

Topological Charge Fluctuations in the Glasma

Pablo Guerrero Rodríguez

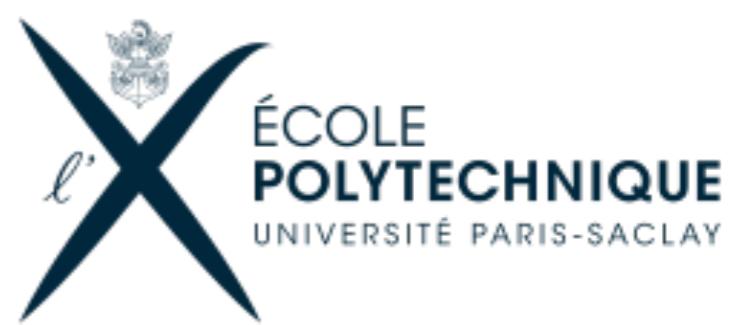
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Low-x
August 26th, 2019
Cyprus

10.1007/JHEP08(2019)026
arXiv:1903.11602



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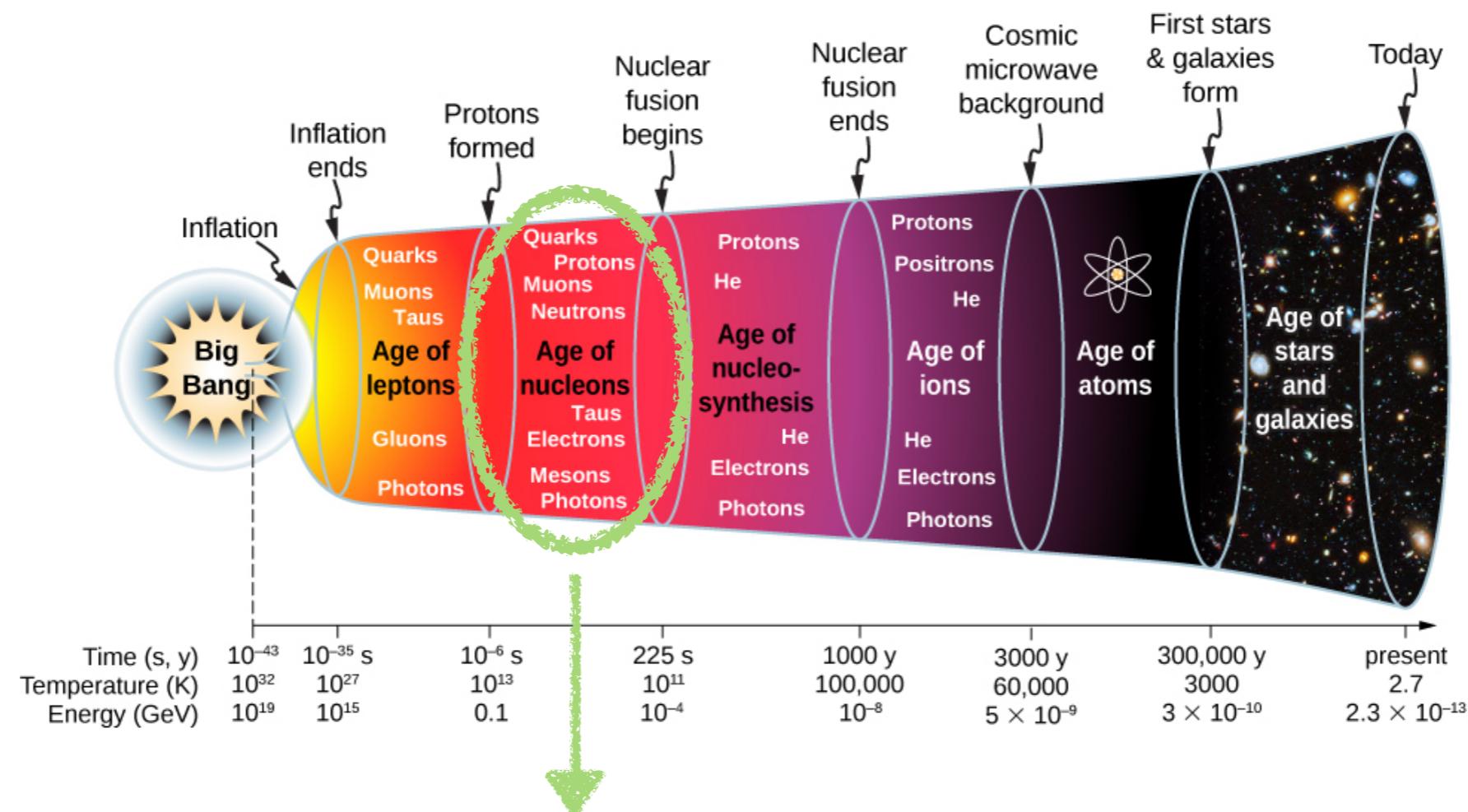


Outline

1. CP violation and the topological structure of QCD
2. CP violation in the Quark Gluon Plasma: The Chiral Magnetic Effect
3. The Color Glass Condensate
4. Correlators of the divergence of the Chern-Simons current
5. Comparison with the Glasma Graph approximation
6. Conclusions

Introduction: The Chiral Magnetic Effect

CP violation and QCD



- **Baryon asymmetry** originated in early **CP-violating fluctuations**
- The search for **CP violation signals**: an ongoing concern of physics
- **Heavy-Ion Collisions** generate physical conditions of temperature and density comparable to those of the Big Bang
- In this context, we must focus on mechanisms of CP violation within **Quantum Chromodynamics**

The chiral symmetry

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_{f,i} (i\gamma_\mu (D^\mu)_{ij} - m_f \delta_{ij}) \psi_{f,j} - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a}$$

- We consider only the light quarks: $f = u, d, s$
- Also, we neglect the mass term:

$$\mathcal{L} = i \sum_f \bar{\psi}_{f,i} \gamma_\mu (D^\mu)_{ij} \psi_{f,j} - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a}$$

- Symmetric under **chiral** transformations, or, equivalently, **vector** and **axial** transformations: $SU(3)_V \times SU(3)_A$

$$\psi \rightarrow \exp \left\{ i \frac{\tau_j \theta^j}{2} \right\} \psi$$

$SU(3)_V$

Observed in nature (ISOSPIN)

$$\psi \rightarrow \exp \left\{ i \gamma^5 \frac{\tau_j \theta^j}{2} \right\} \psi$$

$SU(3)_A$

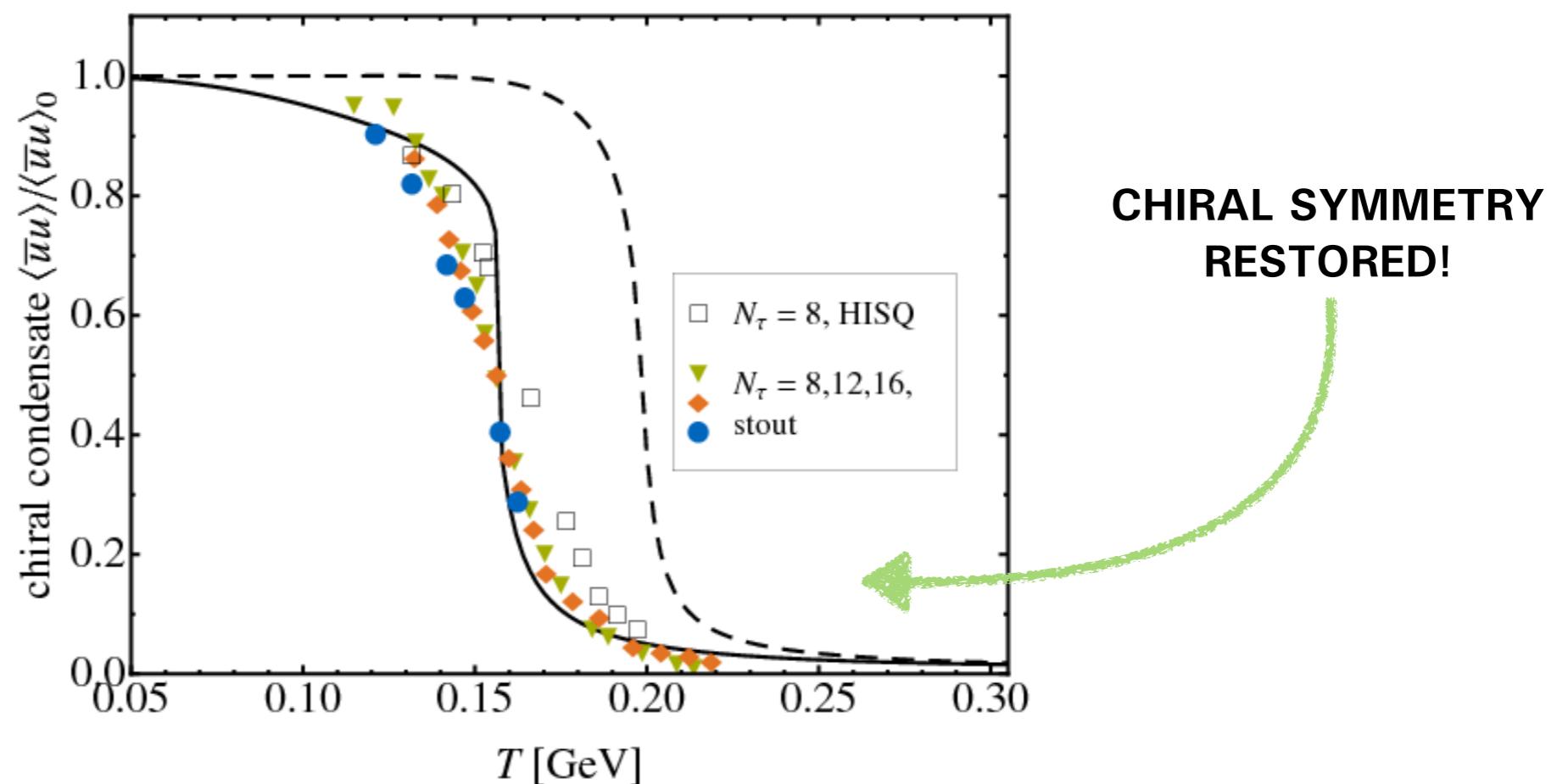
Not observed in nature

Spontaneous breaking and restoration of chiral symmetry

- AXIAL symmetry ($SU(3)_A$) is **spontaneously broken**:

$$\langle \bar{\psi} \psi \rangle \neq 0$$

- Lattice simulations show that at large temperatures the chiral condensate $\langle \bar{\psi} \psi \rangle$ vanishes:

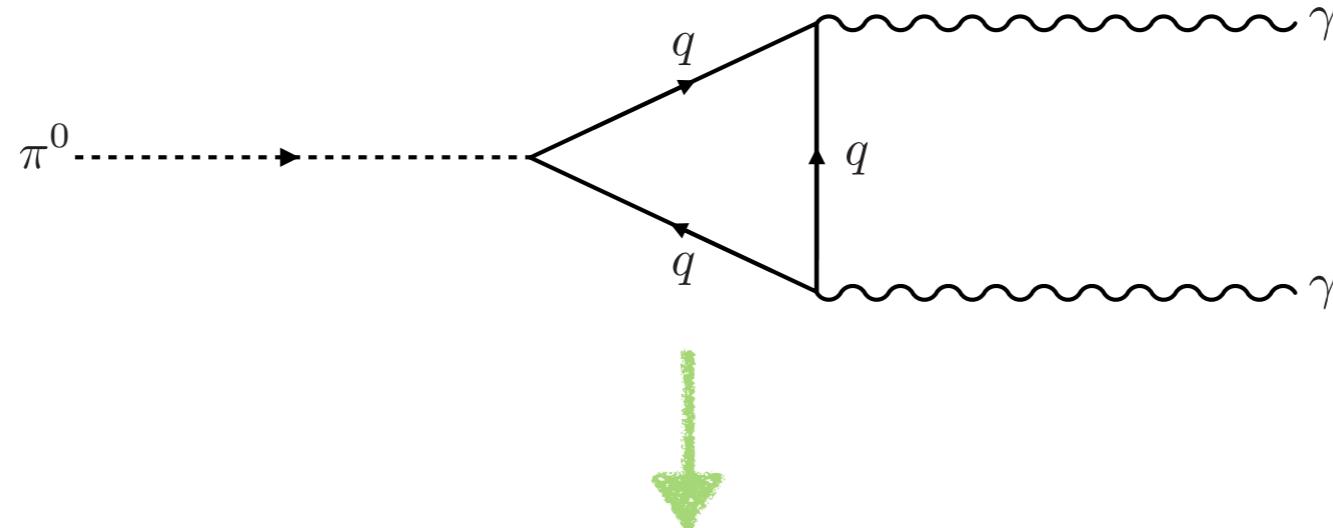


Which signals the onset of a new phase of matter: the **Quark-Gluon Plasma**.

Figure extracted from T. Hell, K. Kashiwa and W. Weise, Phys. Rev. D 83 (Jun, 2011) 114008.

The chiral anomaly

- However, chiral symmetry is only conserved at classical level:



$$\partial_\mu j_A^\mu = -\frac{g^2 N_f}{8\pi^2} \text{Tr} \left\{ F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x) \right\} \quad \text{with} \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

- The **chiral anomaly** induces a transformation of left- into right-handed quarks:

$$\frac{dN_5}{dt} \equiv \frac{d(N_R - N_L)}{dt} = \int d^3x \partial_\mu j_A^\mu = -\frac{g^2 N_f}{8\pi^2} \int d^3x \text{Tr} \left\{ F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x) \right\}$$

- Axial charge production directly related to structure of **gauge fields** through chiral anomaly.
- We focus on contributions stemming from **topological structure** of gluon fields.

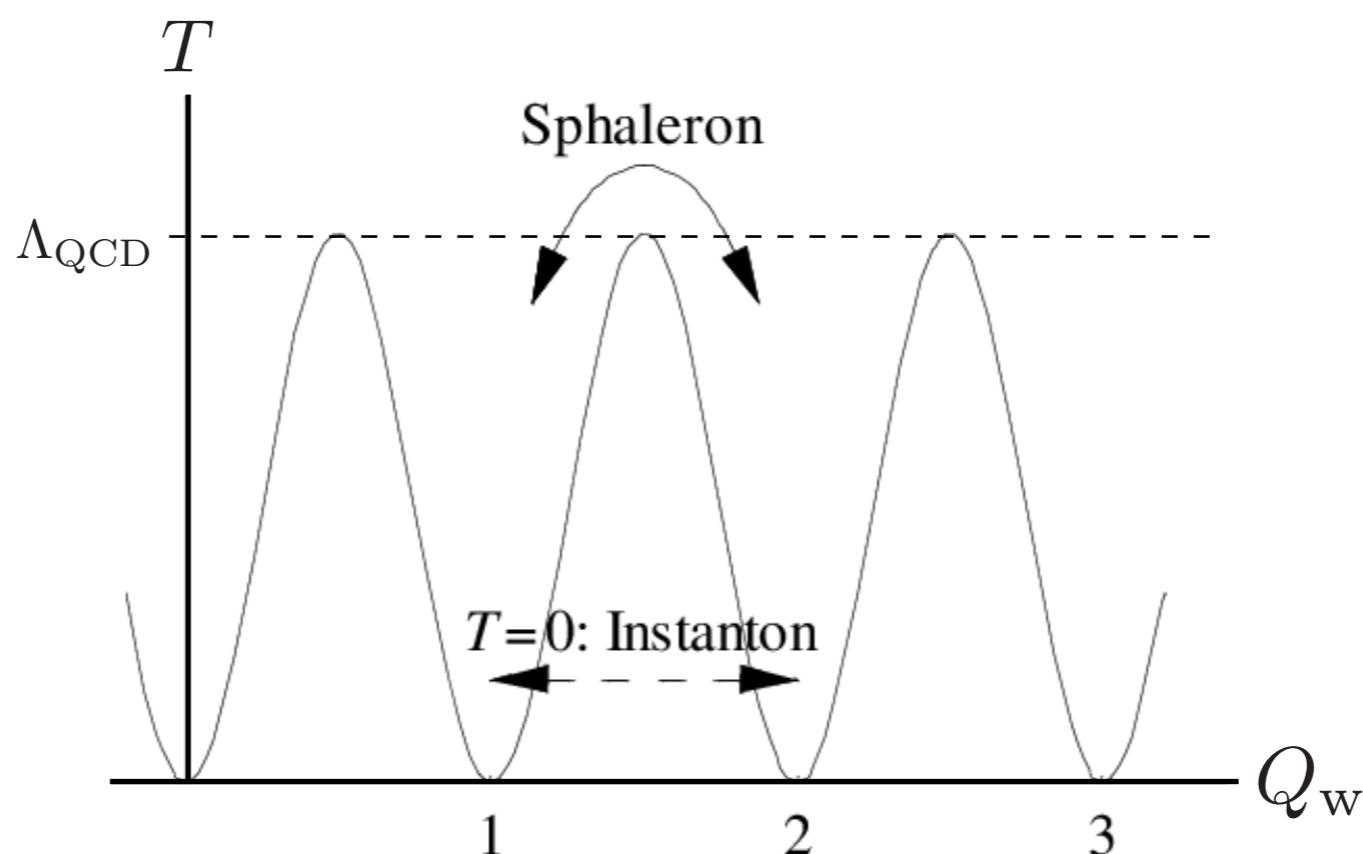
CP violation and the topological structure of QCD

- Gauge field configurations can be classified in distinct classes labeled by their winding number:

$$Q_w = \frac{g^2}{16\pi^2} \int d^4x \text{Tr} \left\{ F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x) \right\}$$

- Q_w fluctuations:

- Low T : requires quantum tunneling, suppressed.
- $T \sim \Lambda_{\text{QCD}}$: leap over potential barriers, not suppressed



CP violation and the topological structure of QCD

- Gauge field configurations can be classified in distinct classes labeled by their winding number:

$$Q_w = \frac{g^2}{16\pi^2} \int d^4x \text{Tr} \left\{ F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x) \right\}$$

- Q_w fluctuations contribute to the generation of axial charge:

$$\frac{dN_5}{dt} = -2N_f \frac{dQ_w}{dt}$$

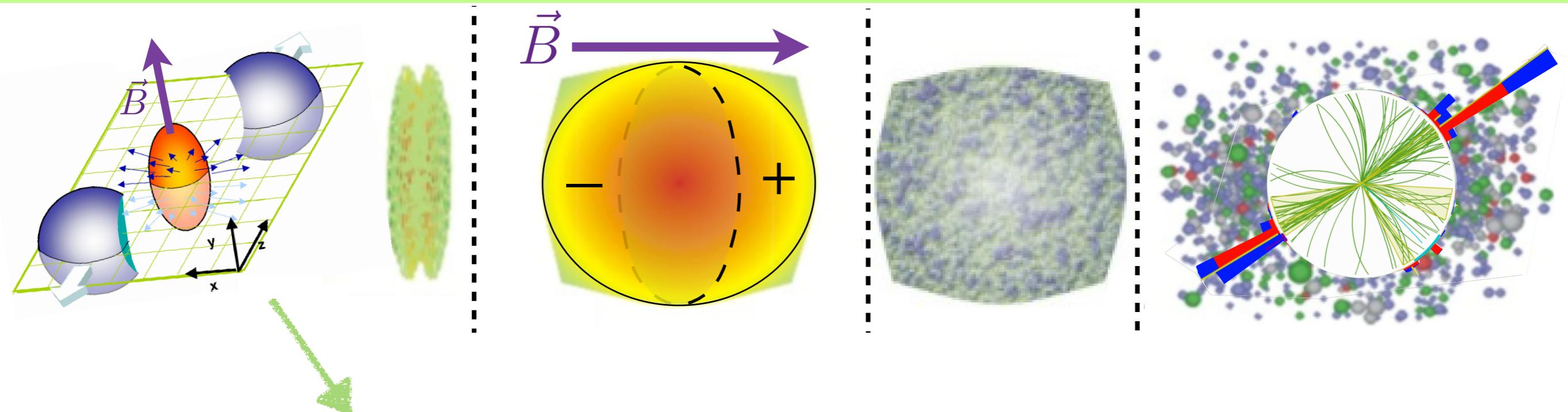
- **NOTATION:** Chern-Simons current $\rightarrow K^\mu = \epsilon^{\mu\nu\rho\sigma} A_\nu^a \left(F_{\rho\sigma}^a + \frac{g}{3} A_\rho^b A_\sigma^c \right)$

Divergence of the Chern-Simons current:

$$\dot{\nu}(x) \equiv \partial_\mu K^\mu = -\frac{1}{4} \text{Tr} \left\{ F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x) \right\}$$

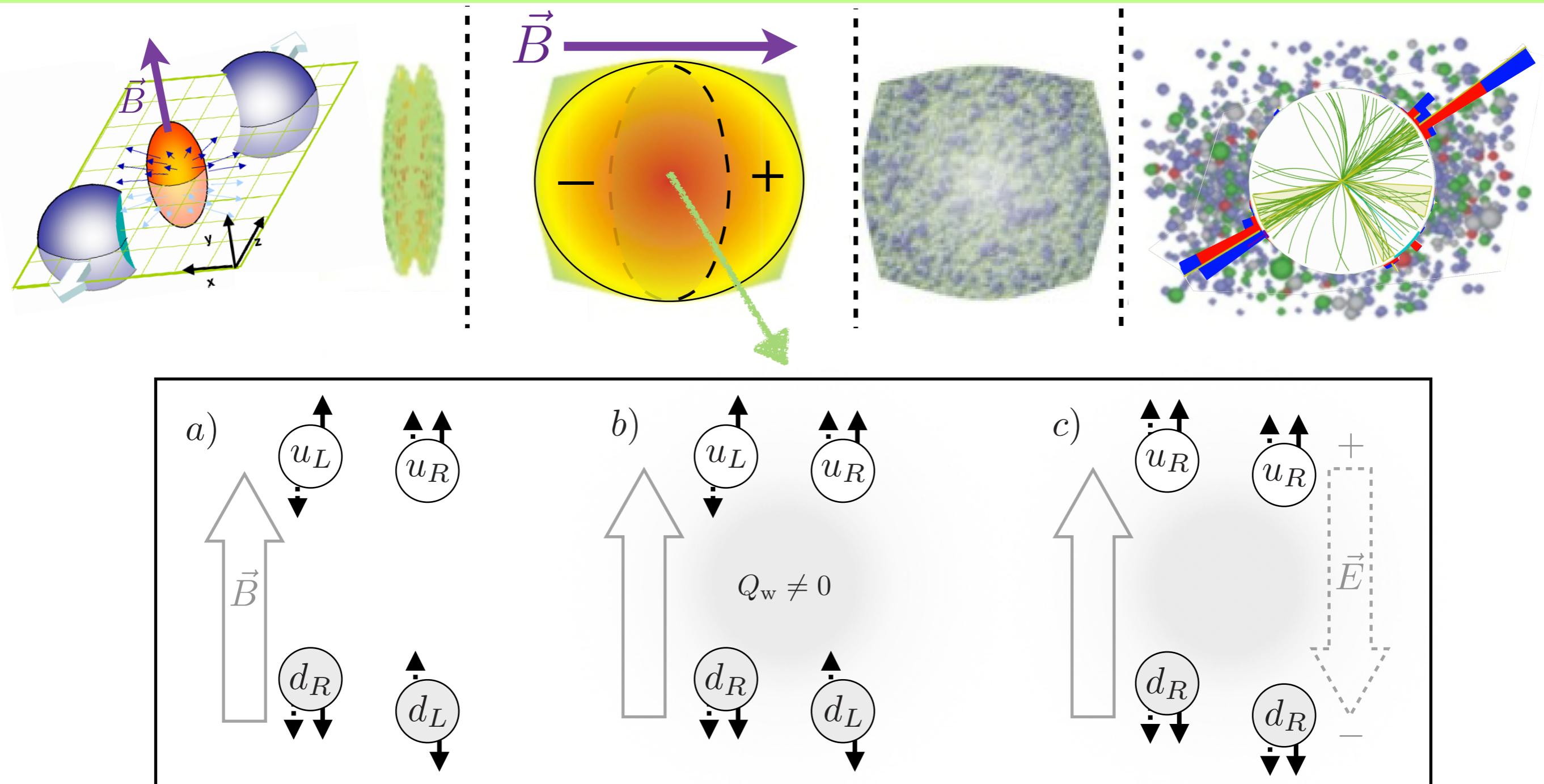
$$\frac{dN_5}{dt} = \frac{g^2 N_f}{2\pi^2} \int d^3x \dot{\nu}(x)$$

CP violation in the Quark Gluon Plasma: the Chiral Magnetic Effect



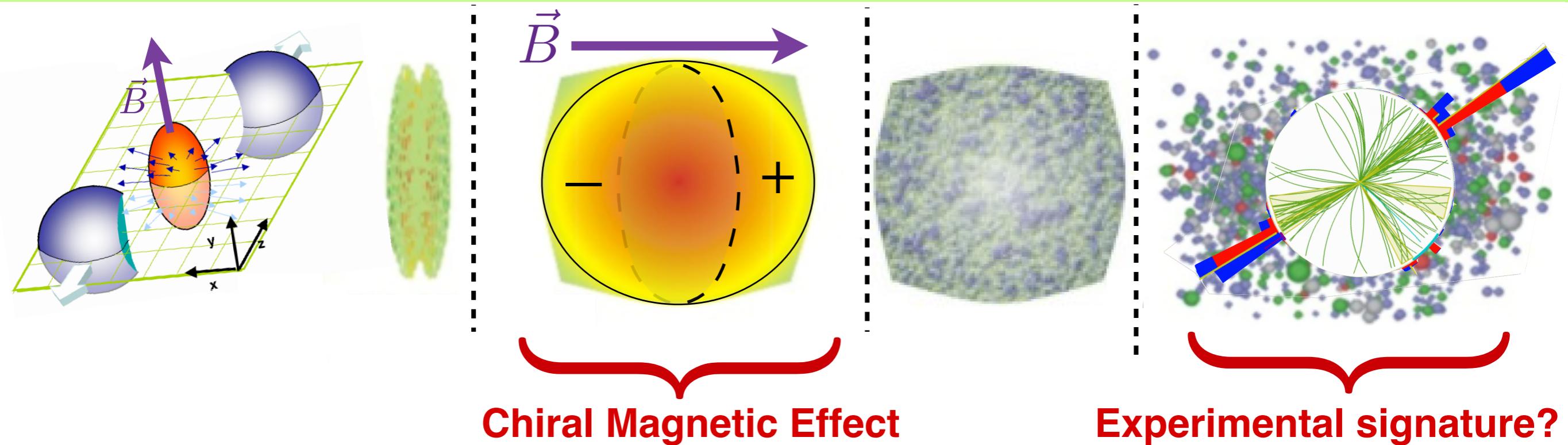
- Off-central HICs give rise to **large background electromagnetic fields**.

CP violation in the Quark Gluon Plasma: the Chiral Magnetic Effect



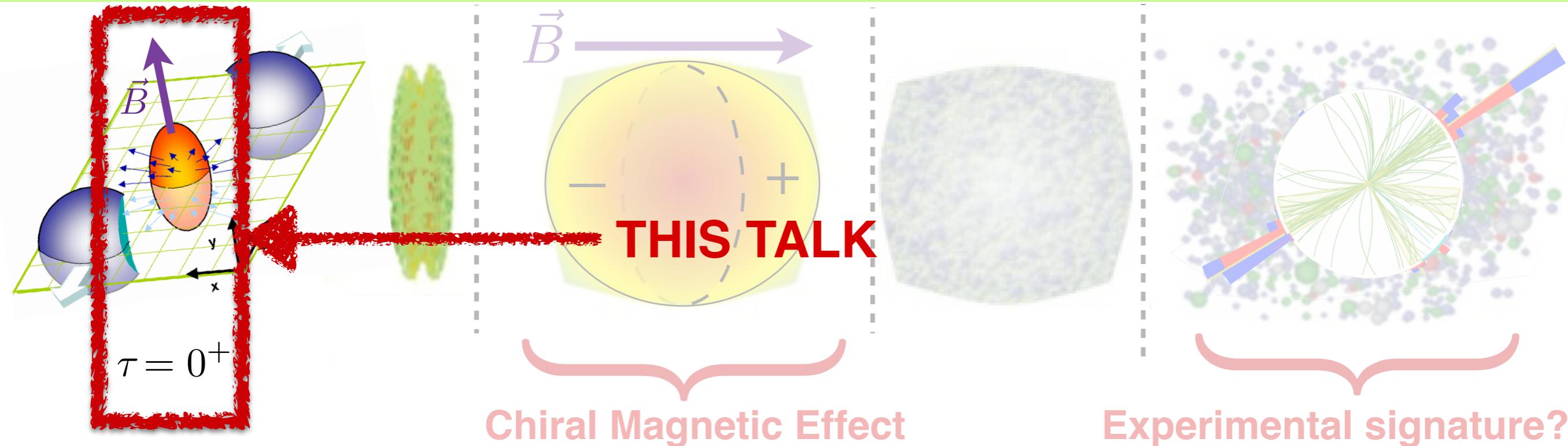
- Chirally-imbalanced matter in the presence of a background magnetic field will induce a separation of positive and negative charges (**Chiral Magnetic Effect**).

CP violation in the Quark Gluon Plasma: the Chiral Magnetic Effect



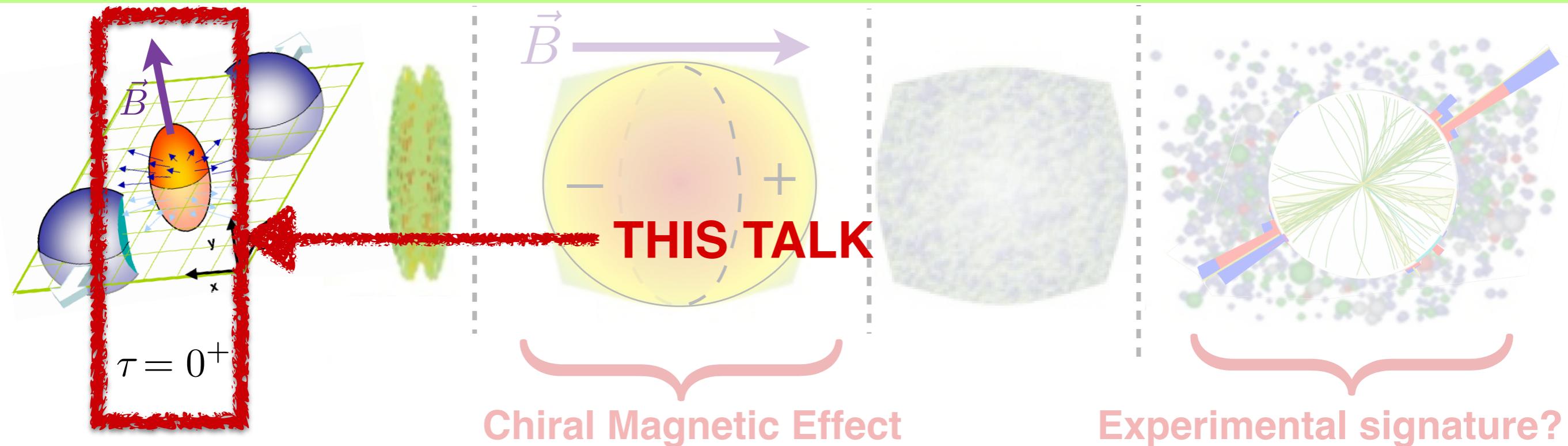
- Off-central HICs give rise to **large background electromagnetic fields**.
- Parity and Charge-Parity violating fluctuations are expected to happen with relatively high probability in the QGP.
- Chirally-imbalanced matter in the presence of a background magnetic field will induce a separation of positive and negative charges (**Chiral Magnetic Effect**).
- The search for signatures of this and other anomalous transport effects is affected by the presence of **large background effects**.

CP violation in the Quark Gluon Plasma: the Chiral Magnetic Effect



- In order to better pinpoint the CME we need a good theoretical description of axial charge production at early times.
- **No theoretical agreement** on the description of this region
- Large degree of **phenomenological modeling**
- **Source of uncertainty** for parameters used in Hydro models

CP violation in the Quark Gluon Plasma: the Chiral Magnetic Effect



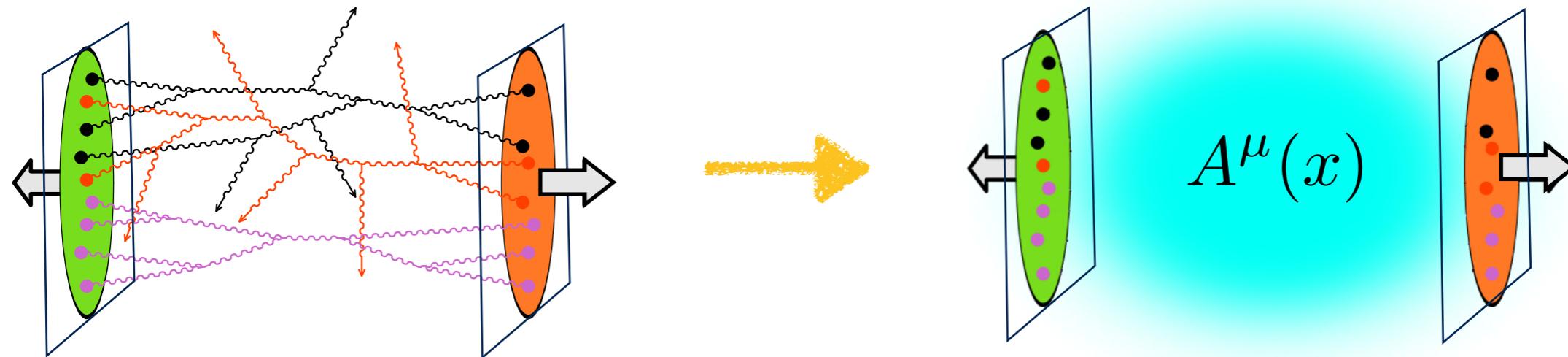
- In order to better pinpoint the CME we need a good theoretical description of axial charge production at early times.
- **No theoretical agreement** on the description of this region
- Large degree of **phenomenological modeling**
- **Source of uncertainty** for parameters used in Hydro models
- We provide **first-principles analytical calculations** of: $\langle \dot{\nu}(x_\perp) \rangle$

$$\langle \dot{\nu}(x_\perp) \dot{\nu}(y_\perp) \rangle$$

The Color Glass Condensate

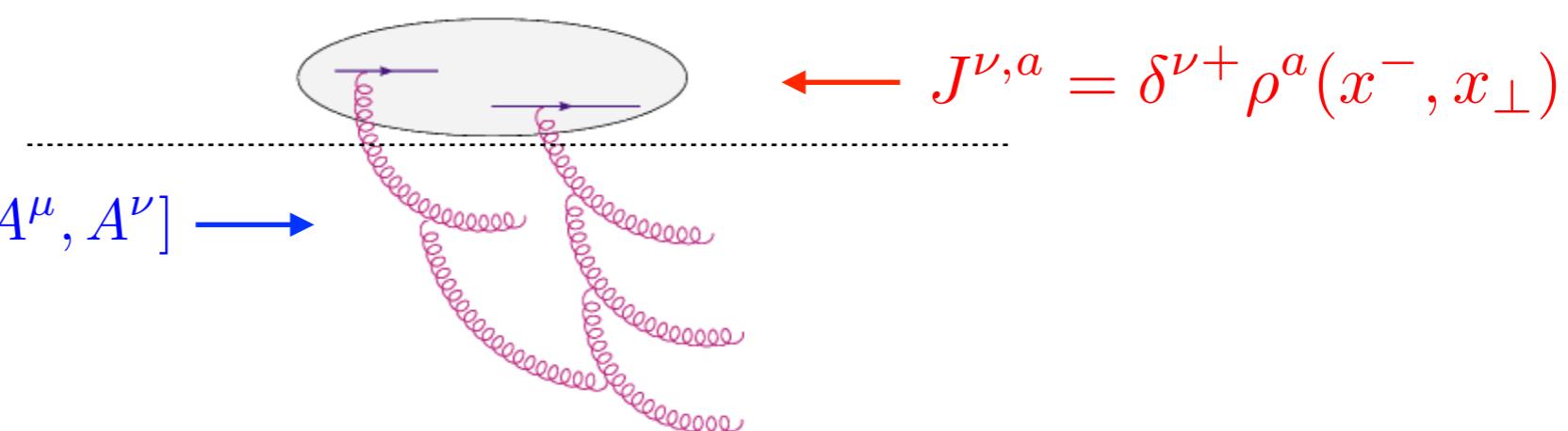
Color Glass Condensate: McLerran-Venugopalan model

- We use an approximation of QCD for high gluon densities where we replace the gluons with a **classical field** generated by the valence quarks



- Dynamics of the field described by **Yang-Mills** classical equations:

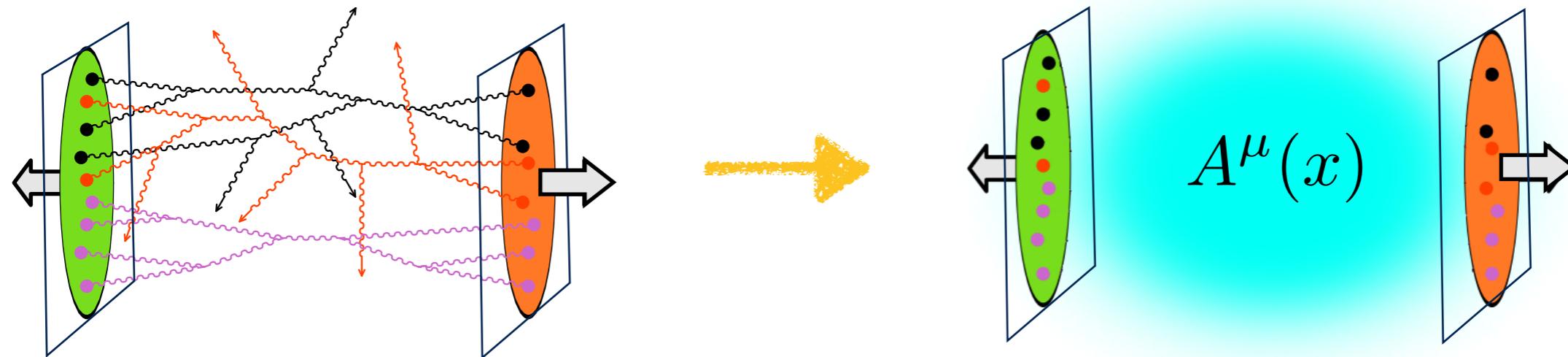
$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho^a(x) t^a$$



$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig [A^\mu, A^\nu] \longrightarrow$$

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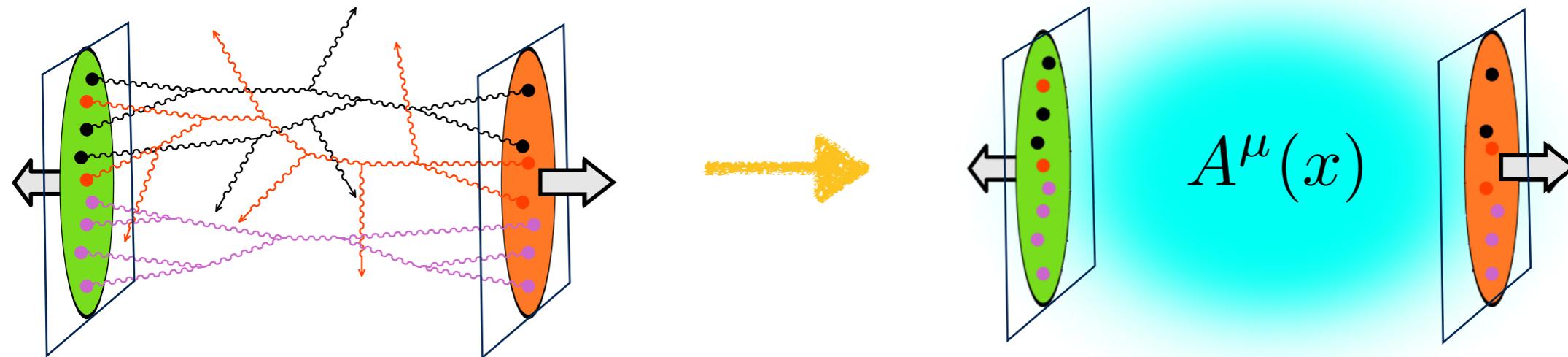
$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho^a(x)t^a$$

- Calculation of observables: **average** over background classical fields

$$\langle \mathcal{O}[\rho] \rangle = \int [d\rho] \exp \left\{ - \int dx \text{Tr} [\rho^2] \right\} \mathcal{O}[\rho]$$

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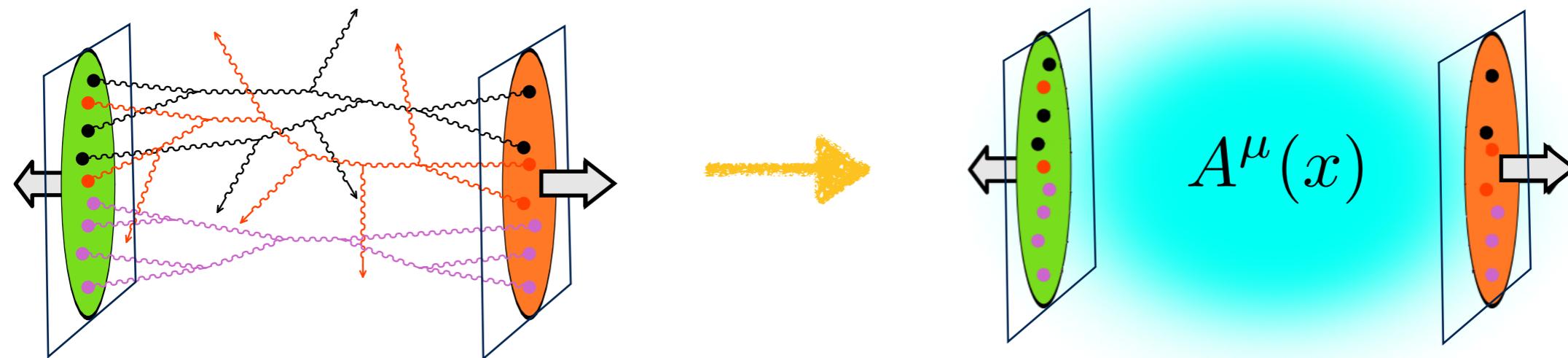
$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho^a(x) t^a$$

- Calculation of observables: **average** over background classical fields
- Basic building block: **2-point correlator (McLerran-Venugopalan)**

$$\langle \rho^a(x^-, x_\perp) \rho^b(y^-, y_\perp) \rangle = \mu^2(x^-) \delta^{ab} \delta(x^- - y^-) \delta^{(2)}(x_\perp - y_\perp)$$

Color Glass Condensate: McLerran-Venugopalan model

- We use an approximation of QCD for high gluon densities where we replace the gluons with a **classical field** generated by the valence quarks



- Dynamics of the field described by **Yang-Mills** classical equations:

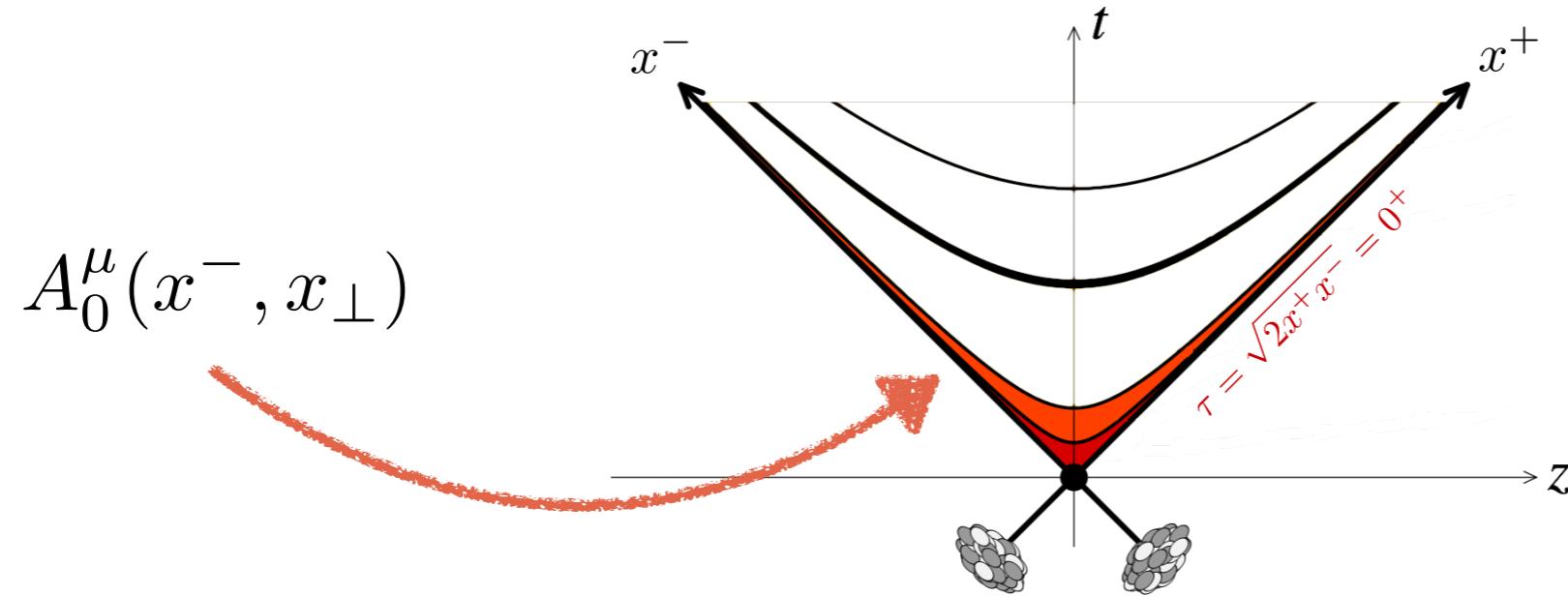
$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho^a(x) t^a$$

- Calculation of observables: **average** over background classical fields
- Basic building block: **(generalized) 2-point correlator**

$$\langle \rho^a(x^-, x_\perp) \rho^b(y^-, y_\perp) \rangle = \mu^2(x^-) h(b_\perp) \delta^{ab} \delta(x^- - y^-) f(x_\perp - y_\perp)$$

Steps for the calculation

1) Calculate the gluon fields at early times in a HIC



2) Build the divergence of the Chern-Simons current

$$\dot{\nu}_0(x_\perp) = -\frac{1}{8}\epsilon^{\mu\nu\rho\sigma}\text{Tr}\{F_{\mu\nu}F_{\rho\sigma}\}_0$$

3) Compute Glasma properties

$$\langle \dot{\nu}_0(x_\perp) \rangle = \int [d\rho_1] W_1[\rho_1] [d\rho_2] W_2[\rho_2] \dot{\nu}_0(x_\perp)[\rho_1, \rho_2]$$

$$\langle \dot{\nu}_0(x_\perp) \dot{\nu}_0(y_\perp) \rangle = \int [d\rho_1] W_1[\rho_1] [d\rho_2] W_2[\rho_2] \dot{\nu}_0(x_\perp) \dot{\nu}_0(y_\perp)[\rho_1, \rho_2]$$

Calculation of the Gluon fields

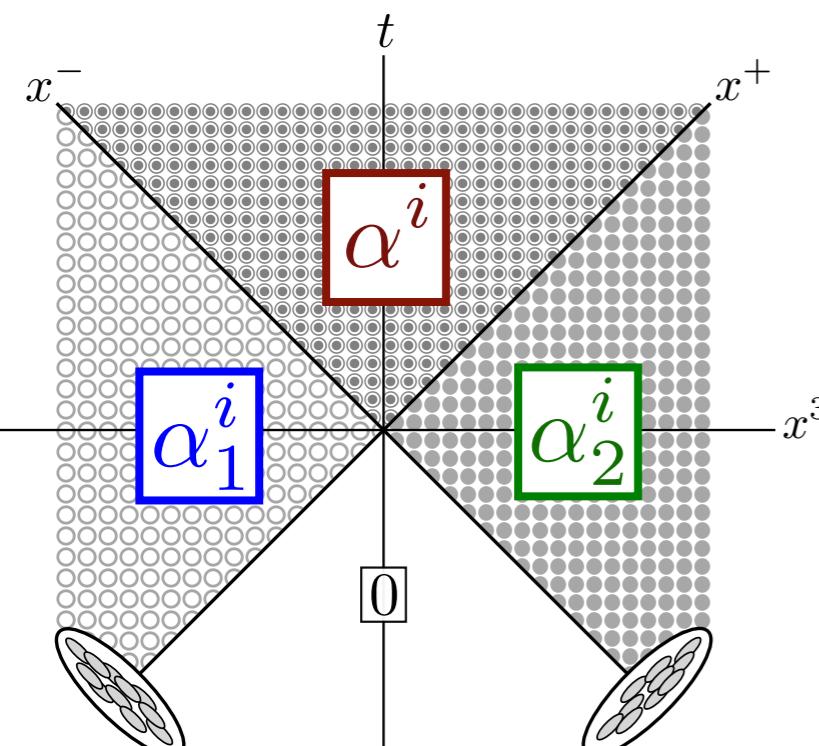
The gluon fields at $\tau = 0^+$ in HICs

[Kovner, McLerran, Weigert 1998]

$$[D_\mu, F^{\mu\nu}] = J_1^\nu + J_2^\nu$$

$$\mathbf{J}_1^\nu = \rho_1(\mathbf{x}_\perp) \delta(\mathbf{x}^-) \delta^{\nu+}$$

$$\mathbf{J}_2^\nu = \rho_2(\mathbf{x}_\perp) \delta(\mathbf{x}^+) \delta^{\nu-}$$



[1, 2] Single nucleus solution

$$A_1^\pm = 0$$

$$A_1^i = \theta(x^-) \int_{-\infty}^{\infty} dz^- \frac{\partial^i \tilde{\rho}_1^a(z^-, z_\perp)}{\nabla^2} U_1^{ab}(z^-, x_\perp) t^b \equiv \theta(x^-) \underline{\alpha_1^{i,b}(x_\perp) t^b}$$

$$U_1^{ab}(x^-, x_\perp) = P^- \exp \left\{ -ig \int_{x_0^-}^{x^-} dz^- \frac{1}{\nabla^2} \tilde{\rho}_1(z^-, x_\perp) \right\}^{ab}$$

[3] Forward light cone $\tau = 0^+$

$$A^\pm = \pm x^\pm \alpha(\tau = 0^+, x_\perp)$$

$$A^i = \alpha^i(\tau = 0^+, x_\perp)$$

$$\alpha^i(\tau = 0^+, x_\perp) = \alpha_1^i(x_\perp) + \alpha_2^i(x_\perp)$$

$$\alpha(\tau = 0^+, x_\perp) = \frac{ig}{2} [\alpha_1^i(x_\perp), \alpha_2^i(x_\perp)]$$

- We can obtain the divergence of the Chern-Simons current as:

$$\begin{aligned} \dot{\nu}_0 &= -g^2 \delta^{ij} \epsilon^{kl} \text{Tr}\{[\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\} \\ &= -g^2 \delta^{ij} \epsilon^{kl} \alpha_1^{i,a} \alpha_2^{j,b} \alpha_1^{k,c} \alpha_2^{l,d} \text{Tr}\{[t^a, t^b][t^c, t^d]\} \\ &= \frac{g^2}{2} \delta^{ij} \epsilon^{kl} f^{abn} f^{cdn} \alpha_1^{i,a} \alpha_2^{j,b} \alpha_1^{k,c} \alpha_2^{l,d} \end{aligned}$$

Correlators of the divergence of the Chern-Simons current at $\tau = 0^+$

$$\langle \dot{\nu}(x_\perp) \rangle$$

- For the 1-point correlator of $\dot{\nu}(x_\perp)$:

$$\begin{aligned}\langle \dot{\nu}(\tau = 0^+, x_\perp) \rangle &\equiv \langle \dot{\nu}_0(x_\perp) \rangle = -g^2 \delta^{ij} \epsilon^{kl} \langle \text{Tr}\{[\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\} \rangle \\ &= -g^2 \delta^{ij} \epsilon^{kl} \langle \alpha_1^{i,a} \alpha_2^{j,b} \alpha_1^{k,c} \alpha_2^{l,d} \rangle \text{Tr}\{[t^a, t^b][t^c, t^d]\} \\ &= \frac{g^2}{2} \delta^{ij} \epsilon^{kl} f^{abn} f^{cdn} \underbrace{\langle \alpha_1^{i,a} \alpha_1^{k,c} \rangle_1 \langle \alpha_2^{j,b} \alpha_2^{l,d} \rangle_2}_{\text{Building block of the calculation}}\end{aligned}$$

$$\langle \dot{\nu}(x_\perp) \rangle$$

- For the 1-point correlator of $\dot{\nu}(x_\perp)$:

$$\begin{aligned}
\langle \dot{\nu}(\tau = 0^+, x_\perp) \rangle &\equiv \langle \dot{\nu}_0(x_\perp) \rangle = -g^2 \delta^{ij} \epsilon^{kl} \langle \text{Tr}\{[\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\} \rangle \\
&= -g^2 \delta^{ij} \epsilon^{kl} \langle \alpha_1^{i,a} \alpha_2^{j,b} \alpha_1^{k,c} \alpha_2^{l,d} \rangle \text{Tr}\{[t^a, t^b][t^c, t^d]\} \\
&= \frac{g^2}{2} \delta^{ij} \epsilon^{kl} f^{abn} f^{cdn} \underbrace{\langle \alpha_1^{i,a} \alpha_1^{k,c} \rangle_1}_{\propto \delta^{ik}} \underbrace{\langle \alpha_2^{j,b} \alpha_2^{l,d} \rangle_2}_{\propto \delta^{jl}} = 0
\end{aligned}$$

In accordance to the fact that there is no global CP violation in the process.

$\langle \dot{\nu}(x_\perp) \rangle$ and $\langle \dot{\nu}(x_\perp) \dot{\nu}(y_\perp) \rangle$

- For the 1-point correlator of $\dot{\nu}(x_\perp)$:

$$\begin{aligned}
\langle \dot{\nu}(\tau = 0^+, x_\perp) \rangle &\equiv \langle \dot{\nu}_0(x_\perp) \rangle = -g^2 \delta^{ij} \epsilon^{kl} \langle \text{Tr}\{[\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\} \rangle \\
&= -g^2 \delta^{ij} \epsilon^{kl} \langle \alpha_1^{i,a} \alpha_2^{j,b} \alpha_1^{k,c} \alpha_2^{l,d} \rangle \text{Tr}\{[t^a, t^b][t^c, t^d]\} \\
&= \frac{g^2}{2} \delta^{ij} \epsilon^{kl} f^{abn} f^{cdn} \underbrace{\langle \alpha_1^{i,a} \alpha_1^{k,c} \rangle_1}_{\propto \delta^{ik}} \underbrace{\langle \alpha_2^{j,b} \alpha_2^{l,d} \rangle_2}_{\propto \delta^{jl}} = 0
\end{aligned}$$

In accordance to the fact that there is no global CP violation in the process.

- For the 2-point correlator:

$$\langle \dot{\nu}_0(x_\perp) \dot{\nu}_0(y_\perp) \rangle = \frac{g^4}{4} \epsilon^{kl} \epsilon^{k'l'} f^{abn} f^{cdn} f^{a'b'm} f^{c'd'm} \langle \alpha_x^{i,a} \alpha_x^{k,c} \alpha_y^{i',a'} \alpha_y^{k',c'} \rangle_1 \langle \alpha_x^{i,b} \alpha_x^{l,d} \alpha_y^{i',b'} \alpha_y^{l',d'} \rangle_2$$

**Building block of
the calculation**

REMINDER:

$$\alpha_1^{i,b}(x_\perp) = \int_{-\infty}^{\infty} dz^- \frac{\partial^i \tilde{\rho}_1^a(z^-, z_\perp)}{\nabla^2} U_1^{ab}(z^-, x_\perp)$$

$\langle \dot{\nu}(x_\perp) \rangle$ and $\langle \dot{\nu}(x_\perp) \dot{\nu}(y_\perp) \rangle$

- For the 1-point correlator of $\dot{\nu}(x_\perp)$:

$$\begin{aligned}
\langle \dot{\nu}(\tau = 0^+, x_\perp) \rangle &\equiv \langle \dot{\nu}_0(x_\perp) \rangle = -g^2 \delta^{ij} \epsilon^{kl} \langle \text{Tr}\{[\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\} \rangle \\
&= -g^2 \delta^{ij} \epsilon^{kl} \langle \alpha_1^{i,a} \alpha_2^{j,b} \alpha_1^{k,c} \alpha_2^{l,d} \rangle \text{Tr}\{[t^a, t^b][t^c, t^d]\} \\
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\end{aligned}$$

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- For the 2-point correlator:

$$\langle \dot{\nu}_0(x_\perp) \dot{\nu}_0(y_\perp) \rangle = \frac{g^4}{4} \epsilon^{kl} \epsilon^{k'l'} f^{abn} f^{cdn} f^{a'b'm} f^{c'd'm} \underbrace{\langle \alpha_x^{i,a} \alpha_x^{k,c} \alpha_y^{i',a'} \alpha_y^{k',c'} \rangle_1}_{\text{red bracket}} \langle \alpha_x^{i,b} \alpha_x^{l,d} \alpha_y^{i',b'} \alpha_y^{l',d'} \rangle_2$$

$$\langle \alpha^{i,a}(x_\perp) \alpha^{k,c}(x_\perp) \alpha^{i',a'}(y_\perp) \alpha^{k',c'}(y_\perp) \rangle = \int_{-\infty}^{\infty} dz^- dw^- dz^{-'} dw^{-'} \left\langle \frac{\partial^i \tilde{\rho}^e(z^-, x_\perp)}{\nabla^2} U^{ea}(z^-, x_\perp) \right. \\
\left. \frac{\partial^k \tilde{\rho}^f(w^-, x_\perp)}{\nabla^2} U^{fc}(w^-, x_\perp) \frac{\partial^{i'} \tilde{\rho}^{e'}(z^{-'}, y_\perp)}{\nabla^2} U^{e'a'}(z^{-'}, y_\perp) \frac{\partial^{k'} \tilde{\rho}^{f'}(w^{-'}, y_\perp)}{\nabla^2} U^{f'c'}(w^{-'}, y_\perp) \right\rangle.$$

$$\langle \dot{\nu}(x_\perp) \dot{\nu}(y_\perp) \rangle$$

$$\begin{aligned}
\langle \dot{\nu}(x_\perp) \dot{\nu}(y_\perp) \rangle = & \frac{16A^4 - B^4}{g^4 \Gamma^4 N_c^2} \left(\left[\frac{N_c^6 + 2N_c^4 - 19N_c^2 + 8}{2(N_c^2 - 1)^2} - 2 \frac{N_c^6 - 3N_c^4 - 26N_c^2 + 16}{N_c^4 - 5N_c^2 + 4} e^{-\frac{Q_{s1}^2 r^2}{4}} \right. \right. \\
& + (N_c^2 - 1) \left(1 - e^{-\frac{Q_{s1}^2 r^2}{4}} \left(1 + \frac{Q_{s1}^2 r^2}{4} \right) \right) \left(1 - e^{-\frac{Q_{s2}^2 r^2}{4}} \left(1 + \frac{Q_{s2}^2 r^2}{4} \right) \right) \\
& + \frac{r^4}{4} Q_{s1}^2 Q_{s2}^2 - 2r^2 Q_{s1}^2 \left(1 - e^{-\frac{Q_{s2}^2 r^2}{4}} \right) + 2 \frac{(N_c^2 - 8)(N_c^2 - 1)(N_c^2 + 4)}{(N_c^2 - 4)^2} e^{-\frac{(Q_{s1}^2 + Q_{s2}^2)r^2}{4}} \\
& + \frac{(N_c - 1)(N_c + 3)N_c^3}{2(N_c + 1)^2(N_c + 2)^2} \left(\frac{N_c}{2} e^{-\frac{(N_c + 1)r^2 Q_{s2}^2}{2N_c}} + (N_c + 2) - 2(N_c + 1)e^{-\frac{Q_{s2}^2 r^2}{4}} \right) e^{-\frac{(N_c + 1)r^2 Q_{s1}^2}{2N_c}} \\
& + \frac{(N_c + 1)(N_c - 3)N_c^3}{2(N_c - 1)^2(N_c - 2)^2} \left(\frac{N_c}{2} e^{-\frac{(N_c - 1)r^2 Q_{s2}^2}{2N_c}} + (N_c - 2) - 2(N_c - 1)e^{-\frac{Q_{s2}^2 r^2}{4}} \right) e^{-\frac{(N_c - 1)r^2 Q_{s1}^2}{2N_c}} \Big] \\
& \left. + [1 \leftrightarrow 2] \right)
\end{aligned}$$

We have: $\Gamma(r_\perp)_{\text{MV}} \approx \frac{r^2}{8\pi} \ln \left(\frac{4}{m^2 r^2} \right)$, $A(r_\perp)_{\text{MV}} \approx \frac{1}{8\pi} \ln \left(\frac{4}{m^2 r^2} \right)$, $B(r_\perp)_{\text{MV}} = \frac{1}{4\pi}$.

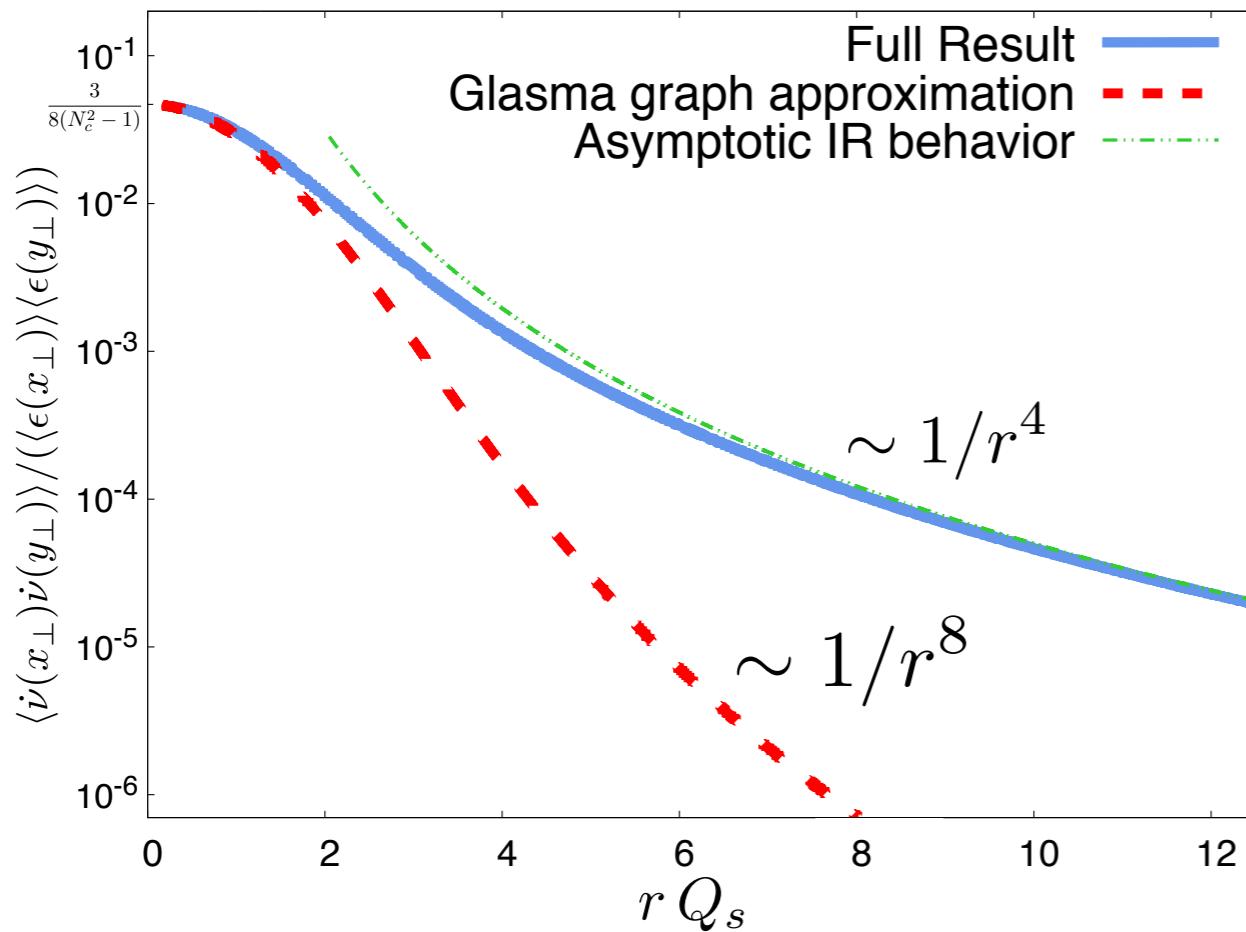
And the saturation scale: $\frac{r^2 Q_s^2}{4} = g^2 \frac{N_c}{2} \Gamma(r_\perp) \bar{\lambda}(b_\perp)$.

Comparison with the ‘Glasma Graph’ approximation

- Glasma Graph approximation [*Lappi & Schlichting 2018, Muller & Schaefer 2012*]. Assume Gaussian distribution of the produced gluon fields:

$$\begin{aligned}\langle \alpha^{i a}(x_\perp) \alpha^{k c}(x_\perp) \alpha^{i' a'}(y_\perp) \alpha^{k' c'}(y_\perp) \rangle_{\text{GG}} = & \langle \alpha^{i a}(x_\perp) \alpha^{k c}(x_\perp) \rangle \langle \alpha^{i' a'}(y_\perp) \alpha^{k' c'}(y_\perp) \rangle \\ & + \langle \alpha^{i a}(x_\perp) \alpha^{i' a'}(y_\perp) \rangle \langle \alpha^{k c}(x_\perp) \alpha^{k' c'}(y_\perp) \rangle \\ & + \langle \alpha^{i a}(x_\perp) \alpha^{k' c'}(y_\perp) \rangle \langle \alpha^{k c}(x_\perp) \alpha^{i' a'}(y_\perp) \rangle.\end{aligned}$$

- Agreement with full result in the $r \rightarrow 0$ limit. **Strong discrepancies** in the $r \rightarrow \infty$ limit



- This slowly decaying behavior could potentially have an impact in both physical interpretations and numerical results for any observable built from this quantity.

Phenomenology

- The expressions presented in this paper provide analytical insight on the early-time local fluctuations of axial charge density in the transverse plane.
- Our formulas can be directly applied in phenomenological studies of anomalous transport phenomena:

$$\frac{dN_5}{d^2x_\perp d\eta} = \int d\tau \tau \partial_\mu j_5^\mu = \frac{g^2 N_f}{2\pi^2} \int d\tau \tau \dot{\nu}(x)$$

$$\left. \frac{dN_5}{d^2x_\perp d\eta} \right|_{\tau=0^+} = \frac{\tau^2}{2} \frac{g^2 N_f}{2\pi^2} \dot{\nu}_0(x_\perp)$$

- From this expression one can straightforwardly relate $\langle \dot{\nu}(x_\perp) \dot{\nu}(y_\perp) \rangle$ to the correlation function:

$$\left\langle \frac{dN_5}{d^2x_\perp d\eta} \frac{dN_5}{d^2y_\perp d\eta} \right\rangle$$

which serves as the fundamental input for the Monte-Carlo modelization of initial conditions of axial charge density (required by hydro simulations).

Conclusions

Conclusions

- We have performed an exact analytical calculation of the covariance of the divergence of the Chern-Simons current of the **Glasma** at $\tau = 0^+$, in the framework of the **Color Glass Condensate**.
 - We find remarkably **long-range correlations** in comparison to naive expectations and previous calculations (such as the one performed in the Glasma Graph approximation).
 - *The modifications introduced in the MV model will prove useful in subsequent phenomenological applications of our results* (not discussed here - see paper).

→ This work presents a wide variety of applications and potential **follow-up projects**:



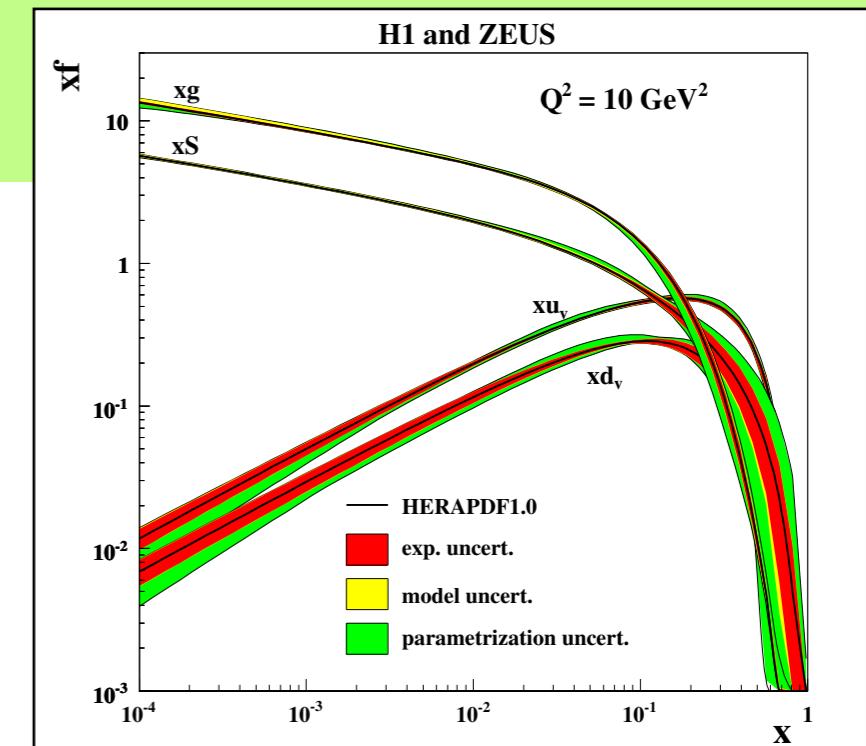
- Modelization of **initial axial charge transversal distributions** for hydro QGP simulations that aim at describing anomalous transport phenomena.
- Computation of **time evolution** of our result towards thermalization time $\tau \sim 1/Q_s$.

$$T^{\mu\nu} = T_0^{\mu\nu} + T_1^{\mu\nu}\tau + T_2^{\mu\nu}\tau^2 + \dots$$

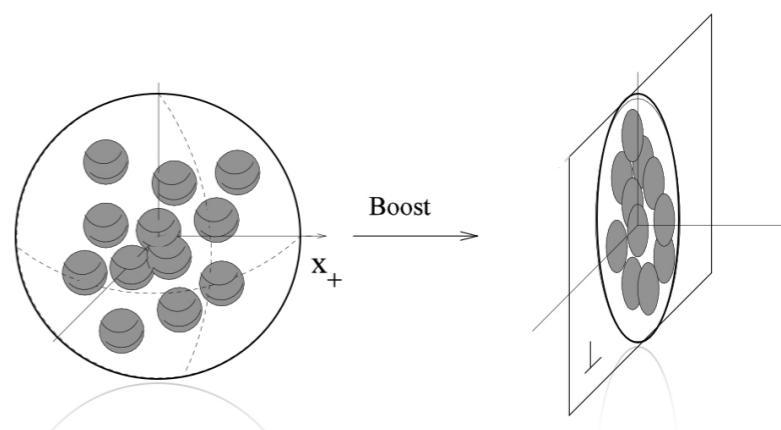
Back-up

Highly Energetic Heavy Ion Collisions

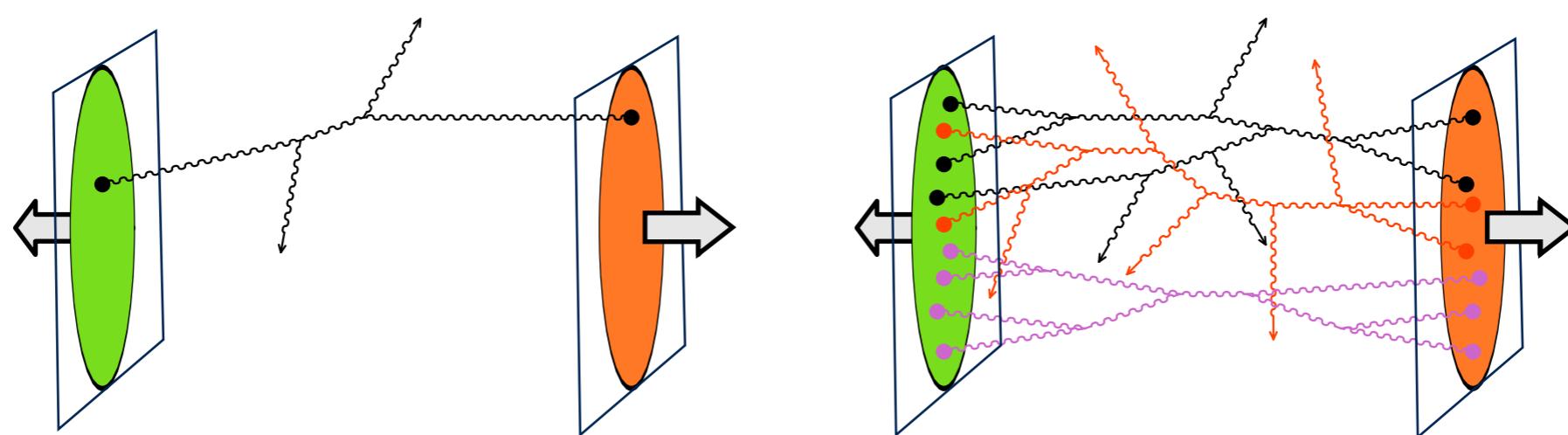
- At high energies (or equivalently, low x) the partonic content of nucleons is **vastly dominated by a high density of gluons**



- Relativistic kinematics: at high energies, the nuclei appear almost two-dimensional in the laboratory frame due to **Lorentz contraction**



- QCD becomes **non-linear** and **non-perturbative!**



$$\langle \alpha^{i,a}(x_\perp) \alpha^{j,b}(x_\perp) \rangle$$

- We momentarily take two different transverse positions:

$$\langle \alpha^{i,a}(x_\perp) \alpha^{j,b}(y_\perp) \rangle = \int_{-\infty}^{\infty} dz^- dz^{-'} \left\langle \frac{\partial^i \tilde{\rho}^{a'}(z^-, x_\perp)}{\nabla_\perp^2} \frac{\partial^j \tilde{\rho}^{b'}(z^{-'}, y_\perp)}{\nabla_\perp^2} \right\rangle \langle U^{a'a}(z^-, x_\perp) U^{b'b}(z^{-'}, y_\perp) \rangle$$

Luckily, in this case Wilson lines and (external) color source densities factorize

REMINDER:

$$\alpha_1^{i,b}(x_\perp) = \int_{-\infty}^{\infty} dz^- \frac{\partial^i \tilde{\rho}_1^a(z^-, z_\perp)}{\nabla^2} U_1^{ab}(z^-, x_\perp)$$

$$\langle \alpha^{i,a}(x_\perp) \alpha^{j,b}(x_\perp) \rangle$$

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$$\langle \alpha^{i,a}(x_\perp) \alpha^{j,b}(y_\perp) \rangle = \int_{-\infty}^{\infty} dz^- dz^- \left\langle \frac{\partial^i \tilde{\rho}^{a'}(z^-, x_\perp)}{\nabla_\perp^2} \frac{\partial^j \tilde{\rho}^{b'}(z'^-, y_\perp)}{\nabla_\perp^2} \right\rangle \langle U^{a'a}(z^-, x_\perp) U^{b'b}(z'^-, y_\perp) \rangle$$

$$\delta^{a'b'} \mu^2(x^-) \delta(x^- - y^-) \partial_x^i \partial_y^j L(x_\perp - y_\perp)$$

Where:

$$L(x_\perp - y_\perp) = \int d^2 z_\perp G(x_\perp - z_\perp) G(y_\perp - z_\perp).$$

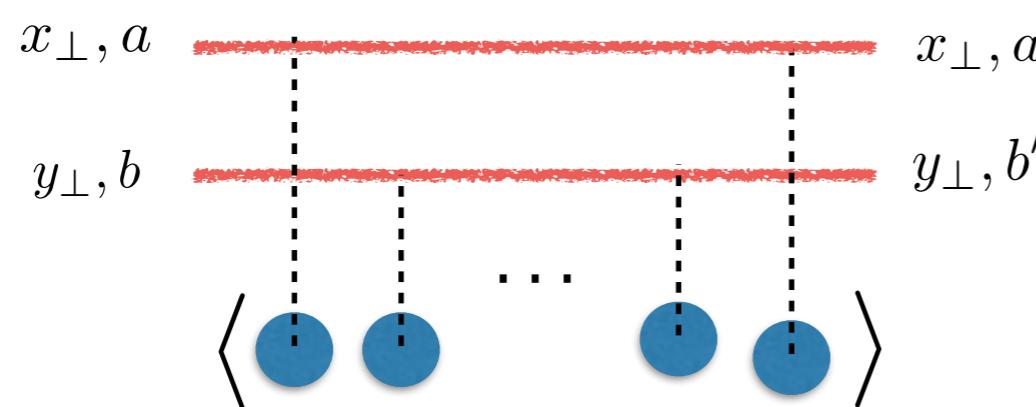
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$$\frac{\delta^{ab} \delta^{a'b'}}{N} \exp \left[-g^2 \frac{N}{2} \Gamma(x_\perp, y_\perp) \bar{\mu}^2(x^-) \right]$$

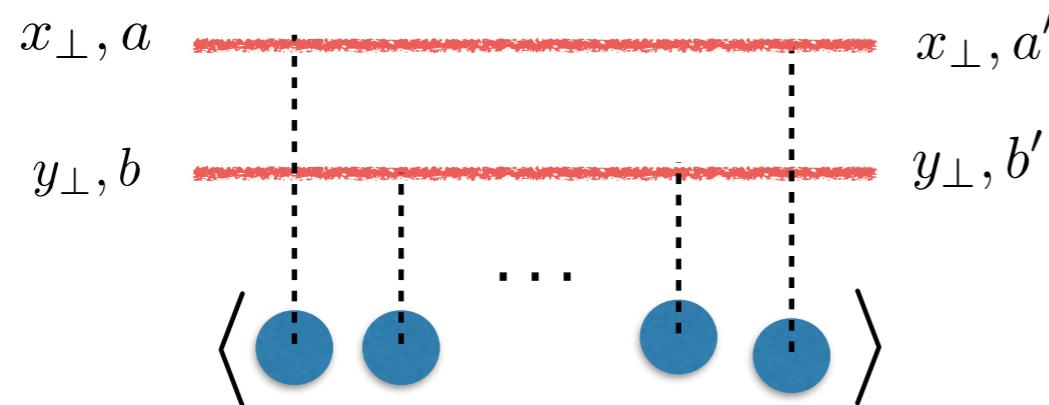
REMINDER:

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$$\frac{\delta^{ab} \delta^{a'b'}}{N} \exp \left[-g^2 \frac{N}{2} \Gamma(x_\perp, y_\perp) \bar{\mu}^2(x^-) \right]$$

Notation:

$$\bar{\mu}^2 = \int_{-\infty}^{\infty} dz^- \mu^2(z^-)$$

- We finally take the limit:

$$\langle \alpha^{i,a}(x_\perp) \alpha^{j,b}(x_\perp) \rangle = \frac{1}{2} \delta^{ab} \delta^{ij} \bar{\mu}^2 (\partial^2 L(0_\perp))$$

- In the MV model the factor $\partial^2 L(0_\perp)$ yields a **logarithmic UV divergence**:

$$\partial_\perp^2 L(0_\perp)_{\text{MV}} = \frac{1}{4\pi} \lim_{r \rightarrow 0} \left[\ln \left(\frac{m^2 r^2}{4} \right) \right]$$