

BFKL resummation in inclusive processes

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based on

F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P., PLB 777 (2018) 141

A.D. Bolognino, F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, A.P., in preparation

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DIPARTIMENTO DI
FISICA



Istituto Nazionale di Fisica Nucleare
GRUPPO COLLEGATO DI COSENZA

Outline

- 1 Introductory remarks
 - Motivation
 - BFKL factorization
 - Search for BFKL dynamics in inclusive processes
- 2 Two new probes: inclusive heavy-quark pair photoproduction ...
 - Theoretical setup
 - Observables and numerical analysis
- 3 ... and inclusive heavy-quark pair hadroproduction
 - Theoretical setup
 - Observables and numerical analysis
- 4 Conclusions and Outlook

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Motivation

- Semihard collision processes, featuring the **scale hierarchy**

$$s \gg Q^2 \gg \Lambda_{\text{QCD}}^2, \quad Q \text{ a hard scale ,}$$

represent a challenge for perturbative QCD,

$$\alpha_s(Q) \log s \sim 1 \implies \text{all-order resummation needed!}$$

- The **Balitsky-Fadin-Kuraev-Lipatov (BFKL)** approach provides a general framework for this resummation: it predicts a peculiar behavior of amplitudes at high energies and is expected to precede the onset of saturation physics.
- However, experimental evidences of the BFKL dynamics are not conclusive, thus motivating the proposal of new probes.
- Here, we suggest a new one in the inclusive (photo/hadro-)production of two rapidity-separated heavy quarks.

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BFKL factorization

Scattering $A + B \rightarrow A' + B'$ in the **Regge kinematical region** $s \rightarrow \infty$, t fixed

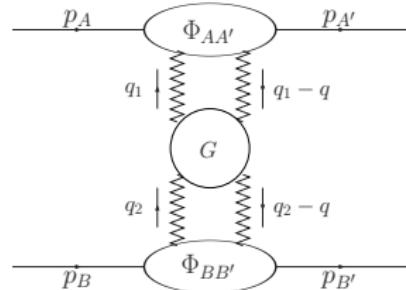
⇒ BFKL factorization for $\text{Im}_s \mathcal{A}$:

convolution of a **Green's function** with the **impact factors** of the colliding particles.

Valid both in

LIA (resummation of all terms $(\alpha_s \ln s)^n$)

NLA (resummation of all terms $\alpha_s (\alpha_s \ln s)^n$).



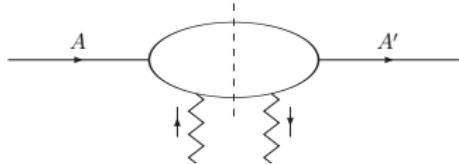
$$\begin{aligned} \text{Im}_s \mathcal{A} = & \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^2} \Phi_{AA'}(\vec{q}_1, \vec{q}; s_0) \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^2} \Phi_{BB'}(-\vec{q}_2, -\vec{q}; s_0) \\ & \times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^\omega G_\omega(\vec{q}_1, \vec{q}_2) \end{aligned}$$

The **Green's function** is process-independent and is determined through the **BFKL equation**.

[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}\vec{q} K(\vec{q}_1, \vec{q}) G_\omega(\vec{q}, \vec{q}_1)$$

Impact factors are process-dependent;
only very few of them known in the NLA ...



- $A = A' = \text{quark}, \quad A = A' = \text{gluon}$ [V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)]
[M. Ciafaloni and G. Rodrigo (2000)]
- $A = \gamma_L^*, A' = V_L$, with $V = \rho^0, \omega, \phi$ (forward) [D.Yu. Ivanov, M.I. Kotsky, A.P. (2004)]
- $A = A' = \gamma^*$ (forward)
 - [J. Bartels, S. Gieseke, C.F. Qiao (2001)]
 - [J. Bartels, S. Gieseke, A. Kyrieleis (2002)]
 - [J. Bartels, D. Colferai, S. Gieseke, A. Kyrieleis (2002)]
 - [V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)]
 - [J. Bartels, A. Kyrieleis (2004)]
 - [I. Balitsky, G.A. Chirilli (2013)]
 - [G.A. Chirilli, Yu.V. Kovchegov (2014)]

... so that only a very limited number of predictions can be built for **exclusive** processes or **total cross sections**, even hardly testable in present colliders.

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Inclusive processes

A lot more possibilities open for **inclusive** processes, with jets or identified particles in the final state, produced in the **fragmentation** regions or in the **central** one.

Straightforward adaptation of the BFKL factorization: just restrict the summation over final states entering the definition of **impact factors** or **Green's function**.

- Mueller-Navelet jet production

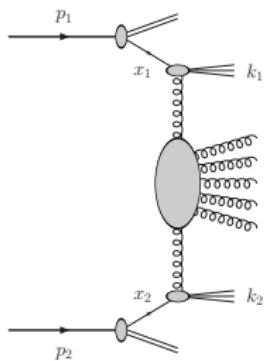
- NLO jet vertex

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P., A. Perri (2011)]
[D.Yu. Ivanov, A.P. (2012)] (small-cone approximation)
[D. Colferai, A. Niccolai (2015)]

- azimuthal correlations (full NLA) and other

[B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P. (2014)]
[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (2015)]

- compatible with CMS (7 TeV)



Inclusive processes

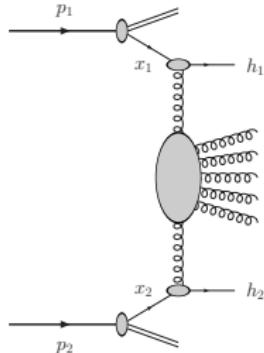
- hadron-hadron production

- NLO hadron vertex

[D.Yu. Ivanov, A.P. (2012)]

- azimuthal correlations (full NLA) and other

[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (2016,2017)]



- hadron-jet production (full NLA)

[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2018)]

- three / four jet production (partial NLA)

[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2016)]

[F. Caporale, F.G. Celiberto, G. Chachamis, A. Sabio Vera (2016)]

[F. Caporale, F.G. Celiberto, G. Chachamis, D.G. Gomez, A. Sabio Vera (2016,2017)]

- J/ Ψ - jet production (partial NLA)

[R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

- Drell-Yan pair - jet (partial NLA)

[K. Golec-Biernat, L. Motyka, T. Stebel (2018)]

- Higgs - jet (partial NLA)

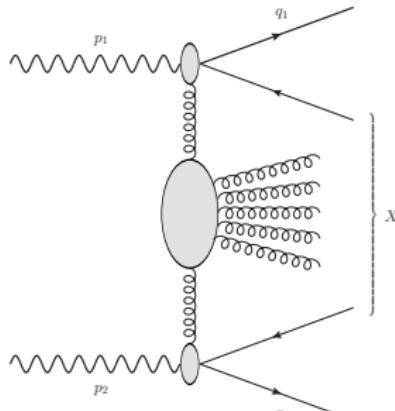
[M.M.A. Mohammed's talk]

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Heavy-quark pair photoproduction: theoretical setup

$$\gamma(p_1) + \gamma(p_2) \longrightarrow Q(q_1) + X + Q(q_2)$$



LO $Q\bar{Q}$ vertex:

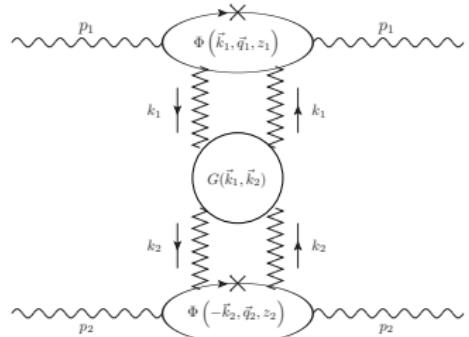
$$\frac{d\Phi}{d^2 q dz} = \frac{\alpha \alpha_s e_Q^2}{\pi} \sqrt{N_c^2 - 1} \left[m^2 R^2 + \vec{P}^2 (z^2 + \bar{z}^2) \right]$$

$$R = \frac{1}{m^2 + \vec{q}^2} - \frac{1}{m^2 + (\vec{q} - \vec{k})^2}$$

$$\vec{P} = \frac{\vec{q}}{m^2 + \vec{q}^2} + \frac{\vec{k} - \vec{q}}{m^2 + (\vec{q} - \vec{k})^2}$$

[I.F. Ginzburg, D.Yu. Ivanov (1996)]

[E.A. Kuraev, A. Schiller, V.G. Serbo (1983)]



Heavy-quark pair photoproduction: kinematics

$$q_{1,2} = z_{1,2} p_{1,2} + \frac{m^2 + \vec{q}_{1,2}^2}{z_{1,2} W^2} p_{2,1} + q_{1,2,\perp}, \quad \bar{z}_i \equiv 1 - z_i$$

$$W^2 = (p_1 + p_2)^2 = 2p_1 p_2 = 4E_{\gamma_1} E_{\gamma_2}$$

$$y_1 = \ln \frac{2E_{\gamma_1} z_1}{\sqrt{m^2 + \vec{q}_1^2}}, \quad y_2 = -\ln \frac{2E_{\gamma_2} z_2}{\sqrt{m^2 + \vec{q}_2^2}}$$

$$\Delta Y \equiv y_1 - y_2 = \ln \frac{W^2 z_1 z_2}{\sqrt{(m^2 + \vec{q}_1^2)(m^2 + \vec{q}_2^2)}}$$

Semihard kinematics:

$$\frac{W^2}{\sqrt{(m^2 + \vec{q}_1^2)(m^2 + \vec{q}_2^2)}} = \frac{e^{\Delta Y}}{z_1 z_2} \gg 1$$

$$dz_1 dz_2 = \frac{e^{y_1 - y_2} \sqrt{m^2 + \vec{q}_1^2} \sqrt{m^2 + \vec{q}_2^2}}{W^2} dy_1 dy_2$$

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Heavy-quark pair photoproduction: observables

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{q}_1| d|\vec{q}_2| d\varphi_1 d\varphi_2} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \mathcal{C}_n \right], \quad \varphi = \varphi_1 - \varphi_2 - \pi$$

$$\begin{aligned} \mathcal{C}_n &= \frac{q_1 q_2 \sqrt{m_1^2 + q_1^2} \sqrt{m_2^2 + q_2^2}}{W^2} e^{\Delta Y} \\ &\times \int_{-\infty}^{+\infty} d\nu \left(\frac{W^2}{s_0} \right)^{\bar{\alpha}_s(\mu_R) \chi(n, \nu) + \bar{\alpha}_s^2(\mu_R) \left(\bar{\chi}(n, \nu) + \frac{\beta_0}{8N_c} \chi(n, \nu) \left(-\chi(n, \nu) + \frac{10}{3} + 2 \ln \frac{\mu_R^2}{\sqrt{s_1 s_2}} \right) \right)} \\ &\quad \times \alpha_s^2(\mu_R) c_1(n, \nu, \vec{q}_1^2, z_1) c_2(n, \nu, \vec{q}_2^2, z_2) \\ &\times \left\{ 1 + \bar{\alpha}_s(\mu_R) \left(\frac{\bar{c}_1^{(1)}}{c_1} + \frac{\bar{c}_2^{(1)}}{c_2} \right) + \bar{\alpha}_s(\mu_R) \frac{\beta_0}{2N_c} \left(\frac{5}{3} + \ln \frac{\mu_R^2}{s_1 s_2} + f(\nu) \right) \right. \\ &\quad \left. + \bar{\alpha}_s^2(\mu_R) \ln \left(\frac{W^2}{s_0} \right) \frac{\beta_0}{4N_c} \chi(n, \nu) f(\nu) \right\} \end{aligned}$$

LO kernel: $\bar{\alpha}_s \chi(n, \nu)$

$$\bar{\alpha}_s \equiv \frac{N_c \alpha_s}{\pi}, \quad \chi(n, \nu) = 2\psi(1) - \psi\left(\frac{n}{2} + \frac{1}{2} + i\nu\right) - \psi\left(\frac{n}{2} + \frac{1}{2} - i\nu\right)$$

NLO kernel contribution: $\bar{\alpha}_s^2$ terms in the exponent

Heavy-quark pair photoproduction: observables

LO impact factors, projected onto the LO kernel eigenfunctions:

$$c_1(n, \nu, \vec{q}_1^2, z_1) = \frac{\alpha e_{Q_1}^2}{\pi} \left[m_1^2 c_{R^2}(n, \nu, \vec{q}_1^2) + \left(z_1^2 + \bar{z}_1^2 \right) c_{\bar{P}^2}(n, \nu, \vec{q}_1^2) \right]$$

$$c_2(n, \nu, \vec{q}_2^2, z_2) = \frac{\alpha e_{Q_2}^2}{\pi} \left[m_2^2 c_{R^2}^*(n, \nu, \vec{q}_2^2) + \left(z_2^2 + \bar{z}_2^2 \right) c_{\bar{P}^2}^*(n, \nu, \vec{q}_2^2) \right]$$

$$e^{in\varphi} c_f(n, \nu, \vec{q}^2, z) \equiv \int \frac{d^2 k}{\pi \sqrt{2}} \left(k^2 \right)^{i\nu - 3/2} e^{in\vartheta} f(\vec{q}, \vec{k}, z), \quad \cos \varphi = \frac{q_x}{|\vec{q}|}, \quad \cos \vartheta = \frac{k_x}{|\vec{k}|}$$

$$i \frac{d}{d\nu} \ln \frac{c_1}{c_2} = 2 \left[f(\nu) - \ln (\sqrt{s_1 s_2}) \right], \quad s_i \equiv m_i^2 + \vec{q}_i^2$$

NLO impact factors (only universal part), e.g.

$$\frac{\bar{c}_1^{(1)}}{c_1} + \frac{\bar{c}_2^{(1)}}{c_2} = \chi(n, \nu) \ln \frac{s_0}{\sqrt{s_1 s_2}}$$

s_0 (arbitrary within NLA) chosen as $\sqrt{s_1 s_2}$

Heavy-quark pair photoproduction: observables

From **photon-photon** to e^+e^- collisions:

$$d\sigma_{e^+e^-} = dn_1 dn_2 d\sigma_{\gamma\gamma}$$

$$dn = \frac{\alpha}{\pi} \frac{dx}{x} \left[1 - x + \frac{x^2}{2} - \frac{m_e^2(1-x)x^2}{\vec{q}^2 + m_e^2 x^2} \right] \frac{d\vec{q}^2}{\vec{q}^2 + m_e^2 x^2}$$

$x = \frac{\omega}{E_e}$ photon energy fraction, \vec{q} photon transverse momentum

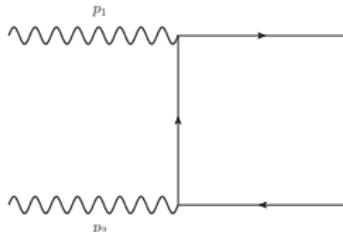
Emission angle $\theta \approx \frac{q_\perp}{E_e(1-x)} \leq \theta_0 \rightarrow (\vec{q})_{\max} = E_e(1-x)\theta_0$:

$$dn = \frac{\alpha}{\pi} \frac{dx}{x} \left[\left(1 - x + \frac{x^2}{2} \right) \ln \left(\frac{E_e^2 \theta_0^2 (1-x)^2 + m_e^2 x^2}{m_e^2 x^2} \right) - (1-x) \right]$$

(neglected some terms $\mathcal{O}(m_e^2/E_e^2)$)

$$\begin{aligned} \frac{d\sigma_{e^+e^-}}{d(\Delta Y)} &= \int dq_1 \int dq_2 \int_{-y_{\max}^{(1)}}^{y_{\max}^{(1)}} dy_1 \int_{-y_{\max}^{(2)}}^{y_{\max}^{(2)}} dy_2 \delta(y_1 - y_2 - \Delta Y) \\ &\times \int_{e^{-\left(y_{\max}^{(1)} - y_1\right)}}^1 \frac{dn_1}{dx_1} dx_1 \int_{e^{-\left(y_{\max}^{(2)} + y_2\right)}}^1 \frac{dn_2}{dx_2} dx_2 d\sigma_{\gamma\gamma}, \quad y_{\max}^{(i)} = \ln \sqrt{\frac{s}{s_i}} \end{aligned}$$

Heavy-quark pair photoproduction: observables



Related process: $\gamma + \gamma \longrightarrow Q + \bar{Q}$

In LO QED

$$\begin{aligned} \frac{d\sigma_{ee}}{d(\Delta Y)} &= \int_0^{\frac{s_{ee}}{2(1+\cosh(\Delta Y))}-m^2} \frac{dq^2}{(m^2+q^2)^2} \frac{2\pi\alpha^2 e_q^4 N_c}{(1+\cosh(\Delta Y))^2} \\ &\times \left[\frac{\cosh(\Delta Y)}{2} + \frac{m^2}{m^2+q^2} - \left(\frac{m^2}{m^2+q^2} \right)^2 \right] \\ &\times \left(\frac{\alpha}{\pi} \right)^2 \left[f(y) \left(\ln \left(\frac{\Lambda^2}{m_e^2 y} \right) - 1 \right)^2 - \frac{1}{3} \left(\ln \frac{1}{y} \right)^3 \right] \end{aligned}$$

$$y = \frac{W^2}{s_{ee}} = \frac{2(1+\cosh(\Delta Y))(m^2+q^2)}{s_{ee}}$$

$$f(y) = \left(1 + \frac{y}{2} \right)^2 \ln \frac{1}{y} - \frac{1}{2} (1-y)(3+y), \quad \Lambda \simeq m^2$$

Heavy-quark pair photoproduction: numerical results

charm quark ($m=1.2$ GeV)

$q_{\min} < q_{1,2} < q_{\max}$, $q_{\min} = 0, 1, 3$ GeV, $q_{\max} = 10$ GeV
 $\theta_0 = 0.0835$ (OPAL)

Table: C_0 [pb], $q_{\min} = 0$, $\sqrt{s} = 200$ GeV; $C = \mu_R^2/(s_1 s_2)$

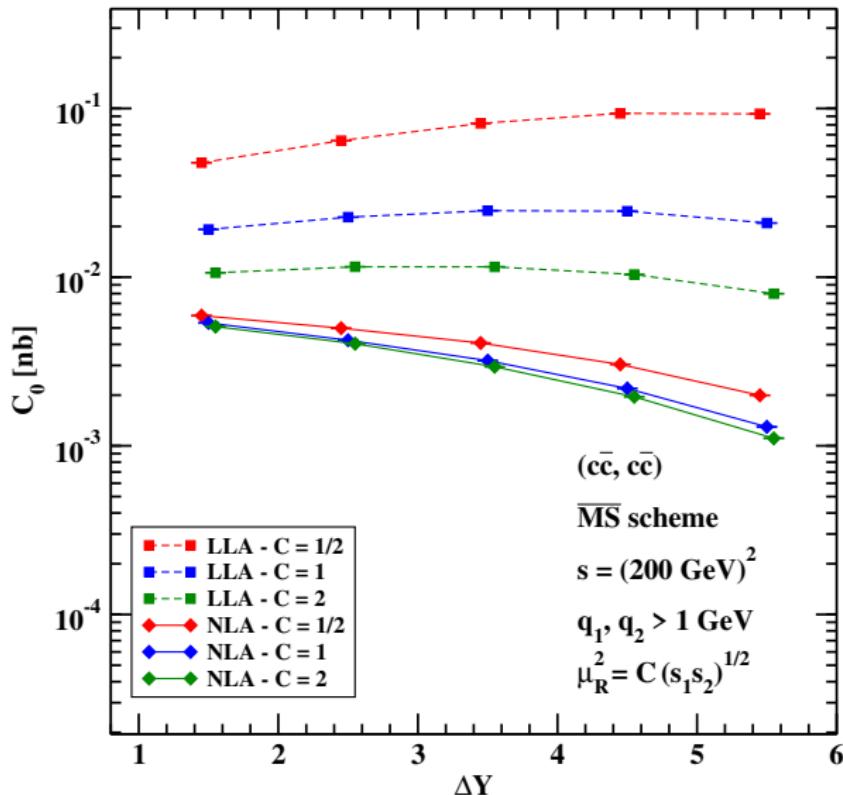
ΔY	Box $q\bar{q}$	LLA $C = 1/2$	LLA $C = 1$	LLA $C = 2$	NLA $C = 1/2$	NLA $C = 1$	NLA $C = 2$
1.5	98.26	415.0(1.3)	65.24(31)	28.94(14)	16.96(10)	11.237(73)	10.289(74)
2.5	42.73	723.7(2.1)	88.64(36)	34.58(17)	17.580(91)	9.581(57)	8.504(56)
3.5	14.077	1203.4(3.4)	113.33(43)	39.01(16)	18.522(92)	7.989(43)	6.637(36)
4.5	3.9497	1851.6(5.0)	133.64(52)	40.42(19)	18.412(90)	6.210(31)	4.893(25)
5.5	0.9862	2559.4(7.1)	140.23(55)	37.18(17)	16.971(83)	4.329(21)	3.138(15)

Table: C_0 [pb], $q_{\min} = 0$, $\sqrt{s} = 3$ TeV; $C = \mu_R^2/(s_1 s_2)$

ΔY	Box $q\bar{q}$	LLA $C = 1/2$	LLA $C = 1$	LLA $C = 2$	NLA $C = 1/2$	NLA $C = 1$	NLA $C = 2$
1.5	280.98	$10.893(49) \cdot 10^3$	530.8(2.4)	195.54(88)	99.57(89)	58.34(58)	52.17(58)
3.5	48.93	$54.84(14) \cdot 10^3$	1568.9(7.6)	439.4(2.1)	184.5(1.1)	65.22(50)	54.38(47)
5.5	4.9819	$254.88(57) \cdot 10^3$	4409(19)	930.6(4.2)	380.2(1.8)	75.83(53)	55.22(36)
7.5	0.4318	$1041.7(2.1) \cdot 10^3$	$10.921(44) \cdot 10^3$	1743.1(8.3)	756.3(3.5)	81.94(44)	51.48(27)
9.5	0.0323	$3429.0(7.8) \cdot 10^3$	$21.530(80) \cdot 10^3$	2618(12)	1267.0(6.1)	72.73(36)	38.86(19)
10.5	0.0081	$5468(14) \cdot 10^3$	$26.23(10) \cdot 10^3$	2761(12)	1443.3(7.2)	59.97(30)	29.21(14)

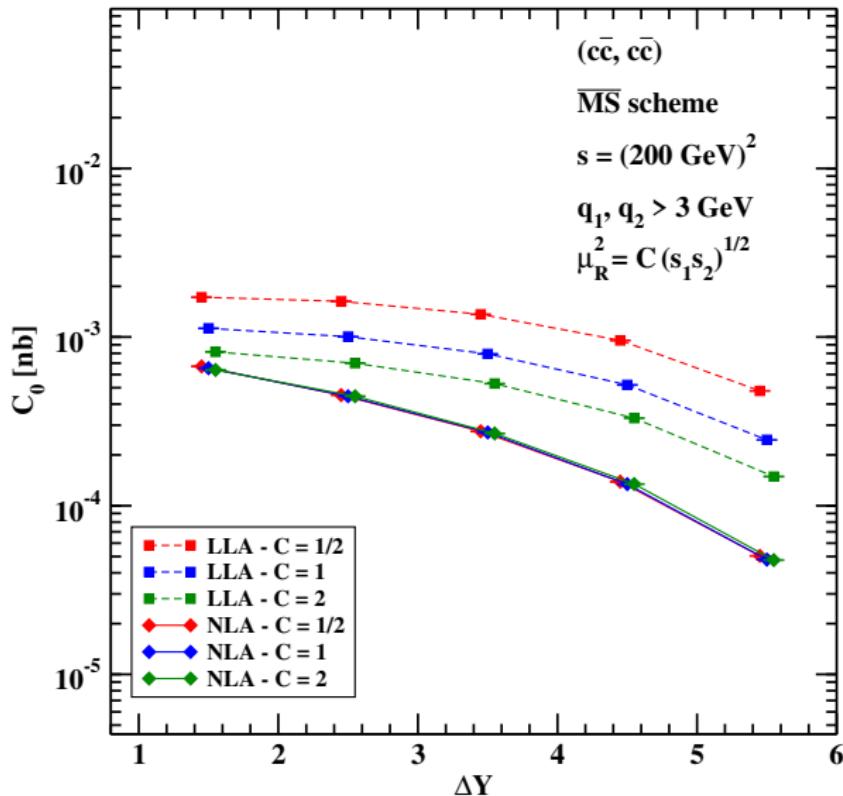
Heavy-quark pair photoproduction: numerical results

C_0 ; $q_{\min} = 1 \text{ GeV}$, $\sqrt{s} = 200 \text{ GeV}$



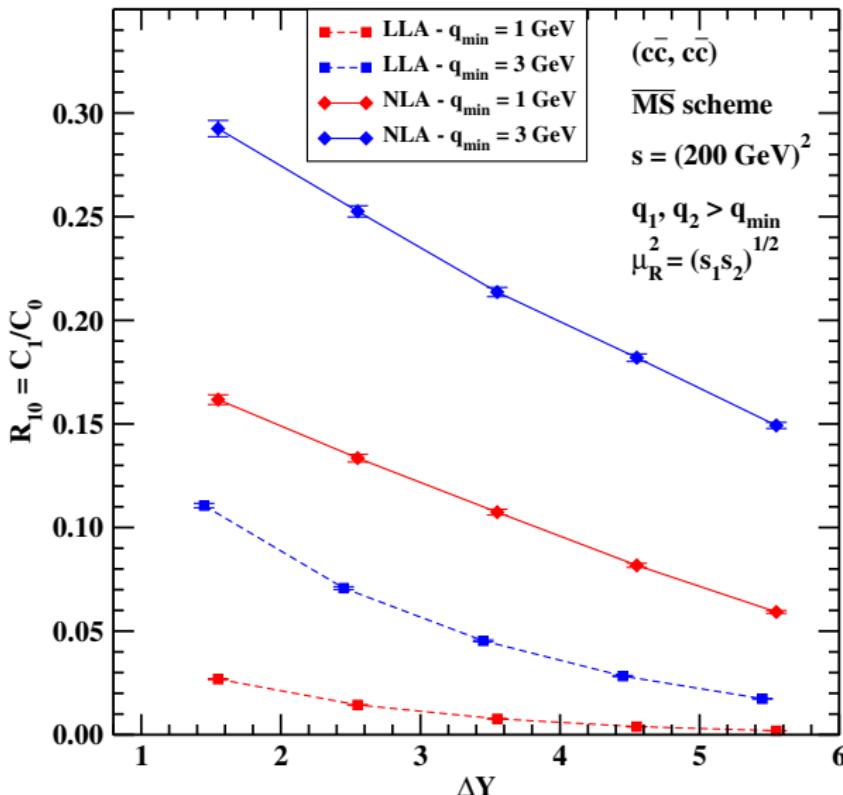
Heavy-quark pair photoproduction: numerical results

C_0 ; $q_{\min} = 3 \text{ GeV}$, $\sqrt{s} = 200 \text{ GeV}$



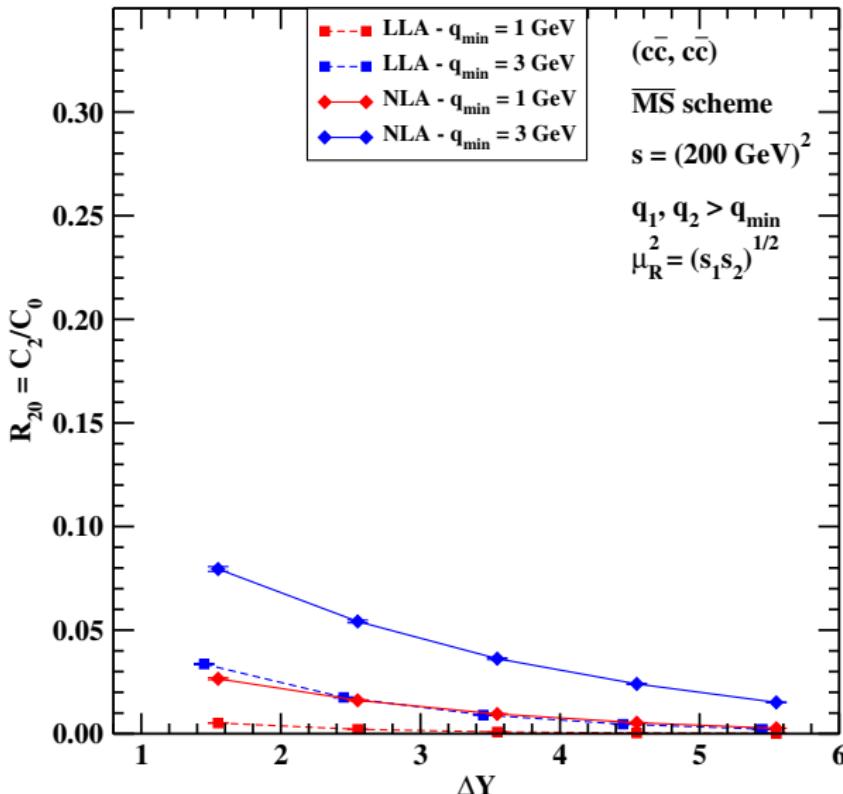
Heavy-quark pair photoproduction: numerical results

C_1/C_0 ; $q_{\min} = 1$ and 3 GeV, $\sqrt{s} = 200$ GeV



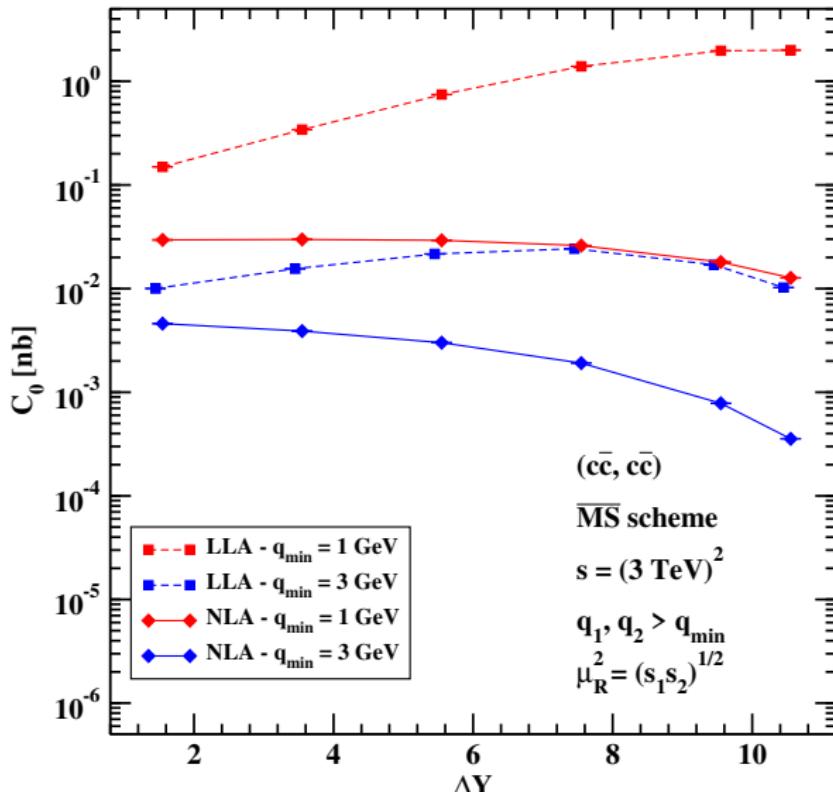
Heavy-quark pair photoproduction: numerical results

C_2/C_0 ; $q_{\min} = 1$ and 3 GeV, $\sqrt{s} = 200$ GeV

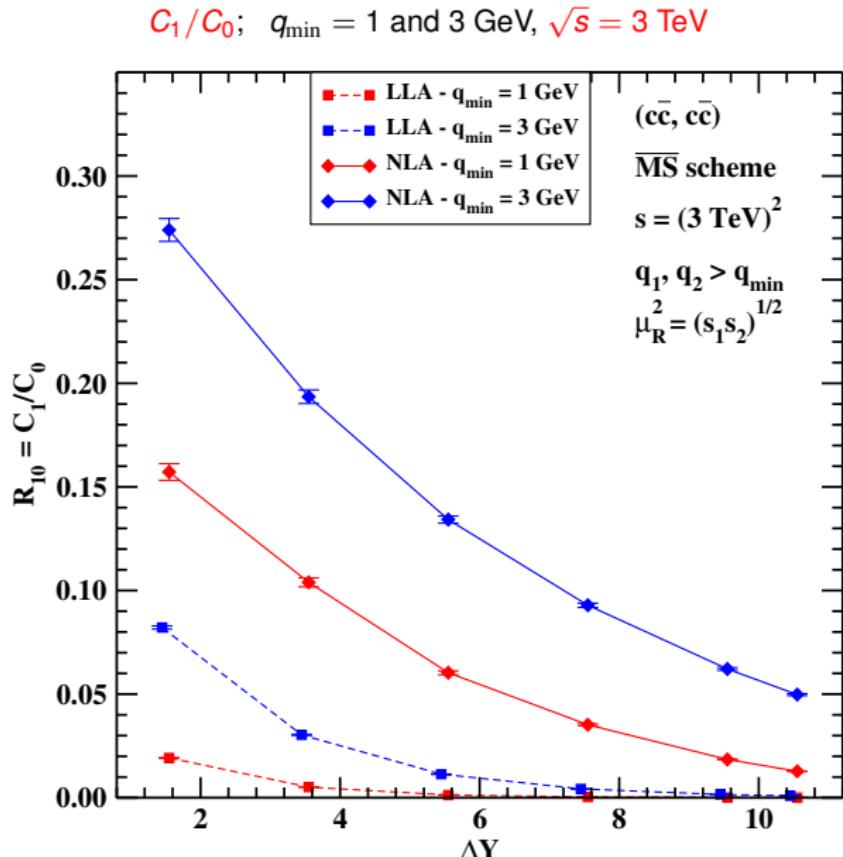


Heavy-quark pair photoproduction: numerical results

C_0 ; $q_{\min} = 1$ and 3 GeV, $\sqrt{s} = 3$ TeV



Heavy-quark pair photoproduction: numerical results



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Heavy-quark pair hadroproduction: theoretical setup



LO $Q\bar{Q}$ vertex:

$$\frac{d\Phi_G}{d^2q dz} = \frac{\alpha_s^2 \sqrt{N_c^2 - 1}}{2\pi N_c} \left[\left(m^2 (R + \bar{R})^2 + (\vec{P} + \vec{\bar{P}})^2 (z^2 + \bar{z}^2) \right) \right.$$
$$\left. - 2 \frac{N_c^2}{N_c^2 - 1} (m^2 R \bar{R} + \vec{P} \vec{\bar{P}} (z^2 + \bar{z}^2)) \right]$$

$$R = \frac{1}{m^2 + \vec{q}^2} - \frac{1}{m^2 + (\vec{q} - \vec{k}z)^2}, \quad \vec{P} = \frac{\vec{q}}{m^2 + \vec{q}^2} + \frac{\vec{k}z - \vec{q}}{m^2 + (\vec{q} - \vec{k}z)^2}$$

$$\bar{R} = \frac{-1}{m^2 + (\vec{q} - \vec{k})^2} + \frac{1}{m^2 + (\vec{q} - \vec{k}z)^2}, \quad \vec{\bar{P}} = \frac{\vec{k} - \vec{q}}{m^2 + (\vec{k} - \vec{q})^2} - \frac{\vec{k}z - \vec{q}}{m^2 + (\vec{q} - \vec{k}z)^2}$$

Same steps as before, with $\frac{dn}{dx}$ (photon flux) $\longrightarrow g(x)$ (gluon PDF)

Color and coupling pre-factors enhance hadro-production cross section by some 10^3 wrt photo-production, but photon flux dn/dx dominates over $g(x)$ for $x \rightarrow 0$ and $x \rightarrow 1$
 \longrightarrow detailed numerical analysis needed!

Outline

- 1 Introductory remarks
 - Motivation
 - BFKL factorization
 - Search for BFKL dynamics in inclusive processes
- 2 Two new probes: inclusive heavy-quark pair photoproduction ...
 - Theoretical setup
 - Observables and numerical analysis
- 3 ... and inclusive heavy-quark pair hadroproduction
 - Theoretical setup
 - **Observables and numerical analysis**
- 4 Conclusions and Outlook

Heavy-quark pair hadroproduction: observables

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{q}_1| d|\vec{q}_2| d\varphi_1 d\varphi_2} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \mathcal{C}_n \right], \quad \varphi = \varphi_1 - \varphi_2 - \pi$$

$$\begin{aligned} \mathcal{C}_n &= \frac{q_1 q_2 \sqrt{m_1^2 + q_1^2} \sqrt{m_2^2 + q_2^2}}{W^2} e^{\Delta Y} \\ &\times \int_{-\infty}^{+\infty} d\nu \left(\frac{W^2}{s_0} \right)^{\bar{\alpha}_s(\mu_R) \chi(n, \nu) + \bar{\alpha}_s^2(\mu_R) \left(\bar{\chi}(n, \nu) + \frac{\beta_0}{8N_c} \chi(n, \nu) \left(-\chi(n, \nu) + \frac{10}{3} + 2 \ln \frac{\mu_R^2}{\sqrt{s_1 s_2}} \right) \right)} \\ &\quad \times \alpha_s^4(\mu_R) c_1(n, \nu, \vec{q}_1^2, z_1) c_2(n, \nu, \vec{q}_2^2, z_2) \\ &\times \left\{ 1 + \bar{\alpha}_s(\mu_R) \left(\frac{\bar{c}_1^{(1)}}{c_1} + \frac{\bar{c}_2^{(1)}}{c_2} \right) + \bar{\alpha}_s(\mu_R) \frac{\beta_0}{2N_c} \left(\frac{5}{3} + \ln \frac{\mu_R^2}{s_1 s_2} + f(\nu) \right) \right. \\ &\quad \left. + \bar{\alpha}_s^2(\mu_R) \ln \left(\frac{W^2}{s_0} \right) \frac{\beta_0}{4N_c} \chi(n, \nu) f(\nu) \right\} \end{aligned}$$

LO kernel: $\bar{\alpha}_s \chi(n, \nu)$

$$\bar{\alpha}_s \equiv \frac{N_c \alpha_s}{\pi}, \quad \chi(n, \nu) = 2\psi(1) - \psi\left(\frac{n}{2} + \frac{1}{2} + i\nu\right) - \psi\left(\frac{n}{2} + \frac{1}{2} - i\nu\right)$$

NLO kernel contribution: $\bar{\alpha}_s^2$ terms in the exponent

Heavy-quark pair hadroproduction: observables

LO impact factors, projected onto the LO kernel eigenfunctions:

$$c_1 \left(n, \nu, \vec{q}_1^2, z_1 \right) = \frac{1}{e^{in\varphi_1} \alpha_s^2} \frac{d\Phi_{gg}^{\{Q\bar{Q}\}} (n, \nu, \vec{q}_1, z_1)}{d^2 \vec{q}_1 dz_1}$$

$$c_2 \left(n, \nu, \vec{q}_2^2, z_2 \right) = \frac{1}{e^{-in(\varphi_2 + \pi)} \alpha_s^2} \left[\frac{d\Phi_{gg}^{\{Q\bar{Q}\}} (n, \nu, \vec{q}_2, z_2)}{d^2 \vec{q}_2 dz_2} \right]^*$$

$$\frac{d\Phi_{gg}^{\{Q\bar{Q}\}} (n, \nu, \vec{q}, z)}{d^2 \vec{q} dz} \equiv \int \frac{d^2 \vec{k}}{\pi \sqrt{2}} (\vec{k}^2)^{i\nu - \frac{3}{2}} e^{in\theta} \frac{d\Phi_{gg}^{\{Q\bar{Q}\}} (\vec{k}, \vec{q}, z)}{d^2 \vec{q} dz}, \quad \cos \varphi = \frac{q_x}{|\vec{q}|}, \quad \cos \vartheta = \frac{k_x}{|\vec{k}|}$$
$$i \frac{d}{d\nu} \ln \frac{c_1}{c_2} = 2 [f(\nu) - \ln(\sqrt{s_1 s_2})], \quad s_i \equiv m_i^2 + \vec{q}_i^2$$

NLO impact factors (only universal part), e.g.

$$\frac{\bar{c}_1^{(1)}}{c_1} + \frac{\bar{c}_2^{(1)}}{c_2} = \chi(n, \nu) \ln \frac{s_0}{\sqrt{s_1 s_2}}$$

s_0 (arbitrary within NLA) chosen as $\sqrt{s_1 s_2}$

Heavy-quark pair hadroproduction: observables

From **gluon-gluon** to **proton-proton** collisions:

$$d\sigma_{pp} = f_{g_1}(x_1, \mu_{F_1}) dx_1 f_{g_2}(x_2, \mu_{F_2}) dx_2 d\sigma_{gg}$$

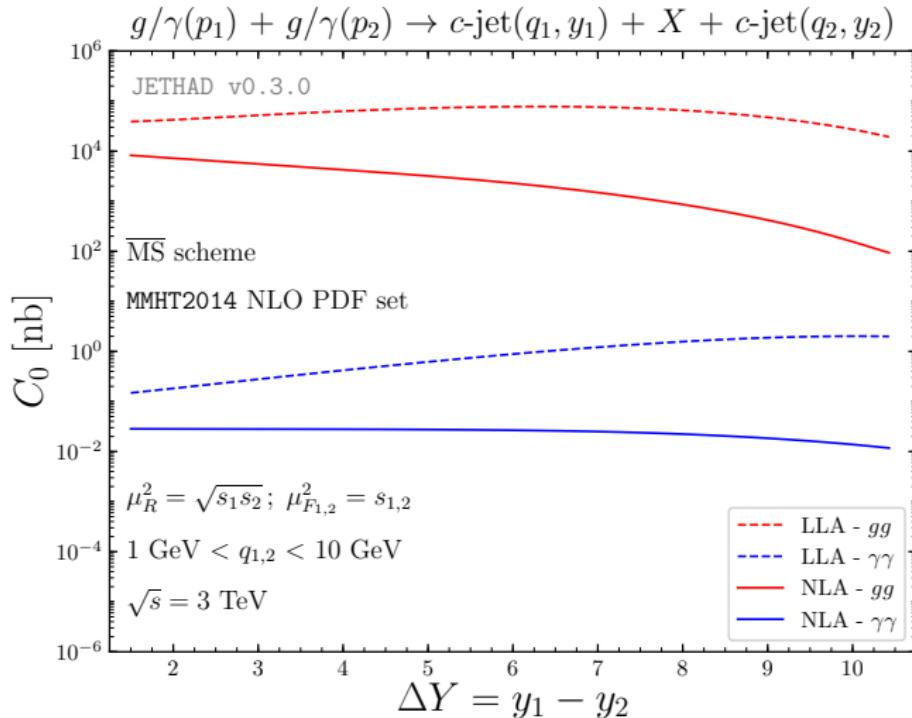
$$\frac{d\sigma_{pp}}{d\Delta Y d\varphi_1 d\varphi_2} = \frac{1}{(2\pi)^2} \left[C_0 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) C_n \right]$$

$$C_n = \int_{q_{1,\min}}^{q_{1,\max}} d|\vec{q}_1| \int_{q_{2,\min}}^{q_{2,\max}} d|\vec{q}_2| \int_{y_{1,\min}}^{y_{1,\max}} dy_1 \int_{y_{2,\min}}^{y_{2,\max}} dy_2 \delta(y_1 - y_2 - \Delta Y) \\ \int_{e^{-(y_{1,\max} - y_1)}}^1 dx_1 f_{g_1}(x_1, \mu_{F_1}) \int_{e^{-(y_{2,\max} + y_2)}}^1 dx_2 f_{g_2}(x_2, \mu_{F_2}) \mathcal{C}_n$$

Heavy-quark pair hadroproduction: numerical results

charm jet ($m_c=1.2$ GeV)

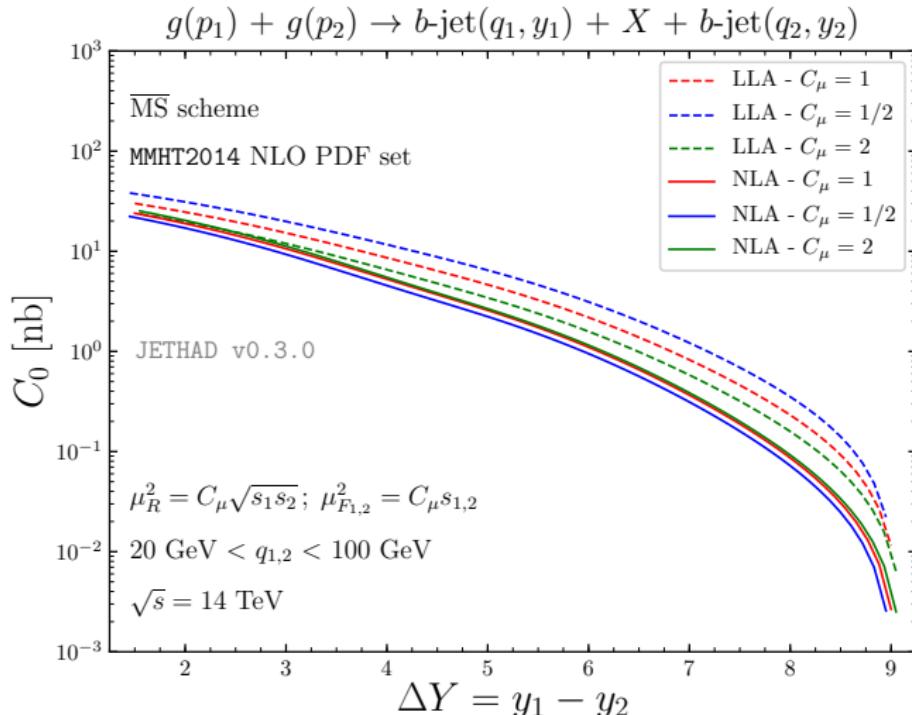
C_0 ; $q_{\min} = 1$ GeV, $q_{\max} = 10$ GeV; $\sqrt{s} = 3$ TeV



Heavy-quark pair hadroproduction: numerical results

bottom jet ($m_b=4.18$ GeV)

C_0 ; $q_{\min} = 10$ GeV, $q_{\max} = 100$ GeV; $\sqrt{s} = 14$ TeV

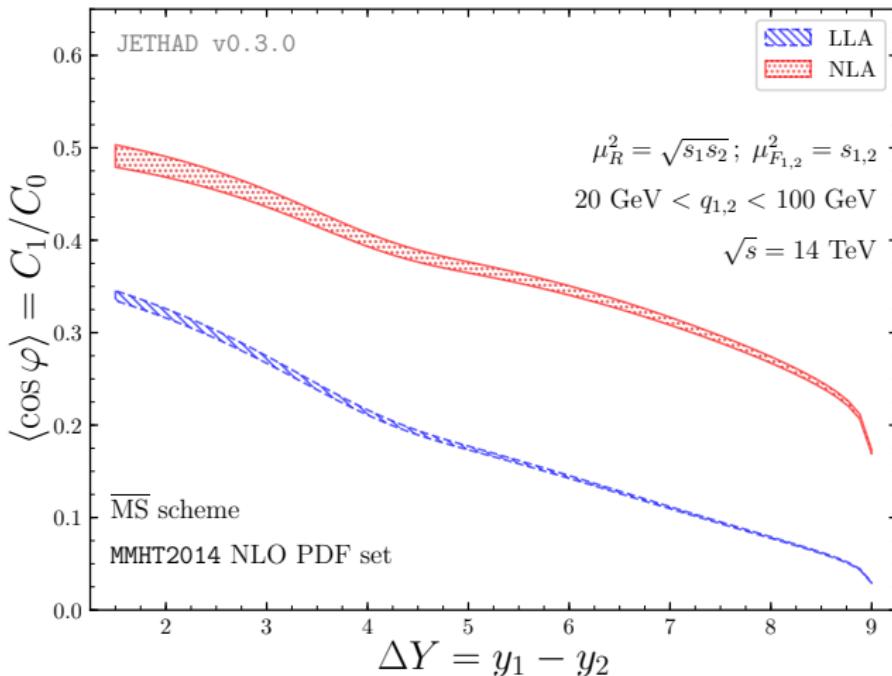


Heavy-quark pair hadroproduction: numerical results

bottom jet ($m_b=4.18$ GeV)

C_1/C_0 ; $q_{\min} = 10$ GeV, $q_{\max} = 100$ GeV; $\sqrt{s} = 14$ TeV

$g(p_1) + g(p_2) \rightarrow b\text{-jet}(q_1, y_1) + X + b\text{-jet}(q_2, y_2)$

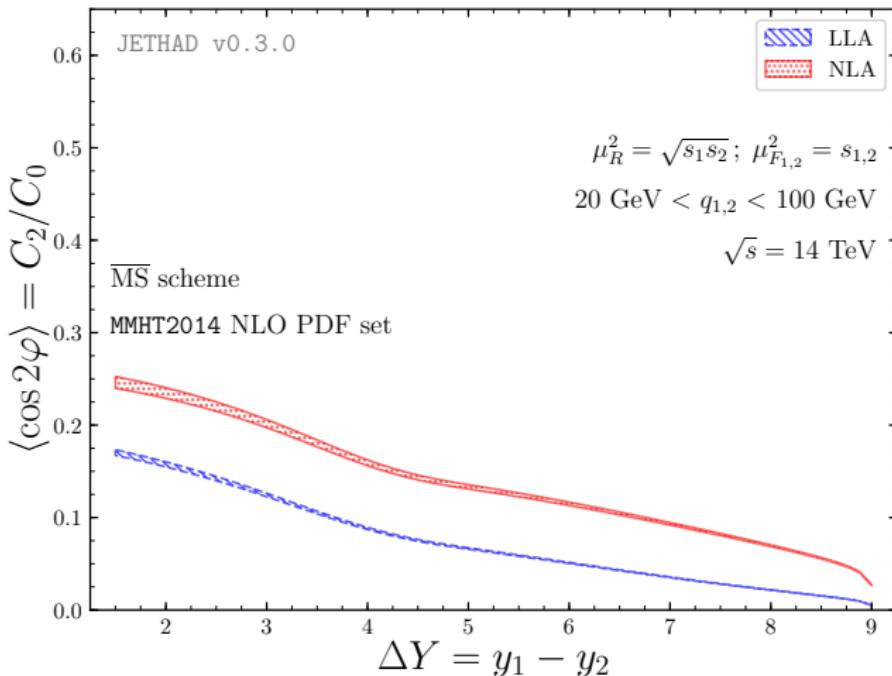


Heavy-quark pair hadroproduction: numerical results

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C_2/C_0 ; $q_{\min} = 10$ GeV, $q_{\max} = 100$ GeV; $\sqrt{s} = 14$ TeV

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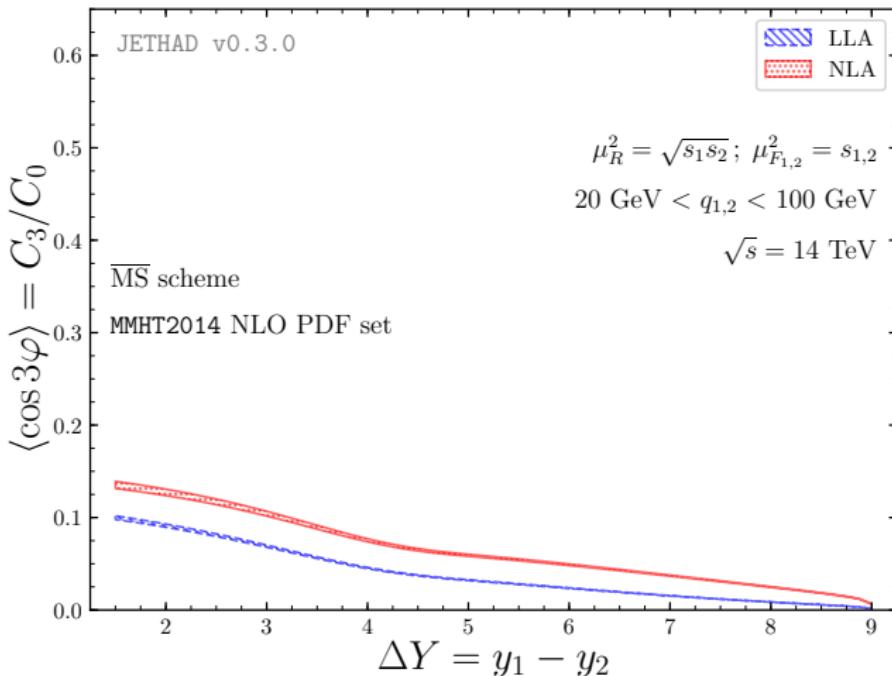


Heavy-quark pair hadroproduction: numerical results

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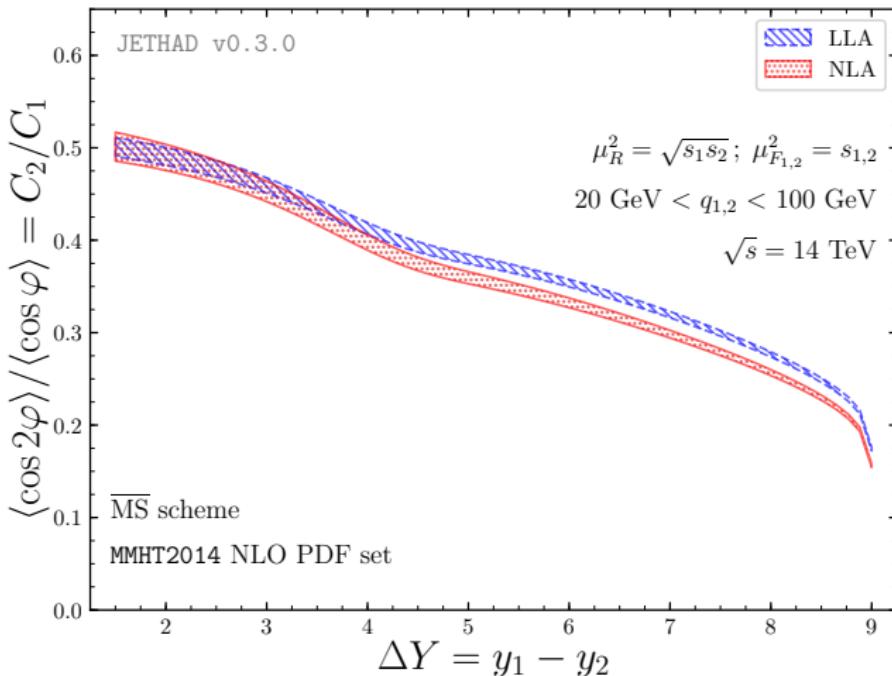


Heavy-quark pair hadroproduction: numerical results

bottom jet ($m_b=4.18$ GeV)

C_2/C_1 ; $q_{\min} = 10$ GeV, $q_{\max} = 100$ GeV; $\sqrt{s} = 14$ TeV

$$g(p_1) + g(p_2) \rightarrow b\text{-jet}(q_1, y_1) + X + b\text{-jet}(q_2, y_2)$$

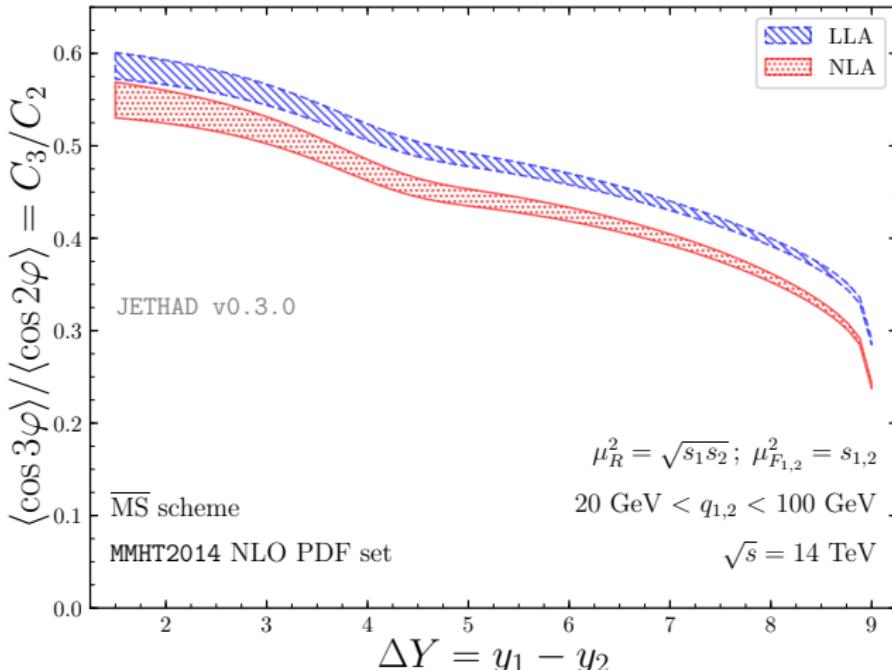


Heavy-quark pair hadroproduction: numerical results

bottom jet ($m_b=4.18$ GeV)

C_3/C_2 ; $q_{\min} = 10$ GeV, $q_{\max} = 100$ GeV; $\sqrt{s} = 14$ TeV

$g(p_1) + g(p_2) \rightarrow b\text{-jet}(q_1, y_1) + X + b\text{-jet}(q_2, y_2)$



Conclusions and Outlook

- Inclusive processes with jets and/or identified hadrons in the final state featuring large rapidity separation are a promising testfield for the search of BFKL dynamics in current and future colliders.
- Among them, the photo- or hadro-production of a pair of heavy quarks is an interesting new possibility.
- Theoretical predictions, including the most relevant part of the energy resummation in the NLA, are available for both photo- and hadro-production cases.
- The inclusion of subleading corrections from the impact factors, needed to get full-fledged NLA predictions, is in our “to do” list.
- The inclusive production of a single forward heavy quark, both in the LLA and in the NLA, can be studied on the same theoretical grounds, via the introduction of the unintegrated gluon distribution