Recent results on central exclusive production within the tensor-pomeron and vector-odderon approach

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Plan

1) Central Exclusive Production (CEP) in *pp* collisions

2) Model for high-energy soft reactions

(tensor-Pomeron and vector-Odderon)

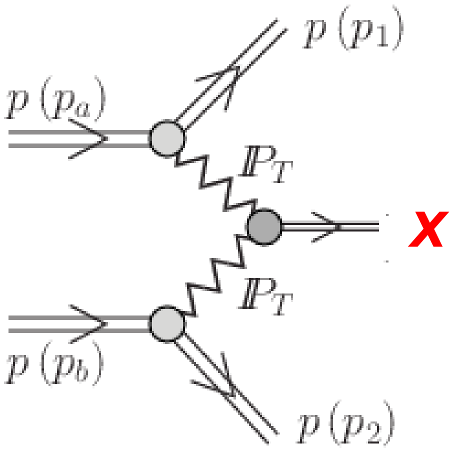
3) Recent results on CEP

- $p\,p \to p\,p\,(\phi \to K^+ K^-)$
- $p\,p \to p\,p\,(\phi \to \mu^+ \mu^-)$
- $p\,p \to p\,p\,K^+ K^- K^+ K^-$ (via intermediate $\phi\phi$ state)

4) Conclusions

Central Exclusive Production (CEP) in *pp* collisions

What can we hope to learn from CEP ?



(1) Properties and the coupling of the exchange objects
Pomeron (IP), Odderon (O), Reggeon (IR)
to the external protons and the system ***X***

(2) Properties of the system ***X***.
Search for and characterisation of resonances,
e.g. glueballs (gluonic bound states)

From the theory point of view the topics (1) - (2) are,
mainly nonperturbative, QCD problems.

We have to resort to models.

Tensor-Pomeron model for high-energy soft reactions

C. Ewerz, M. Maniatis, O. Nachtmann, Ann. Phys. 342 (2014) 31

The main feature of the model is that the Pomeron exchange is described as effective exchange of a symmetric rank 2 tensor:

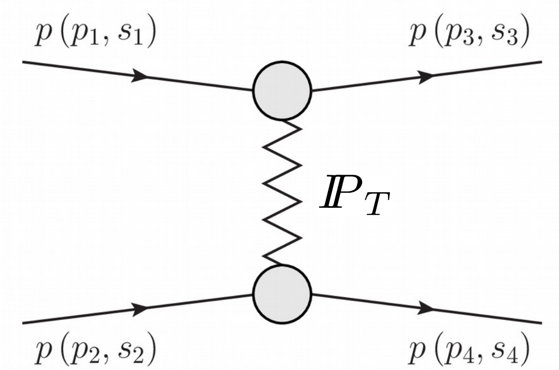
$$i\Delta^{(\mathbb{P})}_{\mu\nu,\kappa\lambda}(s,t) = \frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)(-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}t\,, \quad \alpha_{\mathbb{P}}(0) = 1.0808, \quad \alpha'_{\mathbb{P}} = 0.25\,\mathrm{GeV}^{-2}$$

$$i\Gamma^{(\mathbb{P}pp)}_{\mu\nu}(p',p) = -i3\beta_{\mathbb{P}NN}F_1\big((p'-p)^2\big)\left\{\frac{1}{2}[\gamma_\mu(p'+p)_\nu + \gamma_\nu(p'+p)_\mu] - \frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right\}$$

$$\beta_{\mathbb{P}NN} = 1.87\ \mathrm{GeV}^{-1}$$

***pp* elastic scattering** (helicity amplitudes)



$$\langle 2s_3, 2s_4|\mathcal{T}|2s_1, 2s_2\rangle = (-i)\bar{u}(p_3,s_3)i\Gamma^{(\mathbb{P}pp)}_{\mu\nu}(p_3,p_1)u(p_1,s_1)$$
$$\times i\Delta^{(\mathbb{P})\mu\nu,\kappa\lambda}(s,t)$$
$$\times\bar{u}(p_4,s_4)i\Gamma^{(\mathbb{P}pp)}_{\kappa\lambda}(p_4,p_2)u(p_2,s_2)$$

Only 5 out of 16 helicity amplitudes are independent, e.g.

$\phi_1(s,t) = \langle ++|\mathcal{T}|++\rangle$
$\phi_3(s,t) = \langle +-|\mathcal{T}|+-\rangle$ } helicity-conserving amplitudes

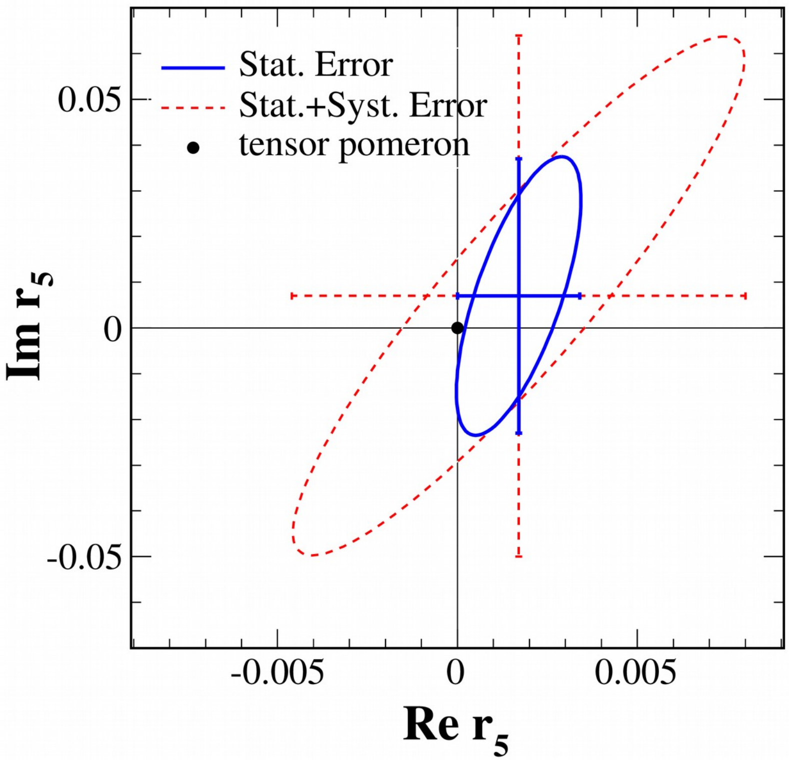
$\phi_5(s,t) = \langle ++|\mathcal{T}|+-\rangle$ single-helicity-flip amplitude

Tensor, vector, or scalar Pomeron ?

C. Ewerz, P. L., O. Nachtmann, A. Szczurek, ***Helicity in proton-proton elastic scattering and the spin structure of the pomeron***, PLB 763 (2016) 382

We choose our ansätze for three the effective *IP* propagators and *IPpp* couplings such that at high energies the ϕ_1 and ϕ_3 are the same for all three cases. This gives the same σ_{tot}^{pp}.
The Donnachie-Landshoff (DL) model treats the Pomeron as effective vector exchange and gives a phenomenologically successful fit to σ_{tot}^{pp} and $d\sigma/dt$.
Our ansätze are chosen such that ϕ_1 and ϕ_3 are as in the DL model.

- Vector exchange IP_V has C = -1 than $\sigma_{tot}^{pp} = -\sigma_{tot}^{\bar{p}p}$ (not a viable option).
- We are left with IP_T and IP_S (both correspond to C=+1 exchanges)
  To decide between them we turn to the **STAR experiment** [PLB 719 (2013)]
  which measured the single spin asymmetry A_N in polarised *pp* elastic scattering.



$$\sqrt{s} = 200 \text{ GeV} \quad 0.003 \leqslant |t| \leqslant 0.035 \text{ GeV}^2$$

$$r_5(s,t) = \frac{2m_p\, \phi_5(s,t)}{\sqrt{-t}\, \text{Im}[\phi_1(s,t)+\phi_3(s,t)]}$$

$$r_5^{I\!P_T}(s,t) = -\frac{m_p^2}{s}\left[i + \tan\left(\frac{\pi}{2}(\alpha_{I\!P}(t)-1)\right)\right]$$

$$r_5^{I\!P_T}(s,0) = (-0.28 - i2.20) \times 10^{-5}$$

$$r_5^{I\!P_S}(s,t) = -\frac{1}{2}\left[i + \tan\left(\frac{\pi}{2}(\alpha_{I\!P}(t)-1)\right)\right]$$

$$r_5^{I\!P_S}(s,0) = -0.064 - i0.500$$ Very far from data !

Only the tensor-Pomeron is compatible with the general rules of QFT and the STAR experimental result

Other applications of the tensor-Pomeron model

$\gamma\, p \to \pi^+ \pi^- p$ *Bolz, Ewerz, Maniatis, Nachtmann, Sauter, Schöning, JHEP 01 (2015) 151*

There will be interference between $\gamma\, p \to (\rho^0 \to \pi^+ \pi^-)\, p$ (IP exchange)
and $\gamma\, p \to (f_2(1270) \to \pi^+ \pi^-)\, p$ (Odderon exchange) processes
and as a consequence $\pi^+ \pi^-$ charge asymmetries.

Photoproduction and low *x* DIS *Britzger, Ewerz, Glazov, Nachtmann, Schmitt, arXiv:1901.08524*

A “vector Pomeron” cannot couple
in the total photoabsorption cross section $\sigma_{\gamma p}$.

Central Exclusive Production

$p\, p \to p\, p$ ***X*** *P.L., Nachtmann, Szczurek:*

X:	η, η', f_0	*Ann. Phys. 344 (2014) 301*
	ρ^0	*PRD91 (2015) 074023*
	$\pi^+ \pi^-$, f_0, f_2	*PRD93 (2016) 054015, and arXiv:1901.07788*
	$\pi^+ \pi^- \pi^+ \pi^-$, $\rho^0 \rho^0$	*PRD94 (2016) 034017*
	ρ^0 with proton diss.	*PRD95 (2017) 034036*
	$K^+ K^-$, f_0, f_2', ϕ	*PRD98 (2018) 014001*
	$p\bar{p}$	*PRD97 (2018) 094027*
	$K^+ K^- K^+ K^-$, $\phi\phi$	*PRD99 (2019) 094034*

For reactions investigated so far the tensor-Pomeron model works well

Searching for Odderon

- Odderon (C = -1 partner of Pomeron)
  L. Łukaszuk and B. Nicolescu, Lett. Nuovo Cim. 8 (1973) 405

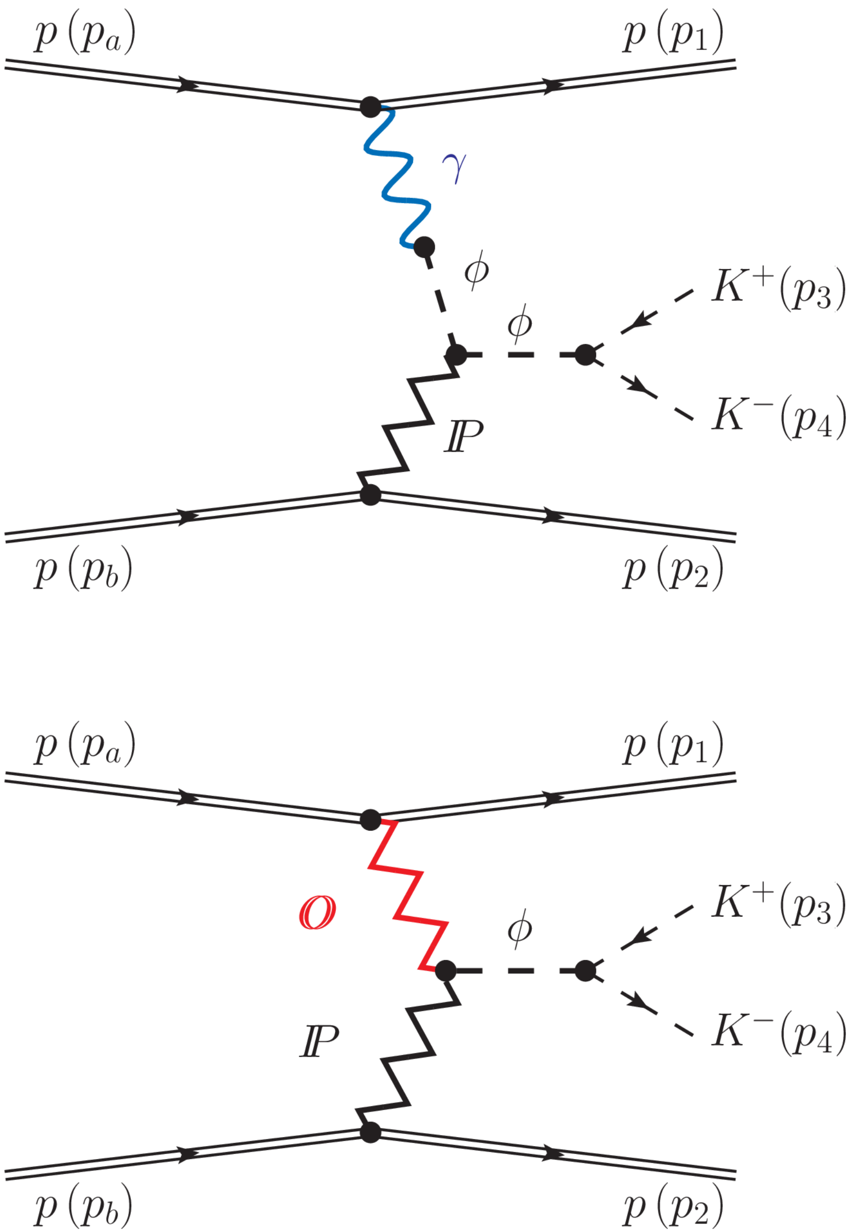
- A hint of the Odderon was seen in ISR results (PRL54 (1985)2180) as a small difference between the differential cross sections of elastic *pp* and $p\bar{p}$ scattering in the diffractive dip region at $\sqrt{s} = 53\ \mathrm{GeV}$.

- New data for *pp* elastic scattering at $\sqrt{s} = 2.76\ \mathrm{TeV}$ of TOTEM collaboration:
  Antchev *et al.* [TOTEM collaboration], arXiv:1812.04732, 1812.08610
  → suggestion of presence of Odderon
  → some last year analyses of Nicolescu *et al.*

- It is of great importance to study possible Odderon effects in other reactions than *pp* elastic scattering:
  → central J/Ψ production in high-energy *pp* and $p\bar{p}$ collisions
  A.Schafer, L.Mankiewicz, O.Nachtmann, PLB272 (1991) 419; A. Bzdak et al. PRD75 (2007) 094023
  → photoproduction of f_2(1270) and a_2(1320), exclusive neutral pseudoscalar mesons
  Berger, Donnachie, Dosch, Nachtmann, EPJC14 (2000) 673
  → photoproduction and electroproduction of heavy C = +1 quarkonia
  → the asymmetry in the fractional energy of charm versus anticharm jets, which is sensitive to Odderon-Pomeron interference Brodsky et al., PLB461 (1999) 114
  → observation of charge asymmetry in the $\pi^+ \pi^-$ production Ginzburg, Ivanov, Nikolaev, EPJC5 (2003) 02
  → ultraperipheral proton-ion collisions Harland-Lang et al., arXiv:1811.12705 Goncalves et al., arXiv: 1811.07622

- Nice review on Odderon physics: C. Ewerz, arXiv: 0306137

$pp \to pp\ (\phi \to K^+K^-)$

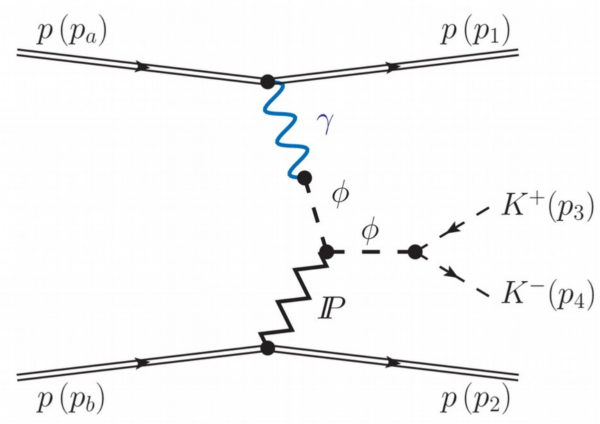


$pp \to pp\,(\phi \to K^+K^-)$

- We consider the 2 → 4 exclusive reaction

$$p(p_a,\lambda_a) + p(p_b,\lambda_b) \to p(p_1,\lambda_1) + \left[\phi(p_{34}) \to K^+(p_3) + K^-(p_4)\right] + p(p_2,\lambda_2)$$

where $p_{a,b}$, $p_{1,2}$ and $\lambda_{a,b}, \lambda_{1,2} = \pm\frac{1}{2}$ denote the four-momenta and helicities of the protons and $p_{3,4}$ denote the four-momenta of the K mesons, respectively



The kinematic variables are

$$p_{34} = p_3 + p_4\,, \quad q_1 = p_a - p_1\,, \quad q_2 = p_b - p_2\,,$$
$$s = (p_a + p_b)^2 = (p_1 + p_2 + p_{34})^2\,,$$
$$t_1 = q_1^2\,, \quad t_2 = q_2^2\,,$$
$$s_1 = (p_1 + p_{34})^2\,, \quad s_2 = (p_2 + p_{34})^2$$

- Born-level amplitude:

$$\mathcal{M}^{(\gamma\mathbb{P})}_{pp\to ppK^+K^-} = (-i)\bar{u}(p_1,\lambda_1) i\Gamma^{(\gamma pp)}_{\mu}(p_1,p_a)u(p_a,\lambda_a)$$
$$\times i\Delta^{(\gamma)\,\mu\sigma}(q_1)\, i\Gamma^{(\gamma\to\phi)}_{\sigma\nu}(q_1)\, i\Delta^{(\phi)\,\nu\rho_1}(q_1)\, i\Gamma^{(\mathbb{P}\phi\phi)}_{\rho_2\rho_1\alpha\beta}(p_{34},q_1)\, i\Delta^{(\phi)\,\rho_2\kappa}(p_{34})\, i\Gamma^{(\phi KK)}_{\kappa}(p_3,p_4)$$
$$\times i\Delta^{(\mathbb{P})\,\alpha\beta,\delta\eta}(s_2,t_2)\,\bar{u}(p_2,\lambda_2) i\Gamma^{(\mathbb{P}pp)}_{\delta\eta}(p_2,p_b)u(p_b,\lambda_b)$$

For the $\mathbb{P}\gamma$-exchange we have the same structure as for the above amplitude with $(p\,(p_a,\lambda_a), p\,(p_1,\lambda_1)) \leftrightarrow (p\,(p_b,\lambda_b), p\,(p_2,\lambda_2))$, $t_1 \leftrightarrow t_2$, $q_1 \leftrightarrow q_2$, $s_1 \leftrightarrow s_2$

$pp \to pp\ (\phi \to K^+K^-)$

- Our ansatz for the effective propagator and proton vertex function

$$i\Delta^{(\mathbb{P})}_{\mu\nu,\kappa\lambda}(s,t) = \frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)(-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$i\Gamma^{(\mathbb{P}pp)}_{\mu\nu}(p',p) = -i3\beta_{\mathbb{P}NN}F_1(t)\left\{\frac{1}{2}\left[\gamma_\mu(p'+p)_\nu + \gamma_\nu(p'+p)_\mu\right] - \frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right\}$$

where $\beta_{\mathbb{P}NN} = 1.87\ \mathrm{GeV}^{-1}$

$$\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}t$$

$$\alpha_{\mathbb{P}}(0) = 1.0808, \qquad \alpha'_{\mathbb{P}} = 0.25\,\mathrm{GeV}^{-2}$$

- For the $\mathbb{P}\phi\phi$ vertex we have

$$i\Gamma^{(\mathbb{P}\phi\phi)}_{\mu\nu\kappa\lambda}(k',k) = iF_M((k'-k)^2)\left[2a_{\mathbb{P}\phi\phi}\,\Gamma^{(0)}_{\mu\nu\kappa\lambda}(k',-k) - b_{\mathbb{P}\phi\phi}\,\Gamma^{(2)}_{\mu\nu\kappa\lambda}(k',-k)\right]$$

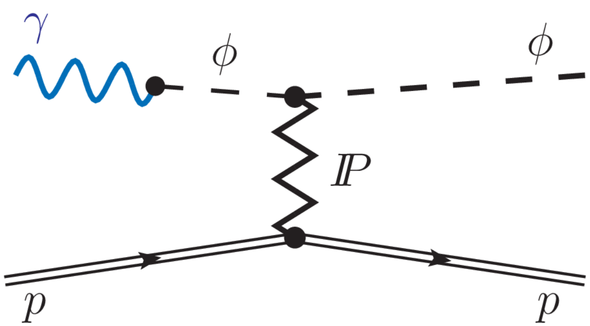
with two rank-four tensor functions: Ewerz, Maniatis, Nachtmann, Ann. Phys. 342 (2014) 31

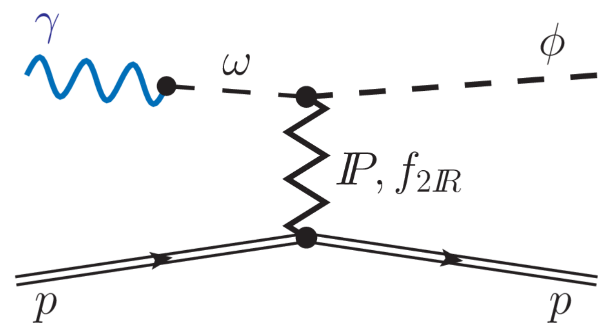
$$\Gamma^{(0)}_{\mu\nu\kappa\lambda}(k_1,k_2) = \left[(k_1\cdot k_2)g_{\mu\nu} - k_{2\mu}k_{1\nu}\right]\left[k_{1\kappa}k_{2\lambda} + k_{2\kappa}k_{1\lambda} - \frac{1}{2}(k_1\cdot k_2)g_{\kappa\lambda}\right]$$

$$\begin{aligned}\Gamma^{(2)}_{\mu\nu\kappa\lambda}(k_1,k_2) = {}& (k_1\cdot k_2)(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa}) + g_{\mu\nu}(k_{1\kappa}k_{2\lambda} + k_{2\kappa}k_{1\lambda}) \\ & - k_{1\nu}k_{2\lambda}g_{\mu\kappa} - k_{1\nu}k_{2\kappa}g_{\mu\lambda} - k_{2\mu}k_{1\lambda}g_{\nu\kappa} - k_{2\mu}k_{1\kappa}g_{\nu\lambda} \\ & - \left[(k_1\cdot k_2)g_{\mu\nu} - k_{2\mu}k_{1\nu}\right]g_{\kappa\lambda}\end{aligned}$$

- We take $F_1(t) = \dfrac{4m_p^2 - 2.79\,t}{(4m_p^2 - t)(1 - t/m_D^2)^2}\,, \quad F_M(t) = \dfrac{1}{1 - t/\Lambda^2_{0,\mathbb{P}\phi\phi}}$

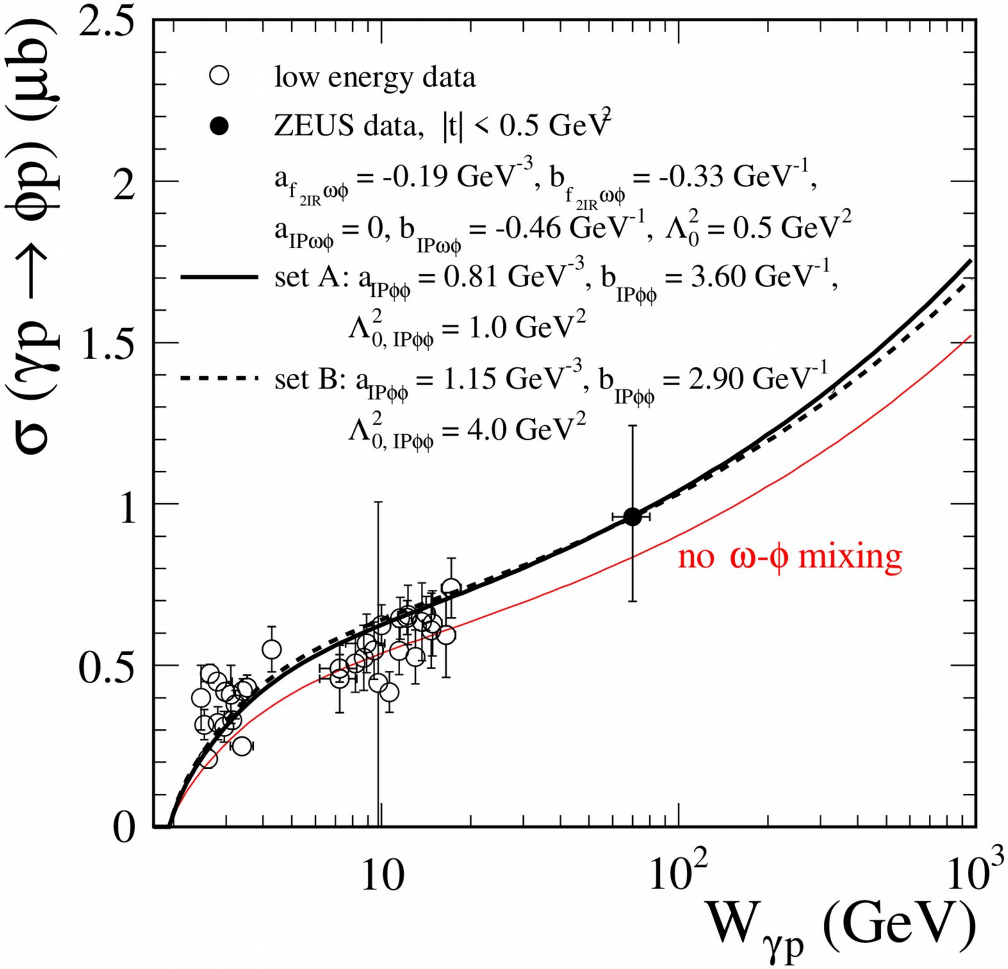
Photoproduction of $\phi(1020)$ meson

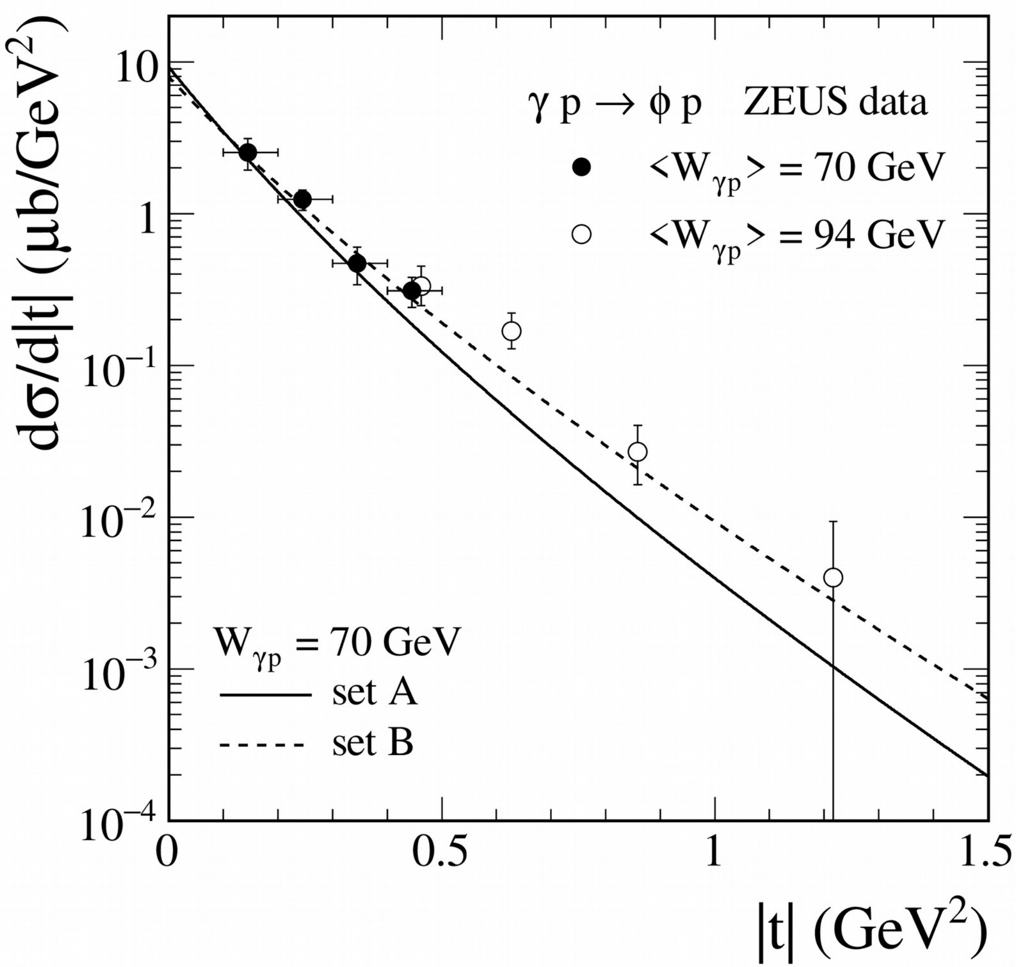




ω - ϕ mixing effect

$$b_{\mathbb{P}\omega\phi} = -b_{\mathbb{P}\omega\omega}\,\tan(\Delta\theta_V)$$
$$b_{\mathbb{P}\omega\omega} = 7.04\ \mathrm{GeV}^{-1}$$
$$\Delta\theta_V = 3.7^o$$

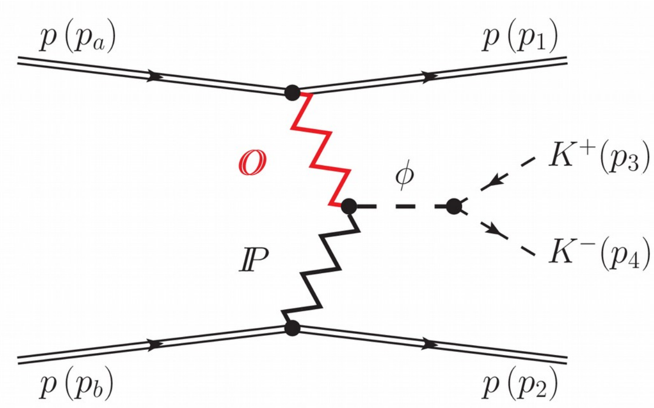




Odderon-Pomeron fusion process

Born-level amplitude:

$$\begin{aligned}\mathcal{M}^{(\mathbb{O}\mathbb{P})}_{pp\to ppK^+K^-} &= (-i)\bar{u}(p_1,\lambda_1) i\Gamma^{(\mathbb{O}pp)}_{\mu}(p_1,p_a)u(p_a,\lambda_a) \\ &\times i\Delta^{(\mathbb{O})\,\mu\rho_1}(s_1,t_1)\, i\Gamma^{(\mathbb{P}\mathbb{O}\phi)}_{\rho_1\rho_2\alpha\beta}(-q_1,p_{34})\, i\Delta^{(\phi)\,\rho_2\kappa}(p_{34})\, i\Gamma^{(\phi KK)}_{\kappa}(p_3,p_4) \\ &\times i\Delta^{(\mathbb{P})\,\alpha\beta,\delta\eta}(s_2,t_2)\,\bar{u}(p_2,\lambda_2) i\Gamma^{(\mathbb{P}pp)}_{\delta\eta}(p_2,p_b)u(p_b,\lambda_b)\end{aligned}$$



Our ansatz for the effective propagator of C = -1 Odderon:

$$i\Delta^{(\mathbb{O})}_{\mu\nu}(s,t) = -ig_{\mu\nu}\frac{\eta_{\mathbb{O}}}{M_0^2}\,(-is\alpha'_{\mathbb{O}})^{\alpha_{\mathbb{O}}(t)-1}$$

$$i\Gamma^{(\mathbb{O}pp)}_{\mu}(p',p) = -i3\beta_{\mathbb{O}pp}\,M_0\,F_1((p'-p)^2)\,\gamma_\mu$$

where $\eta_{\mathbb{O}}$ is a parameter with value $\eta_{\mathbb{O}} = \pm 1$; $M_0 = 1$ GeV is inserted for dimensional reasons; $\alpha_{\mathbb{O}}(t)$ is the odderon trajectory, assumed to be linear in t:

$$\alpha_{\mathbb{O}}(t) = \alpha_{\mathbb{O}}(0) + \alpha'_{\mathbb{O}}\,t$$

The odderon parameters are not yet known from experiment.
In our calculations we shall choose as default values

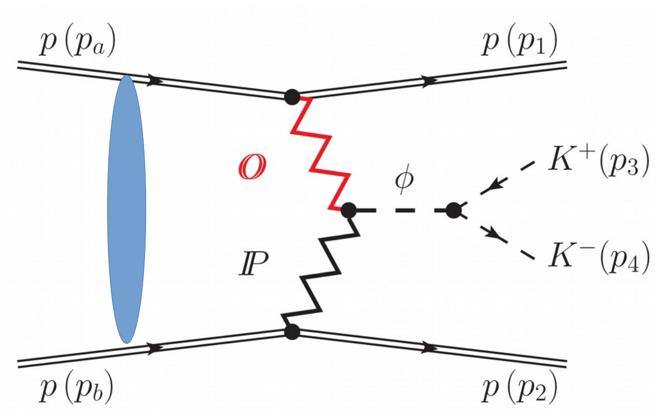
$$\alpha_{\mathbb{O}}(0) = 1.05\,,\quad \alpha'_{\mathbb{O}} = 0.25\ \text{GeV}^{-2}\,,\quad \eta_{\mathbb{O}} = -1\,,\quad \beta_{\mathbb{O}pp} = 0.1\times\beta_{\mathbb{P}NN} \simeq 0.18\ \text{GeV}^{-1}$$

Odderon-Pomeron fusion process

For the $\mathbb{P}\mathbb{O}\phi$ vertex we use an ansatz analogous to the $\mathbb{P}\phi\phi$ vertex:

$$i\Gamma^{(\mathbb{P}\mathbb{O}\phi)}_{\rho_1\rho_2\alpha\beta}(-q_1,p_{34}) = i\left[2\,a_{\mathbb{P}\mathbb{O}\phi}\,\Gamma^{(0)}_{\rho_2\rho_1\alpha\beta}(p_{34},-q_1) - b_{\mathbb{P}\mathbb{O}\phi}\,\Gamma^{(2)}_{\rho_2\rho_1\alpha\beta}(p_{34},-q_1)\right] \times F_M(q_2^2)\,F_M(q_1^2)\,F^{(\phi)}(p_{34}^2)$$

The coupling parameters $a_{\mathbb{P}\mathbb{O}\phi}$, $b_{\mathbb{P}\mathbb{O}\phi}$ and the cut-off parameter $\Lambda^2_{0,\,\mathbb{P}\mathbb{O}\phi}$ in $F_M(q^2) = \frac{1}{1-t/\Lambda^2_{0,\mathbb{P}\mathbb{O}\phi}}$ could be adjusted to experimental data.



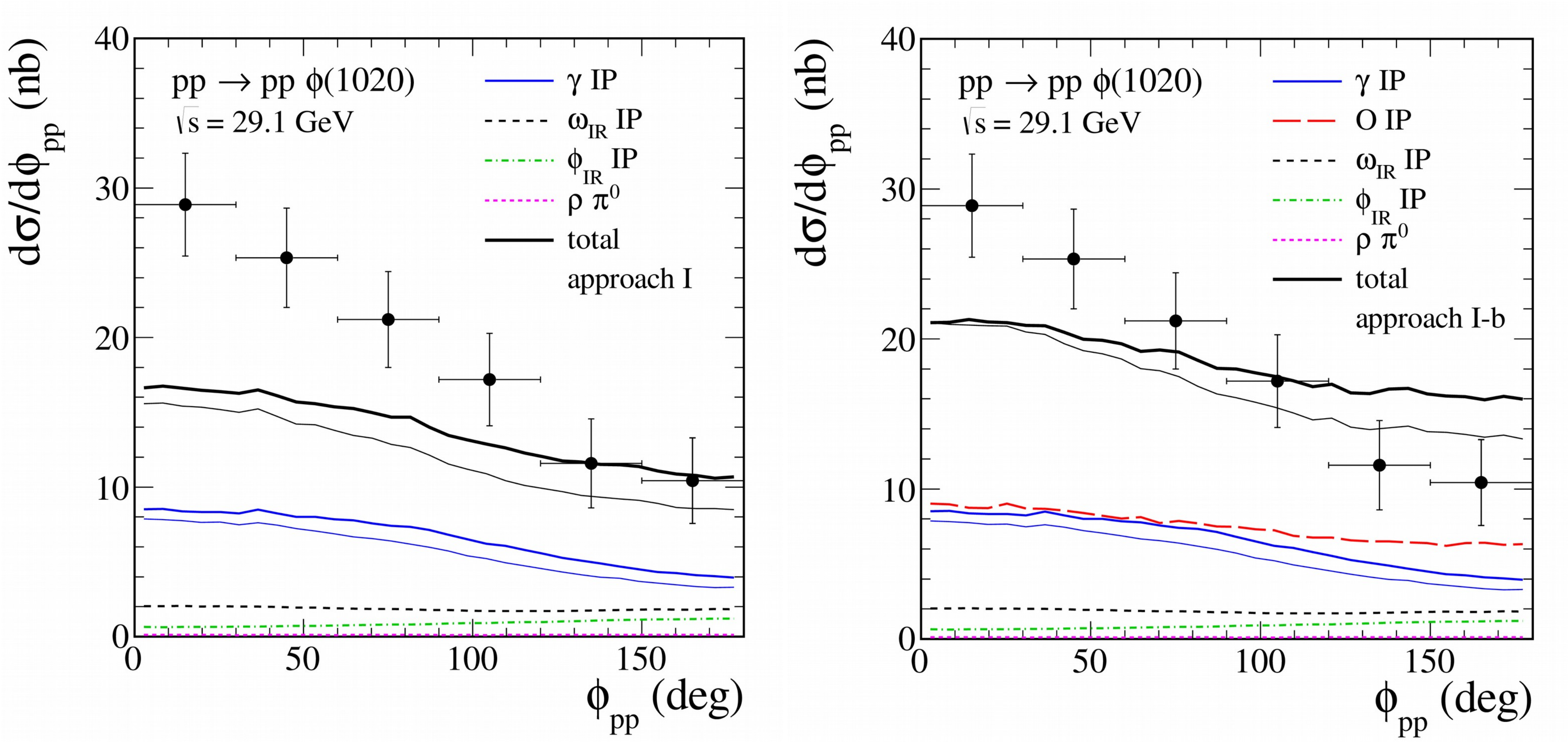
Absorption effects should be included

$$\mathcal{M} = \mathcal{M}^{Born} + \mathcal{M}^{pp-rescattering}$$

$$\mathcal{M}^{pp-rescattering}(s,\vec{p}_{1\perp},\vec{p}_{2\perp}) = \frac{i}{8\pi^2 s}\int d^2\vec{k}_\perp \mathcal{M}^{Born}(s,\vec{p}_{1\perp}-\vec{k}_\perp,\vec{p}_{2\perp}+\vec{k}_\perp)\,\mathcal{M}^{\mathbb{P}-exch.}_{pp\to pp}(s,-\vec{k}_\perp^2)$$

here $\vec{k}_\perp$ is the transverse momentum carried around the loop

Comparison with WA102 data

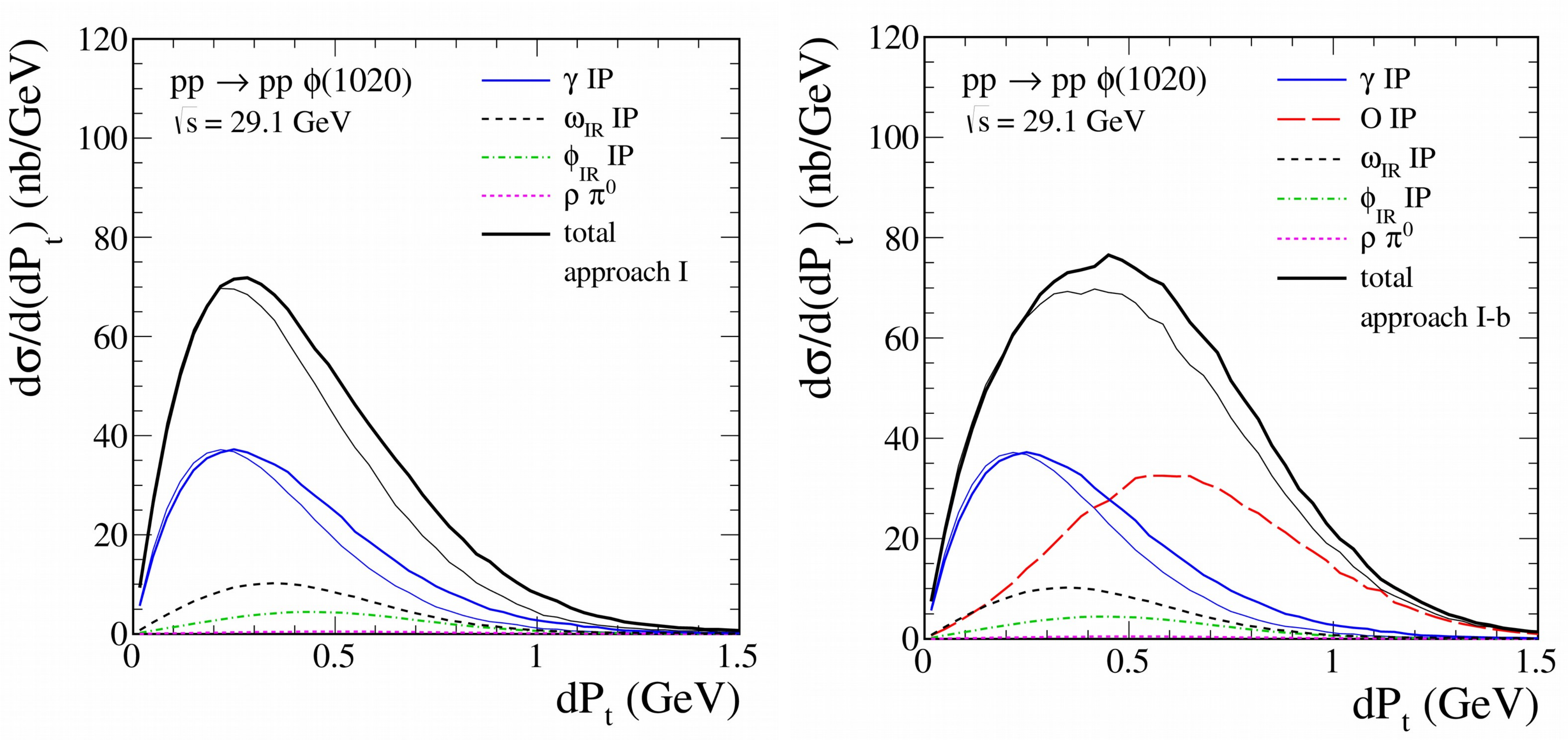


- WA102 data from PLB489 (2000) 29, $\sigma_{\rm exp} = (60 \pm 21)\,{\rm nb}$
- Different fusion processes were considered. Large interference effects.
  The ϕ_{pp} distribution allow us to determine the respective coupling constants

$$a_{\mathbb{P}\mathbb{O}\phi} = -0.8\ {\rm GeV}^{-3}\,, \quad b_{\mathbb{P}\mathbb{O}\phi} = 1.6\ {\rm GeV}^{-1}\,, \quad \Lambda^2_{0,\,\mathbb{P}\mathbb{O}\phi} = 0.5\,{\rm GeV}^2$$

- We obtain the gap survival factor $\langle S^2 \rangle = 0.8$ for the photoproduction term and $\langle S^2 \rangle = 0.4$ for the diffractive contributions

Comparison with WA102 data



$$\mathrm{dP_t} = |\boldsymbol{dP_t}|, \quad \boldsymbol{dP_t} = \boldsymbol{q_{t,1}} - \boldsymbol{q_{t,2}} = \boldsymbol{p_{t,2}} - \boldsymbol{p_{t,1}}$$

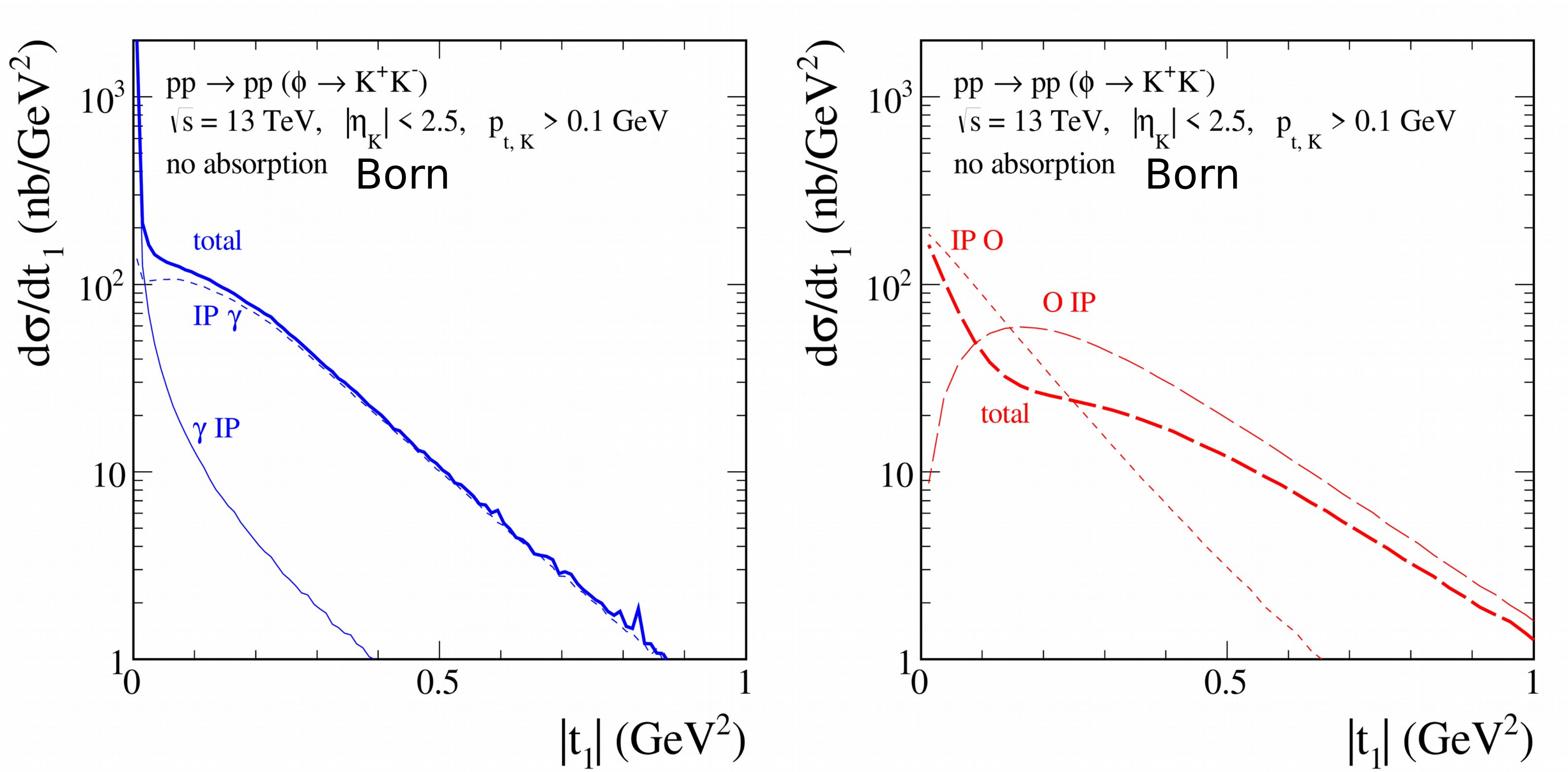
$$R = \frac{d\sigma/d(\mathrm{dP_t} \leqslant 0.2 \ \mathrm{GeV})}{d\sigma/d(\mathrm{dP_t} \geqslant 0.5 \ \mathrm{GeV})}$$

$R_{\mathrm{WA102}} = 0.18 \pm 0.07$ [from PLB489 (2000) 29]

$R = 0.56$ (no Odderon), $R = 0.23$ (with Odderon)

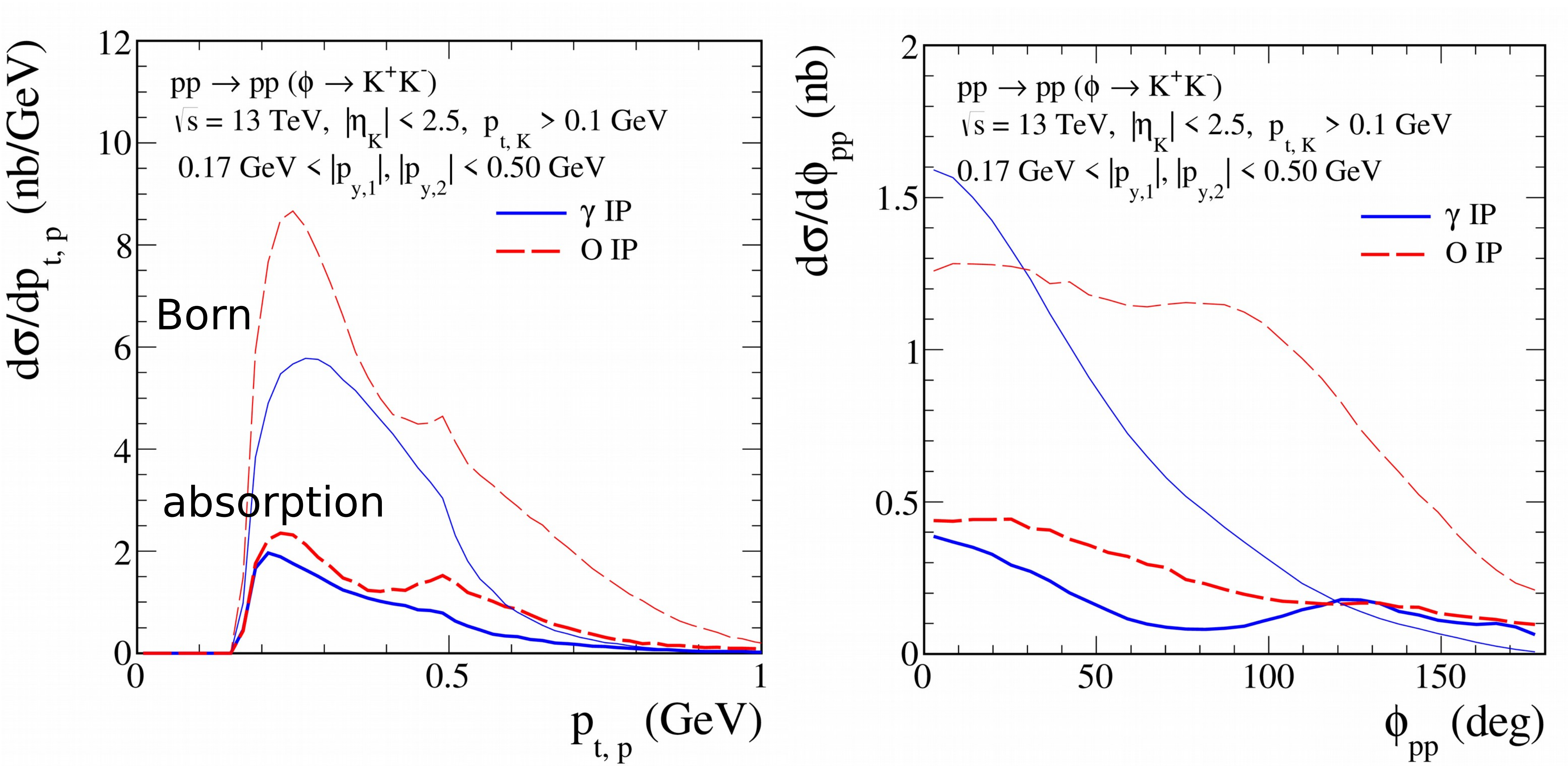
- We find that two couplings $a_{I\!P \mathbb{O} \phi}$ and $b_{I\!P \mathbb{O} \phi}$ are needed
- The WA102 data support the existence of Odderon exchange !
- It would be very useful to measure the outgoing protons at the LHC

Predictions for the *pp → pp K⁺K⁻* reaction



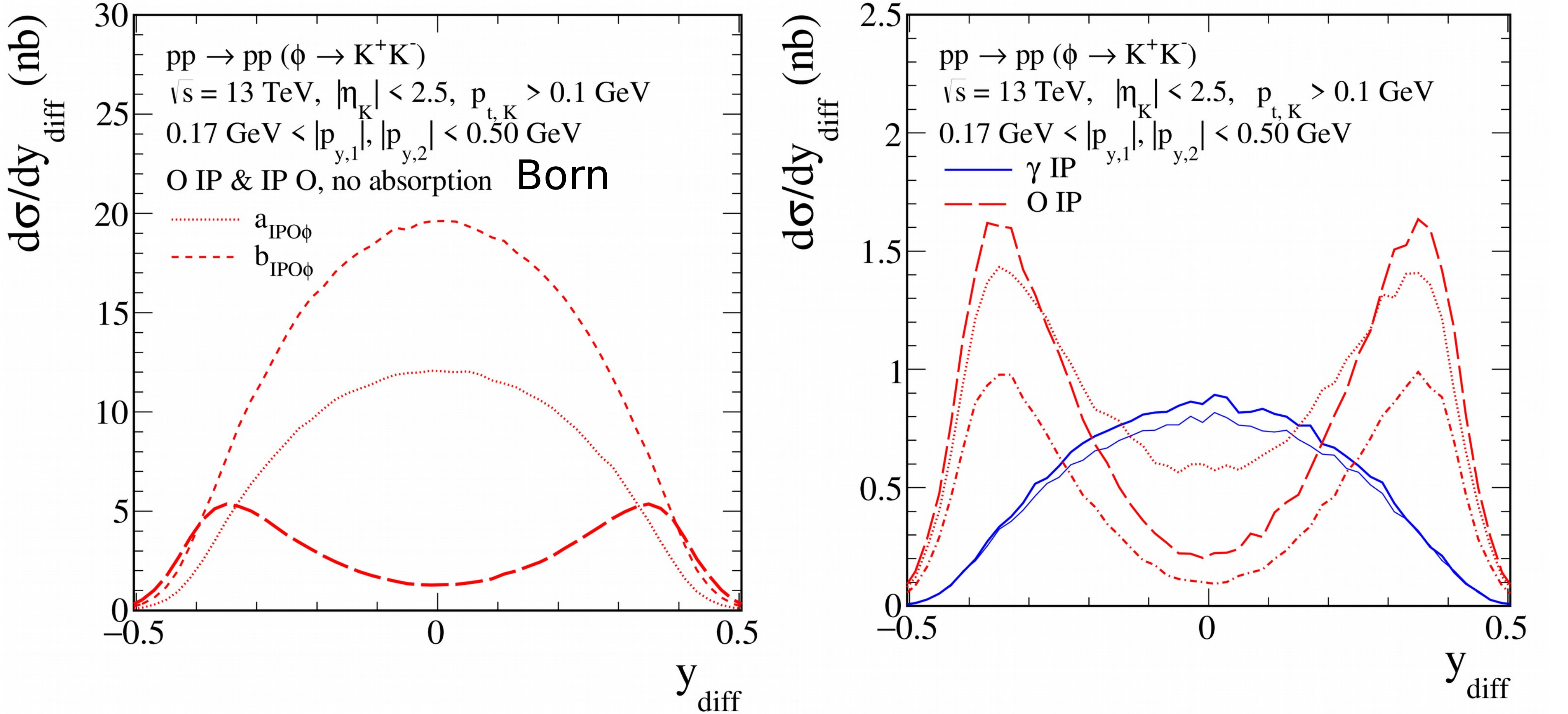
- Shown are the results for two diagrams separately and their coherent sum (total)
- The interference effects between the two diagrams is clearly visible, especially for the OP-fusion mechanism
- Due to the photon exchange the protons are scattered only at small angles and the γP distribution has a singularity for $|t_1| \to 0$. In contrast, the OP-distribution shows a dip for $|t_1| \to 0$

Predictions for the *pp* → *pp* K^+K^- reaction (ATLAS + ALFA)



- Results within (Born) and with (the thick lines) absorption effects are shown. For the ATLAS-ALFA kinematics the absorption effects lead to a large damping of the cross section both for the purely diffractive and for the photoproduction mechanisms.
- This effect could be verified in future experiments when both protons are measured, e.g. by the ATLAS-ALFA and CMS-TOTEM experimental groups.

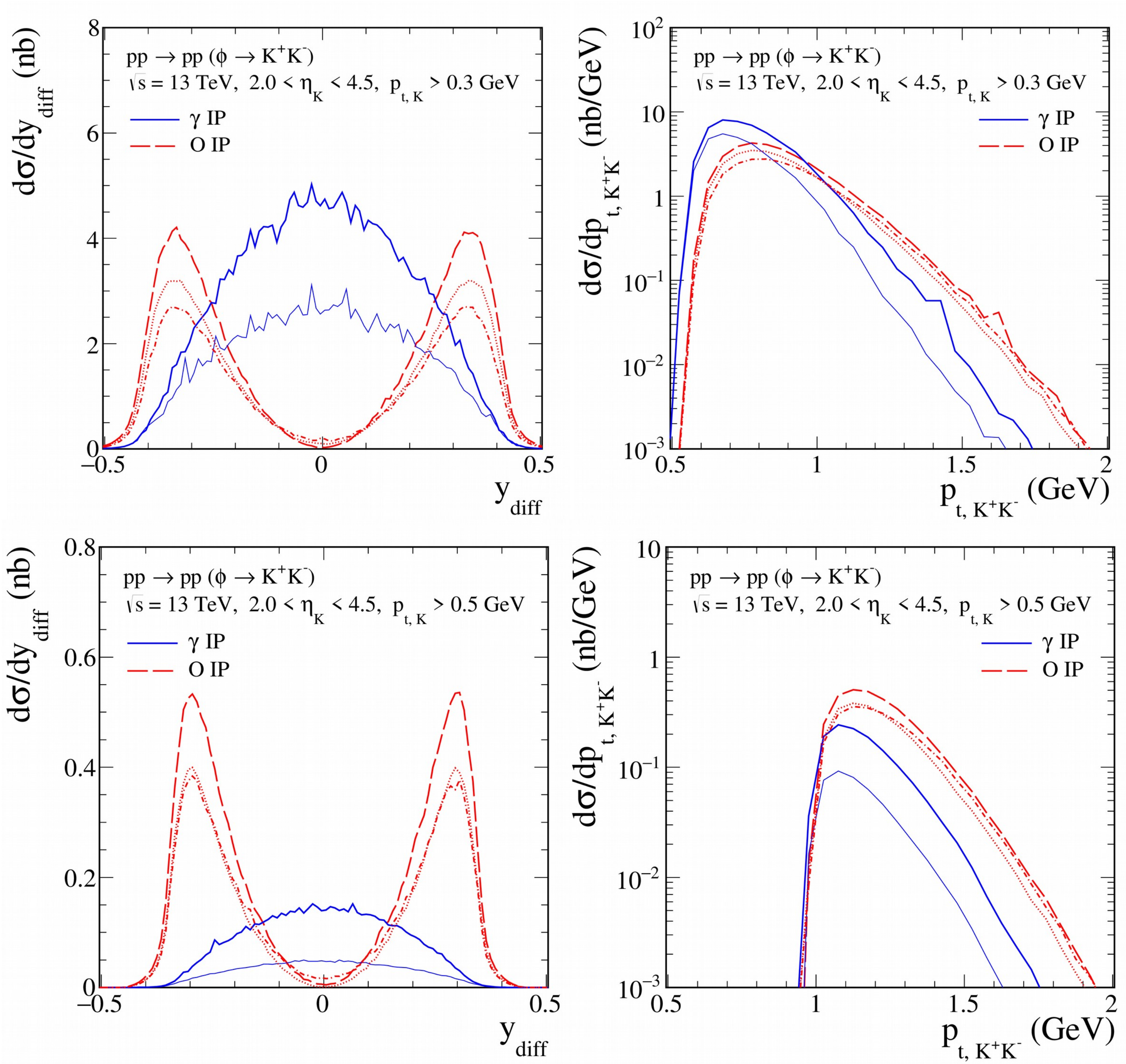
Predictions for the *pp* → *pp* K^+K^- reaction (ATLAS + ALFA)



- (left panel) For the OP-fusion process the complete result indicates a large (destructive) interference effect of the two type of couplings, a and b, in the POϕ vertex → minimum at $y_{diff} = y_3 - y_4 = 0$

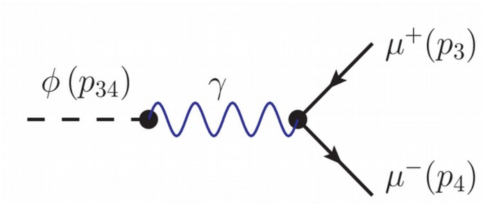
- (right panel) Different behaviour is seen for γP and OP contributions.
The results for different choices of parameters are shown.
The red dash-dotted line, the long-dashed line, and the dotted line correspond to (a [GeV^{-3}], b [GeV^{-1}]) = (-0.8, 1.0), (-0.8,1.6), and (-0.6,1.6), respectively.

Predictions for the *pp* → *pp* K^+K^- reaction (LHCb)



Predictions for the $pp \to pp\, \mu^+\mu^-$ reaction (LHCb)

The amplitudes for the $pp \to pp\mu^+\mu^-$ reaction through ϕ resonance production can be obtained from the $pp \to ppK^+K^-$ amplitudes with $i\Gamma_\kappa^{(\phi KK)}(p_3, p_4)$ replaced by $\bar{u}(p_4, \lambda_4) i\Gamma_\kappa^{(\phi\mu\mu)}(p_3, p_4) v(p_3, \lambda_3)$.



Here we describe the transition $\phi \to \gamma \to \mu^+\mu^-$ by an effective vertex:

$$i\Gamma_\kappa^{(\phi\mu\mu)}(p_3, p_4) = i g_{\phi\mu^+\mu^-}\, \gamma_\kappa$$

The decay rate $\phi \to \mu^+\mu^-$ is calculated from the diagram (neglecting radiative corrections) as

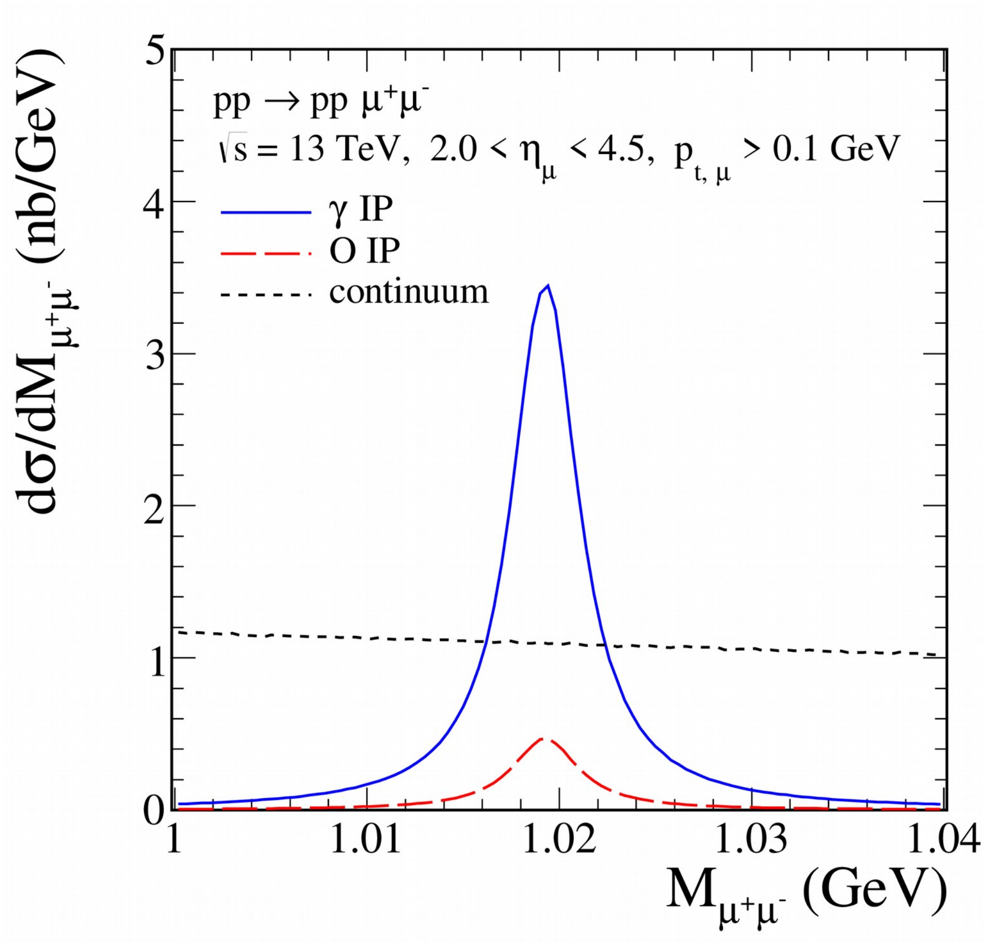
$$\Gamma(\phi \to \mu^+\mu^-) = \frac{1}{12\pi} |g_{\phi\to\mu^+\mu^-}|^2\, m_\phi \left(1 + \frac{2m_\mu^2}{m_\phi^2}\right) \left(1 - \frac{4m_\mu^2}{m_\phi^2}\right)^{1/2}$$

From the experimental values (PDG)

$$\begin{aligned} m_\phi &= (1019.461 \pm 0.016)\ \text{MeV}\,, \\ \Gamma(\phi \to \mu^+\mu^-)/\Gamma_\phi &= (2.86 \pm 0.19) \times 10^{-4}\,, \\ \Gamma_\phi &= (4.249 \pm 0.013)\ \text{MeV} \end{aligned}$$

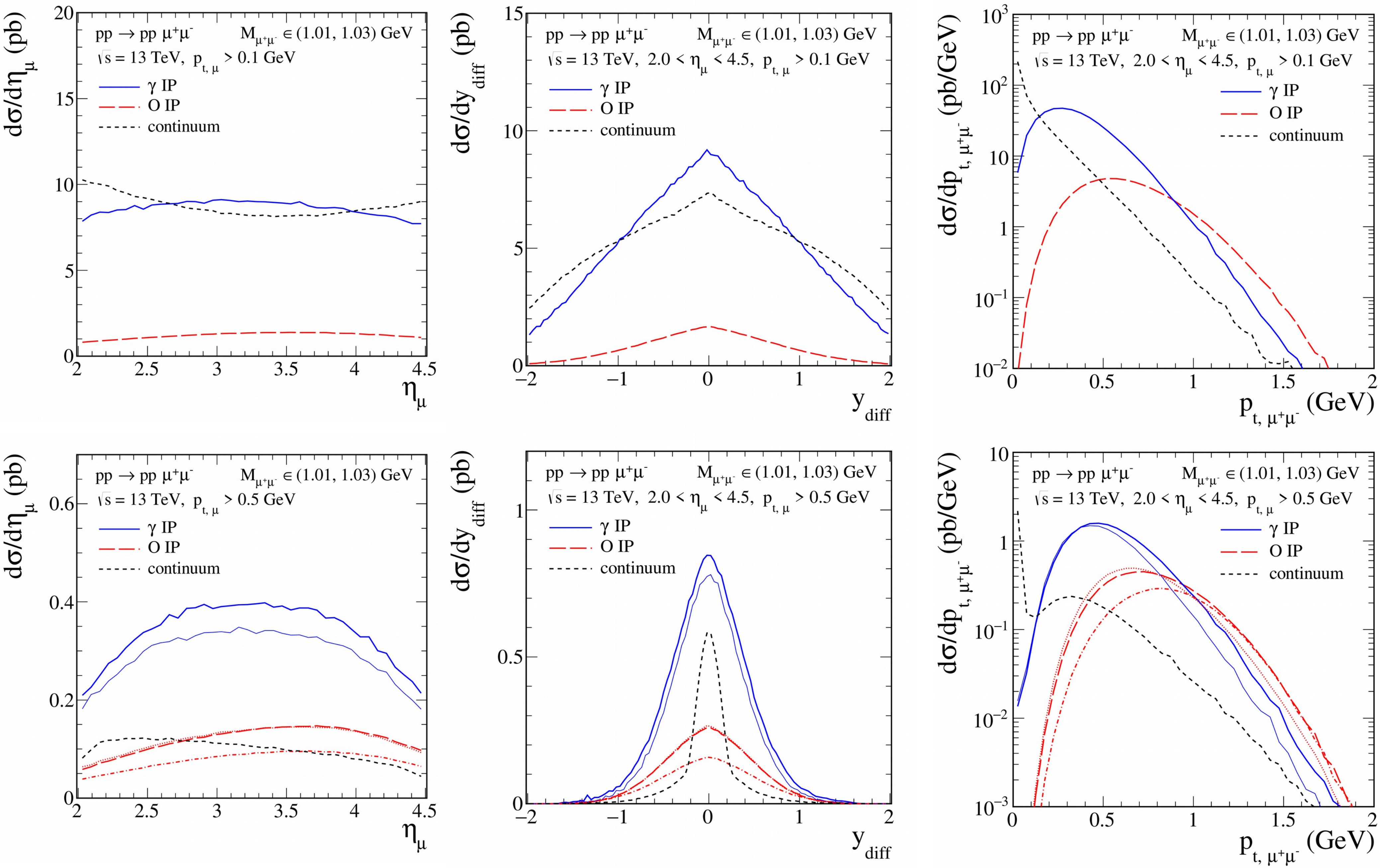
we get $g_{\phi\mu^+\mu^-} = (6.71 \pm 0.22) \times 10^{-3}$

Predictions for the $pp \to pp\, \mu^+\mu^-$ reaction (LHCb)



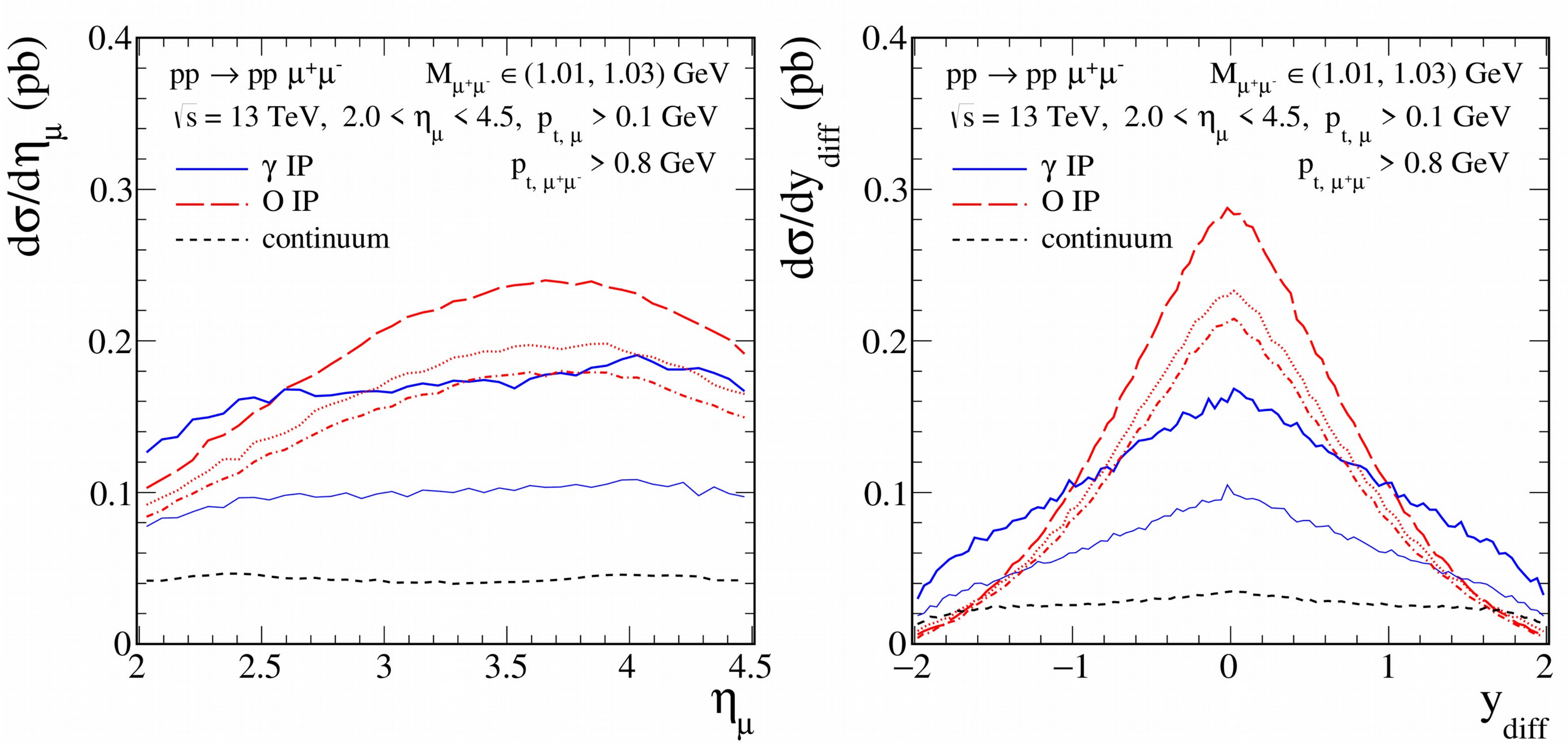
We show the contributions from the $\gamma\mathbb{P}$- and $\mathbb{O}\mathbb{P}$-fusion processes and the continuum $\gamma\gamma \to \mu^+\mu^-$ term.

Predictions for the *pp* → *pp* $\mu^+\mu^-$ reaction (LHCb)



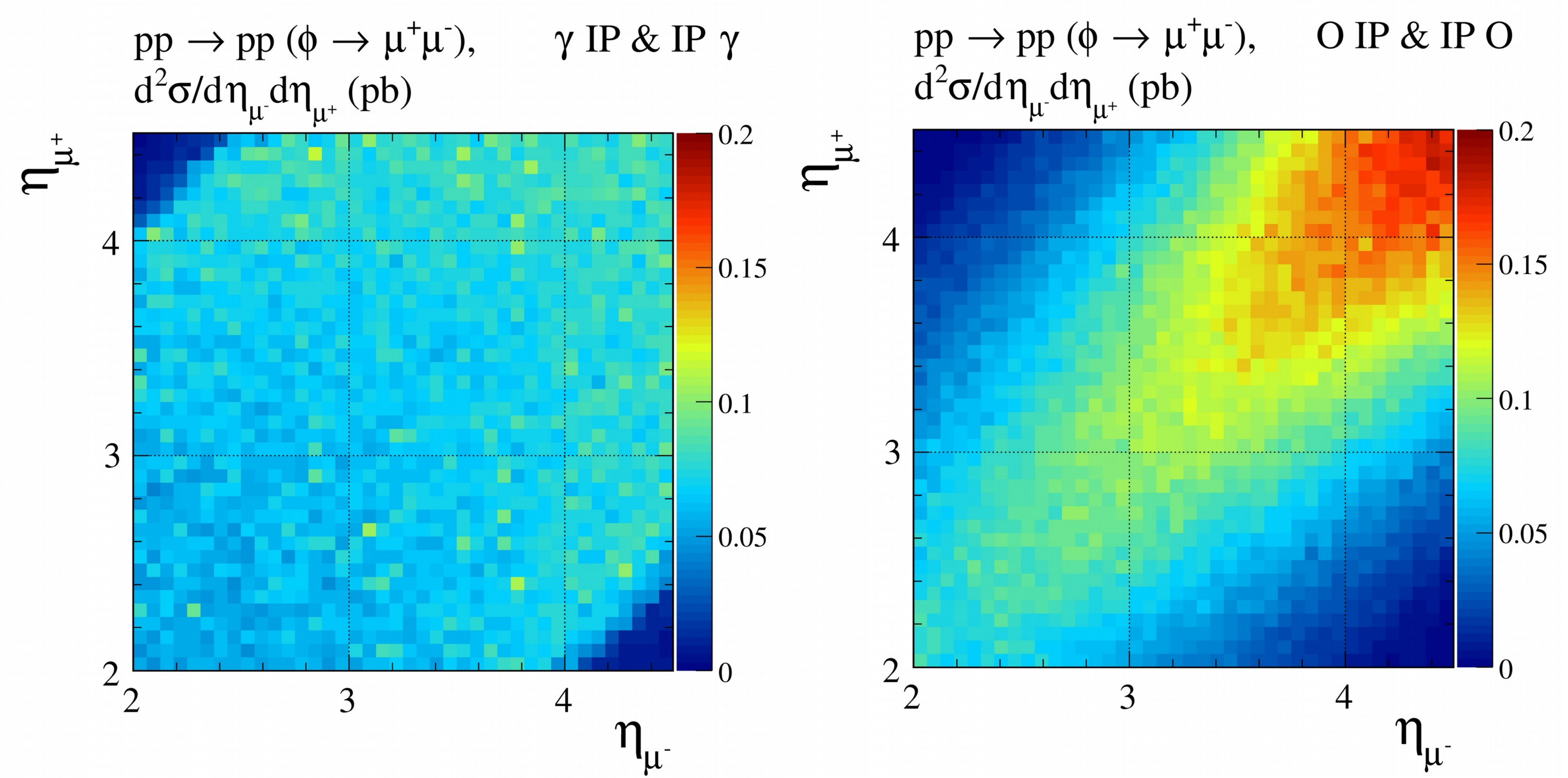
- $p_{t,\mu+\mu-}$ > 0.8 GeV cut can be helpful to reduce the γγ → $\mu^+\mu^-$ continuum and γP-fusion contributions

Predictions for the $pp \to pp\, \mu^+\mu^-$ reaction (LHCb)



- In general, the $\mu^+\mu^-$ channel seems to be less promising in identifying the Odderon exchange. In this case the absolute normalization of the cross section or detailed studies of shapes of distributions should provide a hint whether one observes the Odderon effect

Predictions for the *pp* → *pp* $\mu^+\mu^-$ reaction (LHCb)



- The calculations were done for $\sqrt{s} = 13$ TeV and with cuts on $2.0 < \eta_\mu < 4.5$, $p_{t,\mu} > 0.1$ GeV, and $p_{t,\mu^+\mu^-} > 0.8$ GeV

- The Odderon-exchange contribution shows an enhancement at $\eta_{\mu^+} \sim \eta_{\mu^-} \sim (4.0 - 4.5)$

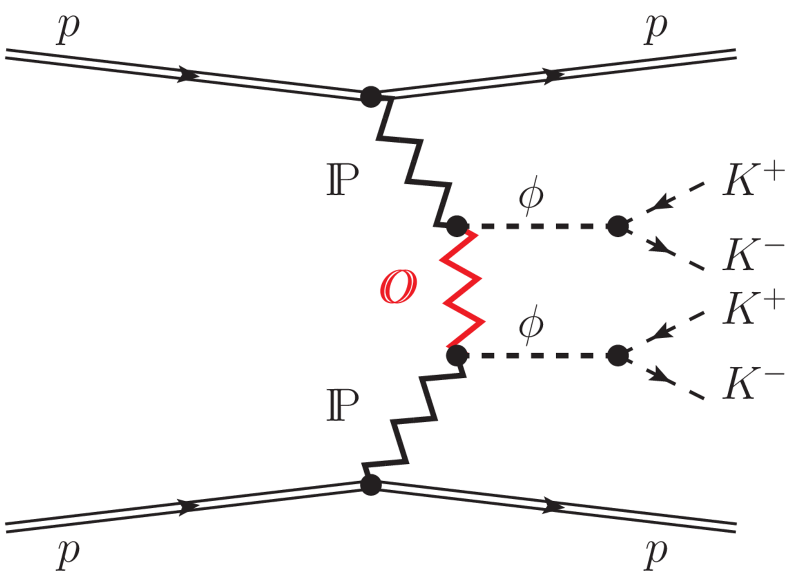
Cross sections

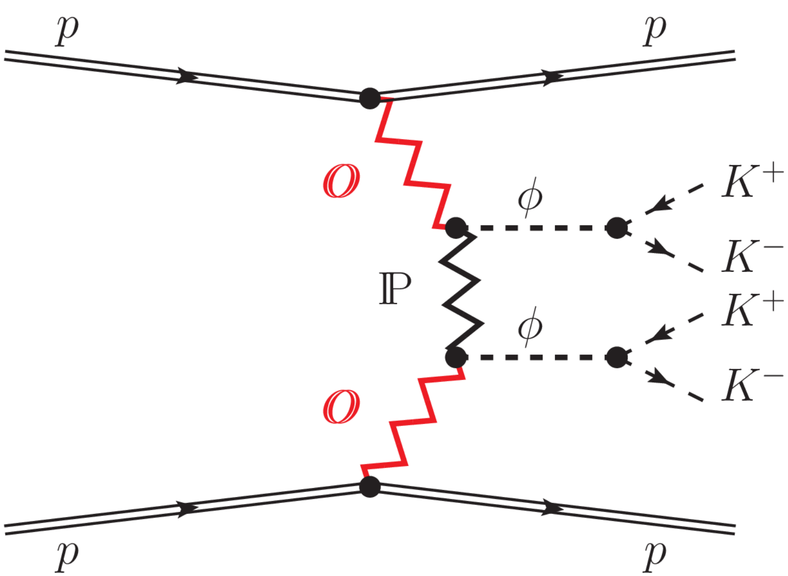
Table 1: The integrated cross sections in nb for the central exclusive K^+K^- and $\mu^+\mu^-$ production of ϕ meson in proton-proton collisions. The results have been calculated for $\sqrt{s} = 13$ TeV in the dikaon/dimuon invariant mass region $M_{34} \in (1.01, 1.03)$ GeV and for some typical experimental cuts. The ratios of full and Born cross sections $\langle S^2 \rangle$ (the gap survival factors) are also shown.

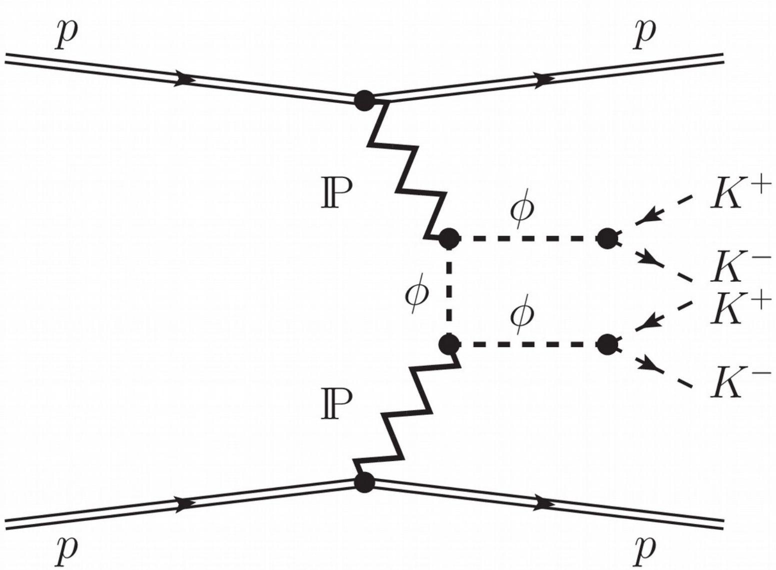
Cuts	Process	$\sigma^{(\mathrm{Born})}$ (nb)	$\sigma^{(\mathrm{full})}$ (nb)	$\langle S^2 \rangle$
$\lvert\eta_K\rvert < 2.5$, $p_{t,K} > 0.1$ GeV, 0.17 GeV $< \lvert p_{y,1}\rvert, \lvert p_{y,2}\rvert < 0.5$ GeV	$\gamma\mathbb{P}$-fusion	1.77	0.52	0.3
	$\mathbb{OP}$-fusion	2.91	0.79	0.3
$2.0 < \eta_K < 4.5$, $p_{t,K} > 0.1$ GeV	$\gamma\mathbb{P}$-fusion	43.18	40.07	0.9
	$\mathbb{OP}$-fusion	16.73	4.70	0.3
$2.0 < \eta_K < 4.5$, $p_{t,K} > 0.3$ GeV	$\gamma\mathbb{P}$-fusion	3.09	2.57	0.8
	$\mathbb{OP}$-fusion	6.57	1.64	0.3
$2.0 < \eta_K < 4.5$, $p_{t,K} > 0.5$ GeV	$\gamma\mathbb{P}$-fusion	0.93×10^{-1}	0.66×10^{-1}	0.7
	$\mathbb{OP}$-fusion	0.88	0.16	0.2
$2.0 < \eta_\mu < 4.5$, $p_{t,\mu} > 0.1$ GeV	$\gamma\mathbb{P}$-fusion	23.93×10^{-3}	20.96×10^{-3}	0.9
	$\mathbb{OP}$-fusion	10.06×10^{-3}	3.02×10^{-3}	0.3
$2.0 < \eta_\mu < 4.5$, $p_{t,\mu} > 0.5$ GeV	$\gamma\mathbb{P}$-fusion	1.21×10^{-3}	0.85×10^{-3}	0.7
	$\mathbb{OP}$-fusion	1.49×10^{-3}	0.45×10^{-3}	0.2
$2.0 < \eta_\mu < 4.5$, $p_{t,\mu} > 0.1$ GeV, $p_{t,\mu^+\mu^-} > 0.8$ GeV	$\gamma\mathbb{P}$-fusion	0.70×10^{-3}	0.42×10^{-3}	0.6
	$\mathbb{OP}$-fusion	2.46×10^{-3}	0.49×10^{-3}	0.2

$pp \to pp\phi\phi$

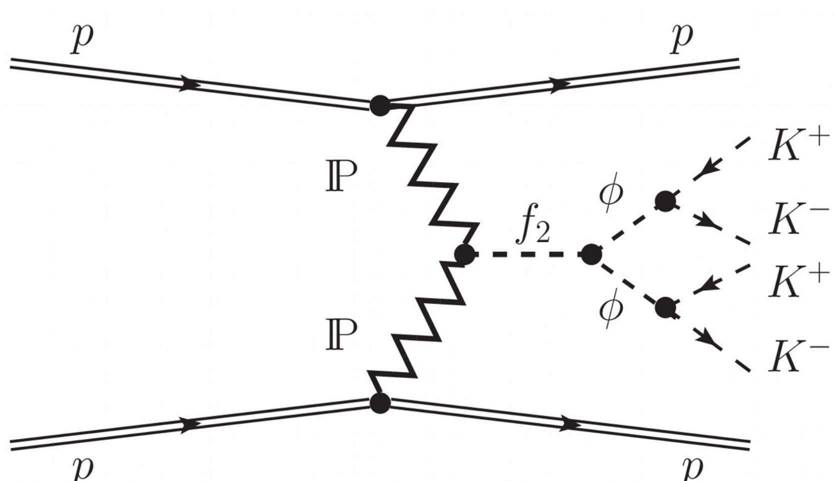
- Central Exclusive Production (CEP) of $\phi\phi$ state offers a very nice way to look for Odderon effects *P.L., O. Nachtmann, A. Szczurek, PRD99 (2019) 094034*





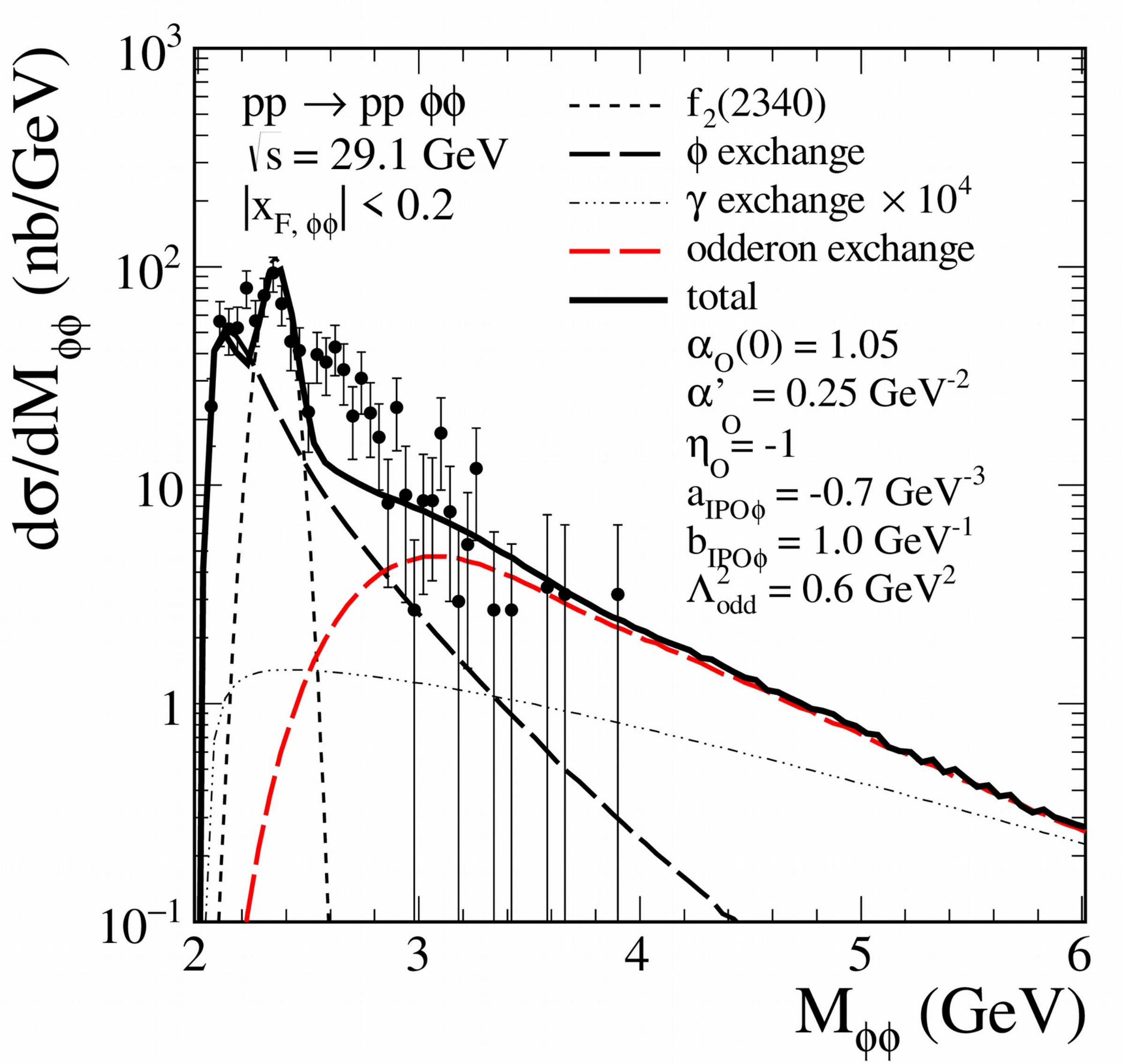


reggeized ϕ-exchange contribution



e.g. $f_2(2340)$, tensor glueball production

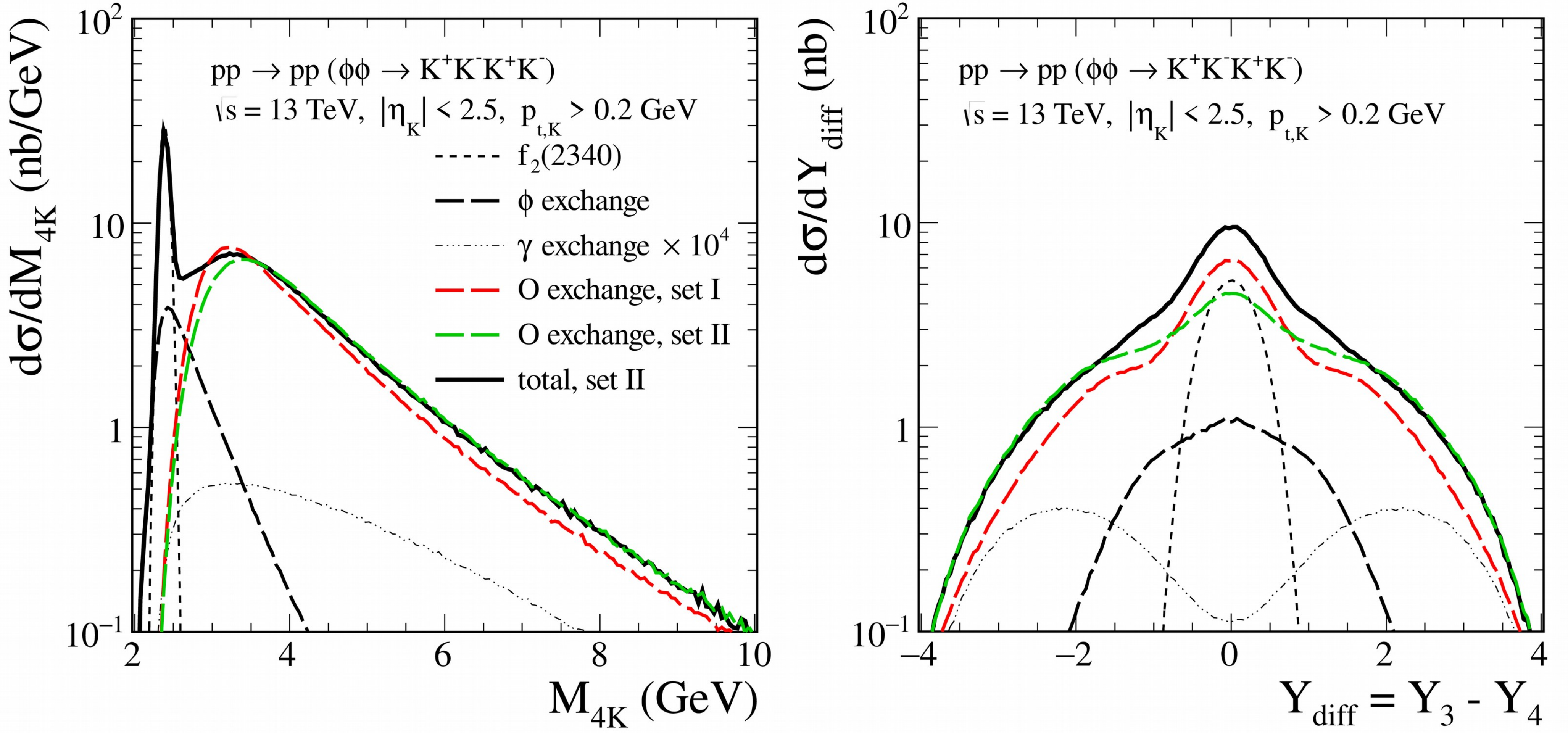
$pp \to pp\phi\phi$



WA102 data
PLB432 (1998) 436
PLB489 (2000) 29

- An upper limit for the Odderon exchange has been established based on the WA102 data

$pp \to pp\ (\phi\phi \to K^+K^-K^+K^-)$



The small intercept of the ϕ reggeon exchange, $\alpha_\phi(0) = 0.1$ makes the ϕ-exchange contribution steeply falling with increasing M_{4K} and $|Y_{\rm diff}|$. Therefore, an Odderon with an intercept $\alpha(0) \sim 1.0$ should be clearly visible in these distributions.

Conclusions

- The $pp \to pp\ \phi$ and $pp \to pp\ \phi\phi$ reactions have been studied within the tensor-Pomeron and vector-Odderon approach. All amplitudes are formulated in terms of effective propagators and vertices respecting the standard crossing and charge conjugation relations of QFT

  $pp \to pp\ \phi$
  - WA102 data give an indication for Odderon-exchange contribution
  - We have presented distributions which are sensitive to the Odderon-exchange contributions and where the background from the photon-Pomeron fusion can be rather reliably calculated
  - To observe a sizeable deviation from photoproduction a $p_{t,\mu+\mu-} > 0.8$ GeV cut on transverse momentum of the $\mu^+\mu^-$ pair seems necessary

  $pp \to pp\ \phi\phi$
  - The $\phi\phi$ invariant mass distribution has a rich structure (continuum, resonances, interference effects)
  - The Odderon exchange contribution should be distinguishable from other contributions for large rapidity distance between the ϕ mesons and in the region of large four-kaon invariant masses

- Comparison with ‘exclusive’ data expected from LHCb, ALICE, CMS+TOTEM, ATLAS+ALFA, and STAR experiments should provide further information on CEP and should be very valuable for clarifying the status of the Odderon

Backup

$pp \to pp\ (\phi\phi \to K^+K^-K^+K^-)$

- Some modifications are needed to simulate 2 → 6 reaction
  (e.g. smearing of ϕ masses due to their resonance distribution)

$$\sigma_{2\to6} = [\mathcal{B}(\phi \to K^+K^-)]^2 \int_{2m_K} \int_{2m_K} \sigma_{2\to4}(..., m_{X_3}, m_{X_4})\, f_\phi(m_{X_3})\, f_\phi(m_{X_4})\, dm_{X_3}\, dm_{X_4}$$

with the branching fraction $\mathcal{B}(\phi(1020) \to K^+K^-) = 0.492$ [PDG]
and the spectral function of ϕ meson:

$$f_\phi(m_{X_i}) = C_\phi \left(1 - \frac{4m_K^2}{m_{X_i}^2}\right)^{3/2} \frac{\frac{2}{\pi} m_\phi^2 \Gamma_\phi}{(m_{X_i}^2 - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2}$$

C_ϕ is found from the condition $\int_{2m_K}^{\infty} f_\phi(m_{X_i}) dm_{X_i} = 1$

- Any differential distribution can be calculated.

  To include experimental cuts on produced kaons we perform
  the decays of ϕ mesons isotropically in the ϕ rest frames
  and then use relativistic transformations to the overall c.m. frame

$pp \to pp\phi\phi$ (ϕ- exchange continnum)

We consider the 2 → 4 exclusive reaction:

$$p(p_a,\lambda_a)+p(p_b,\lambda_b)\to p(p_1,\lambda_1)+\phi(p_3,\lambda_3)+\phi(p_4,\lambda_4)+p(p_2,\lambda_2)$$

The Born-level amplitude (ϕ-exchange continuum) can be written as the sum

$$\mathcal{M}^{(\phi-\text{exchange})\,\rho_3\rho_4}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\phi\phi}=\mathcal{M}^{(\hat{t})\,\rho_3\rho_4}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\phi\phi}+\mathcal{M}^{(\hat{u})\,\rho_3\rho_4}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\phi\phi}$$

with the $\hat{t}$-channel amplitude:

$$\begin{aligned}\mathcal{M}^{(\hat{t})}_{\rho_3\rho_4}=&(-i)\bar{u}(p_1,\lambda_1)i\Gamma^{(I\!\!Ppp)}_{\mu_1\nu_1}(p_1,p_a)u(p_a,\lambda_a)\,i\Delta^{(I\!\!P)\,\mu_1\nu_1,\alpha_1\beta_1}(s_{13},t_1)\\&\times i\Gamma^{(I\!\!P\phi\phi)}_{\rho_1\rho_3\alpha_1\beta_1}(\hat{p}_t,-p_3)\,i\Delta^{(\phi)\,\rho_1\rho_2}(\hat{p}_t)\,i\Gamma^{(I\!\!P\phi\phi)}_{\rho_4\rho_2\alpha_2\beta_2}(p_4,\hat{p}_t)\\&\times i\Delta^{(I\!\!P)\,\alpha_2\beta_2,\mu_2\nu_2}(s_{24},t_2)\,\bar{u}(p_2,\lambda_2)i\Gamma^{(I\!\!Ppp)}_{\mu_2\nu_2}(p_2,p_b)u(p_b,\lambda_b)\end{aligned}$$

where $\hat{p}_t=p_a-p_1-p_3$, $s_{ij}=(p_i+p_j)^2$, $t_1=(p_1-p_a)^2$, $t_2=(p_2-p_b)^2$

Absorptive corrections should be included:

$$\mathcal{M}_{pp\to pp\phi\phi}=\mathcal{M}^{Born}_{pp\to pp\phi\phi}+\mathcal{M}^{pp-rescattering}_{pp\to pp\phi\phi}$$

$pp \to pp\phi\phi$ (ϕ-exchange continnum)

In the high-energy approximation we can write

$$\mathcal{M}^{(\phi-\text{exchange})\,\rho_3\rho_4}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\phi\phi} = 2(p_1+p_a)_{\mu_1}(p_1+p_a)_{\nu_1}\,\delta_{\lambda_1\lambda_a}\,F_1(t_1)\,F_M(t_1)$$
$$\times\Big\{V^{\rho_3\rho_1\mu_1\nu_1}(s_{13},t_1,\hat{p}_t,p_3)\;\Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p}_t)\;V^{\rho_4\rho_2\mu_2\nu_2}(s_{24},t_2,-\hat{p}_t,p_4)\;\Big[\hat{F}_\phi(\hat{p}_t^2)\Big]^2$$
$$+\,V^{\rho_4\rho_1\mu_1\nu_1}(s_{14},t_1,-\hat{p}_u,p_4)\;\Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p}_u)\;V^{\rho_3\rho_2\mu_2\nu_2}(s_{23},t_2,\hat{p}_u,p_3)\;\Big[\hat{F}_\phi(\hat{p}_u^2)\Big]^2\Big\}$$
$$\times\,2(p_2+p_b)_{\mu_2}(p_2+p_b)_{\nu_2}\,\delta_{\lambda_2\lambda_b}\,F_1(t_2)\,F_M(t_2)$$

where

$$V_{\mu\nu\kappa\lambda}(s,t,k_2,k_1) = \frac{1}{4s}\,3\beta_{I\!PNN}\,(-is\alpha'_{I\!P})^{\alpha_{I\!P}(t)-1}\Big[2a_{I\!P\phi\phi}\Gamma^{(0)}_{\mu\nu\kappa\lambda}(k_1,k_2)-b_{I\!P\phi\phi}\Gamma^{(2)}_{\mu\nu\kappa\lambda}(k_1,k_2)\Big]$$

The amplitude contains a form factor taking into account the off-shell dependences of the intermediate ϕ-mesons

$$\hat{F}_\phi(\hat{p}^2) = \exp\left(\frac{\hat{p}^2-m_\phi^2}{\Lambda^2_{off,E}}\right)$$

where the cut-off parameter $\Lambda_{off,E}$ could be adjusted to experimental data.

$pp \to pp\phi\phi$ (ϕ-exchange continnum)

We should take into account the fact that the exchanged intermediate object is not a simple spin-1 particle (ϕ meson) but may correspond to a Regge exchange, that is, the reggeization of the intermediate ϕ meson is necessary.

$$\Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p}) \to \Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p}) \left(\exp(i\phi(s_{34})) \, \frac{s_{34}}{s_{\rm thr}} \right)^{\alpha_\phi(\hat{p}^2)-1}$$

Eq. (3.21)

in Figure

where $s_{34} = (p_3 + p_4)^2 = \mathrm{M}^2_{\phi\phi}$, $s_{\rm thr} = 4m^2_\phi$

We assume for the ϕ Regge trajectory (from Collins book)

$$\alpha_\phi(\hat{p}^2) = \alpha_\phi(0) + \alpha'_\phi \, \hat{p}^2$$
$$\alpha_\phi(0) = 0.1 \, , \quad \alpha'_\phi = 0.9 \ \mathrm{GeV}^{-2}$$

In order to have the correct phase behaviour we introduced the function $\exp(i\phi(s_{34}))$ with

$$\phi(s_{34}) = \frac{\pi}{2} \exp\left(\frac{s_{\rm thr} - s_{34}}{s_{\rm thr}} \right) - \frac{\pi}{2}$$

This procedure of reggeization assures agreement with mesonic physics in the $\phi\phi$ system close to threshold, $s_{34} = 4m^2_\phi$ (no suppression), and it gives the Regge behaviour at large s_{34}.

$pp \to pp\phi\phi$ (ϕ-exchange continnum)

- Regge formalism applies when $|\hat{p}_t^2|, |\hat{p}_u^2| \ll s_{34}$

  At the threshold ($M_{\phi\phi} = 2m_\phi$) both $|\hat{p}_t^2|$ and $|\hat{p}_u^2|$ are not very small

- Another idea of reggeization:
  At $\mathrm{Y_{diff}} = \mathrm{Y}_3 - \mathrm{Y}_4 = 0$ (i.e. for $|\hat{p}^2| \sim s_{34}$) reproduce meson physics, suggested by Harland-Lang, Khoze, Ryskin

We propose a formula for the ϕ propagator which interpolates between the regions of low $\mathrm{Y_{diff}}$, where we use the standard ϕ propagator, and of high $\mathrm{Y_{diff}}$ where we use the reggeized form:

$$\Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p}) \to \Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p})\, F(\mathrm{Y_{diff}}) + \Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p})\, [1 - F(\mathrm{Y_{diff}})] \left(\exp(i\phi(s_{34}))\, \frac{s_{34}}{s_{\mathrm{thr}}}\right)^{\alpha_\phi(\hat{p}^2)-1}$$

Eq. (3.25)

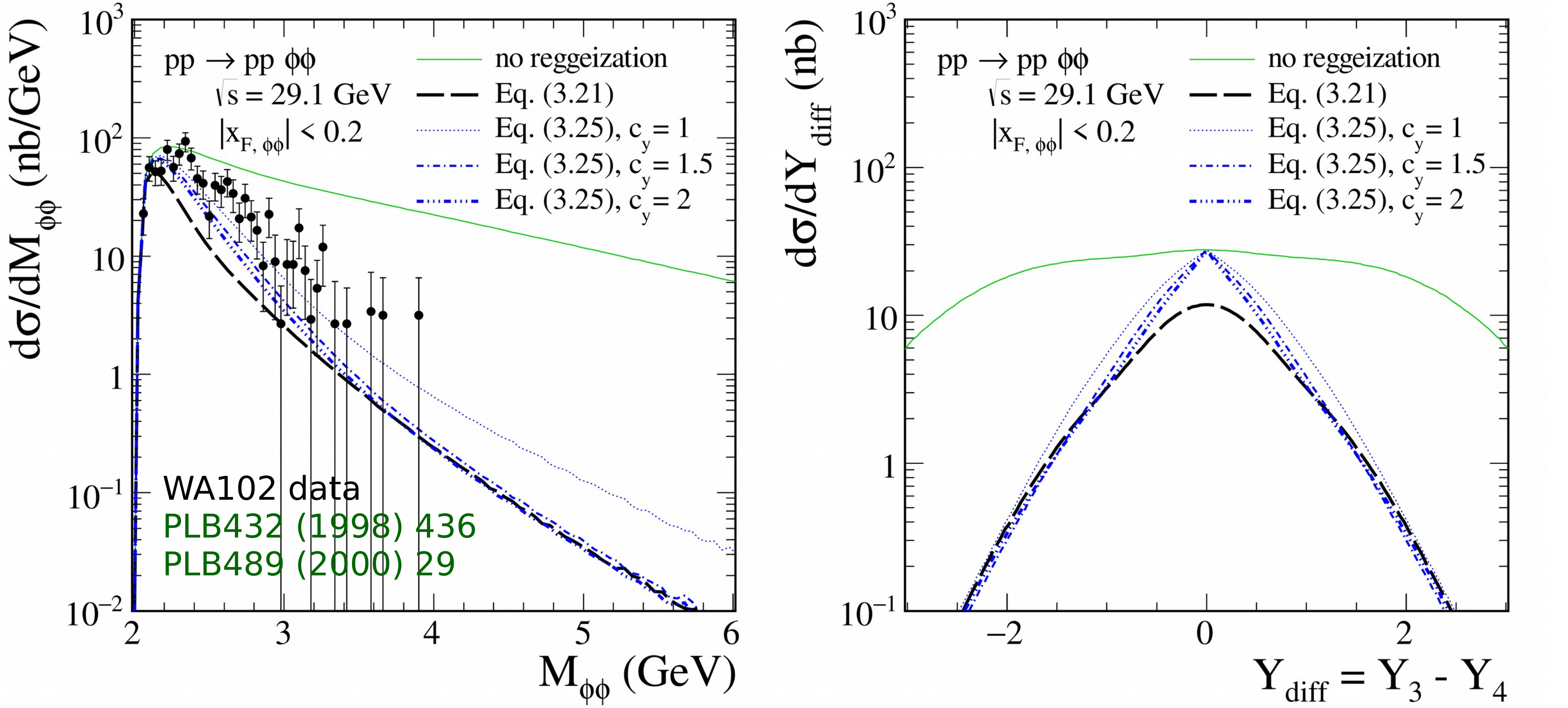
in Figure

with a simple function

$$F(\mathrm{Y_{diff}}) = \exp\left(-\mathrm{c_y}|\mathrm{Y_{diff}}|\right)$$

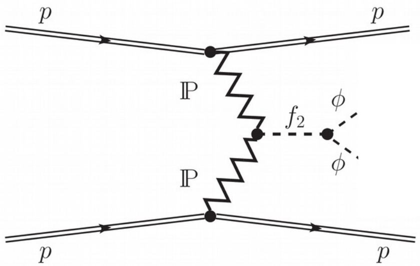
Here $\mathrm{c_y}$ is an unknown parameter which measures how fast one approaches to the Regge regime.

$pp \to pp\phi\phi$ (ϕ-exchange continnum)



- The distributions in $\phi\phi$ invariant mass and in Y_{diff}, the rapidity distance between the two $\phi\phi$ mesons, for the ϕ-exchange continuum contribution
- The green solid line corresponds to the non-reggeized contribution.
  The results for the two prescription of reggeization are shown by the black and blue lines
- The absorption effects calculated at the amplitude level (*pp* nonperturbative interactions) are included

Table 1: A list of resonances, up to a mass of 2500 MeV, that decay into a vector meson pair. The meson masses m and their total widths Γ are taken from PDG. For $\eta(2100)$ and $X(2500)$ the information is taken from BESIII experiment (arXiv:1602.01523). In the columns: • indicates rather established particles, (?) denotes the states that need further experimental confirmation.



Meson	$I^G J^{PC}$	m (MeV)	Γ (MeV)	$\phi\phi$	$K^{*0}\bar{K}^{*0}$	$\rho^0\rho^0$	$\omega\omega$
• $f_1(1285)$	0^+1^{++}	1281.9 ± 0.5	22.7 ± 1.1			Seen	
• $f_0(1370)$	0^+0^{++}	$1200 - 1500$	$200 - 500$			Dominant	Not seen
• $f_0(1500)$	0^+0^{++}	1504 ± 6	109 ± 7			Seen	
$f_2(1565)$	0^+2^{++}	1562 ± 13	134 ± 8			Seen	Seen
$f_2(1640)$	0^+2^{++}	1639 ± 6	99^{+60}_{-40}				Seen
• $f_0(1710)$	0^+0^{++}	1723^{+6}_{-5}	139 ± 8				Seen
$\eta(1760)$	0^+0^{-+}	1751 ± 15	240 ± 30			Seen	Seen
$f_2(1910)$	0^+2^{++}	1903 ± 9	196 ± 31			Seen	Seen
• $f_2(1950)$	0^+2^{++}	1944 ± 12	472 ± 18		Seen		
• $f_2(2010)$	0^+2^{++}	2011^{+60}_{-80}	202 ± 60	Seen			
$f_0(2020)$	0^+0^{++}	1992 ± 16	442 ± 60			Seen	Seen
$f_0(2100)$	0^+0^{++}	2101 ± 7	224^{+23}_{-21}	Seen (?)			
$\eta(2100)$	0^+0^{-+}	2050^{+30+75}_{-24-26}	$250^{+36+181}_{-30-164}$	Seen (?)			
• $f_4(2050)$	0^+4^{++}	2018 ± 11	237 ± 18				Seen
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$	2231.1 ± 3.5	23^{+8}_{-7}	Not seen			
$\eta(2225)$	0^+0^{-+}	2221^{+13}_{-10}	185^{+40}_{-20}	Seen (?)			
• $f_2(2300)$	0^+2^{++}	2297 ± 28	149 ± 40	Seen			
$f_4(2300)$	0^+4^{++}	2320 ± 60	250 ± 80			Seen	Seen
• $f_2(2340)$	0^+2^{++}	2345^{+50}_{-40}	322^{+70}_{-60}	Seen			
$X(2500)$	0^+0^{-+}	$2470^{+15+101}_{-19-23}$	230^{+64+56}_{-35-33}	Seen (?)			

The nature of these resonances is not understood at present and a tensor glueball has still not been clearly identified. According to lattice-QCD simulations, the lightest tensor glueball has a mass between 2.2 and 2.4 GeV.

$pp \to pp\phi\phi$ *via* IP IP $\to f_2 \to \phi\phi$

- Now we consider the amplitude through *s*-channel f_2 meson exchange
- $f_2(2010)$, $f_2(2300)$ and $f_2(2340)$ mesons could be considered as potential candidates

$$\mathcal{M}^{(\mathbb{P}\mathbb{P}\to f_2 \to \phi\phi)\,\rho_3\rho_4}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\phi\phi} = (-i)\,\bar{u}(p_1,\lambda_1) i\Gamma^{(\mathbb{P}pp)\,\mu_1\nu_1}(p_1,p_a) u(p_a,\lambda_a)\; i\Delta^{(\mathbb{P})}_{\mu_1\nu_1,\alpha_1\beta_1}(s_1,t_1)$$
$$\times\, i\Gamma^{(\mathbb{P}\mathbb{P}f_2)\,\alpha_1\beta_1,\alpha_2\beta_2,\rho\sigma}(q_1,q_2)\; i\Delta^{(f_2)}_{\rho\sigma,\alpha\beta}(p_{34})\; i\Gamma^{(f_2\phi\phi)\,\alpha\beta\rho_3\rho_4}(p_3,p_4)$$
$$\times\, i\Delta^{(\mathbb{P})}_{\alpha_2\beta_2,\mu_2\nu_2}(s_2,t_2)\; \bar{u}(p_2,\lambda_2) i\Gamma^{(\mathbb{P}pp)\,\mu_2\nu_2}(p_2,p_b) u(p_b,\lambda_b)$$

where $s_1 = (p_1+p_3+p_4)^2$, $s_2 = (p_2+p_3+p_4)^2$, $q_1 = p_a - p_1$, $q_2 = p_b - p_2$, $t_1 = q_1^2$, $t_2 = q_2^2$, and $p_{34} = q_1 + q_2 = p_3 + p_4$.

P.L., Nachtmann, Szczurek
PRD93 (2016) 054015

- The $I\!P I\!P f_2$ vertex, including a form factor, can be written as

$$i\Gamma^{(\mathbb{P}\mathbb{P}f_2)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = \left(i\Gamma^{(\mathbb{P}\mathbb{P}f_2)(1)}_{\mu\nu,\kappa\lambda,\rho\sigma}\,|_{\rm bare} + \sum_{j=2}^{7} i\Gamma^{(\mathbb{P}\mathbb{P}f_2)(j)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2)\,|_{\rm bare} \right) \tilde{F}^{(\mathbb{P}\mathbb{P}f_2)}(q_1^2,q_2^2,p_{34}^2)$$

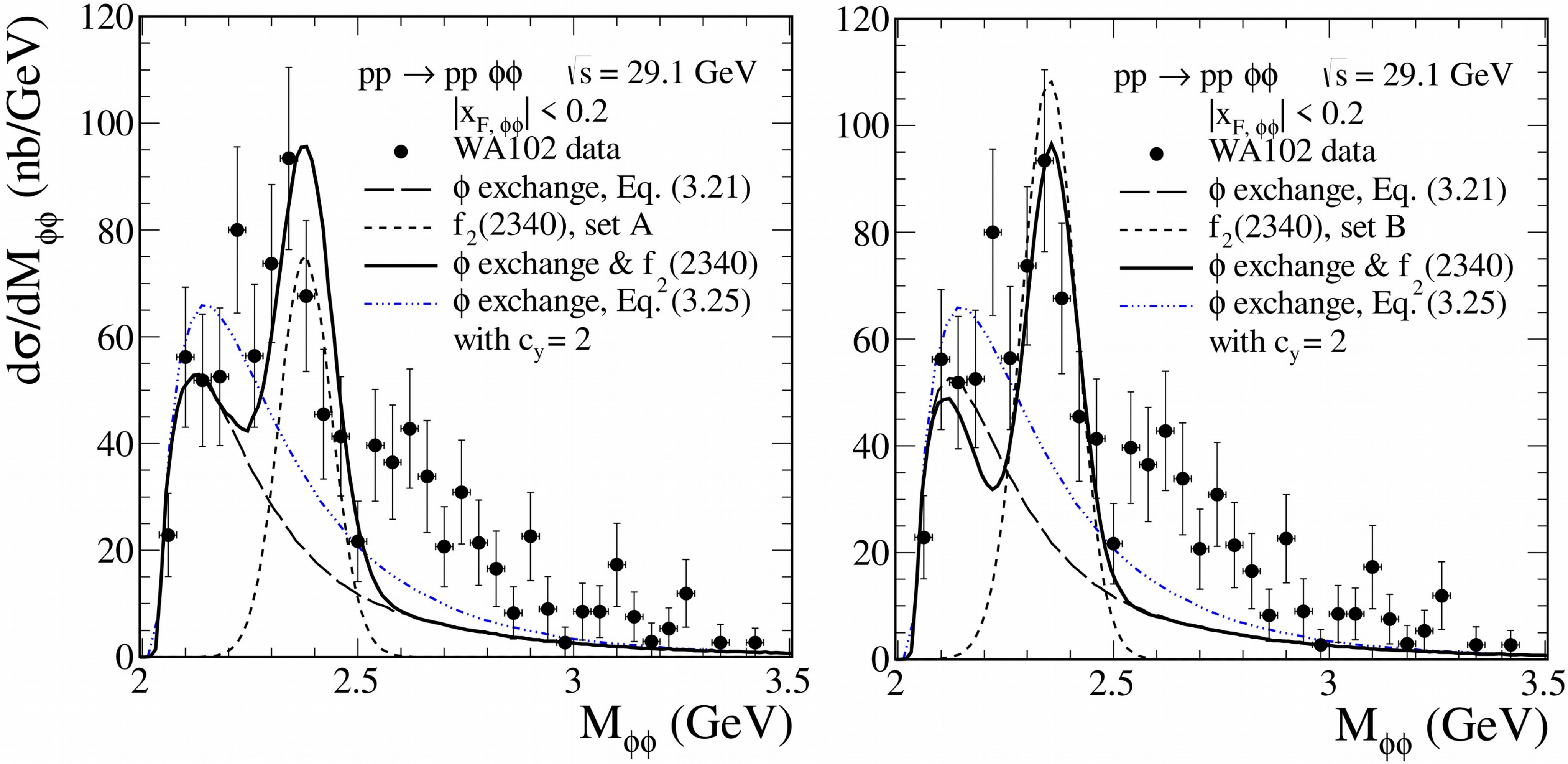
$$i\Gamma^{(I\!P I\!P f_2)(1)}_{\mu\nu,\kappa\lambda,\rho\sigma} = 2i\, g^{(1)}_{I\!P I\!P f_2} M_0\, R_{\mu\nu\mu_1\nu_1}\, R_{\kappa\lambda\alpha_1\lambda_1}\, R_{\rho\sigma\rho_1\sigma_1}\, g^{\nu_1\alpha_1}\, g^{\lambda_1\rho_1}\, g^{\sigma_1\mu_1}$$

$$R_{\mu\nu\kappa\lambda} = \frac{1}{2}g_{\mu\kappa}g_{\nu\lambda} + \frac{1}{2}g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{4}g_{\mu\nu}g_{\kappa\lambda}$$

- For the $f_2\phi\phi$ vertex we take (in analogy to $f_2\gamma\gamma$ vertex)

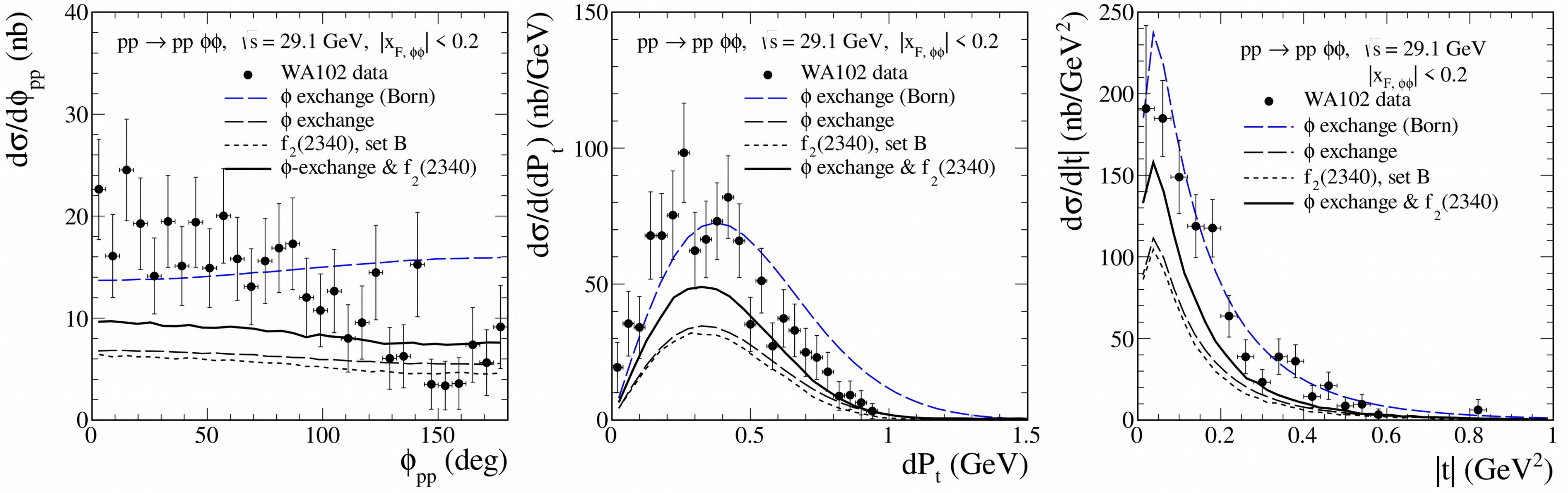
$$i\Gamma^{(f_2\phi\phi)}_{\mu\nu\kappa\lambda}(p_3,p_4) = i\frac{2}{M_0^3} g'_{f_2\phi\phi}\,\Gamma^{(0)}_{\mu\nu\kappa\lambda}(p_3,p_4)\,F'^{(f_2\phi\phi)}(p_{34}^2)$$
$$- i\frac{1}{M_0} g''_{f_2\phi\phi}\,\Gamma^{(2)}_{\mu\nu\kappa\lambda}(p_3,p_4)\,F''^{(f_2\phi\phi)}(p_{34}^2)$$

$pp \to pp\phi\phi$



- We show results for two sets of parameters:
  set A (left panel) $g^{(1)}_{I\!PI\!Pf_2} g'_{f_2\phi\phi} \neq 0$ and set B (right panel) $g^{(1)}_{I\!PI\!Pf_2} g''_{f_2\phi\phi} \neq 0$
- The interference of the continuum and resonance contributions depends on subtle details (choice of the couplings for resonant term, reggeization)

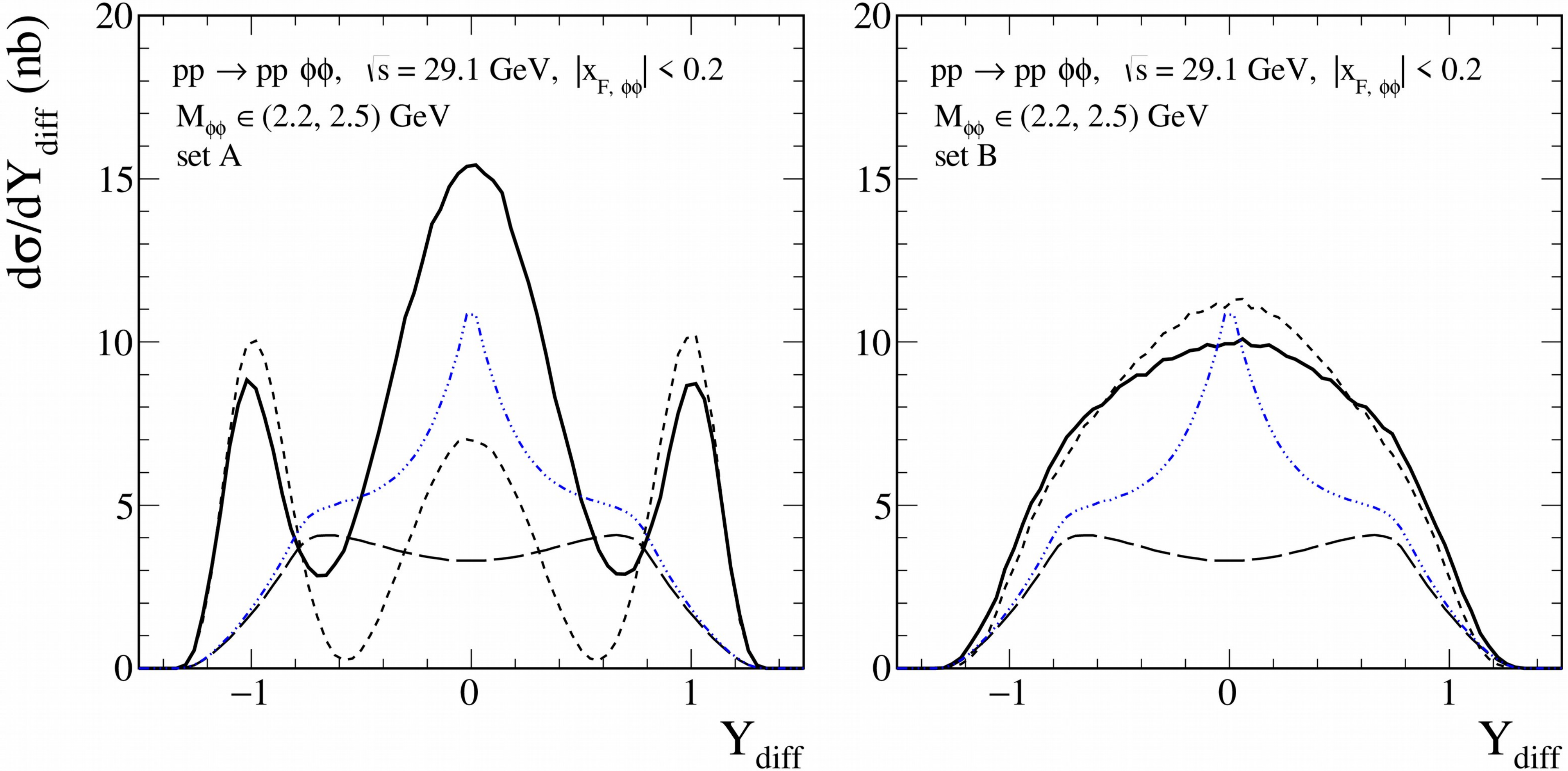
$pp \to pp\phi\phi$



Here $\mathrm{dP_t}$ is the "glueball-filter variable" $\boldsymbol{dP_t} = \boldsymbol{q_{t,1}} - \boldsymbol{q_{t,2}} = \boldsymbol{p_{t,2}} - \boldsymbol{p_{t,1}}$, $\mathrm{dP_t} = |\boldsymbol{dP_t}|$ and ϕ_{pp} is the azimuthal angle between the transverse momentum vectors $\boldsymbol{p_{t,1}}$, $\boldsymbol{p_{t,2}}$ of the outgoing protons.

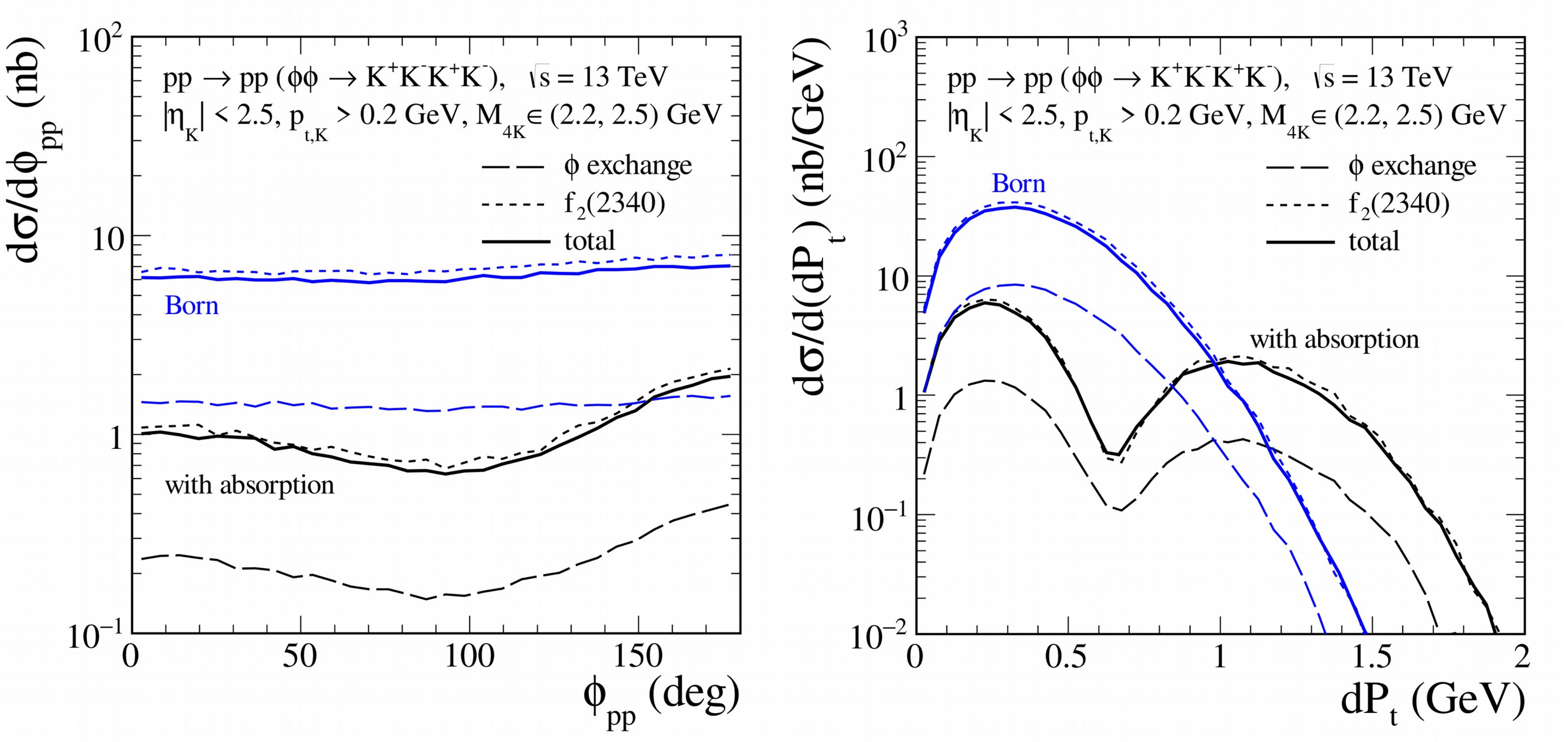
- Quite a different pattern can be seen for the Born case and for the case with absorption. The ratio of full and Born cross sections is $\langle S^2 \rangle \sim 0.4$ (WA102 kinematics).
- Glueball candidates should be prominent for $\mathrm{dP_t} \to 0$.

$pp \to pp\phi\phi$



- We have checked that the shapes of Y_{diff} distributions do not depend significantly on the choice of the IP IP f_2 vertex coupling
- The distribution in Y_{diff} can be used to determine the $f_2(2340) \to \phi\phi$ coupling → using results expected from LHC measurements, in particular, if they cover a wider range of rapidities

$pp \to pp$ ($\phi\phi \to K^+K^-K^+K^-$)



The ratio of full and Born cross sections is S_g ~0.2 (LHC kinematics).

Absorption effect leads to significant modification of these distributions.

$pp \to pp\ (\phi\phi \to K^+K^-K^+K^-)$

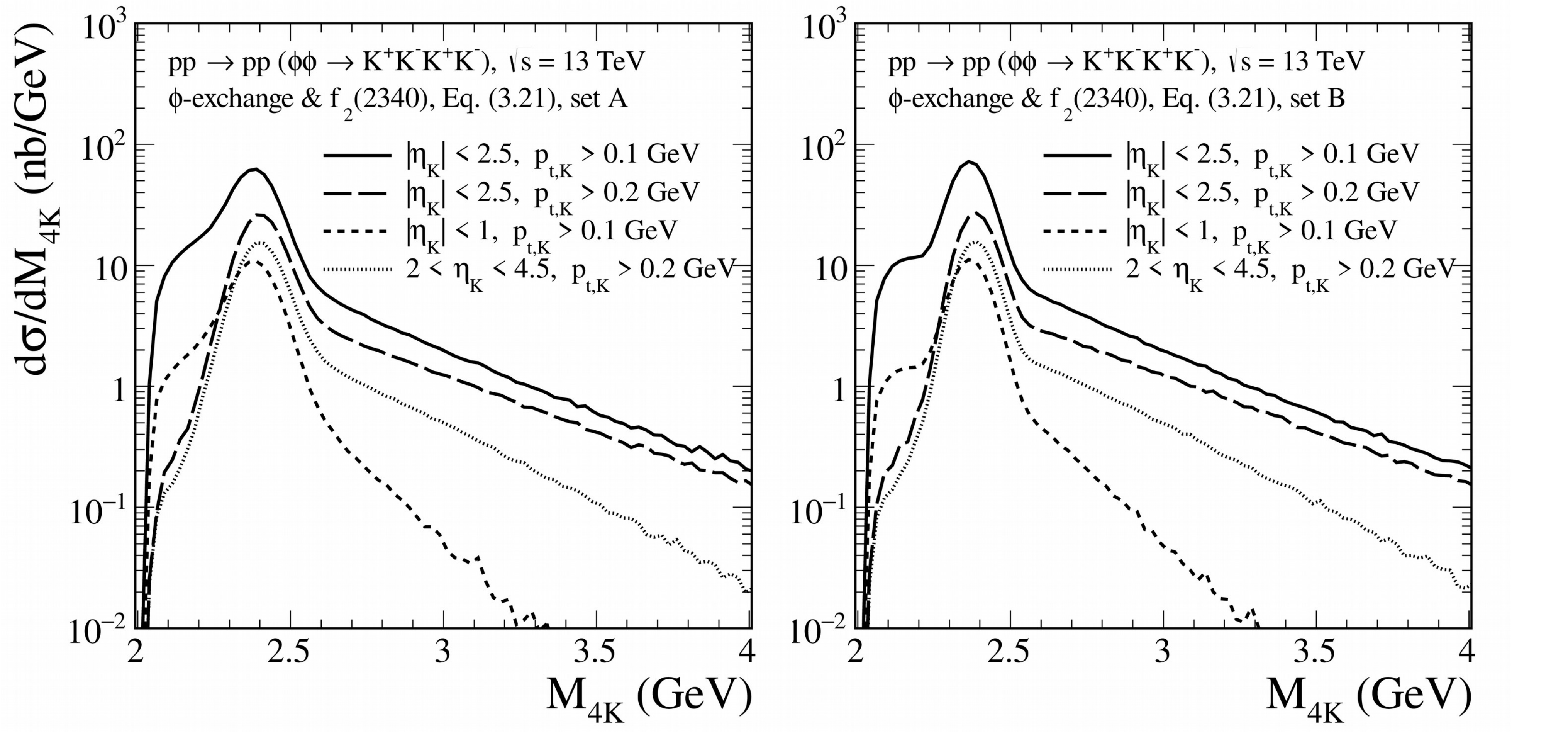


Table 1: The integrated **cross sections in nb** for the $pp \to pp(4K)$ reaction. The absorption effects are included here.

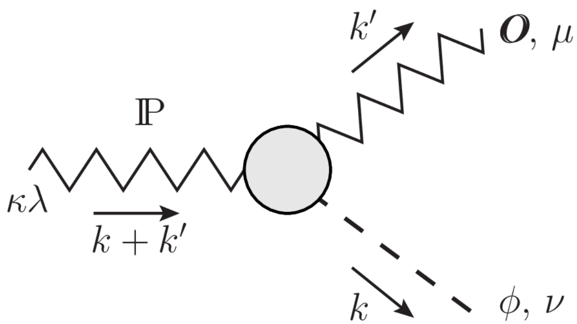
$\sqrt{s}$, TeV	Cuts	Total	ϕ exchange	$f_2(2340)$ (set B)
13	$\lvert\eta_K\rvert < 1$, $p_{t,K} > 0.1$ GeV	2.11	0.83	2.00
13	$\lvert\eta_K\rvert < 2.5$, $p_{t,K} > 0.1$ GeV	16.16	8.30	12.80
13	$\lvert\eta_K\rvert < 2.5$, $p_{t,K} > 0.2$ GeV	5.75	2.67	4.47
13	$2 < \eta_K < 4.5$, $p_{t,K} > 0.2$ GeV	3.06	1.26	2.62

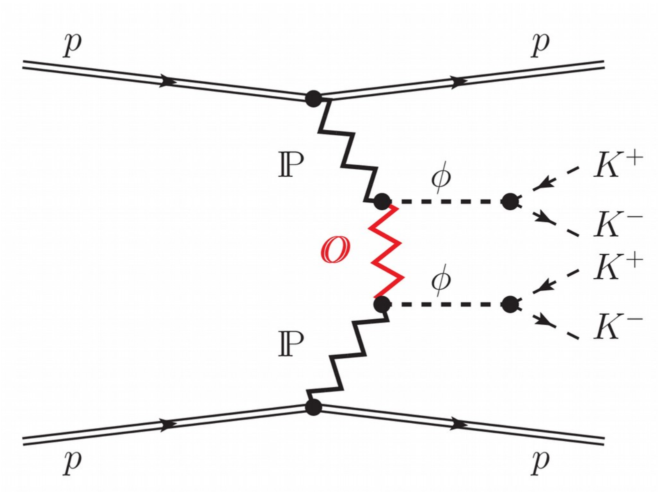
Continuum with Odderon exchange

- The amplitude as for ϕ-exchange contribution, but we have to make:

$$i\Delta^{(\phi)}_{\mu\nu}(\hat{p}) \to i\Delta^{(\mathbb{O})}_{\mu\nu}(s_{34},\hat{p}^2)$$

$$i\Gamma^{(I\!P\phi\phi)}_{\mu\nu\kappa\lambda}(k',k) \to i\Gamma^{(I\!P\mathbb{O}\phi)}_{\mu\nu\kappa\lambda}(k',k)$$





- For the $I\!P\mathbb{O}\phi$ vertex we use an ansatz analogous to $I\!P\phi\phi$vertex:

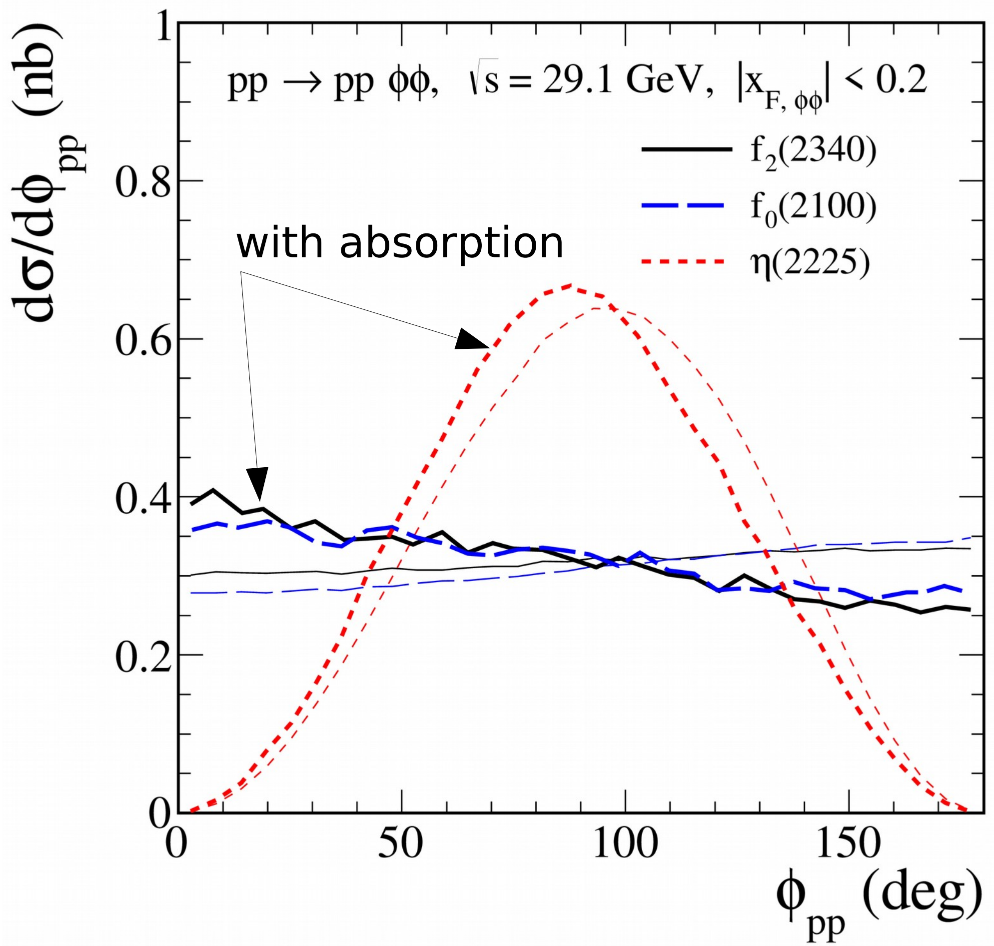
$$i\Gamma^{(I\!P\mathbb{O}\phi)}_{\mu\nu\kappa\lambda}(k',k) = iF^{(I\!P\mathbb{O}\phi)}((k+k')^2,k'^2,k^2)\left[2\,a_{I\!P\mathbb{O}\phi}\,\Gamma^{(0)}_{\mu\nu\kappa\lambda}(k',k) - b_{I\!P\mathbb{O}\phi}\,\Gamma^{(2)}_{\mu\nu\kappa\lambda}(k',k)\right]$$

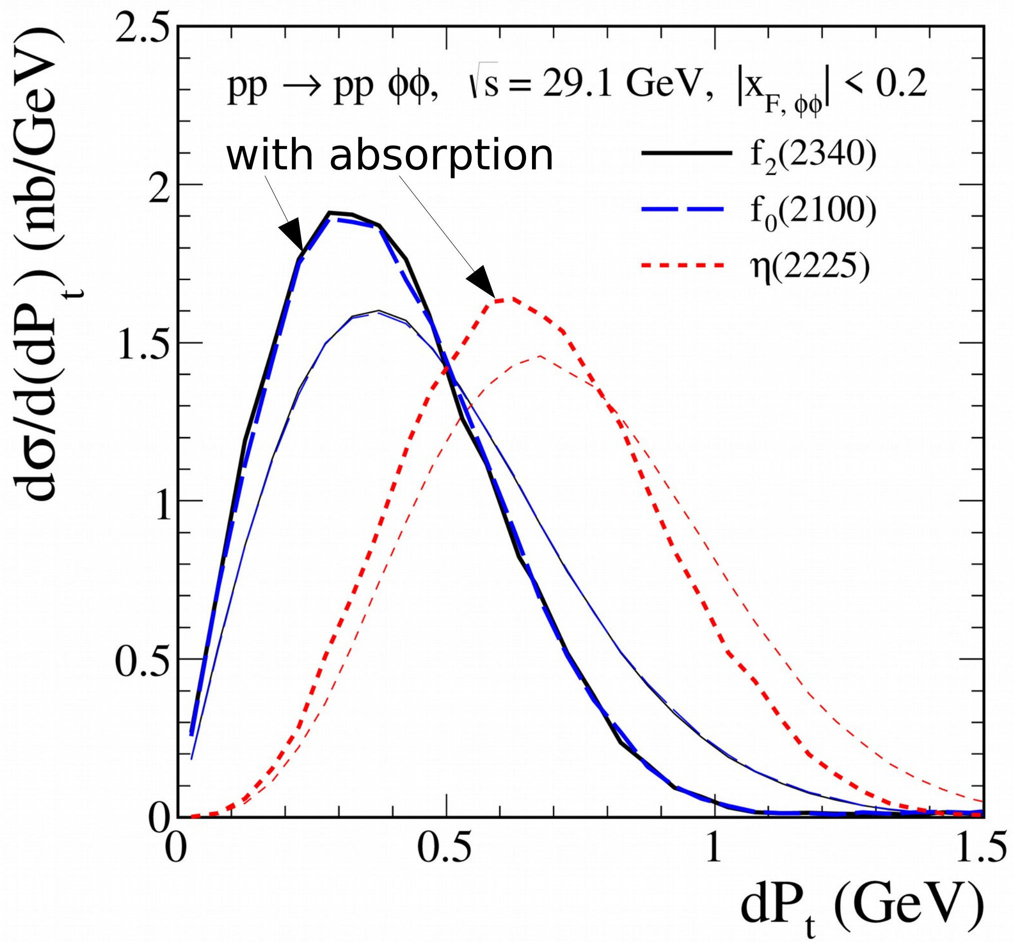
In practical calculations we take the factorized form for the $I\!P\mathbb{O}\phi$ form factor

$$F^{(I\!P\mathbb{O}\phi)}((k+k')^2,k'^2,k^2) = F((k+k')^2)\,F(k'^2)\,F^{(I\!P\mathbb{O}\phi)}(k^2)\,,$$

$$F(k^2) = \frac{1}{1-k^2/\Lambda^2_{\rm odd}}, \quad F^{(I\!P\mathbb{O}\phi)}(0,0,m^2_\phi) = 1$$

$pp \to pp\phi\phi$





The distribution in $\mathrm{dP_t}$ and in ϕ_{pp} for the central exclusive $\phi\phi$ production at $\sqrt{s} = 29.1$ GeV and $|x_{F,\phi\phi}| \leqslant 0.2$. The results for scalar, pseudoscalar and tensor resonances without (the thin lines) and with (the thick lines) absorptive corrections are shown. Because here we are interested only in the shape of the distributions we normalised the differential distributions arbitrarily to 1 nb for both cases, with and without absorption corrections.

IP IP M couplings

P. L., O. Nachtmann, A. Szczurek,
Annals Phys. 344 (2014) 301;
Phys. Rev. D93 (2016) 054015

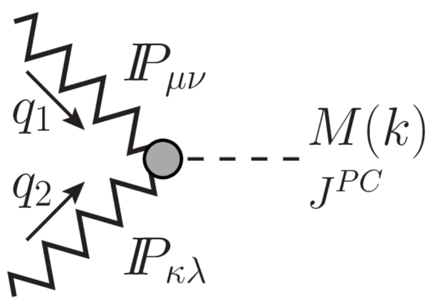
l – orbital angular momentum
S – total spin, we have $S \in \{0, 1, 2, 3, 4\}$
J – total angular momentum (spin of the produced meson)
P – parity of meson
and Bose symmetry requires $l - S$ to be even

In table we list the values of *J* and *P* of mesons which can be produced in annihilation of two "real tensor pomerons".

For each value of *l*, *S*, *J*, and *P* we can construct a covariant Lagrangian density coupling *L*' the field operator for the meson *M* to the pomeron fields and then we can obtain the "bare" vertices corresponding to the *l* and *S*.

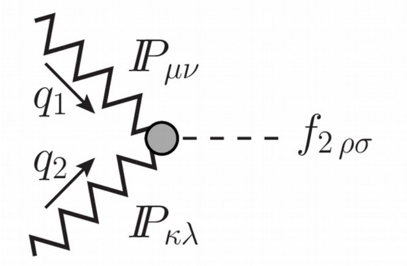
The lowest (*l*,*S*) term for a scalar meson $J^{PC} = 0^{++}$ is (0,0) while for a tensor meson $J^{PC} = 2^{++}$ is (0,2).

l	S	$\vert l-S\vert \leqslant J \leqslant l+S$	$P = (-1)^l$
0	0	0	+
	2	2	
	4	4	
1	1	0, 1, 2	−
	3	2, 3, 4	
2	0	2	+
	2	0,1,2,3,4	
	4	2,3,4,5,6	
3	1	2,3,4	−
	3	0,1,2,3,4,5,6	
4	0	4	+
	2	2,3,4,5,6	
	4	0,1,2,3,4,5,6,7,8	
5	1	4.5.6	−
	3	2,3,4,5,6,7,8	
6	0	6	+
	2	4,5,6,7,8	
	4	2,3,4,5,6,7,8,9,10	



There, *l* is related to the number of derivatives in *L*' thus giving an indication of the angular momentum barrier in the production of *M*.

$I\!P$-$I\!P$-f_2 couplings



In order to write the corresponding formulae of vertices in a compact and convenient form we find it useful to define the tensor $R_{\mu\nu\kappa\lambda} = \frac{1}{2} g_{\mu\kappa} g_{\nu\lambda} + \frac{1}{2} g_{\mu\lambda} g_{\nu\kappa} - \frac{1}{4} g_{\mu\nu} g_{\kappa\lambda}$

$$i\Gamma^{(I\!PI\!Pf_2)(1)}_{\mu\nu,\kappa\lambda,\rho\sigma} = 2i\, g^{(1)}_{I\!PI\!Pf_2} M_0\, R_{\mu\nu\mu_1\nu_1}\, R_{\kappa\lambda\alpha_1\lambda_1}\, R_{\rho\sigma\rho_1\sigma_1}\, g^{\nu_1\alpha_1}\, g^{\lambda_1\rho_1}\, g^{\sigma_1\mu_1}$$

$$i\Gamma^{(I\!PI\!Pf_2)(2)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = -\frac{2i}{M_0}\, g^{(2)}_{I\!PI\!Pf_2} \Big((q_1\cdot q_2)\, R_{\mu\nu\rho_1\alpha}\, R_{\kappa\lambda\sigma_1}{}^{\alpha} - q_{1\rho_1}\, q_2^{\mu_1}\, R_{\mu\nu\mu_1\alpha}\, R_{\kappa\lambda\sigma_1}{}^{\alpha} - q_1^{\mu_1}\, q_{2\sigma_1}\, R_{\mu\nu\rho_1\alpha}\, R_{\kappa\lambda\mu_1}{}^{\alpha} + q_{1\rho_1}\, q_{2\sigma_1}\, R_{\mu\nu\kappa\lambda} \Big) R_{\rho\sigma}{}^{\rho_1\sigma_1}$$

$$i\Gamma^{(I\!PI\!Pf_2)(3)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = -\frac{2i}{M_0}\, g^{(3)}_{I\!PI\!Pf_2} \Big((q_1\cdot q_2)\, R_{\mu\nu\rho_1\alpha}\, R_{\kappa\lambda\sigma_1}{}^{\alpha} + q_{1\rho_1}\, q_2^{\mu_1}\, R_{\mu\nu\mu_1\alpha}\, R_{\kappa\lambda\sigma_1}{}^{\alpha} + q_1^{\mu_1}\, q_{2\sigma_1}\, R_{\mu\nu\rho_1\alpha}\, R_{\kappa\lambda\mu_1}{}^{\alpha} + q_{1\rho_1}\, q_{2\sigma_1}\, R_{\mu\nu\kappa\lambda} \Big) R_{\rho\sigma}{}^{\rho_1\sigma_1}$$

$$i\Gamma^{(I\!PI\!Pf_2)(4)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = -\frac{i}{M_0}\, g^{(4)}_{I\!PI\!Pf_2} \Big(q_1^{\alpha_1}\, q_2^{\mu_1}\, R_{\mu\nu\mu_1\nu_1}\, R_{\kappa\lambda\alpha_1\lambda_1} + q_2^{\alpha_1}\, q_1^{\mu_1}\, R_{\mu\nu\alpha_1\lambda_1}\, R_{\kappa\lambda\mu_1\nu_1} \Big) R^{\nu_1\lambda_1}{}_{\rho\sigma}$$

$$i\Gamma^{(I\!PI\!Pf_2)(5)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = -\frac{2i}{M_0^3}\, g^{(5)}_{I\!PI\!Pf_2} \Big(q_1^{\mu_1}\, q_2^{\nu_1}\, R_{\mu\nu\nu_1\alpha}\, R_{\kappa\lambda\mu_1}{}^{\alpha} + q_1^{\nu_1}\, q_2^{\mu_1}\, R_{\mu\nu\mu_1\alpha}\, R_{\kappa\lambda\nu_1}{}^{\alpha} - 2(q_1\cdot q_2)\, R_{\mu\nu\kappa\lambda} \Big) q_{1\alpha_1}\, q_{2\lambda_1}\, R^{\alpha_1\lambda_1}{}_{\rho\sigma}$$

$$i\Gamma^{(I\!PI\!Pf_2)(6)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = \frac{i}{M_0^3}\, g^{(6)}_{I\!PI\!Pf_2} \Big(q_1^{\alpha_1}\, q_1^{\lambda_1}\, q_2^{\mu_1}\, q_{2\rho_1}\, R_{\mu\nu\mu_1\nu_1}\, R_{\kappa\lambda\alpha_1\lambda_1} + q_2^{\alpha_1}\, q_2^{\lambda_1}\, q_1^{\mu_1}\, q_{1\rho_1}\, R_{\mu\nu\alpha_1\lambda_1}\, R_{\kappa\lambda\mu_1\nu_1} \Big) R^{\nu_1\rho_1}{}_{\rho\sigma}$$

$$i\Gamma^{(I\!PI\!Pf_2)(7)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = -\frac{2i}{M_0^5}\, g^{(7)}_{I\!PI\!Pf_2}\, q_1^{\rho_1}\, q_1^{\alpha_1}\, q_1^{\lambda_1}\, q_2^{\sigma_1}\, q_2^{\mu_1}\, q_2^{\nu_1}\, R_{\mu\nu\mu_1\nu_1}\, R_{\kappa\lambda\alpha_1\lambda_1}\, R_{\rho\sigma\rho_1\sigma_1}$$

We can associate the couplings j = 1, ..., 7 with (*l*,*S*) values:
(0,2), (2,0) – (2,2), (2,0) + (2,2), (2,4), (4,2), (4,4), (6,4), respectively.

see P. L., O. Nachtmann, A. Szczurek, Phys. Rev. D93 (2016) 054015