The Mueller-Tang impact factors in the soft-gluon approximation

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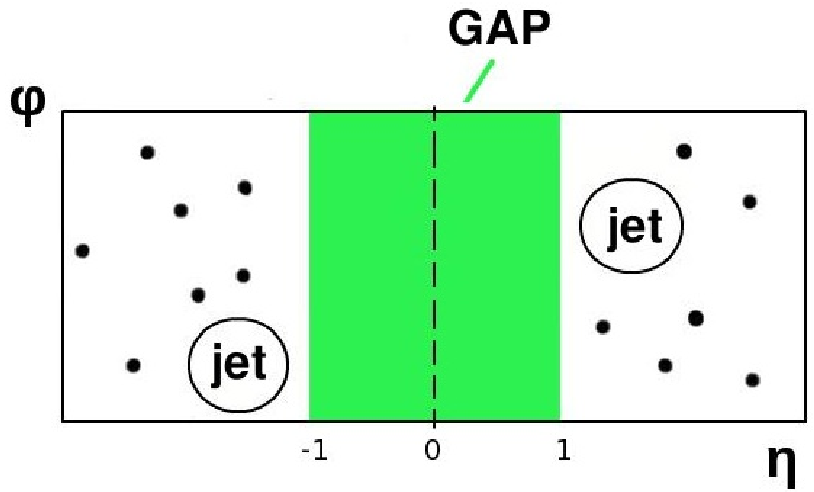
Outline

- Introduction:
  - Motivations for study of NL Mueller-Tang Impact Factors in soft approx
  - Some details of original calculation

    [Hentchinski, Madrigal Martinez, Murdaca, Sabio Vera, '14]
- Discussion on some unexpected features of NL MT IF:
  - theoretically puzzling
  - numerically problematic
- For a first understanding on this issue:
  - Calculation of NL MT IF in soft-gluon approximation
  - Comparison with original expression
- Conclusion:
  - Suggestion of an independent calculation of NL MT IF
  - Prompt a discussion: which phase-space constraints are needed (or advisable) for a good IR safe definition of MT jets?

Mueller-Tang jets



An important process for studying PT high-energy QCD and the Pomeron at hadron colliders *[Mueller, Tang '87]*

Final states with two jets with similar p_T and large rapidity gap between them

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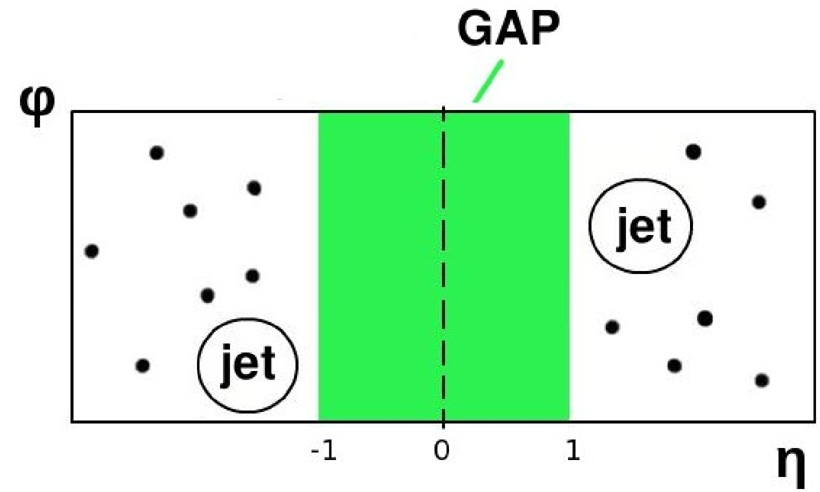

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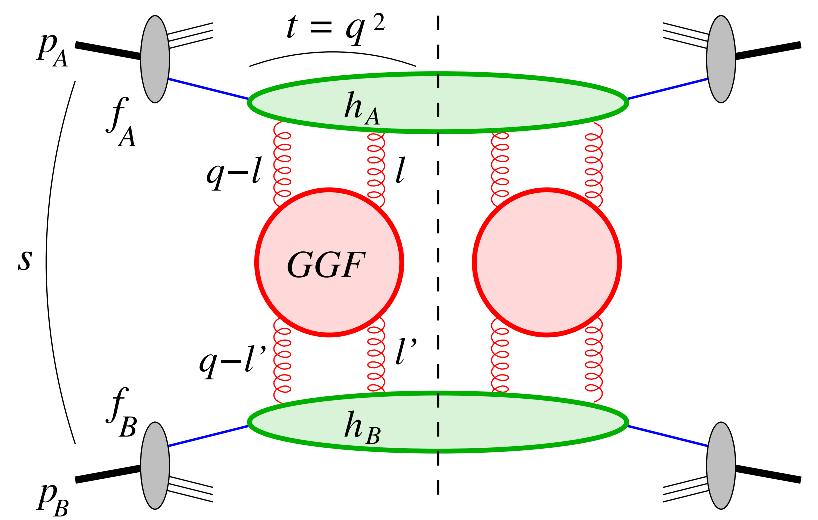

Final states with two jets with similar p_T and large rapidity gap between them

A NL calculation of MT jets requires NLO PDFs, NLL GGFs and NLO IFs

[see F. Deganutti's talk]

- Problem in integration of some terms in IF, due to unexpected soft singularities, causing divergent cross section, if a constraint on invariant mass of particles in each region is not imposed
- MT jet observable does not require such constraint, though admissible

Rewiev of MT IF calculation

- To solve this issue, study MT jets in the limit of soft-gluon emission
- Non-trivial check of *[HMMS]* computation (no independent calculation yet)
- To this purpose, I review derivation of high-energy factorization formula

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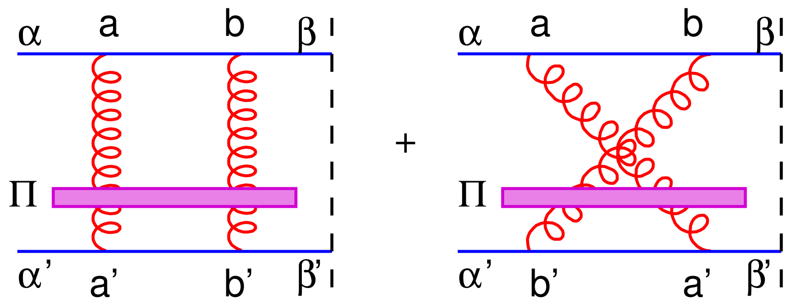
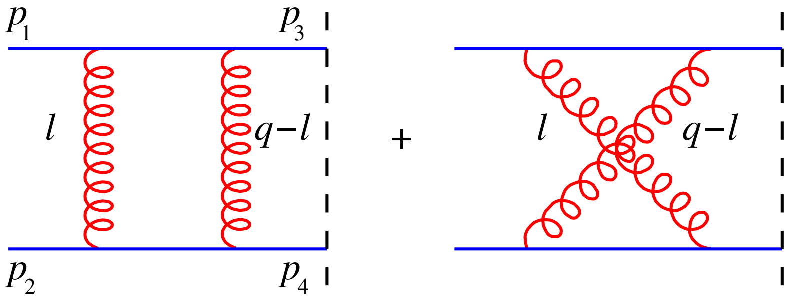
MT jets at lowest order (quark-induced jet)



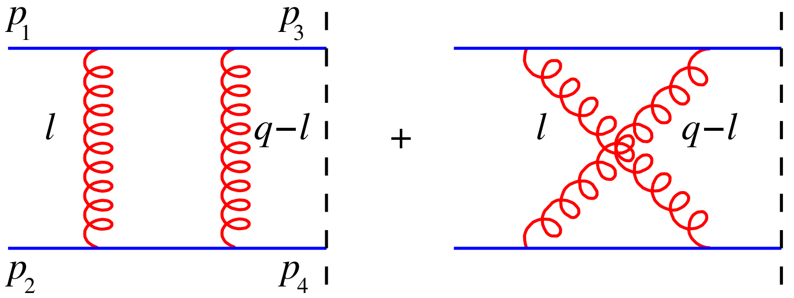
- Large rapidity gap expected
  to occur for colour-singlet exchange
- LO diagrams involve two gluons: $8 \otimes 8 = 1 \oplus 8 + \cdots$
  For quarks $(t^a t^b)_{\alpha\beta} = \frac{1}{2N_c}\delta_{\alpha\beta}\delta^{ab} + \frac{1}{2}\left(\mathrm{i}f^{abc} + d^{abc}\right) t^c_{\alpha\beta}$
- Singlet projector: $\Pi^{ab,a'b'} = \delta^{ab}\delta^{a'b'}/(N_c^2 - 1) \;\Rightarrow$ colour structure
  $(t^a t^b)_{\alpha\beta}\Pi^{ab,a'b'}(t^{a'} t^{b'})_{\alpha'\beta'} = \left(\frac{C_F}{\sqrt{N_c^2-1}}\delta_{\alpha\beta}\right)\left(\frac{C_F}{\sqrt{N_c^2-1}}\delta_{\alpha'\beta'}\right)$

Rewiev of MT IF calculation



- Kinematic factor: exploit high-energy (Regge) limit $-t = -q^2 \simeq \boldsymbol{q}^2 \ll s$ $\Rightarrow$ small deflection angles $\iff$ large $|y|$'s
- Eikonal approximation of vertices: $\bar{u}_3 \gamma^\mu (\hat{p}_1 + \hat{l}) \gamma^\nu u_1 \simeq (2p_1^\mu)(2p_1^\nu)\delta_{\lambda_3\lambda_1}$
- l^+ and l^- integrations of loop integral carried out by enclosing poles of Feynman propagators of quarks.

Rewiev of MT IF calculation



$$\mathrm{i}\mathcal{M}_0 = \Phi_1 \Phi_2 \int \frac{\mathrm{d}^2 \boldsymbol{l}}{(2\pi)^2} \frac{1}{\boldsymbol{l}^2 (\boldsymbol{q}-\boldsymbol{l})^2}$$

$$\Phi_A = \frac{g^2 C_F \sqrt{2\hat{s}}\, \delta_{\alpha\beta} \delta_{\lambda_3 \lambda_1}}{\sqrt{N_c^2 - 1}}$$

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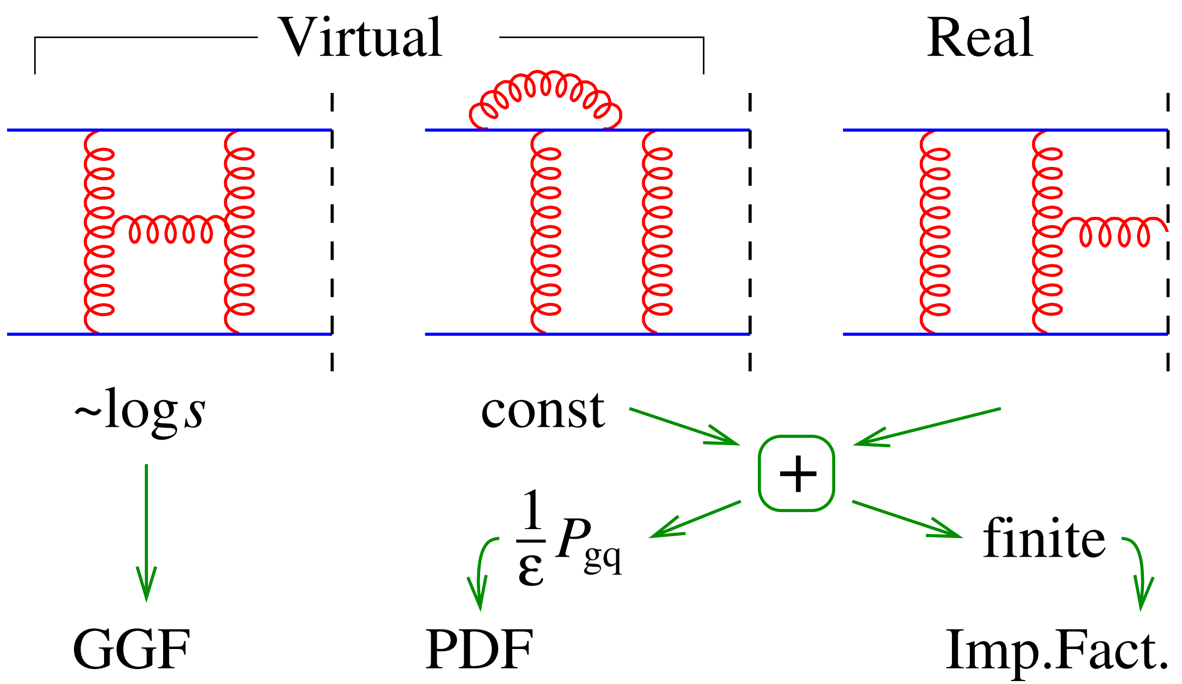
- l^+ and l^- integrations of loop integral carried out by enclosing poles of Feynman propagators of quarks. Summing the two diagrams:

LO partonic differential cross section: (constant in s)

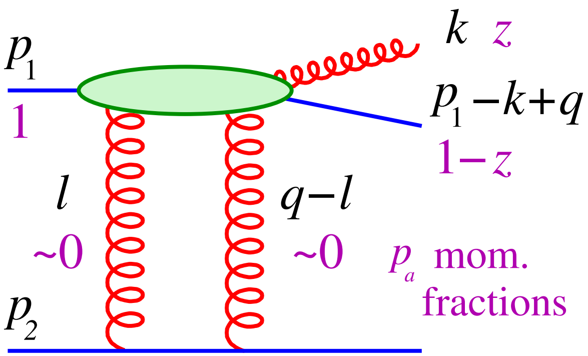
$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}^2\boldsymbol{q}} = h_0\, G_0(\boldsymbol{q})^2\, h_0 \,, \quad h_0 = \frac{\alpha_s^2 C_F^2}{N_c^2 - 1} \,, \quad G_0(\boldsymbol{q}) = \int \frac{\mathrm{d}^2 \boldsymbol{l}}{(2\pi)^2} \frac{1}{\boldsymbol{l}^2 (\boldsymbol{q}-\boldsymbol{l})^2}$$

NL contributions



- Non-trivial task of assessing high-energy factorization formula at NL: cancellation of IR singularities
  (apart from collinear ones to be absorbed in PDFs)
- This is what *[HMMS]* wanted to achieve by taking virtual correction to IF from *[Fadin,Fiore,Kotsky,Papa '00]* and by computing real corrections

NL Impact Factor



$$h_1(\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{q}) = \frac{\alpha_s^3}{2\pi(N_c^2-1)} \int_0^1 \mathrm{d}z \int \mathrm{d}\boldsymbol{k}$$

$$\times\, \boldsymbol{S_J}(\boldsymbol{k}, \boldsymbol{q}, z)\, C_F \frac{1+(1-z)^2}{z}$$

$$\times \left\{ C_F^2 \frac{z^2 \boldsymbol{q}^2}{\boldsymbol{k}^2(\boldsymbol{k}-z\boldsymbol{q})^2} + C_F C_A\, f_1(\boldsymbol{l}_{1,2}, \boldsymbol{k}, \boldsymbol{q}, z) + C_A^2\, f_2(\boldsymbol{l}_{1,2}, \boldsymbol{k}, \boldsymbol{q}) \right\}$$

- Phase-space integration restricted by IR-safe jet algorithm (e.g., kt $\simeq$ cone)
- In these diffractive processes, “lower” quark p_2 is the “backward” jet
  Other two partons are in the forward hemisphere and form (at least) one jet:
  - $\Delta\Omega \equiv \sqrt{\Delta y^2 + \Delta\phi^2} < R \quad \Rightarrow J = \{qg\}$ composite jet
  - $\Delta\Omega > R \Rightarrow J = \{g\}$ and q outside jet cone or $J = \{q\}$ and g outside
- Such restricted phase-space integration is collinear and soft divergent
- Combining with virtual corrections and extracting PDF’s collinear poles…

NL Impact Factor

…do they obtain a finite result? (= NL IF)

- *[HMMS]* claimed that this is the case, if one also requires the invariant mass of the q-g system bounded from above
- Problem in the C_A^2 term when $J = \{q\}$ and the gluon becomes soft; it can be emitted in any direction $\Rightarrow$ arbitrary low rapidity
- At fixed $\boldsymbol{k}$, the integral $\int_0^1 \mathrm{d}z/z = \int_{-\infty}^{\log\sqrt{s}/\boldsymbol{k}} \mathrm{d}y$ flat in y, formally infinite
  This is why the numerical integration of the NLIF does not converge
- If we believe the IF calculation to be reliable at least in the forward hemisphere ($y > 0$) $\Rightarrow$ $\int_0^{\log\sqrt{s}/\boldsymbol{k}} \mathrm{d}y = \int_{\boldsymbol{k}/\sqrt{s}}^1 \mathrm{d}z/z = \frac{1}{2}\log(s/\boldsymbol{k}^2)$
- Hence we would have a $\log(s)$ from IFs!
  This is not acceptable within the spirit of BFKL factorization

Constraint on q-g invariant mass

In order to solve this problem, *[HMMS]* constrain invariant mass $\hat{M}_{qg}^2 \equiv (p_1 + k)^2 = (\boldsymbol{k} - z\boldsymbol{q})^2/z(1-z) < M_{\rm max}^2$

- In this case $z \gtrsim \boldsymbol{k}^2/M_{\rm max}^2 \;\Rightarrow$ finite z-integral $\sim \log(M_{\rm max}^2/\boldsymbol{k}^2)$
- the remaining $\boldsymbol{k}$ integration yields $\mathrm{d}\sigma \propto M_{\rm max}^2/\boldsymbol{p}_J^2$ (for $M_{\rm max} \lesssim \boldsymbol{p}_J$)

Crucial question: do we really need to impose a cut on the diffractive mass?

- Theoretically no: singlet exchange should forbid gluons in central region, no $\log s$ contribution from gluon integration in c.r. (contrary to octet, MNJ)
- Experimentally no: gluon $\boldsymbol{k} < \boldsymbol{k}_{\rm soft}$ indistinguishable from soft radiation, hadron remnants, or even undetected $\Rightarrow$ no way to impose bound on $\hat{M}_{qg}$
- Maybe C_A^2 term of IF incorrect? This doubt could be proved or disproved by independent calculation (std QCD) of the NLIF (*[HMMS]* used Lipatov's effective action for QCD at high energy, powerful but delicate technique)

Soft theorem

- “Impeached” C_A^2 term shows “strange” behaviour when gluon is soft
  $k = zp_1 + \frac{\boldsymbol{k}^2}{zs} p_2 + \boldsymbol{k} = z(p_1 + \frac{\boldsymbol{K}^2}{s} p_2 + \boldsymbol{K}) \xrightarrow{z \to 0} 0$
- Idea: compute real emission part of IF in soft gluon approx (std tool)

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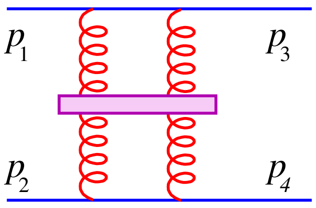
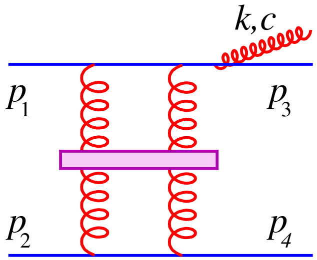
Start from LO amplitude $\mathcal{M}_0$

Emission of soft gluon given by

generalization of *[Weinberg '65]*





$$\mathcal{M}_1\big(\{p_i\}, (k,\lambda,c)\big) = g \sum_{i\in\text{ext legs}} T_i^c \frac{p_i\cdot\epsilon_\lambda(k)}{p_i\cdot k}\mathcal{M}_0(\{p_i\})$$

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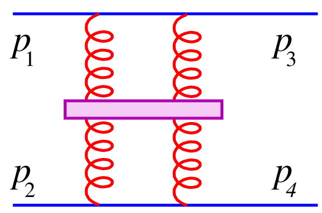

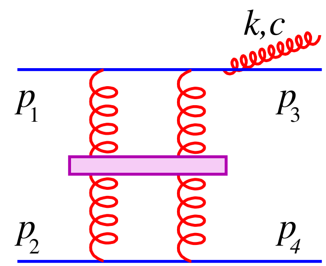

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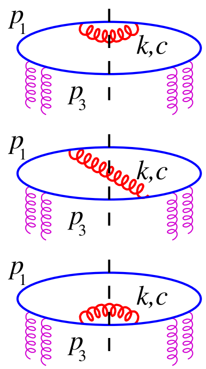
squared matrix element
$$\sum_{\substack{\text{spin}\\ \text{colours}}} \langle\mathcal{M}_1|\mathcal{M}_1\rangle = g^2 \sum_{i,j} \langle\mathcal{M}_0|T_i^c T_j^c|\mathcal{M}_0\rangle \underbrace{\frac{p_i^\mu d_{\mu\nu}(k) p_j^\nu}{p_i\cdot k \;\; p_j\cdot k}}_{S_{ij}}$$

- $d_{\mu\nu}(k) = -g_{\mu\nu} + \frac{p_{2\,\mu}k_\nu + k_\mu p_{2\nu}}{p_2\cdot k}$ gauge dependent, $\sum_{i,j}$ gauge invariant
- In light-cone gauge $p_2\cdot A = 0$ no gluon emission from "lower" quark line

Soft real emission (singlet exchange)

i	j	$t_i^c t_j^c$	$\mathcal{T}_i^c \mathcal{T}_j^c$	$\frac{1}{2} S_{ij}$
1	1	C_F	C_A	$-\frac{2}{\boldsymbol{k}^2}$
1	3	$-C_F$	$-C_A$	$\frac{z^2 \boldsymbol{p}_2^2}{\boldsymbol{k}^2(\boldsymbol{k}-z\boldsymbol{q})^2} - \frac{1}{\boldsymbol{k}^2} - \frac{(1-z)^2}{(\boldsymbol{k}-z\boldsymbol{q})^2}$
3	3	C_F	C_A	$-2\frac{(1-z)^2}{(\boldsymbol{k}-z\boldsymbol{q})^2}$



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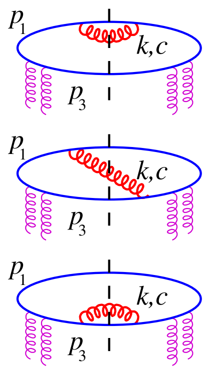
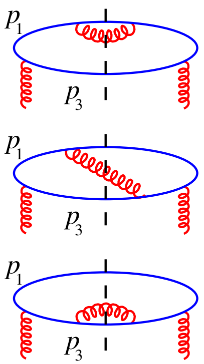


$$\rightsquigarrow \qquad h_1^{\text{soft}} = \frac{\alpha_s^3}{2\pi(N_c^2-1)} \int \mathrm{d}z\, \mathrm{d}\boldsymbol{k}\; S_J(\boldsymbol{k},\boldsymbol{q},z)\, \frac{2C_F}{z}\; C_F^2 \frac{z^2\boldsymbol{q}^2}{\boldsymbol{k}^2(\boldsymbol{k}-z\boldsymbol{q})^2}$$

- C_F/z pole is the soft remnant of $P_{gq}(z)$ splitting function
- Only the C_F^2 term is present. It matches exactly C_F^2 term of full IF
- Other colour structure $C_F C_A$, C_A^2 appear from gluon insertions on *internal* lines $\Rightarrow$ finite in the soft limit
- Gluon-induced IF yields analogous result ($C_F \to C_A$)

Soft real emission (octet exchange)

i	j	$t_i^c t_j^c$	$\mathcal{T}_i^c \mathcal{T}_j^c$	$\frac{1}{2} S_{ij}$
1	1	C_F	C_A	$-\frac{2}{\boldsymbol{k}^2}$
1	3	$\frac{1}{2}C_A - C_F$	$-\frac{1}{2}C_A$	$\frac{z^2 \boldsymbol{q}^2}{\boldsymbol{k}^2 (\boldsymbol{k}-z\boldsymbol{q})^2} - \frac{1}{\boldsymbol{k}^2} - \frac{(1-z)^2}{(\boldsymbol{k}-z\boldsymbol{q})^2}$
3	3	C_F	C_A	$-\frac{2(1-z)^2}{(\boldsymbol{k}-z\boldsymbol{q})^2}$



$$h_1^{\text{soft}} = \frac{\alpha_s^2}{\boldsymbol{q}^2} \int \mathrm{d}z \, \mathrm{d}^2\boldsymbol{k} \, \frac{2C_F}{z} \left\{ C_A \frac{(1-z)\boldsymbol{k}\cdot(\boldsymbol{k}-z\boldsymbol{q})}{\boldsymbol{k}^2 \, (\boldsymbol{k}-z\boldsymbol{q})^2} + C_F \frac{z^2 \boldsymbol{q}^2}{\boldsymbol{k}^2 \, (\boldsymbol{k}-z\boldsymbol{q})^2} \right\}$$

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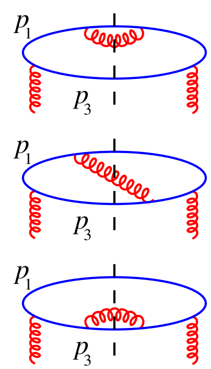

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To be compared with the full result *[Ciafaloni, DC '98]*, the basis for MN IF

$$h_1 = \frac{\alpha_s^2}{\boldsymbol{q}^2} \int \mathrm{d}z \, \mathrm{d}^2\boldsymbol{k} \, P_{gq}(z) \left\{ C_A \frac{(1-z)\boldsymbol{k}\cdot(\boldsymbol{k}-z\boldsymbol{q})}{\boldsymbol{k}^2 \, (\boldsymbol{k}-z\boldsymbol{q})^2} + C_F \frac{z^2\boldsymbol{q}^2}{\boldsymbol{k}^2 \, (\boldsymbol{k}-z\boldsymbol{q})^2} \right\}$$

- C_F/z pole is the soft remnant of $P_{gq}(z)$ splitting function
- The other factors are reproduced exactly!

Conclusions

I want to bring to your attention two issues:

- It is important to have an independent calculation of NL IF for Mueller-Tang jets in order to be sure to have the correct expressions
- When doing phenomenology (calculations or measurements) of Mueller-Tang jets,
  - does it make sense to impose an upper bound on the invariant mass of the particles in each hemisphere?
  - is it compulsory at theoretical or experimental level?

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I hope to have enlightening discussions (at least on the last point) and hopefully to see the result of an independent calculation of NL impact factors in the near future.