Regge cuts in the Reggeized gluon channel

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Introduction

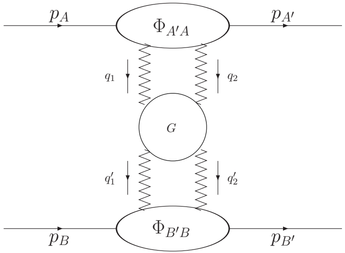
One of remarkable properties of QCD is the Reggeization of all elementary particles in perturbation theory, which is very important for theoretical description of high energy processes. The gluon Reggeization is especially important because it determines the high energy behaviour of non-decreasing with energy cross sections. In particular, it appears to be the basis of the BFKL (Balitskii-Fadin-Kuraev-Lipatov) equation, which was first derived in non-Abelian theories with spontaneously broken symmetry
F. V.S., Kuraev E.A., Lipatov L.N., 1975
and whose applicability in QCD was then shown
Balitsky I.I., Lipatov L.N., 1978.
The equation was derived using unitarity and analiyicity.

Introduction

In the Regge and multi-Regge kinematics in each order of perturbation theory dominant (having the largest $\ln s$ degrees) are amplitudes with gluon quantum numbers and negative signatures in cross-channels. They determine the s-channel discontinuites of amplitudes with the same and all other possible quantum numbers.

It is extremely important that both in the leading logarithmic approximation (LLA) and in the next-to-leading one (NLLA) the amplitudes used in the unitarity relations are determined by the Regge pole contributions and have a simple factorized form (pole Regge form). Due to this, the Reggeization provides a simple derivation of the BFKL equation in the LLA and in the NLLA.

The *s*-channel discontinuities of elastic amplitudes are presented by the picture



and have the form :

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B}.$$

Impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describe transitions $A \to A'$ $B \to B'$,
G – Green's function for two interacting Reggeized gluons,

$$\hat{\mathcal{G}} = e^{Y\hat{\mathcal{K}}},$$

$\hat{\mathcal{K}}$ – BFKL kernel, $Y = \ln(s/s_0)$,

$$\hat{\mathcal{K}} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_r$$

$$\hat{\mathcal{K}}_r = \hat{\mathcal{K}}_G + \hat{\mathcal{K}}_{Q\bar{Q}} + \hat{\mathcal{K}}_{GG}$$

Energy dependence of scattering amplitudes is determined by the BFKL kernel.
The BFKL kernel and the impact factors are expressed in terms of the Reggeon vertices and trajectory.
The kernel is universal (process independent).

Introduction

Validity of the pole Regge form is proved now in all orders of perturbation theory in the coupling constant g both in the LLA and in the NLLA.

The pole Regge form is violated in the NNLLA.

The first observation of the violation was done

Del Duca V., Glover E.W.N., 2001

at consideration of the high-energy limit of the two-loop amplitudes for gg, gq and qq scattering. The discrepancy appears in non-logarithmic terms.

If the pole Regge form would be correct in the NNLLA, they should satisfy a definite condition (factorization condition), because three amplitudes should be expressed in terms of two Reggeon-Particle-Particle vertices.

Using the infrared factorizationm techniques, consideration of the terms responsible for breaking of the pole Regge form in amplitudes of elastic scattering in QCD was performed by Del Duca V., Falcioni G., Magnea L., Vernazza L., 2013-2015.

In particular, the non-logarithmic terms not satisfying the factorization condition at two-loops were recovered and single-logarithmic terms at three loops violating the pole Regge form were found.

It is necessary to say that, in general, breaking the pole Regge form is not a surprise.

It is well known that Regge poles in the complex angular momenta plane generate Regge cuts. Moreover, in amplitudes with positive signature the Regge cuts appear already in the LLA. In particular, the BFKL Pomeron is the two-Reggeon cut in the complex angular momenta plane. But in amplitudes with negative signature Regge cuts must be at least three-Reggeon ones and can appear only in the NNLLA.

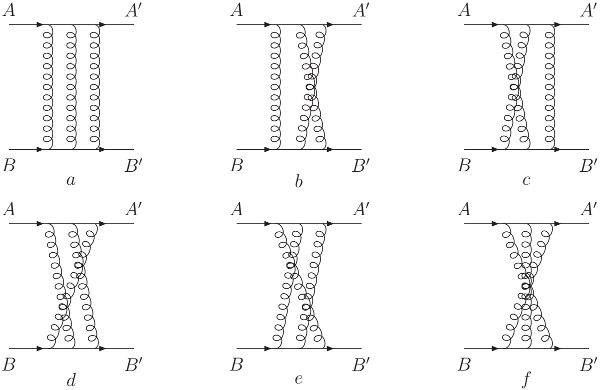
It was natural to expect that the observed violation of the pole Regge form can be explained by existence of the cut.

Indeed, all known cases of breaking of the pole Regge form are now explained by the three-Reggeon cuts
F. V.S., 2016; F. V.S., Lipatov L.N., 2017,
Caron-Huot S., Gardi E., Vernazza L., 2017
Unfortunately, the approaches used and the explanations given in these papers are different.

Their results coincide in three loops but may diverge in more loops.
It requires further investigation.

Appearance of three-Reggeon cuts

Due to the signature conservation the cut with negative signature must be the three-Reggeon one.
Since our Reggeon is the Reggeized gluon, the three-Reggeon cut first contribute to amplitudes corresponding to the diagrams



Appearance of three-Reggeon cuts

The amplitude of the process $\mathcal{A}_{AB}^{A'B'}$ can be written as the sum over permutations σ of products of colour factors and colour-independent matrix elements:

$$\mathcal{A}_{AB}^{A'B'} = \sum_{\sigma} \left(C_{AB}^{(0)\sigma} \right)_{\alpha\beta}^{\alpha'\beta'} M_{AB}^{(0)\sigma}(s,t) ,$$

where α and β (α' and β') are colour indices of incoming (outgoing) projectile A and target B respectively. We use the same letters for quark and gluon colour indices; it should be remembered, however, that there is no difference between upper and lower indices (running from 1 to $N_c^2 - 1$) for gluons, whereas for quarks lower and upper indices (running from 1 to N_c) refer to mutually related representations.
The colour factors can be decomposed into irreducible representations $\mathcal{R}$ of the colour group in the t-channel:

Appearance of three-Reggeon cuts

$$\left(C_{AB}^{(0)\sigma}\right)^{\alpha'\beta'}_{\alpha\beta} = \sum_{R} [\mathcal{P}_{AB}^{R}]^{\alpha'\beta'}_{\alpha\beta} \sum_{\sigma} \mathcal{G}(R)_{AB}^{(0)\sigma},$$

where

$$[\mathcal{P}_{AB}^{R}]^{\alpha'\beta'}_{\alpha\beta} = \sum_{n} [\mathcal{P}_{A}^{R,n}]^{\alpha'}_{\alpha} [\mathcal{P}_{B}^{R,n}]^{\beta'}_{\beta}$$

$\hat{\mathcal{P}}^{R,n}$ is the projection operator on the state n in the representation $\mathcal{R}$,

$$\mathcal{G}(R)_{AB}^{(0)\sigma} = \frac{1}{N_R \; T_A \; T_B} \left(\mathcal{T}_A^{c_1} \mathcal{T}_A^{c_2} \mathcal{T}_A^{c_3}\right)^{\alpha'}_{\alpha}$$

$$\times \left(\mathcal{T}_B^{c_1^\sigma} \mathcal{T}_B^{c_2^\sigma} \mathcal{T}_B^{c_3^\sigma}\right)^{\beta'}_{\beta} [\mathcal{P}_{AB}^{R}]^{\alpha\beta}_{\alpha'\beta'} \; ,$$

N_R is the dimension of the representation R, $\mathcal{T}^a$ are the colour group generators in the corresponding representations,

Appearance of three-Reggeon cuts

$[\mathcal{T}^a, \mathcal{T}^b] = i f_{abc}\mathcal{T}^c$; for all representations,
$(\mathcal{T}^a)^b{}_c = -i f_{abc}$ for gluons,
$(\mathcal{T}^a)^{\alpha'}{}_{\alpha} = (t^a)^{\alpha'}{}_{\alpha}$ for quarks;
$\mathrm{Tr}(\mathcal{T}^a_A \mathcal{T}^b_B) = T_A \delta_{AB}\delta_{ab}, \quad T_q = 1/2, \quad T_g = N_c$.

In contrast to the Reggeon, which contributes only to amplitudes with the adjoint representation of the colour group (colour octet in QCD) in the *t*-channel, the cut can contribute to various representations.

Possible representations for quark-quark and quark-gluon scattering are only singlet (**1**) and octet (**8**), whereas for the gluon-gluon scattering there are also **10**, $\mathbf{10}^*$ and **27**.

Additional restrictions are imposed by signature.

Taking into account Bose statistic for gluons, symmetry of the representations **1** and **27**, antisymmetry **10** and $\mathbf{10}^*$ and existence both symmetric $\mathbf{8_s}$ and antisymmetric $\mathbf{8_a}$ representations for them, gives that

Appearance of three-Reggeon cuts

amplitudes with negative signature there are in the representations besides the Reggeon channel **8** amplitudes with negative signature **1** for quark-quark-scattering and in the representation **10** and **10*** for the gluon-gluon scattering. Corresponding projection operators are

$$[\mathcal{P}_{gg}^{\mathbf{8_a}}]^{ab}_{a'b'} = \frac{f_{aa'c}f_{bb'c}}{N_C} \ ,$$

$$[\mathcal{P}_{gg}^{\mathbf{10}}]^{ab}_{a'b''} = \frac{1}{4}\Big(\delta_{ab}\delta_{a'b'} - \delta_{ab'}\delta_{a'b} - 2\frac{f_{aa'c}f_{bb'c}}{N_C}$$

$$+if_{ba'c}d_{b'ac} + if_{ba'c}d_{b'ac}\Big) \ ,$$

$$[\mathcal{P}_{gg}^{\mathbf{10}^*}]^{ab}_{a'b''} = \left([\mathcal{P}_{gg}^{\mathbf{10}}]^{ab}_{a'b''}\right)^* \ ,$$

Appearance of three-Reggeon cuts

$$[\mathcal{P}^{\mathbf{8}}_{qq}]^{\alpha'\beta'}_{\alpha\beta} = 2(t^c)^{\alpha'}_{\alpha}(t^c)^{\beta'}_{\beta} \ ,$$

$$[\mathcal{P}^{\mathbf{1}}_{qq}]^{\alpha'\beta'}_{\alpha\beta} = \sqrt{\frac{2}{N_c}}\delta^{\alpha'}_{\alpha}\delta^{\beta'}_{\beta} \ ,$$

$$[\mathcal{P}^{\mathbf{8_a}}_{gq}]^{a'\beta'}_{a\beta} = \frac{1}{N_c} f^c_{aa'}(t^c)^{\beta'}_{\beta} \ ,$$

It turns out that for the representations R different from the Reggeized gluon one the colour coefficients $\mathcal{G}(R)^{(0)\sigma}_{AB}$ do not depend on σ, so that momentum dependent factors for them summed up to the eikonal amplitude

$$\sum_{\sigma} M^{(0)\sigma}_{AB}(s,t) = A^{eik} = g^6 \frac{s}{t}\left(\frac{-4\pi^2}{3}\right)\vec{q}^{\,2} A_2(q_\perp) \ ,$$

Appearance of three-Reggeon cuts

where $A_2(q_\perp)$ is depicted by the diagram

and is written as

$$A_2(q_\perp) = \int \frac{d^{2+2\epsilon}l_1\, d^{2+2\epsilon}l_2}{(2\pi)^{2(3+2\epsilon)}\vec{l}_1^{\,2}\vec{l}_2^{\,2}(\vec{q}-\vec{l}_1-\vec{l}_2)^2} \, .$$

Note that the "infrared" ϵ is used, $\epsilon = (D-4)/2$, D is the space-time dimension.

This result is very important, because contribution of the cut must be gauge invariant, whereas $M_{AB}^{(0)\sigma}$ taken separately are gauge dependent.

Appearance of three-Reggeon cuts

In the Reggeized gluon channel the colour coefficients $\mathcal{G}(R)_{AB}^{(0)\sigma}$ depend on σ. However, this dependence has a specific form

$$\mathcal{G}(\mathbf{8_a})_{AB}^{(0)b} = \mathcal{G}(\mathbf{8_a})_{AB}^{(0)c} = \mathcal{G}(\mathbf{8_a})_{AB}^{(0)d}\mathcal{G}(\mathbf{8_a})_{AB}^{(0)e} \equiv \mathcal{G}(\mathbf{8_a})_{AB}^{(0)} ,$$

$$\mathcal{G}(\mathbf{8_a})_{gg}^{(0)a} = \mathcal{G}(\mathbf{8_a})_{gg}^{(0)f} ,$$

$$\mathcal{G}(\mathbf{8_a})_{gq}^{(0)a} = \mathcal{G}(\mathbf{8_a})_{gq}^{(0)f} ,$$

$$\frac{1}{2}\left[\mathcal{G}(\mathbf{8_a})_{AB}^{(0)a} + \mathcal{G}(\mathbf{8_a})_{AB}^{(0)f}\right] = \mathcal{G}(\mathbf{8_a})_{AB}^{(0)} + \frac{N_c^2}{8} .$$

The last equality means that the terms violating the pole factorization have σ-independent colour coefficients, so that momentum factors for them summed up to the eikonal amplitude.

Three loops

Separation of the pole and cut contributions is impossible in the two-loop approximation because of the ambiguity of the allocation of the part of the amplitudes violating the factorization. The separation becomes possible in higher loops, due to different energy dependence of the pole and cut contributions. Energy dependence of the pole contribution is determined by the Regge factor of the Reggeized gluon $\exp(\omega(t)\ln s)$, where $\omega(t)$ is the gluon trajectory, whereas for the three-Reggeon cut it is

$$e^{[(\hat{\omega}_1+\hat{\omega}_2+\hat{\omega}_3+\hat{\mathcal{K}}_r(1,2)+\hat{\mathcal{K}}_r(1,3)+\hat{\mathcal{K}}_r(2,3))\ln s]}\ ,$$

where $\hat{\mathcal{K}}_r(m,n)$ is the real part of the BFKL kernel describing interaction between Reggeons m and n.

Three loops

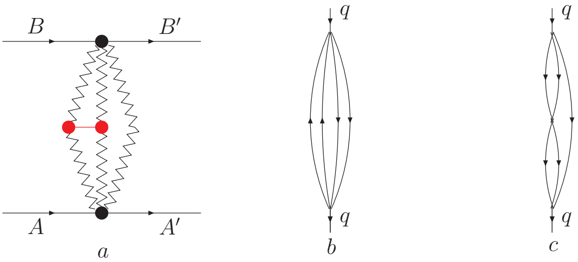
With the help of the integral representation of the trajectory

$$\omega(t) = -g^2 N_c \vec{q}^{\,2} \int \frac{d^{2+2\epsilon} l}{2(2\pi)^{(3+2\epsilon)} \vec{l}^{\,2} (\vec{q} - \vec{l})^2}$$

and the explicit form of the real part of the kernel describing interaction between two Reggeons with transverse momenta $\vec{l}_1$ and $\vec{l}_2$ and colour indices c_1 and c_2

$$\left[\mathcal{K}_r(\vec{q}_1, \vec{q}_2; \vec{k})\right]_{c_1 c_2}^{c_1' c_2'} = -T^a_{c_1 c_1'} T^a_{c_2 c_2'} \frac{g^2}{(2\pi)^{D-1}} \left[\frac{\vec{q}_1^{\,2} \vec{q}_2^{\,\prime\, 2} + \vec{q}_2^{\,2} \vec{q}_1^{\,\prime\, 2}}{\vec{k}^{\,2}} - \vec{q}^{\,2}\right],$$

the first order corrections are expressed through the diagrams b and c.



Three loops

The calculation of the three-loop corrections
F.V.S.
shows that the violation of the pole Regge form, analysed in this approximation with the help of the infrared factorization, can be explained by the pole and cut contributions. In other words, the restrictions imposed by the infrared factorization on the parton scattering amplitudes with the adjoint representation of the colour group in the t-channel and negative signature can be fulfilled in the NNLLA at two and three loops if besides the Regge pole contribution there is the Regge cut contribution

$$\mathcal{G}(\mathbf{8_a})_{AB}^{(cut)} \left(A^{eik} + g^2 N_c \ln s \left(\frac{1}{2} A_3^b(q_\perp) - A_3^c(q_\perp) \right) \right) ,$$

Three loops

$$A_3^b(q_\perp) = -\int \frac{d^{2+2\epsilon}l_1\, d^{2+2\epsilon}l_2 d^{2+2\epsilon}l_3}{(2\pi)^{3(3+2\epsilon)}\vec{l}_1^{\,2}\vec{l}_2^{\,2}\vec{l}_3^{\,2}(\vec{q}-\vec{l}_1-\vec{l}_2-\vec{l}_3)^2}\,,$$

$$A_3^c(q_\perp) = \int \frac{d^{2+2\epsilon}l_1\, d^{2+2\epsilon}l_2 d^{2+2\epsilon}l_3(\vec{q}-l_1)^2}{(2\pi)^{3(3+2\epsilon)}\vec{l}_1^{\,2}\vec{l}_2^{\,2}\vec{l}_3^{\,2}(\vec{q}-\vec{l}_1-\vec{l}_2)^2(\vec{q}-\vec{l}_1-\vec{l}_3)^2}\,.$$

$$\mathcal{G}(\mathbf{8_a})_{gg}^{(cut)} = -\frac{3}{2}\,, \quad \mathcal{G}(\mathbf{8_a})_{gq}^{(cut)} = --\frac{3}{2}\,,$$

$$\mathcal{G}(\mathbf{8_a})_{qq}^{(cut)} = \frac{3(1-N-c^2)}{4N_c^2}\,.$$

Alterntive approach

Of course, this result is limited to three loops and can not be considered as a proof that in the NNLLA the only singularities in the J plane are the Regge pole and the three-Reggeon cut. Moreover, the explanation of the violation of the pole Regge form given in
Caron-Huot S., Gardi E., Vernazza L., 2017
differs from described above. In this paper, besides the cut with the vertex of interaction with particles i having the colour structure

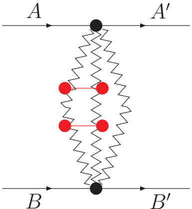
$$C^{(0)c}_{a'a} = \mathcal{T}^{c}_{a'a} \frac{1}{3!(N_c^2-1)T_i T_j} \operatorname{Tr} \sum_{\sigma} \left(\mathcal{T}^{c_1^\sigma} \mathcal{T}^{c_2^\sigma} \mathcal{T}^{c_3^\sigma} \mathcal{T}^{c} \right) ,$$

the Reggeon-cut mixing is introduced.

Alterntive approach

To my mind, the approach used in
Caron-Huot S., Gardi E., Vernazza L., 2017
has some weak points.
– The parton-parton-cut vertex was in fact postulated, without any derivation and justification.
– No connection with Feynman diagrams was established.
– The cut contribution is not suppressed at large N_c, i.e. it exists in the planar $N = 4$ SYM, in contradiction with the common wisdom.
In our approach, the Reggeon-cut mixing is not necessary in the three-loop approximation.
Whether mixing is necessary can be verified in the four-loop approximation.

In the four loops there are three types of corrections. The first (simplest) ones come from account of the Regge factors of each of three Reggeons. The second type of the corrections are given by the products of the trajectories and real parts of the BFKL kernel, and the third come from account of Reggeon-Reggeon interactions.



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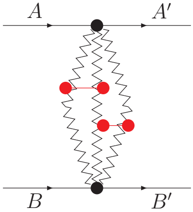
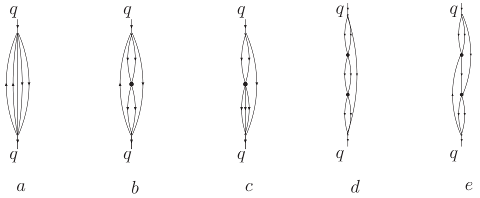


Four loops

All the types of the corrections are expressed through the integrals in the transverse momentum space corresponding to the diagrams



Four loops

$$I_i = \int \frac{d^{2+2\epsilon} l_1 \, d^{2+2\epsilon} l_2 \, d^{2+2\epsilon} l_3}{(2\pi)^{3(3+2\epsilon)} \vec{l}_1^{\,2} \vec{l}_2^{\,2} \vec{l}_3^{\,2}} F_i \delta^{2+2\epsilon}(\vec{q} - \vec{l}_1 - \vec{l}_2 - \vec{l}_3) \, ,$$

$$F_a = f_1(\vec{l}_1) f_1(\vec{l}_2) \, , \quad F_b = f_1(\vec{l}_1) f_1(\vec{l}_1) \, , \quad F_c = f_2(\vec{l}_1 + \vec{l}_2) \, ,$$

$$F_d = f_1(\vec{l}_1 + \vec{l}_2) f_1(\vec{l}_1 + \vec{l}_2) \, , \quad F_e = f_1(\vec{q} - \vec{l}_1) f_1(\vec{q} - \vec{l}_3) \, ,$$

$$f_1(\vec{k}) = \vec{k}^{\,2} \int \frac{d^{2+2\epsilon} l}{(2\pi)^{(3+2\epsilon)} \vec{l}^{\,2} (\vec{l} - \vec{k})^2} \, , \quad f_2(\vec{k}) = \int \frac{d^{2+2\epsilon} l f_1(\vec{l})}{(2\pi)^{(3+2\epsilon)} \vec{l}^{\,2} (\vec{l} - \vec{k})^2} \, .$$

These integrals enter in the total four-loop correction with different colour factors in the approaches with or without the Reggeon-cut mixing.

Four loops

Calculation of the colour factors $\mathcal{G}(\mathbf{8_a})^{(2)\sigma}_{AB}$ shows that as well as in the two and three loops the terms violating the pole factorization have σ-independent colour coefficients.
The question of whether the four-loop amplitudes of elastic scattering in QCD are given by the Regge pole and cut contributions, with or without mixing, can be solved by comparing of these corrections with the result obtained using the infrared factorization.

Summary

- The pole Regge form of amplitudes with gluon quantum numbers in cross channels and negative signature, which is the basis of the BFKL equation, is violated in the NNLLA.
- It was shown that the observed violation can be explained by the cut contributions.
- But the assertion that the QCD amplitudes with gluon quantum numbers in cross-channels and negative signature are given in the NNLLA by the contributions of the Regge pole and the three-Reggeon cut is only a hypotheses, and as yet there is no general proof of it, it should be checked in each order of the perturbation theory.