{\boldsymbol{x},m} \nu^{jb}_{\boldsymbol{y},n} \rangle = \frac{1}{\epsilon} \delta^{ij} \delta^{ab} \delta_{mn} \delta_{\boldsymbol{x}\boldsymbol{y}} \quad \longleftrightarrow \quad \tilde{\nu}^{i,a}_{\boldsymbol{z},n} \equiv U^{\dagger ab}_{\boldsymbol{z},n} \nu^{i,b}_{\boldsymbol{z},n}$$



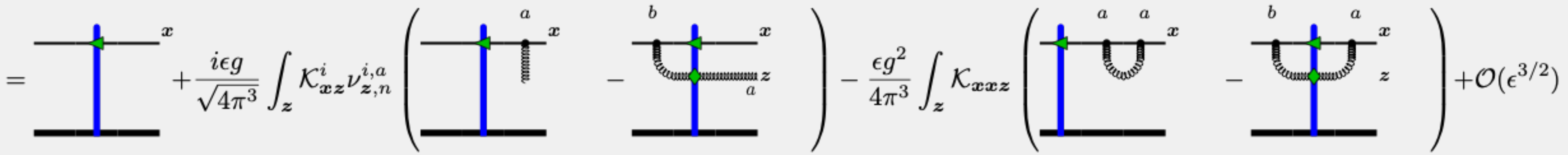
First expansion: in step size

Expand the Langevin equation

$$U^\dagger_{\boldsymbol{x},n+1} = \exp\left\{i\epsilon g \alpha^L_{\boldsymbol{x},n}\right\} U^\dagger_{\boldsymbol{x},n} \exp\left\{-i\epsilon g \alpha^R_{\boldsymbol{x},n}\right\}$$

in epsilon (recall $Y - Y_0 = N\epsilon, \quad \epsilon \to 0$)

$$U^\dagger_{\boldsymbol{x},n+1} = U^\dagger_{\boldsymbol{x},n} + \frac{i\epsilon g}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}} \nu^{i,a}_{\boldsymbol{z},n} \left(t^a U^\dagger_{\boldsymbol{x},n} - U^\dagger_{\boldsymbol{x},n} U^{\dagger ab}_{\boldsymbol{z},n} t^b\right) - \frac{\epsilon g^2}{4\pi^3} \int_{\boldsymbol{z}} \mathcal{K}_{\boldsymbol{x}\boldsymbol{x}\boldsymbol{z}} t^a \left(t^a U^\dagger_{\boldsymbol{x},n} - U^\dagger_{\boldsymbol{x},n} U^{\dagger ab}_{\boldsymbol{z},n} t^b\right) + \mathcal{O}(\epsilon^{3/2})$$



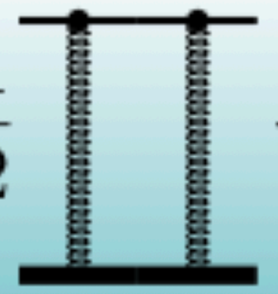
Second expansion: in gluons

Expand a second time in elements of the **group algebra**

$$U^{\dagger}_{\boldsymbol{x},n} \equiv e^{i\lambda_{\boldsymbol{x},n}} = \mathbb{1} + i\lambda_{\boldsymbol{x},n} - \frac{1}{2}\lambda^2_{\boldsymbol{x},n} + \mathcal{O}(\lambda^3) =$$

$$+\, i$$

$$-\frac{1}{2}$$

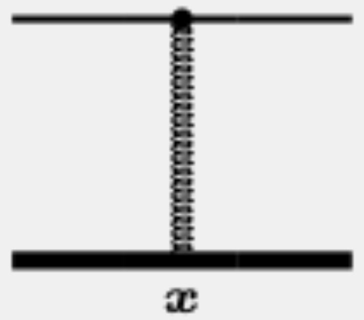


$$+\,\mathcal{O}(\lambda^3)$$

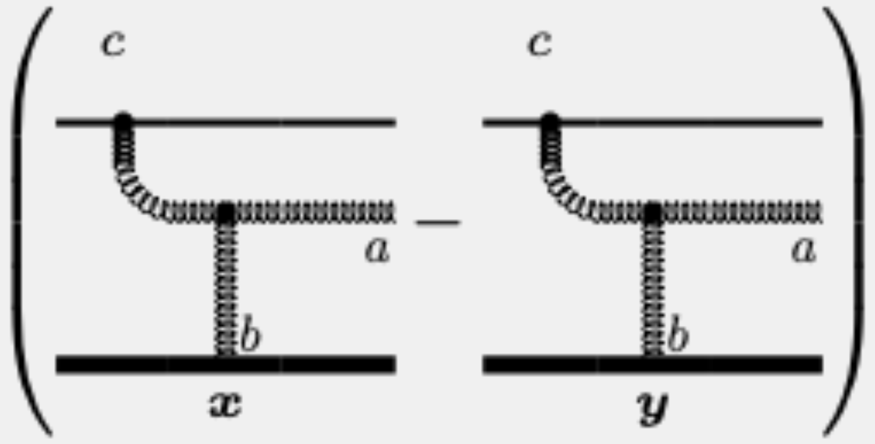
The (twice) expanded Langevin equation in the **dilute limit** becomes

$$\lambda_{\boldsymbol{x},n+1} = \lambda_{\boldsymbol{x},n} + \frac{i\epsilon g}{\sqrt{4\pi^3}} \int_{\boldsymbol{y}} \mathcal{K}^i_{\boldsymbol{xy}} \nu^{i,a}_{\boldsymbol{y},n} i f^{abc} t^c (\lambda^b_{\boldsymbol{x},n} - \lambda^b_{\boldsymbol{y},n}) - \frac{\epsilon g^2}{4\pi^3} \int_{\boldsymbol{y}} \mathcal{K}_{\boldsymbol{xxy}} t^a i f^{abc} t^c (\lambda^b_{\boldsymbol{x},n} - \lambda^b_{\boldsymbol{y},n}) + \mathcal{O}(\epsilon^{3/2}, \lambda^2)$$

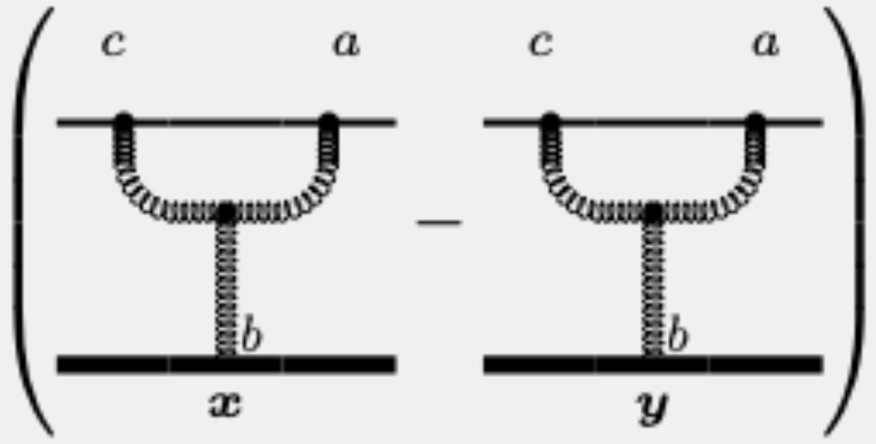
$$=$$



$$+\frac{i\epsilon g}{\sqrt{4\pi^3}} \int_{\boldsymbol{y}} \mathcal{K}^i_{\boldsymbol{xy}} \nu^{i,a}_{\boldsymbol{y},n} i$$



$$-\frac{\epsilon g^2}{4\pi^3} \int_{\boldsymbol{y}} \mathcal{K}_{\boldsymbol{xxy}}$$



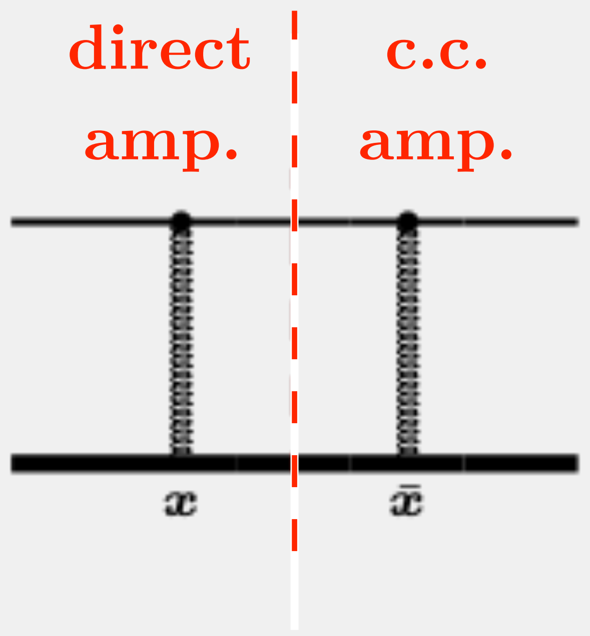
$$+\,\mathcal{O}(\epsilon^{3/2}, \lambda^2)$$

(this Reggeises)

BFKL Evolution

Define the unintegrated gluon distribution

$$\phi^{n}_{\boldsymbol{x}\bar{\boldsymbol{x}}} \equiv \langle \lambda^{a}_{\boldsymbol{x},n} \bar{\lambda}^{a}_{\bar{\boldsymbol{x}},n} \rangle \sim$$



The evolution equation for lambda (one-gluon exchange) gives

$$\phi^{n+1}_{\boldsymbol{x}\bar{\boldsymbol{x}}} - \phi^{n}_{\boldsymbol{x}\bar{\boldsymbol{x}}} = -\frac{N_c}{2}\frac{\epsilon\alpha_s}{\pi^2}\int_{\boldsymbol{z}} [\mathcal{K}_{\boldsymbol{x}\boldsymbol{x}\boldsymbol{z}}(\phi^{n}_{\boldsymbol{x}\bar{\boldsymbol{x}}} - \phi^{n}_{\boldsymbol{z}\bar{\boldsymbol{x}}}) + \mathcal{K}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}\boldsymbol{z}}(\phi^{n}_{\boldsymbol{x}\bar{\boldsymbol{x}}} - \phi^{n}_{\boldsymbol{x}\boldsymbol{z}}) - 2\mathcal{K}_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{z}}(\phi^{n}_{\boldsymbol{x}\bar{\boldsymbol{x}}} - \phi^{n}_{\boldsymbol{x}\boldsymbol{z}} - \phi^{n}_{\boldsymbol{z}\bar{\boldsymbol{x}}} + \phi^{n}_{\boldsymbol{z}\boldsymbol{z}})] + \mathcal{O}(\epsilon^{3/2}, \phi^{3/2})$$

$$\phi^{n+1}(\boldsymbol{q}) - \phi^{n}(\boldsymbol{q}) = +4N_c\epsilon\alpha_s \int_{\boldsymbol{p}} \frac{1}{(\boldsymbol{q}-\boldsymbol{p})^2}\left(\frac{\phi^n(\boldsymbol{p})\boldsymbol{p}^2}{\boldsymbol{q}^2} - \frac{1}{2}\frac{\phi^n(\boldsymbol{q})\boldsymbol{q}^2}{\boldsymbol{p}^2}\right) + \mathcal{O}(\epsilon^{3/2}, \phi^{3/2})$$

This is the colour-singlet, zero momentum transfer BFKL equation (not Mueller's BFKL)

Two-particle production

Iancu & Triantafyllopoulos [JHEP 1311 (2013) 067]

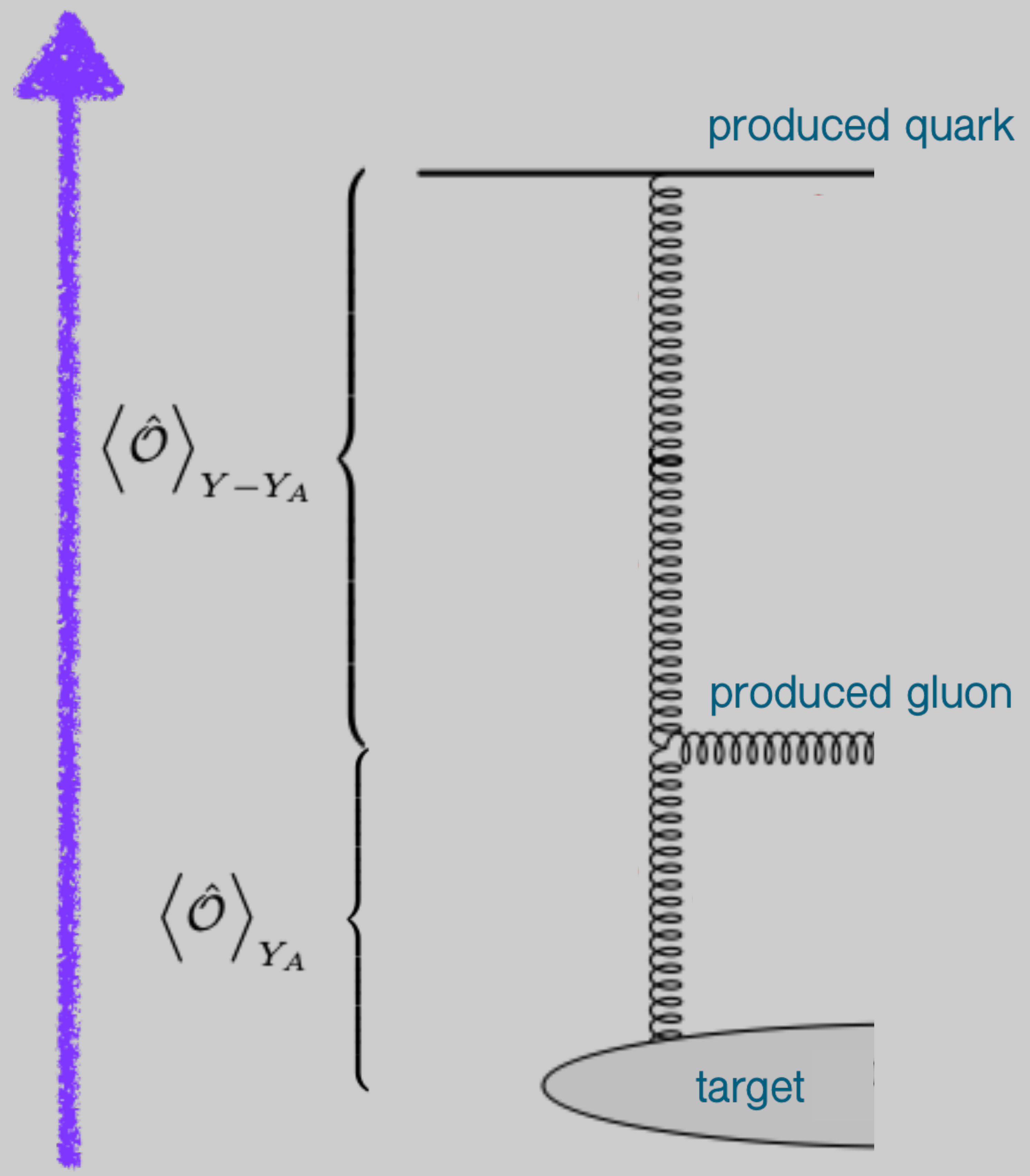
Consider double inclusive quark-gluon production at unequal rapidities.

The correlator gets modified:

$$\langle \hat{O} \rangle_Y \equiv \int [DU] W_Y[U] \hat{O} \longrightarrow \langle \hat{O} \rangle_{Y-Y_A} \equiv \int [DU D\bar{U}] W_{Y-Y_A}[U,\bar{U}|U_A,\bar{U}_A] \hat{O}$$

The differential equation gets modified:

$$\frac{\partial}{\partial Y} W_Y[U] = H W_Y[U] \longrightarrow \frac{\partial}{\partial Y} W_{Y-Y_A}[U,\bar{U}|U_A,\bar{U}_A] = H_{\rm evol} W_{Y-Y_A}[U,\bar{U}|U_A,\bar{U}_A]$$



Two-particle production

Iancu & Triantafyllopoulos [JHEP 1311 (2013) 067]

The cross section contains two different averages

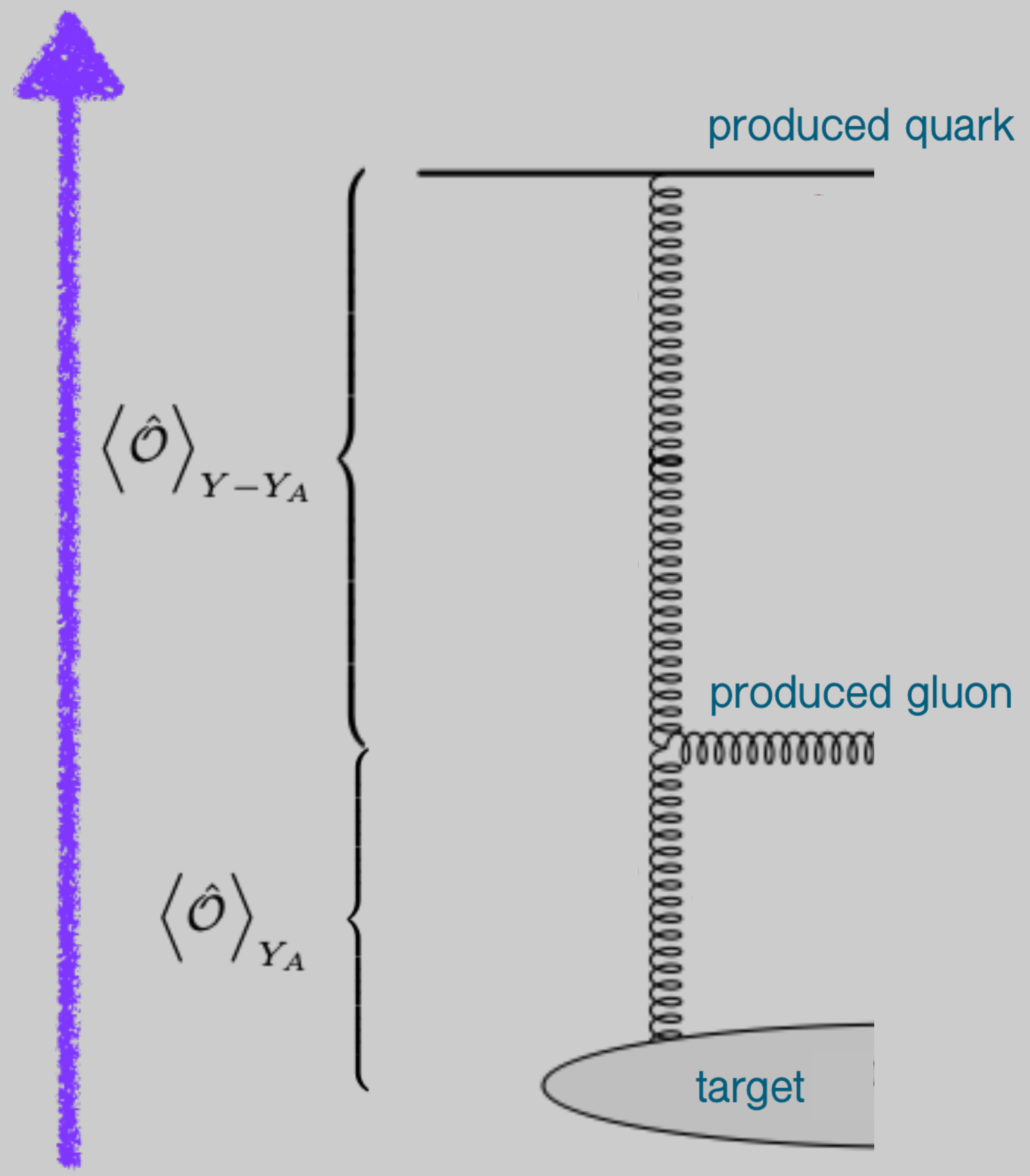
$$\left\langle \hat{\mathcal{O}} \right\rangle_{Y_A} \equiv \int [DU] W_{Y_A}[U] \hat{\mathcal{O}}$$

$$\frac{d\sigma_{qg}}{dY\, d^2\boldsymbol{p}\, dY_A d^2\boldsymbol{k}_A} = \frac{1}{(2\pi)^4} \int_{\boldsymbol{x}\bar{\boldsymbol{x}}} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})} \left\langle H_{\rm prod}(\boldsymbol{k}_A) \left\langle \hat{S}_{\boldsymbol{x}\bar{\boldsymbol{x}}} \right\rangle_{Y-Y_A} \Big|_{\bar{U}_A = U_A} \right\rangle_{Y_A}$$

$$\left\langle \hat{\mathcal{O}} \right\rangle_{Y-Y_A} \equiv \int [DU D\bar{U}] W_{Y-Y_A}[U,\bar{U}|U_A,\bar{U}_A] \hat{\mathcal{O}}$$

The production Hamilton produces the gluon:

$$H_{\rm prod}(\boldsymbol{k}) = \frac{1}{4\pi^3} \int_{\boldsymbol{y}\bar{\boldsymbol{y}}} e^{-i\boldsymbol{k}\cdot(\boldsymbol{y}-\bar{\boldsymbol{y}})} \int_{\boldsymbol{u}\bar{\boldsymbol{u}}} \mathcal{K}^i_{\boldsymbol{y}\boldsymbol{u}} \mathcal{K}^i_{\bar{\boldsymbol{y}}\bar{\boldsymbol{u}}} (L^a_{\boldsymbol{u},0} - U^{\dagger ab}_{\boldsymbol{y},0} R^b_{\boldsymbol{u},0})(\bar{L}^a_{\bar{\boldsymbol{u}},0} - \bar{U}^{\dagger ac}_{\bar{\boldsymbol{y}},0} \bar{R}^c_{\bar{\boldsymbol{u}},0})$$



Bilocal Langevin Equations

The production Hamiltonian requires Lie-differentiated evolution equations

$$R^a_{\boldsymbol{u},0}U^\dagger_{\boldsymbol{x},n+1} = \exp\left\{i\epsilon g \alpha^L_{\boldsymbol{x},n}\right\} R^a_{\boldsymbol{u},0}U^\dagger_{\boldsymbol{x},n} \exp\left\{-i\epsilon g \alpha^R_{\boldsymbol{x},n}\right\} - \frac{i\epsilon g}{\sqrt{4\pi^3}} \exp\left\{i\epsilon g \alpha^L_{\boldsymbol{x},n}\right\} U^\dagger_{\boldsymbol{x},n} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \times [U_{\boldsymbol{z},n}\nu^i_{\boldsymbol{z},n}U^\dagger_{\boldsymbol{z},n}, U_{\boldsymbol{z},n}R^a_{\boldsymbol{u},0}U^\dagger_{\boldsymbol{z},n}]$$

Define the group algebra element $R^a_{\boldsymbol{ux},n} \equiv U_{\boldsymbol{x},n}R^a_{\boldsymbol{u},0}U^\dagger_{\boldsymbol{x},n}$, then

$$\frac{1}{\epsilon}(R^a_{\boldsymbol{ux},n+1} - R^a_{\boldsymbol{ux},n}) = \frac{ig}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}}[\tilde{\nu}^i_{\boldsymbol{z},n}, R^a_{\boldsymbol{ux},n} - R^a_{\boldsymbol{uz},n}] - \frac{N_c}{2}\frac{g^2}{4\pi^3} \int_{\boldsymbol{z}} \mathcal{K}_{\boldsymbol{xxz}}(R^a_{\boldsymbol{ux},n} - R^a_{\boldsymbol{uz},n}) + \mathcal{O}(\epsilon^{3/2})$$

This is independent of the unrotated noise and of explicit Wilson lines.

No need for full non-linear numerics!

(But the cross section remains with nonlinear elements)

$$\sigma \sim \mathrm{tr}\left\{\bar{L}^a_{\boldsymbol{v\bar{x}},N}U_{\boldsymbol{\bar{x}},N}U^\dagger_{\boldsymbol{x},N}L^a_{\boldsymbol{ux},N}\right\} - \bar{\tilde{U}}^{\dagger ac}_{\boldsymbol{\bar{y}},0}\mathrm{tr}\left\{\bar{R}^c_{\boldsymbol{v\bar{x}},N}U_{\boldsymbol{\bar{x}},N}U^\dagger_{\boldsymbol{x},N}L^a_{\boldsymbol{ux},N}\right\} - U^{\dagger ab}_{\boldsymbol{y},0}\mathrm{tr}\left\{\bar{L}^a_{\boldsymbol{v\bar{x}},N}U_{\boldsymbol{\bar{x}},N}U^\dagger_{\boldsymbol{x},N}R^b_{\boldsymbol{ux},N}\right\} + U^{\dagger ab}_{\boldsymbol{y},0}\bar{\tilde{U}}^{\dagger ac}_{\boldsymbol{\bar{y}},0}\mathrm{tr}\left\{\bar{R}^c_{\boldsymbol{v\bar{x}},N}U_{\boldsymbol{\bar{x}},N}U^\dagger_{\boldsymbol{x},N}R^b_{\boldsymbol{ux},N}\right\}$$

Bilocal Langevin Equations in the Dilute Limit

The bilocal Langevin equation becomes

$$R^a_{\boldsymbol{u},0}\lambda_{\boldsymbol{x},n+1} = R^a_{\boldsymbol{u},0}\lambda_{\boldsymbol{x},n} + \int_{\boldsymbol{z}} \left(\frac{i\epsilon g}{\sqrt{4\pi^3}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{i,d}_{\boldsymbol{z},n} - \frac{\epsilon g^2}{4\pi^3} \mathcal{K}_{\boldsymbol{xxz}} t^d \right) i f^{dbc} t^c R^a_{\boldsymbol{u},0} (\lambda^b_{\boldsymbol{x},n} - \lambda^b_{\boldsymbol{z},n}) + \mathcal{O}(\epsilon^{3/2}, \lambda^2)$$

The expanded production Hamiltonian becomes:

$$H_{\text{prod}}(\boldsymbol{k}) = \frac{1}{4\pi^3} \int_{\boldsymbol{y}\bar{\boldsymbol{y}}} e^{-i\boldsymbol{k}\cdot(\boldsymbol{y}-\bar{\boldsymbol{y}})} \int_{\boldsymbol{u}\bar{\boldsymbol{u}}} \mathcal{K}^i_{\boldsymbol{yu}} \mathcal{K}^i_{\bar{\boldsymbol{y}}\bar{\boldsymbol{u}}} \underbrace{(L^a_{\boldsymbol{u},0} - U^{\dagger ab}_{\boldsymbol{y},0} R^b_{\boldsymbol{u},0})(\bar{L}^a_{\bar{\boldsymbol{u}},0} - \bar{U}^{\dagger ac}_{\bar{\boldsymbol{y}},0} \bar{R}^c_{\bar{\boldsymbol{u}},0})}$$

$$= g^2 [f^{abc} f^{ade} (\bar{\lambda}^e_{\bar{\boldsymbol{u}},0} - \bar{\lambda}^e_{\bar{\boldsymbol{y}},0})(\lambda^c_{\boldsymbol{u},0} - \lambda^c_{\boldsymbol{y},0}) + \mathcal{O}(\lambda^3)] \frac{\delta}{\delta \bar{\lambda}^d_{\bar{\boldsymbol{u}},0}} \frac{\delta}{\delta \lambda^b_{\boldsymbol{u},0}}$$

The expanded cross section becomes

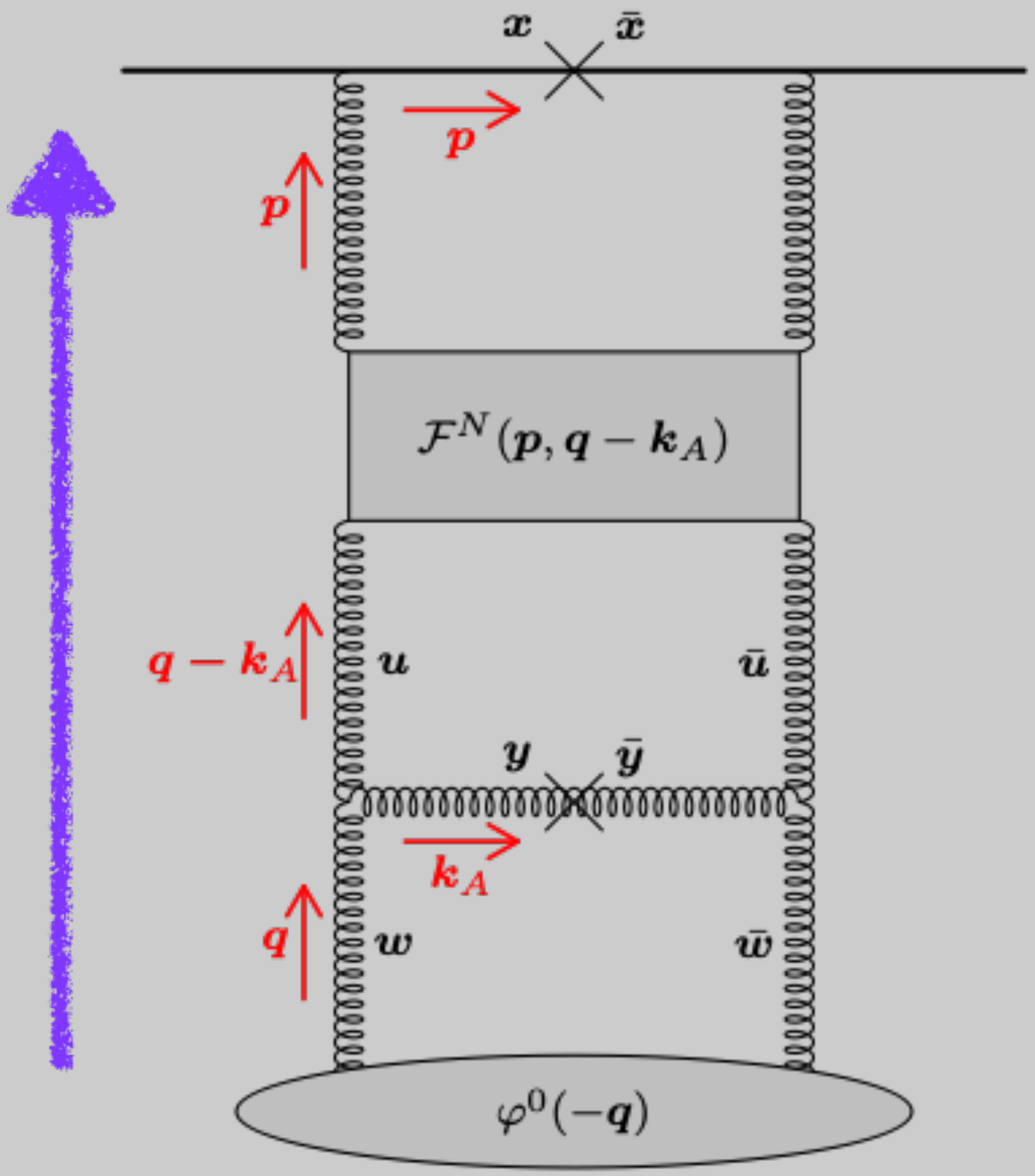
$$\frac{d\sigma_{qg}}{dY\, d^2\boldsymbol{p}\, dY_A\, d^2\boldsymbol{k}_A} = \frac{1}{(2\pi)^4} \frac{1}{2N_c} \frac{\alpha_s}{\pi^2} \int_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{y}\bar{\boldsymbol{y}}\boldsymbol{u}\bar{\boldsymbol{u}}} \mathcal{K}^i_{\boldsymbol{yu}} \mathcal{K}^i_{\bar{\boldsymbol{y}}\bar{\boldsymbol{u}}} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}}) - i\boldsymbol{k}_A\cdot(\boldsymbol{y}-\bar{\boldsymbol{y}})} (\phi^0_{\bar{\boldsymbol{u}}\boldsymbol{u}} - \phi^0_{\bar{\boldsymbol{u}}\boldsymbol{y}} - \phi^0_{\bar{\boldsymbol{y}}\boldsymbol{u}} + \phi^0_{\bar{\boldsymbol{y}}\boldsymbol{y}}) \underbrace{\mathcal{F}^N_{\boldsymbol{x},\bar{\boldsymbol{x}},\boldsymbol{u},\bar{\boldsymbol{u}}}} + \mathcal{O}(\phi^{3/2})$$

$$\mathcal{F}^n_{\boldsymbol{x},\bar{\boldsymbol{x}},\boldsymbol{u},\bar{\boldsymbol{u}}} \equiv \frac{\delta}{\delta \bar{\lambda}^a_{\bar{\boldsymbol{u}},0}} \frac{\delta}{\delta \lambda^a_{\boldsymbol{u},0}} \bar{\lambda}^b_{\bar{\boldsymbol{x}},n} \lambda^b_{\boldsymbol{x},n}$$

BFKL Ladder Diagrams

The final k_T-factorised cross section for a quark and a gluon produced at unequal rapidities is

$$\frac{d\sigma_{qg}}{dY\, d^2\boldsymbol{p}\, dY_A d^2\boldsymbol{k}_A} = -\frac{\alpha_s}{N_c}\int_{\boldsymbol{q}} \frac{\boldsymbol{q}^2}{(\boldsymbol{q}-\boldsymbol{k}_A)^2\boldsymbol{k}_A^2}\mathcal{F}^N(-\boldsymbol{p},\boldsymbol{p},\boldsymbol{q}-\boldsymbol{k}_A,-\boldsymbol{q}+\boldsymbol{k}_A)\phi^0(-\boldsymbol{q}) + \mathcal{O}(\varphi^{3/2})$$



Summary

- Studied Langevin picture of JIMWLK evolution → alternative formulation of evolution as stochastic diffusion
- Two expansions → epsilon (rapidity step), lambda (group algebra element)
- Bilocal Langevin evolution equation is linear (full dense case)
- BFKL dynamics emerge in dilute limit
- Particle production cross section simplifies somewhat → no need for full nonlinear numerics

(work in progress, with Tuomas Lappi,

Mark Mace & Soeren Schlichting)