The Mueller-Tang impact factors
in the soft-gluon approximation

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Outline

■ Introduction:
  • Motivations for study of NL Mueller-Tang Impact Factors in soft approx
  • Some details of original calculation
    
    [Hentchinski, Madrigal Martínez, Murdaca, Sabio Vera, ’14]

■ Discussion on some unexpected features of NL MT IF:
  • theoretically puzzling
  • numerically problematic

■ For a first understanding on this issue:
  • Calculation of NL MT IF in soft-gluon approximation
  • Comparison with original expression

■ Conclusion:
  • Suggestion of an independent calculation of NL MT IF
  • Prompt a discussion: which phase-space constraints are needed
    (or advisable) for a good IR safe definition of MT jets?
Mueller-Tang jets

An important process for studying PT high-energy QCD and the Pomeron at hadron colliders [Mueller, Tang ’87]

Final states with two jets with similar $p_T$ and large rapidity gap between them
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An important process for studying PT high-energy QCD and the Pomeron at hadron colliders \([\text{Mueller, Tang '87}]\)

Final states with two jets with similar \(p_T\) and large rapidity gap between them

A NL calculation of MT jets requires NLO PDFs, NLL GGFs and NLO IFs \([\text{see F. Deganutti's talk}]\)

- **Problem** in integration of some terms in IF, due to unexpected soft singularities, causing divergent cross section, if a constraint on invariant mass of particles in each region is not imposed

- MT jet observable does not require such constraint, though admissible
Rewiew of MT IF calculation

- To solve this issue, study MT jets in the limit of soft-gluon emission
- Non-trivial check of [HMMS] computation (no independent calculation yet)
- To this purpose, I review derivation of high-energy factorization formula
To solve this issue, study MT jets in the limit of soft-gluon emission

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MT jets at lowest order (quark-induced jet)

Large rapidity gap expected to occur for colour-singlet exchange

LO diagrams involve two gluons: $8 \otimes 8 = 1 \oplus 8 + \cdots$

For quarks $(t^a t^b)_{\alpha \beta} = \frac{1}{2N_c} \delta_{\alpha \beta} \delta^{ab} + \frac{1}{2} (i f^{abc} + d^{abc}) t^c_{\alpha \beta}$

Singlet projector: $\Pi^{ab, a'b'} = \delta^{ab} \delta^{a'b'}/(N_c^2 - 1) \Rightarrow$ colour structure

$$(t^a t^b)_{\alpha \beta} \Pi^{ab, a'b'} (t^{a'} t^{b'})_{\alpha' \beta'} = \left( \frac{C_F}{\sqrt{N_c^2 - 1}} \delta_{\alpha \beta} \right) \left( \frac{C_F}{\sqrt{N_c^2 - 1}} \delta_{\alpha' \beta'} \right)$$
Kinematic factor: exploit high-energy (Regge) limit \(-t = -q^2 \simeq q^2 \ll s\) ⇒ small deflection angles ⇐⇒ large \(|y|’s\)

Eikonal approximation of vertices: \(\bar{u}_3 \gamma^\mu (\hat{p}_1 + \hat{l}) \gamma^\nu u_1 \simeq (2p_1^\mu)(2p_1^\nu) \delta_{\lambda_3 \lambda_1}\)

\(l^+\) and \(l^-\) integrations of loop integral carried out by enclosing poles of Feynman propagators of quarks.
Review of MT IF calculation

\[ i\mathcal{M}_0 = \Phi_1 \Phi_2 \int \frac{d^2l}{(2\pi)^2} \frac{1}{l^2(q-l)^2} \]

\[ \Phi_A = \frac{g^2 C_F \sqrt{2\delta} \delta_{\alpha\beta} \delta_{\lambda_3\lambda_1}}{\sqrt{N_c^2 - 1}} \]

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- \(l^+\) and \(l^-\) integrations of loop integral carried out by enclosing poles of Feynman propagators of quarks. Summing the two diagrams:

LO partonic differential cross section: \((\text{constant in } s)\)

\[ \frac{d\sigma_0}{d^2q} = h_0 G_0(q)^2 h_0 , \quad h_0 = \frac{\alpha_s^2 C_F^2}{N_c^2 - 1} , \quad G_0(q) = \int \frac{d^2l}{(2\pi)^2} \frac{1}{l^2(q-l)^2} \]
Non-trivial task of assessing high-energy factorization formula at NL: cancellation of IR singularities (apart from collinear ones to be absorbed in PDFs)

This is what [HMMS] wanted to achieve by taking virtual correction to IF from [Fadin,Fiore,Kotsky,Papa '00] and by computing real corrections.
\[ h_1(l_1, l_2, q) = \frac{\alpha_s^3}{2\pi(N_c^2 - 1)} \int_0^1 dz \int dk \]
\[ \times S_J(k, q, z) C_F \frac{1 + (1 - z)^2}{z} \]
\[ \times \left\{ C_F^2 \frac{z^2 q^2}{k^2(k - zq)^2} + C_F C_A f_1(l_1,2, k, q, z) + C_A^2 f_2(l_1,2, k, q) \right\} \]

- Phase-space integration restricted by IR-safe jet algorithm (e.g., \( kt \simeq \text{cone} \))
- In these diffractive processes, “lower” quark \( p_2 \) is the “backward” jet
  Other two partons are in the forward hemisphere and form (at least) one jet:
  - \( \Delta\Omega \equiv \sqrt{\Delta y^2 + \Delta \phi^2} < R \) ⇒ \( J = \{ qg \} \) composite jet
  - \( \Delta\Omega > R \) ⇒ \( J = \{ g \} \) and \( q \) outside jet cone or \( J = \{ q \} \) and \( g \) outside
- Such restricted phase-space integration is collinear and soft divergent
- Combining with virtual corrections and extracting PDF’s collinear poles…
NL Impact Factor

...do they obtain a finite result? (= NL IF)

- [HMMS] claimed that this is the case, if one also requires the invariant mass of the q-g system bounded from above

- Problem in the $C_A^2$ term when $J = \{q\}$ and the gluon becomes soft; it can be emitted in any direction $\Rightarrow$ arbitrary low rapidity

- At fixed $k$, the integral $\int_0^1 \frac{dz}{z} = \int_{-\infty}^{\log \sqrt{s}/k} dy$ flat in $y$, formally infinite
  This is why the numerical integration of the NLIF does not converge

- If we believe the IF calculation to be reliable at least in the forward hemisphere ($y > 0$) $\Rightarrow \int_0^{\log \sqrt{s}/k} dy = \int_{k/\sqrt{s}}^1 \frac{dz}{z} = \frac{1}{2} \log(s/k^2)$

- Hence we would have a $\log(s)$ from IFs!
  This is not acceptable within the spirit of BFKL factorization
Constraint on q-g invariant mass

In order to solve this problem, [HMMS] constrain invariant mass
\[ \hat{M}_{qg}^2 \equiv (p_1 + k)^2 = (k - zq)^2 / z(1 - z) < M_{\text{max}}^2 \]

- In this case \( z \gtrsim k^2 / M_{\text{max}}^2 \Rightarrow \) finite \( z \)-integral \( \sim \log(M_{\text{max}}^2 / k^2) \)
- the remaining \( k \) integration yields \( d\sigma \propto M_{\text{max}}^2 / p_J^2 \) (for \( M_{\text{max}} \lesssim p_J \))

Crucial question: do we really need to impose a cut on the diffractive mass?

- Theoretically no: singlet exchange should forbid gluons in central region, no \( \log s \) contribution from gluon integration in c.r. (contrary to octet, MNJ)
- Experimentally no: gluon \( k < k_{\text{soft}} \) indistinguishable from soft radiation, hadron remnants, or even undetected \( \Rightarrow \) no way to impose bound on \( \hat{M}_{qg} \)
- Maybe \( C_A^2 \) term of IF incorrect? This doubt could be proved or disproved by independent calculation (std QCD) of the NLIF ([HMMS] used Lipatov’s effective action for QCD at high energy, powerful but delicate technique)
Soft theorem

- “Impeached” $C_A^2$ term shows “strange” behaviour when gluon is soft
  \[ k = z p_1 + \frac{k^2}{z s} p_2 + k = z(p_1 + \frac{K^2}{s} p_2 + K) \xrightarrow{z \to 0} 0 \]

- Idea: compute real emission part of IF in soft gluon approx (std tool)
Soft theorem

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Start from LO amplitude $\mathcal{M}_0$

Emission of soft gluon given by generalization of [Weinberg ’65]

$$\mathcal{M}_1(\{p_i\}, (k, \lambda, c)) = g \sum_{i \in \text{ext legs}} T^c_i \frac{p_i \cdot \epsilon_\lambda(k)}{p_i \cdot k} \mathcal{M}_0(\{p_i\})$$
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\]

Squared matrix element

$$\sum \langle \mathcal{M}_1 | \mathcal{M}_1 \rangle = g^2 \sum_{i,j} \langle \mathcal{M}_0 | T_i^c T_j^c | \mathcal{M}_0 \rangle \frac{p_i^\mu d_{\mu\nu}(k) p_j^\nu}{p_i \cdot k p_j \cdot k}$$

- $d_{\mu\nu}(k) = -g_{\mu\nu} + \frac{p_{2\mu} k_{\nu} + k_{\mu} p_{2\nu}}{p_2 \cdot k}$ gauge dependent, $\sum_{i,j}$ gauge invariant

- In light-cone gauge $p_2 \cdot A = 0$ no gluon emission from "lower" quark line
Soft real emission (singlet exchange)

<table>
<thead>
<tr>
<th>$i$</th>
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\[ P_3 \]

\[ p_1 \]

\[ P_2 \]

\[ k, c \]

\[ p_3 \]
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\[ \Rightarrow \quad h_1^{\text{soft}} = \frac{\alpha_s^3}{2\pi (N_c^2 - 1)} \int dz \, dk \, S_J(k, q, z) \frac{2C_F}{z} C_F^2 \frac{z^2 q^2}{k^2 (k-zq)^2} \]

- $C_F / z$ pole is the soft remnant of $P_{gq}(z)$ splitting function
- Only the $C_F^2$ term is present. It matches exactly $C_F^2$ term of full IF
- Other colour structure $C_F C_A$, $C_A^2$ appear from gluon insertions on internal lines $\Rightarrow$ finite in the soft limit
- Gluon-induced IF yields analogous result ($C_F \rightarrow C_A$)
## Soft real emission (octet exchange)

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\[
\alpha_s^2 \frac{1}{q^2} \int dz \, d^2 k \, \frac{2C_F}{z} \left\{ C_A \frac{(1-z) \mathbf{k} \cdot (\mathbf{k} - z\mathbf{q})}{k^2 (k - zq)^2} + C_F \frac{z^2 q^2}{k^2 (k - zq)^2} \right\}
\]
### Soft real emission (octet exchange)

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$$h_1^{\text{soft}} = \frac{\alpha_s^2}{q^2} \int d z \, d^2 k \, \frac{2 C_F}{z} \left\{ C_A \frac{(1-z)k \cdot (k - z q)}{k^2 (k - z q)^2} + C_F \frac{z^2 q^2}{k^2 (k - z q)^2} \right\}$$

To be compared with the full result [Ciafaloni, DC '98], the basis for MN IF

$$h_1 = \frac{\alpha_s^2}{q^2} \int d z \, d^2 k \, P_{gq}(z) \left\{ C_A \frac{(1-z)k \cdot (k - z q)}{k^2 (k - z q)^2} + C_F \frac{z^2 q^2}{k^2 (k - z q)^2} \right\}$$

- $C_F/z$ pole is the soft remnant of $P_{gq}(z)$ splitting function
- The other factors are reproduced exactly!

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Conclusions

I want to bring to your attention two issues:

- It is important to have an independent calculation of NL IF for Mueller-Tang jets in order to be sure to have the correct expressions.

- When doing phenomenology (calculations or measurements) of Mueller-Tang jets,
  - does it make sense to impose an upper bound on the invariant mass of the particles in each hemisphere?
  - is it compulsory at theoretical or experimental level?
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  - is it compulsory at theoretical or experimental level?

I hope to have enlightening discussions (at least on the last point) and hopefully to see the result of an independent calculation of NL impact factors in the near future.