## Towards Mueller-Tang jets at Next-to-leading order

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## Outline

- Motivations and background
- BFKL approach
- Mueller-Tang jets vs Mueller-Navelet jets process
- From LL to NLL
- Non-forward eigenfunction in momentum space
- NLO impact factors
- Result (incomplete)
- Future outcomes

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### Regge theory and the Pomeron

Regge Theory grew out of pre-QCD S-matrix theory of the 50's and 60's. Amplitudes are seen as unitary, Lorentz invariant functions of analytic momenta. (doesn't assume an underlying theory) At asymptotic energies  $s \gg -t$ , using partial wave analysis, the interaction is seen as an exchange of an *entire trajectory* of particles. The Pomeron is the dominant trajectory.  $\sigma_{tot} \sim s^{\alpha_{\mathcal{P}}(t)-1}$ This soft Pomeron has been used to fit to p-p total cross sections since '70s.



Authors	$\alpha_{\mathcal{P}}(0)$
Donnachie-Landshoff (1992)	1.0808
Cudell, Kang and Kim (1997)	$1.096^{+0.012}_{-0.009}$
Cudell <i>et al.</i> (2000)	$1.093\pm0.003$
COMPETE Collaboration (2002)	$1.0959\pm0.0021$
Luna and Menon (2003)	1.085 - 1.104
Menon and Silva (2013)	$1.0926\pm0.0016$

Promising tool to interpolate between perturbative and unperturbative regimes. What QCD has to say about the Pomeron?

## High energy limit of QCD

Modern colliders accelerate particles at a center-of-mass energy way larger than the characteristic transferred momentum. Semi-hard regimes:  $s \gg -t \gg \Lambda_{QCD}$ Despite the smallness of the coupling the convergence of the perturbative expansion is jeopardized by the appearance of log  $\left(\frac{s}{-t}\right) \gg 1$ .

The powers of logs grow with the approximation order.

Radiative corrections of order n to the partonic cross sections

$$d\hat{\sigma} \simeq \alpha_s^n \log^n\left(\frac{s}{-t}\right) A + \alpha_s^n \log^{n-1}\left(\frac{s}{-t}\right) B + \dots$$

The perturbative expansion

breaks down when  $\alpha_s \log s/t \simeq 1$ .  $\alpha_s^2 \log^2 s/t \simeq \alpha_s \log s/t$ The perturbative

expansion must be rearranged according to a new hierarchy.

• Leading logarithmic approximation (LL):

$$\alpha_s^n \log^n \left(\frac{s}{-t}\right), \ n = 1, 2, \dots$$

• Next-to-leading logarithmic approximation (NLL):

$$\alpha_s^n \log^{n-1}\left(\frac{s}{-t}\right), \ n=1,2,\ldots$$



Figure: Example of a diagram enhanced by one power of the large logarithms.

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## DGLAP resummation

BFKL effects compete with another limit of the perturbative expansion.

- The DGLAP approach resums terms proportional to  $\log \frac{t}{\lambda} \gg 1$ . It is the relevant approach when  $s \simeq t$ ,  $\lambda$  is the factorization scale.
- Even in semi-hard regimes DGLAP contributions can be important.



• Phase space DGLAP LO



 $y_1 \simeq y_2 \simeq \cdots \simeq y_n$ 



• Phase space BFKL LL

 $y_1 \gg y_2 \gg \cdots \gg y_n$  $\mathbf{k}_1 \simeq \mathbf{k}_2 \simeq \cdots \simeq \mathbf{k}_n$ 

#### MN and MT jets

## Looking at jets to see the Pomeron

Semi-*hard* regimes  $s \gg -t \gg \Lambda_{QCD}$  QCDp  $\rightarrow$  BFKL resummation  $\rightarrow$  power-like growth of cross-section in *s*.

## Mueller Navelet jets

 $p + p \rightarrow jet_1 + jet_2 + anything else$  Tagged jets far apart in rapidity. Preferred testing ground for BFKL dynamics. Phenomenology study at NLL available [arXiv:1010.0160] But.. at intermediate energies  $\alpha_s \log(s/-t) < 1$  large contaminations from other limits (DGLAP, finite order) tend to hide the Pomeron signature. Need tuning of renorm. scale (BLM) and/or asymmetric observables

$$\Delta Y = \log(\frac{s}{-t})$$

Mueller Tang jets  $p + p \rightarrow jet_1 + jet_2 + gap$ Dijets separated by a large rapidity gap can probe the finite-momentum structure of the Pomeron. Subleading compared to M-N ( $d\hat{\sigma} \propto \alpha_s^4$ ) but the lower background promises a cleaner recognition of the Pomeron contribution. cross-section lowered by proton remnant rescattering, I'll say few more words later...





- The absence of any additional emission over a large rapidity region suggests that the color-singlet exchange contributes substantially to the jet-gap-jet cross section.
- The BFKL predictions for these processes have been studied at LL accuracy and partially also at NLL order
- The last ingredients that have to be taken into account to complete the approximation order are the NLO impact factors



- Fixed rapidity gap |η| < 1, no charged particles and no photons or neutral hadrons with p<sub>T</sub> > 0.2 GeV.
- Dijet events with at least 2 hard jets with  $p_T^{jet} > 40 \text{ GeV}$  and  $|\eta^{jet}| > 1.5$

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## Previous fits and analysis



- Charged-particle multiplicity in the gap region between the tagged jets compared to PYTHIA and HERWIG predictions.
- HERWIG 6: include contributions from color singlet exchange (CSE), based on BFKL at LL.
- PYTHIA 6: inclusive dijets (tune  $Z2^*$ ), no-CSE.



considering the size of the NLL corrections.

[CMS-PAS-FSQ-12-001]

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## Previous fits and analysis



Left: LL & NLL BFKL at Tevatron [hep-ph/1012.3849].

• Ratio  $R = \frac{NLL^* BFKL}{NLOQCD}$  of jet-gap-jet events to inclusive dijet events as a function of  $p_t$  and the rapidity gap Y.



NLL\* BFKL calculations different implementations of the soft rescattering processes (EEI models), describe many features of the data, but none of the implementations is able to simultaneously describe all the features of the measurement. Ekstedt, Enberg, Ingelman, [1703.10919]

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# incorporating NLO jet vertex

A full NLL/O calculation is within reach. NLO MT impact factors recently calculated. Very complicated! (not in a factorizable form!)

But...only certain combinations of jet vertex and Green's function approximation orders contribute effectively to the NL order of the cross section.



• In general the cross section for these processes is given as a multiple convolution between the the jet vertices and the GGFs.

$$\begin{aligned} \frac{d\hat{\sigma}}{dJ_1 dJ_2 d^2 \mathbf{q}} &= \int d^2 \mathbf{k}_1 d^2 \mathbf{k'}_1 d^2 \mathbf{k}_2 d^2 \mathbf{k'}_2 V_a(\mathbf{k}_1, \mathbf{k}_2, J_1, \mathbf{q}) \times \\ & \quad G(\mathbf{k}_1, \mathbf{k'}_1, \mathbf{q}, Y) G(\mathbf{k}_2, \mathbf{k'}_2, \mathbf{q}, Y) V_b(\mathbf{k'}_1, \mathbf{k'}_2, J_2, \mathbf{q}), \qquad J = \{\mathbf{k}_J, x_J\}. \end{aligned}$$

• The explicit form of the jet vertex and the Green function depends on the approximation level.

NLL vertex is known [Hentschinski, Madrigal Martinez, Murdaca, Sabio Vera].

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## Pomeron exchange amplitude

With the leading order vertex independent from the loop momenta the partonic cross section can be expressed as the square modulus of an amplitude that is nothing more than the GGF integrated in its transverse momenta.



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$$A(Y,q) = h_a^0 h_b^0 \int d^2 k d^2 k' G(\mathbf{k},\mathbf{k}',q,Y), \quad \frac{d\hat{\sigma}}{dqdY} = \frac{1}{16\pi} |A(Y,q)|^2$$

The integration over the transverse momenta greatly simplifies the calculation of the Fourier transform of the eigenfunctions canceling the Lipatov term contribution. The Pomeron exchange amplitude is given by the simple expression

$$A(Y,q) = h_a^0 h_b^0 \sum_{n=-\inf}^{+\inf} \int_{-\inf}^{+\inf} d\nu e^{Y\omega_{n\nu}} R_{n\nu}\left(\frac{4}{\mathbf{q}^2}\right)$$

This can be easily extended to include the GGF at NLL  $\omega^{LL}(n,\nu) \rightarrow \omega^{NLL}(n,\nu)$ .

The decision to keep just the pure NL contribution brings some simplification

$$\frac{\frac{d\hat{\sigma}}{dJ_{1}dJ_{2}d^{2}\mathbf{q}} = \int d^{2}k_{1}d^{2}k_{2}V^{1}(\mathbf{k}_{1},\mathbf{k}_{2},\mathbf{q};J_{1}) \times \underbrace{\int d^{2}k'_{1}G(\mathbf{k}_{1},\mathbf{k}'_{1},\mathbf{q},Y)}_{\overline{G}(\mathbf{k}_{1},\mathbf{q},Y)} \underbrace{\int d^{2}k'_{2}G(\mathbf{k}_{2},\mathbf{k}'_{2},\mathbf{q},Y)}_{\overline{G}(\mathbf{k}_{2},\mathbf{q},Y)}V^{0}(J_{2},\mathbf{q})$$



• Large increase in computation time due to the high-dimensional multiple integration. The average  $\bar{G}$  is [Bartels, Braun, Colferai, Vacca]

$$\bar{G}(x_1x_2, q, \Delta\theta, k) = \propto \left[k^{*\bar{h}-2} k'^{*h-2} {}_{2}F_1(1-h, 2-h; 2, -\frac{k}{k'}) {}_{2}F_1(1-\bar{h}, 2-\bar{h}; 2, -\frac{k'^*}{k_1^*}) + \{1 \to 2\}\right]$$

• The momentum dependence of the eigenfunction is expressed through Gauss hypergeometric functions.

Hypergeometric functions are hard to compute.  ${}_{2}F_{1}(a, b; c, z) = \sum_{n} \frac{(a)_{n}(b)_{n}}{(c)_{n}} \frac{z^{n}}{n!}$ Michel and Stoitsov [arXiv:0708.0116, math-ph],Doornik, [Math. Comp. 84 (2015), 1813-1833]. Conformal spins n > 10 require more then quadrupole precision. *[F. Johansson, Arb: a C library for ball arithmetic]* 

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### Numerical analysis

None of the cited codes for the hypergeometric function work for  $\Im(a), \Im(b) \gtrsim 5$ .

$$\bar{G}(x_1x_2, q, \Delta\theta, \frac{k}{k'}) \propto \sum_{m}^{m \text{ even}} \int d\nu \left[ k^{*\bar{h}-2} k'^{h-2} {}_2F_1\left(1-h, 2-h, 2; -\frac{k}{k'}\right) {}_2F_1\left(1-\bar{h}, 2-\bar{h}, 2; -\frac{k'^*}{k^*}\right) + \{1 \leftrightarrow 2\} \right] \right]$$

- Integrand is highly oscillatory and slowly falling with  $\nu$ . Need to integrate up to very large values of  $\nu \ (\gtrsim 10)$ .  $h = \frac{1+n}{2} + i\nu$
- Bending the contour helps but for some z the two terms rise with ν and convergence is met only after the sum is taken.



## GGF LL + NLO jet vertex



## Details of NLO impact factor

$$\begin{split} \frac{d\hat{V}^{(1)}(\mathbf{x}, k, l_1, l_2; \mathbf{x}_j, k_j; M_{\mathbf{X}, \max}, \mathbf{s}_0)}{dJ} &= \\ &= v_q^{(0)} \frac{\alpha \mathbf{s}}{2\pi} \left[ S_j^{(2)}(\mathbf{k}, \mathbf{x}) \cdot \left[ -\frac{\beta_0}{4} \left[ \left\{ \ln \left( \frac{l_1^2}{\mu^2} \right) + \ln \left( \frac{(l_1 - k)^2}{\mu^2} \right) + \{1 \leftrightarrow 2\} \right\} - \frac{20}{3} \right] - 8C_f \\ &+ \frac{C_3}{2} \left[ \left\{ \frac{3}{2k^2} \left\{ l_1^2 \ln \left( \frac{(l_1 - k)^2}{l_1^2} \right) + (l_1 - k)^2 \ln \left( \frac{l_1^2}{(l_1 - k)^2} \right) - 4|l_1||l_1 - k|\phi_1 \sin \phi_1 \right\} \right. \\ &- \frac{3}{2} \left[ \ln \left( \frac{l_1^2}{k^2} \right) + \ln \left( \frac{(l_1 - k)^2}{k^2} \right) \right] - \ln \left( \frac{l_1^2}{k^2} \right) \ln \left( \frac{(l_1 - k)^2}{s_0} \right) - \ln \left( \frac{(l_1 - k)^2}{k^2} \right) \ln \left( \frac{l_1^2}{s_0} \right) - 2\phi_1^2 + \{1 \leftrightarrow 2\} \right\} + 2\pi^2 + \frac{14}{3} \right] \right] \\ &+ \int_{20}^{1} dz \left\{ \ln \frac{\lambda^2}{\mu_F^2} S_j^{(2)}(\mathbf{k}, z\mathbf{x}) \left[ P_{qq}(z) + \frac{C_g^2}{c_f^2} P_{gq}(z) \right] + \left[ (1 - z) \left[ 1 - \frac{2}{z} \frac{C_g^2}{c_f^2} \right] + 2(1 + z^2) \left( \frac{\ln(1 - z)}{1 - z} \right)_+ \right] S_j^{(2)}(\mathbf{k}, z\mathbf{x}) + 4S_j^{(2)}(\mathbf{k}, x) \right\} \\ &+ \int_0^1 dz \int \frac{d^2q}{\pi} \left[ P_{qq}(z) \Theta \left( \frac{|q|}{1 - z} - \lambda \right) \times \frac{k^2}{q^2(p - zk)^2} S_j^{(3)}(p, q, (1 - z)\mathbf{x}, \mathbf{x}) \right. \\ &+ S_j^{(3)}(p, q, z\mathbf{x}, \mathbf{x}) P_{gq}(z) \times \left\{ \frac{C_g}{c_f} \left[ J_1(q, \mathbf{k}, l_1, z) + J_1(q, \mathbf{k}, l_2, z) \right] + \frac{C_g^2}{c_f^2} J_2(q, \mathbf{k}, l_1, l_2) \Theta(p^2 - \lambda^2) \right\} \right] \right] \end{split}$$

• The residual finite part of the absorbed IF divergences is colored in *skyblue*.

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### Details of NLO jet vertex



#### Conclusions

## Conclusions

- QCD predictions even in the perturbative regimes are not fully understood.
- BFKL is the relevant framework to study processes in semi-hard regimes.
- NLL corrections are large and must be taken into account.
- The search for BFKL signature via Mueller-Navelet jet process is contaminated by important DGLAP contributions.
- Look for more exclusive observables that show a pronounced DGLAP suppression.
- BFKL predictions for Mueller-Tang observables are in fair agreement with the measurement.
- An even better agreement could be found completing a phenomenological analysis with the full NLL order.

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#### Outcomes

#### What has been done ..

- Implementation of Gauss hypergeometric function is finally general enough to be used for the non-forward BFKL eigenfunction.
- Code up and running for generation of gluon-Green function grids to interpolate with the NLL impact factors.

#### What's next..

- $\bullet~{\sf Run}~{\sf code}~{\sf on}~{\sf HPC}$  at KU (40 cores at 2.4 GHz with 192 GB 2666 MHz DDR4 memory) .
- Look at the NLL vertex corrections and play with the observable definition to identify physical region where cross-section is "more factorizable".
- Include full corrections into MC generator with *ad hoc* parametrization.
- Where else can be useful the momentum space BFKL eigenfunction?

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## Numerical analysis

Peculiar characteristics of the NLO the jet vertex.

- Non trivial dependence from the reggeon momenta  $\rightarrow$  "connects" the two GGFs over the cut.
- Up to two partons emitted by the same vertex  $\rightarrow$  dependence from the jet reconstruction algorithm. (1) The two partons form the same jet or (2) one of the two has energy lower than the calorimeter threshold and so it is not detected.
- The parton emission below threshold in the prohibited region alter the alignment between the forward and the backward jet. Jets not back-to-back anymore  $\hat{\sigma}(q, Y) \rightarrow \hat{\sigma}(k_{J_1}, k_{J_2}, \theta_{J_2, J_2}, Y)$ .
- Calculation of the partonic cross section.
  (1) G
   as a grid of its parameters {k<sub>i</sub>, q<sub>j</sub>, θ<sub>l</sub>, Y<sub>m</sub>}. It involves a numerical integration over ν and a sum over n for each set of the parameters.
  (2) Partonic cross section as the interpolation of G
   grids

 $q_{i+1}$ 

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and the NLO vertex.

 $\frac{\hat{\sigma}(k_{J_1},k_{J_2},\theta_{J_1},J_2,Y)}{\frac{dk_JdY}{dk_JdY}} \propto \sum V(k_{1_i},k_{2_j},\theta_{1_n},\theta_{2_m},J)\bar{G}(k_{1_i},q_r,\theta_{1_n},Y_l)\bar{G}(k_{2_j},q_r,\theta_{2_m},Y_l)$ 

#### Numerical analysis

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The non-forward BFKL eigenfunction at LL is given in terms of Gauss hypergeometric functions.

$$\begin{split} \tilde{E}_{n\nu}(k,k') &\propto \left[ k^{*\bar{h}-2} k'^{*h-2} \,_{2}F_{1}(1-h,2-h;2,-\frac{k}{k'}) \,_{2}F_{1}(1-\bar{h},2-\bar{h};2,-\frac{k'^{*}}{k_{1}^{*}}) + \{1 \rightarrow 2\} \right] \\ &= q-k, \ h = \left(\frac{1+n}{2} + i\nu\right), \ \bar{h} = \left(\frac{1-n}{2} + i\nu\right) \end{split}$$

Hypergeometric functions are hard to compute.  $_2F_1(a, b; c, z) = \sum_n \frac{(a)_n(b)_n}{(c)_n} \frac{z^n}{n!}$ What about the rest of the complex plane?

Several transformations connect the various regions of the conplex plane:  $z, \frac{1}{z}, \frac{1}{z-z_0}, \frac{z-1}{z}$ . Special care must be taken for  $b-a \simeq \mathbb{Z}^-$  and near  $\overline{\mathbb{E}} = \{0, 1, \inf, \exp(\pm i\pi/3)\}$ . Michel and Stoitsov [arXiv:0708.0116, math-ph], Doornik, [Math. Comp. 84 (2015), 1813-1833].



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## LL approximation: Non forward gluon Green function

The GGF is given by the Mellin transform of the function  $f_{\omega}$  which is the solution of the BFKL equation. The solution of the non forward BFKL equation is more naturally expressed in the impact parameter space.

$$G(\mathbf{k}, \mathbf{k}', \mathbf{q}, Y) = \int_{-i \inf}^{+i \inf} \frac{d\omega}{2\pi i} e^{Y\omega} f_{\omega}(\mathbf{k}, \mathbf{k}', \mathbf{q})$$

$$f_{\omega}(\rho_{1}, \rho_{2}, \rho_{1}', \rho_{2}') = \frac{1}{(2\pi)^{6}} \sum_{n=-\inf}^{+\inf} \int_{-\inf}^{+inf} d\nu \frac{R_{n\nu}}{\omega - \omega(n, \nu)} E_{n\nu}^{*}(\rho_{1}', \rho_{2}') E_{n\nu}(\rho_{1}, \rho_{2})$$

$$E_{n\nu}(\rho_{1}, \rho_{2}) = \underbrace{\left(\frac{\rho_{1} - \rho_{2}}{\rho_{1}\rho_{2}}\right)^{h} \left(\frac{\rho_{1}^{*} - \rho_{2}^{*}}{\rho_{1}^{*}\rho_{2}^{*}}\right)^{\bar{h}}}_{\text{Lipatov term}} \underbrace{-\left(\frac{1}{\rho_{2}}\right)^{h} \left(\frac{1}{\rho_{2}^{*}}\right)^{\bar{h}} - \left(\frac{-1}{\rho_{1}}\right)^{h} \left(\frac{-1}{\rho_{1}^{*}}\right)^{\bar{h}}}_{\text{Mueller-Tang correction}}$$

 $E_{n\nu}$  are the eigenfunctions in the impact parameter space.

The GGF in momentum space is recovered applying a Fourier transformation to the eigenfunctions.

$$\tilde{E}_{n\nu}(\mathbf{k},\mathbf{q}) = \int \frac{d^2 r_1 d^2 r_2}{(2\pi)^4} E_{n\nu}(\rho_1,\rho_2) e^{i(\mathbf{k}\cdot\mathbf{r}_1 + (\mathbf{q}-\mathbf{k})\cdot\mathbf{r}_2)}$$

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