THE UNIVERSITY OF MELBOURNE

Radiative neutrino mass models and the flavour anomalies

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1. Summary of the flavour anomalies

2. Systematic survey of radiative neutrino mass models

3. Brief review of the minimal Bauer-Neubert proposal

4. A next-to-minimal model & *exploding!* operators

5. Final remarks

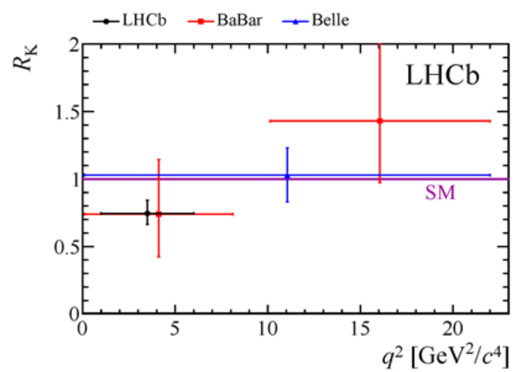
1. Summary of the flavour anomalies

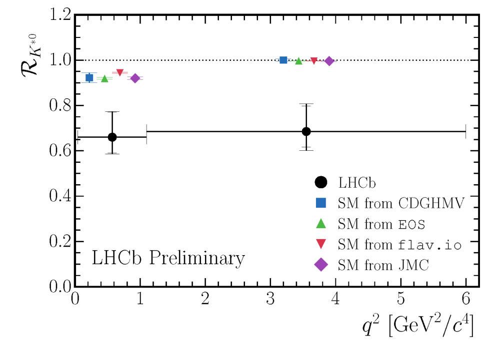
A) Hints for μ vs e universality violation in $b \to s$ transitions

$$R_{K^{(*)}} = \frac{\Gamma(B \to K^{(*)}\mu^+\mu^-)}{\Gamma(B \to K^{(*)}e^+e^-)}$$

hadronic uncertainties cancel in the ratio

Pre-Moriond 2019:





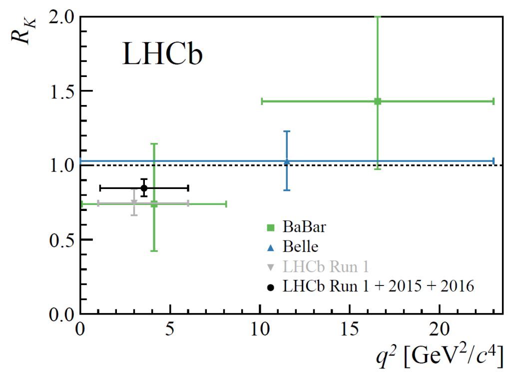
Post-Moriond 2019:

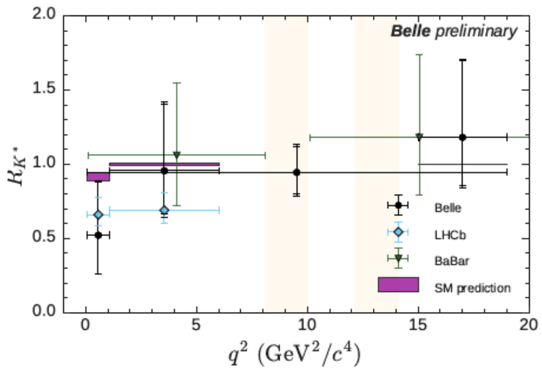
LHCb:

$$R_K = 0.846\ ^{+0.060}_{-0.054}\ ^{+0.016}_{-0.014} \quad 0.1 < q^2/\text{GeV}^2 < 6$$

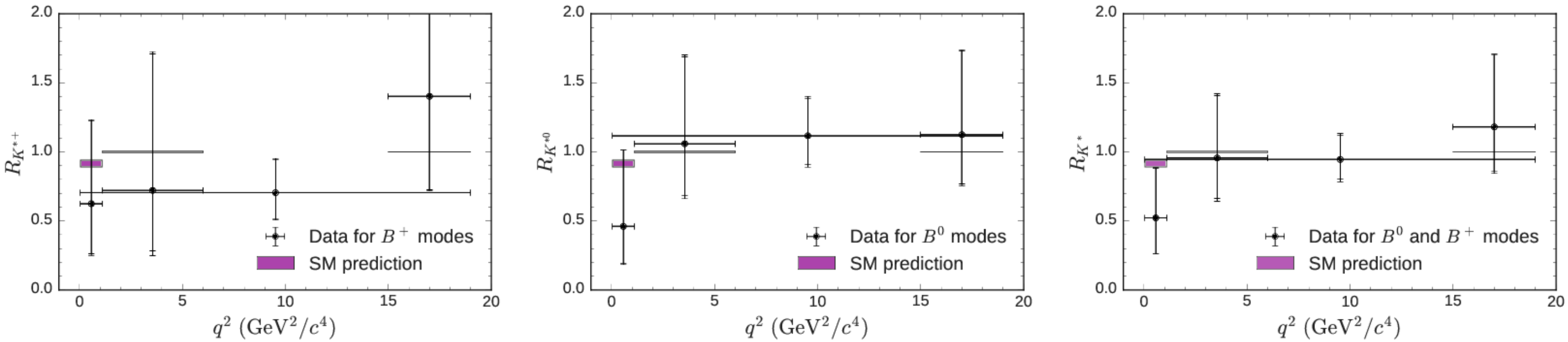
Belle R_{K^*}:

q^2 in GeV^2/c^4	All modes	B^0 modes	B^+ modes
[0.045, 1.1]	$0.52^{+0.36}_{-0.26} \pm 0.05$	$0.46^{+0.55}_{-0.27} \pm 0.07$	$0.62^{+0.60}_{-0.36} \pm 0.10$
[1.1, 6]	$0.96^{+0.45}_{-0.29} \pm 0.11$	$1.06^{+0.63}_{-0.38} \pm 0.13$	$0.72^{+0.99}_{-0.44} \pm 0.18$
[0.1, 8]	$0.90^{+0.27}_{-0.21} \pm 0.10$	$0.86^{+0.33}_{-0.24} \pm 0.08$	$0.96^{+0.56}_{-0.35} \pm 0.14$
[15, 19]	$1.18^{+0.52}_{-0.32} \pm 0.10$	$1.12^{+0.61}_{-0.36} \pm 0.10$	$1.40^{+1.99}_{-0.68} \pm 0.11$
[0.045,]	$0.94^{+0.17}_{-0.14} \pm 0.08$	$1.12^{+0.27}_{-0.21} \pm 0.09$	$0.70^{+0.24}_{-0.19} \pm 0.07$





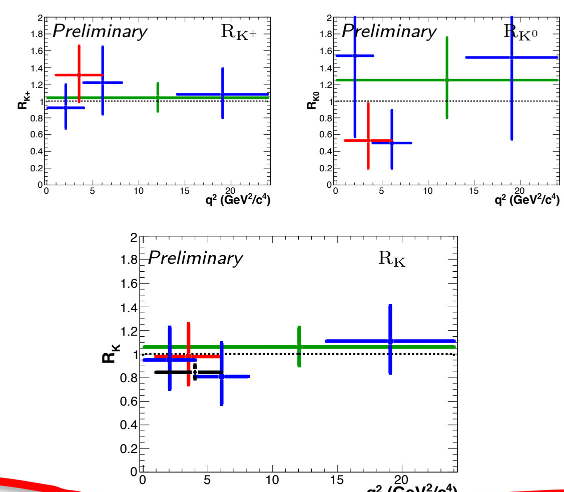
Belle 1904.02440 on R_{K^*}



Belle EPS HEP July 2019, talk by S. Choudhury

R_K, R_{K^+} and R_{K^0} results from Belle **[New]**

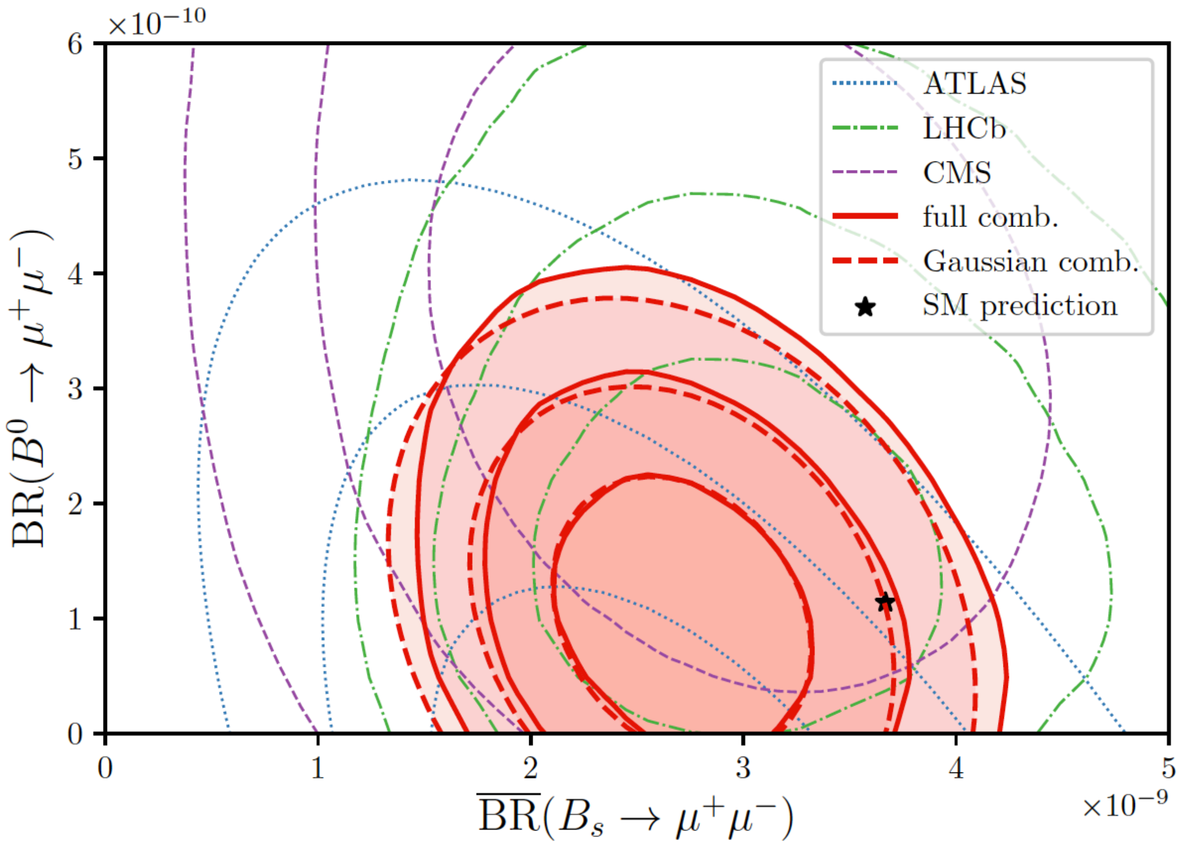
- R_{K^+}, R_{K^0} and R_K are measured for [0.1 , 4.0], [4.0 , 8.12], [1.0 , 6.0], $>$ 14.18 and $>$ 0.1 q^2 bins.
- R_K is taken as weighted average of R_{K^+} and R_{K^0}.



- The measurements are found to be consistent with SM prediction as well as LHCb result.

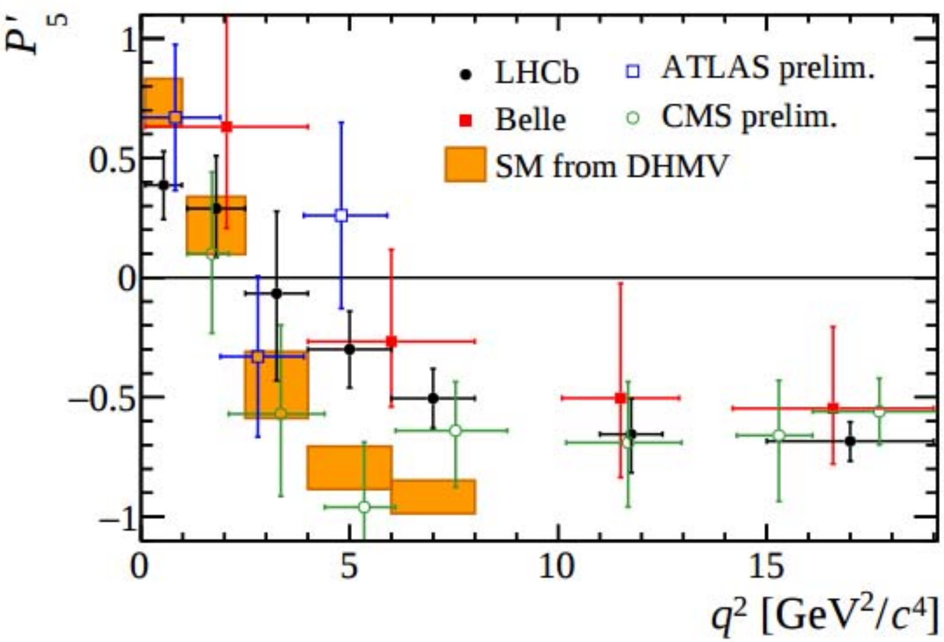
Slide from David Straub's talk at Moriond 2019:

$B_q \to \mu^+\mu^-$: combination of LHCb, ATLAS, CMS



- Including new ATLAS measurement Talk by O. Igonkina
- $\overline{\mathrm{BR}}(B_s \to \mu^+\mu^-)$ roughly 2σ below the SM prediction

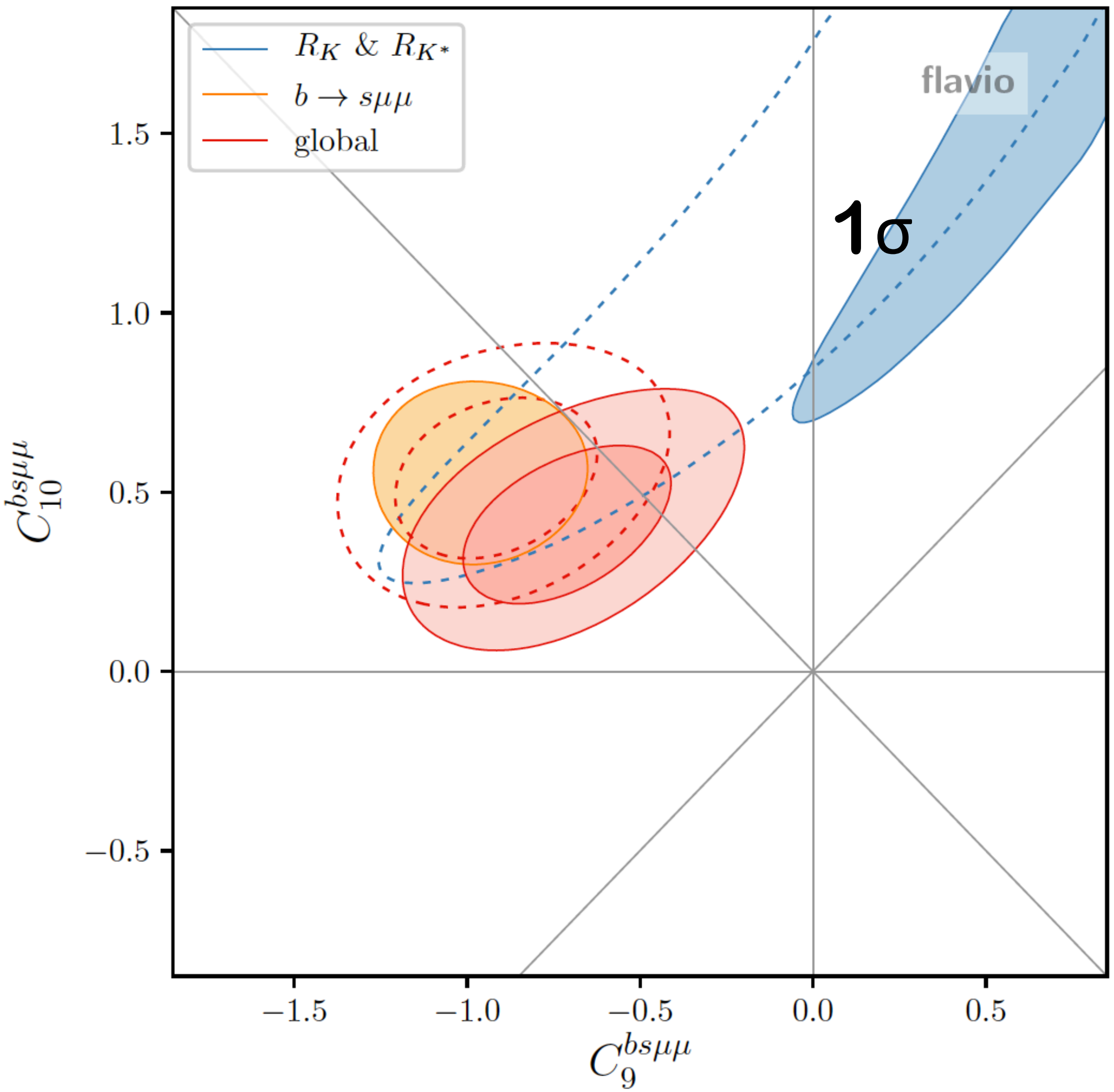
B) Hint of non-standard angular observable in $B \to K^* \mu^+ \mu^-$



Note: CMS is consistent with SM, and there are hadronic uncertainties.

Slide from Moriond talk of David Straub

Muonic C_9 vs. C_{10}



$C_9^{bs\mu\mu}\ (\bar{s}_L \gamma^\mu b_L)(\mu \gamma_\mu \mu)$

$C_{10}^{bs\mu\mu}\ (\bar{s}_L \gamma^\mu b_L)(\mu \gamma_\mu \gamma_5 \mu)$

Pre-Moriond

- Perfect agreement between $R_{K(*)}$ & $b \to s\mu\mu$
- Pull towards $C_{10} > 0$ mostly from $B_s \to \mu^+\mu^-$
- Excellent for models with LH leptons ($C_9 = -C_{10}$)

Now

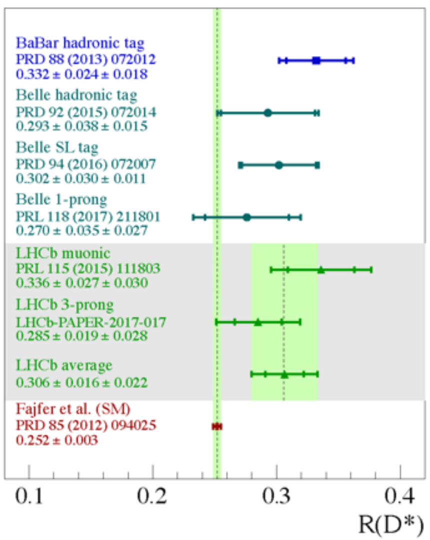
- Agreement between $R_{K(*)}$ & $b \to s\mu\mu$ no longer perfect
- Fit closer to SM, $C_9 = -C_{10}$ still preferred

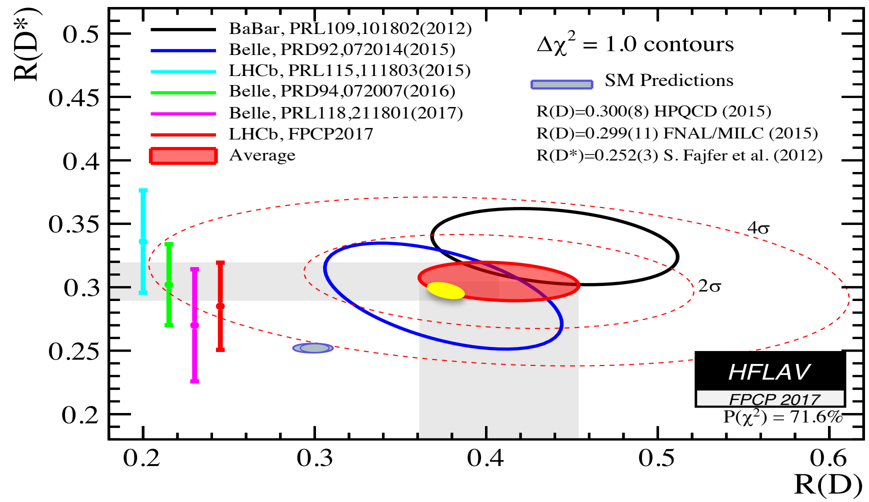
C) Hints for τ vs μ/e universality violation in b→c transitions

$$R_{D^{(*)}} \equiv \frac{\Gamma(\bar{B} \to D^{(*)} \tau \bar{\nu})}{\Gamma(\bar{B} \to D^{(*)} \ell \bar{\nu})}$$

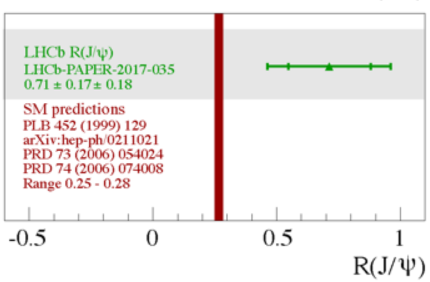
hadronic uncertainties cancel in the ratio

Pre-Moriond 2019:





= approx. Belle II projection



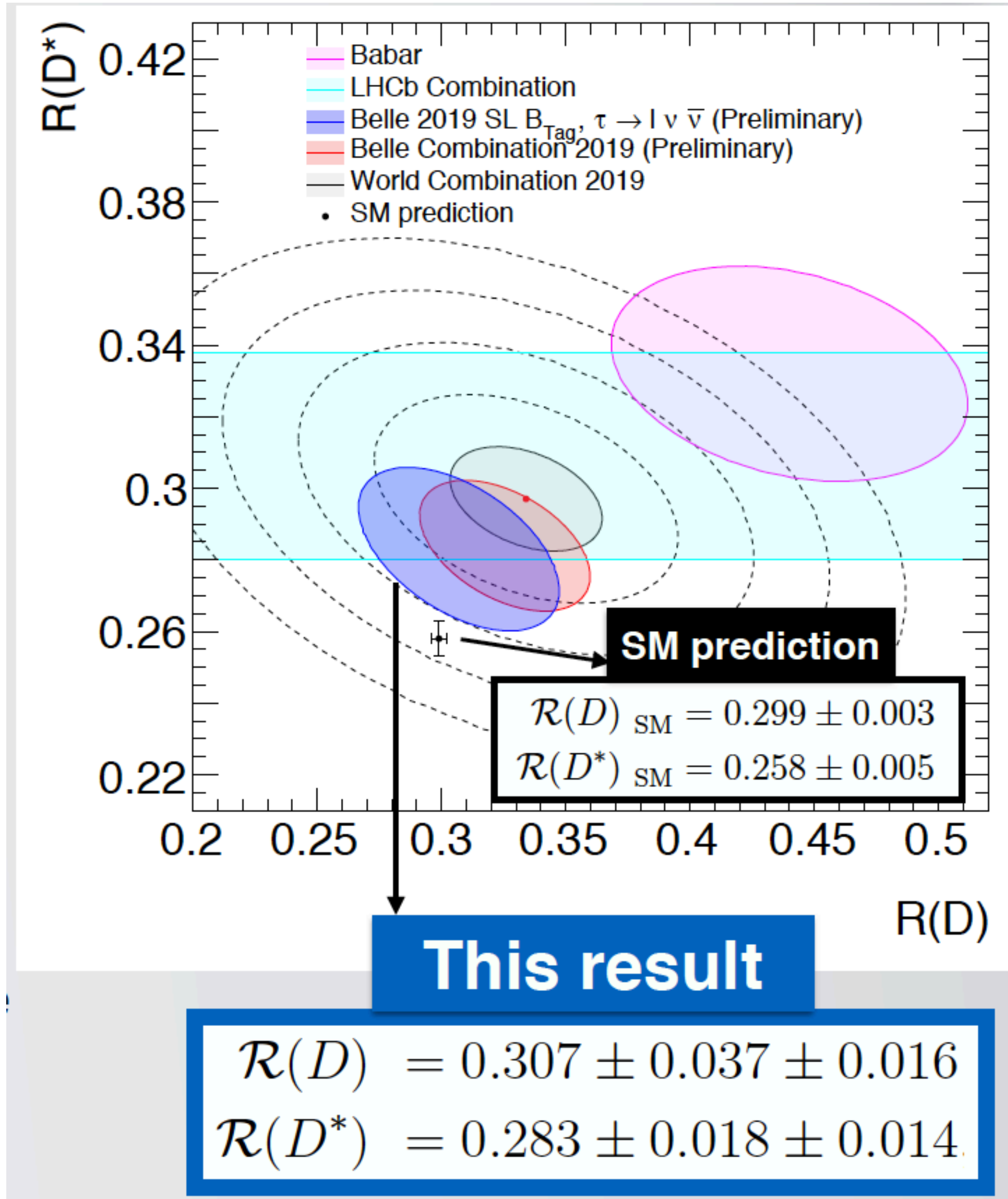
$$R_{J/\psi} = \frac{\Gamma(B_c^+ \to J/\psi\, \tau^+ \nu_\tau)}{\Gamma(B_c^+ \to J/\psi\, \mu^+ \nu_\mu)} = 0.71 \pm 0.17 \pm 0.18$$

SM prediction is 0.25 – 0.28

hadronic uncertainties cancel in the ratio

Post Moriond 2019:

Talk by Giacomo Caria



2. Systematic survey of radiative neutrino mass models

See recent long review article: Cai+ (2017) “From the trees to the forest …”

A connection with radiative neutrino mass models can be made through scalar leptoquarks whose interactions with b,s,c,τ,μ could resolve the anomalies and also generate neutrino masses.

Radiative neutrino mass models are interesting in their own right, even if the anomalies go away.

Restrict to Majorana models with no RH neutrinos.

Early models by Zee and Zee&Babu are quite well known, but there are actually *many, many models* …

Systematic classification and model-construction schemes are needed. There are two complementary approaches based on opening-up $\Delta L=2$ effective operators systematically.

One approach is based on opening-up the ΔL=2 Weinberg operator and higher mass dimension generalisations:

$$\frac{1}{M}LLHH, \quad \frac{1}{M^3}LLHH(H^\dagger H), \quad \frac{1}{M^5}LLHH(H^\dagger H)^2, \ldots$$

O_1 $\quad$ O_1' $\quad$ O_1''

O_1 at tree-level ⇒ the Type-I,II,III seesaw models (next slide).

O_1 at 1-loop: Ma (1998); Bonnet, Hirsch, Ota, Winter (2012).

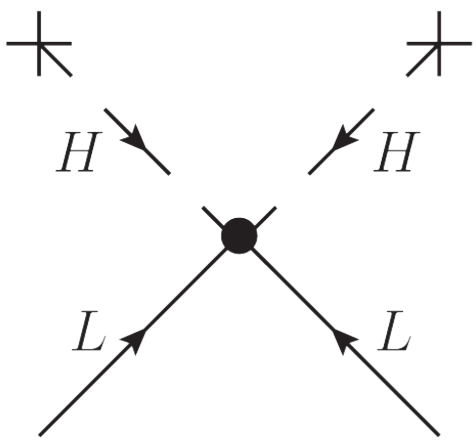
O_1 at 2-loops: Aristizabal Sierra, Degee, Dorame, Hirsch (2015).

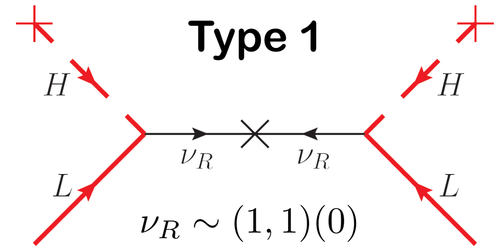
O_1' at 1-loop: Cepedello, Hirsch, Helo (2017).

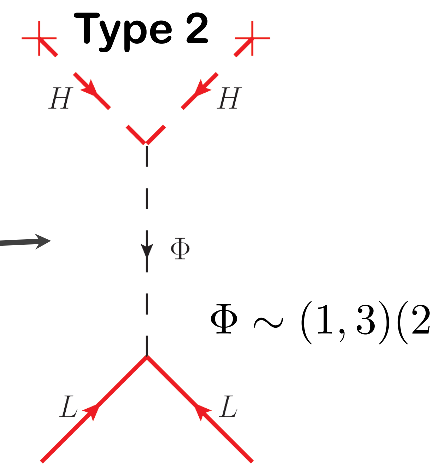
See back-up slide for some diagram topologies.

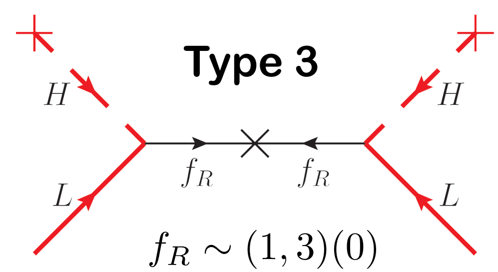
Type-1,2,3 seesaw models:

"Open up" dim-5 LLHH in all minimal, tree-level ways: get all the minimal seesaw models systematically.









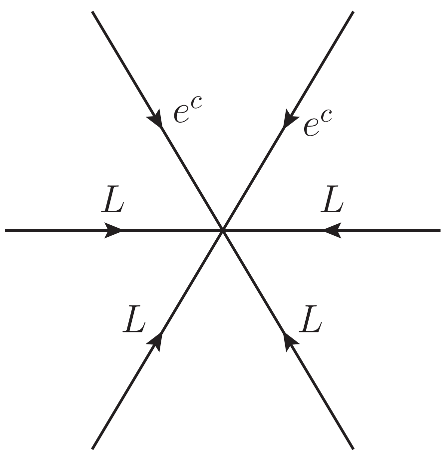
The advantage of this approach to constructing models is that you don't miss any (within the constraints of your assumptions about the general nature of the new physics).

<u>Another approach</u> is based on opening-up the ΔL=2 non-Weinberg ops.

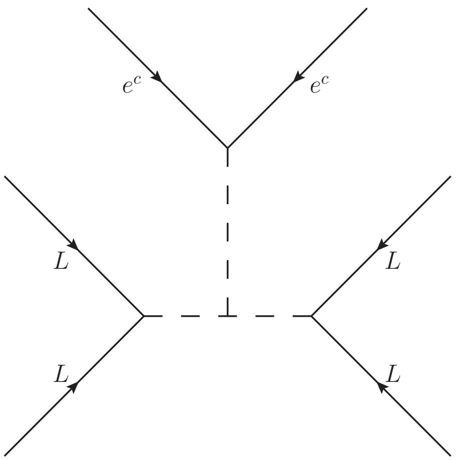
Babu & Leung wrote down the operators constructed from SM fermions (no RH neutrinos) and a single Higgs doublet for mass dimensions 7, 9 and 11. (Higher-d operators don't work.)

Babu, Leung (2001)
See also: de Gouvêa, Jenkins (2008)
With gauge fields: Bhattacharya, Wudka (2016)
Henning+ (2015)

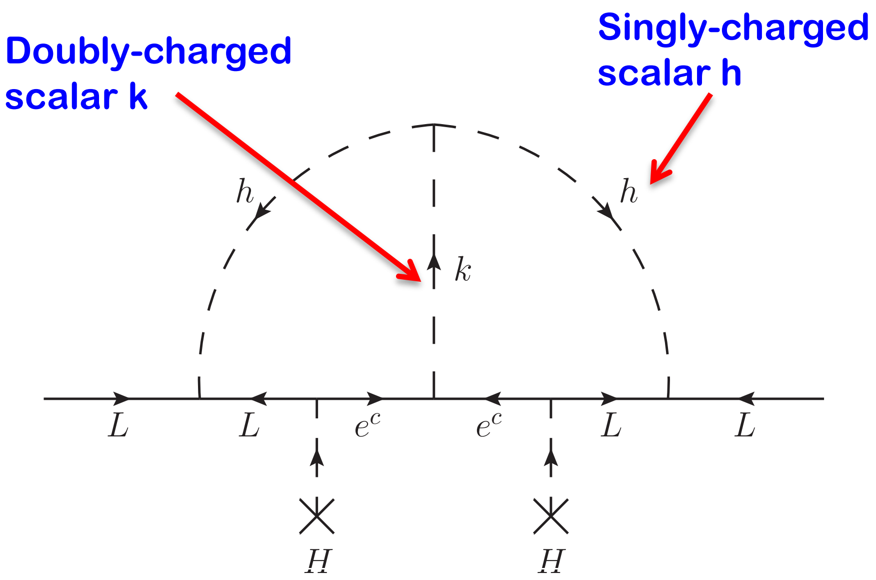
Famous example: Zee-Babu model



Effective op
$O_9 = LLLe^cLe^c$



Opening it up at tree-level



2-loop nu mass diagram

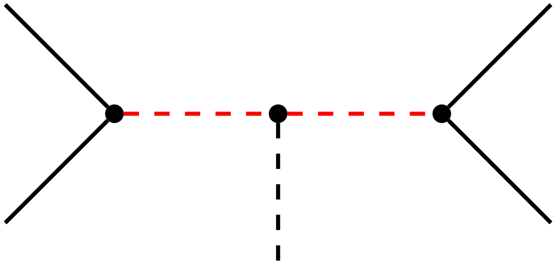
The exotics (k, h in this case) can be searched for at the LHC.

In Cai+ (2015), we finished the job of constructing all minimal models from d = 7 ops:

$$\mathcal{O}'_1 = LL\tilde{H}HHH$$

$$\mathcal{O}_2 = LLL\bar{e}H, \quad \mathcal{O}_3 = LLQ\bar{d}H, \quad \mathcal{O}_4 = LLQ^\dagger\bar{u}^\dagger H, \quad \mathcal{O}_8 = L\bar{d}\bar{e}^\dagger\bar{u}^\dagger H$$

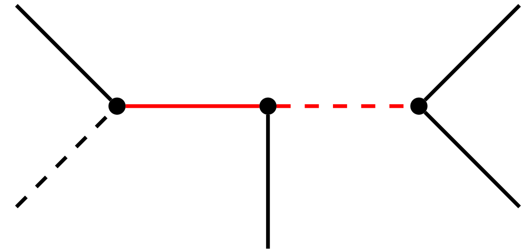
Scalar-only extension:



Scalar	Scalar	Operator	
(1,2,1/2)	(1,1,1)	$O_{2,3,4}$	Zee
(3,2,1/6)	(3,1,-1/3)	$O_{3,8}$	Babu, Leung, Julio
(3,2,1/6)	(3,3,-1/3)	O_3	

The LQ scalars highlighted in red are well-known candidates for explaining the flavour anomalies! (Note: they are not the only ones – see back-up slide for a list of other proposals.)

Scalar + fermion extension:



Dirac fermion	Scalar	Operator
(1,2,-3/2)	(1,1,1)	O_2
(3,2,-5/6)	(1,1,1)	O_3
(3,1,2/3)	(1,1,1)	O_3
(3,1,2/3)	(3,2,1/6)	O_3
(3,2,-5/6)	(3,1,-1/3)	$O_{3,8}$
(3,2,-5/6)	(3,3,-1/3)	O_3
(3,3,2/3)	(3,2,1/6)	O_3
(3,2,7/6)	(1,1,1)	O_4
(3,1,-1/3)	(1,1,1)	O_4
(3,2,7/6)	(3,2,1/6)	O_8
(1,2,-1/2)	(3,2,1/6)	O_8

Babu, Julio

Cai, Clarke, Schmidt, RV

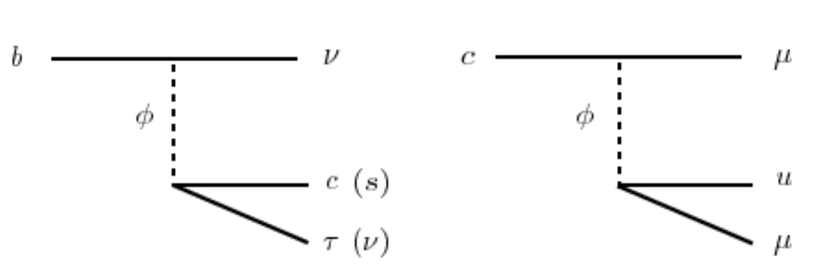
The LQ scalars highlighted in red are well-known candidates for explaining the flavour anomalies!

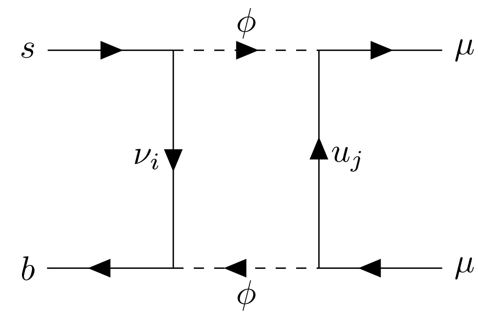
3. Brief review of the minimal Bauer-Neubert proposal

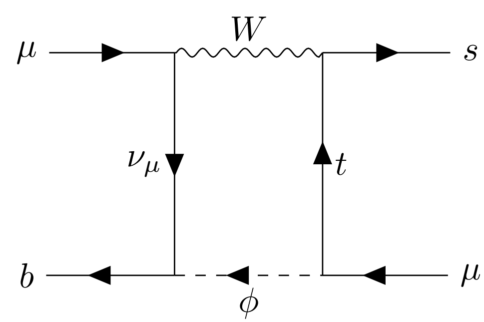
Cai, Gargalionis, Schmidt, RV (2017) – the plots to follow have been updated post Moriond

The Φ~(3,1,-1/3) scalar leptoquark

Proposed by Bauer & Neubert (2016) to explain $R_{D^{(*)}}$ at tree-level and R_K at 1-loop.

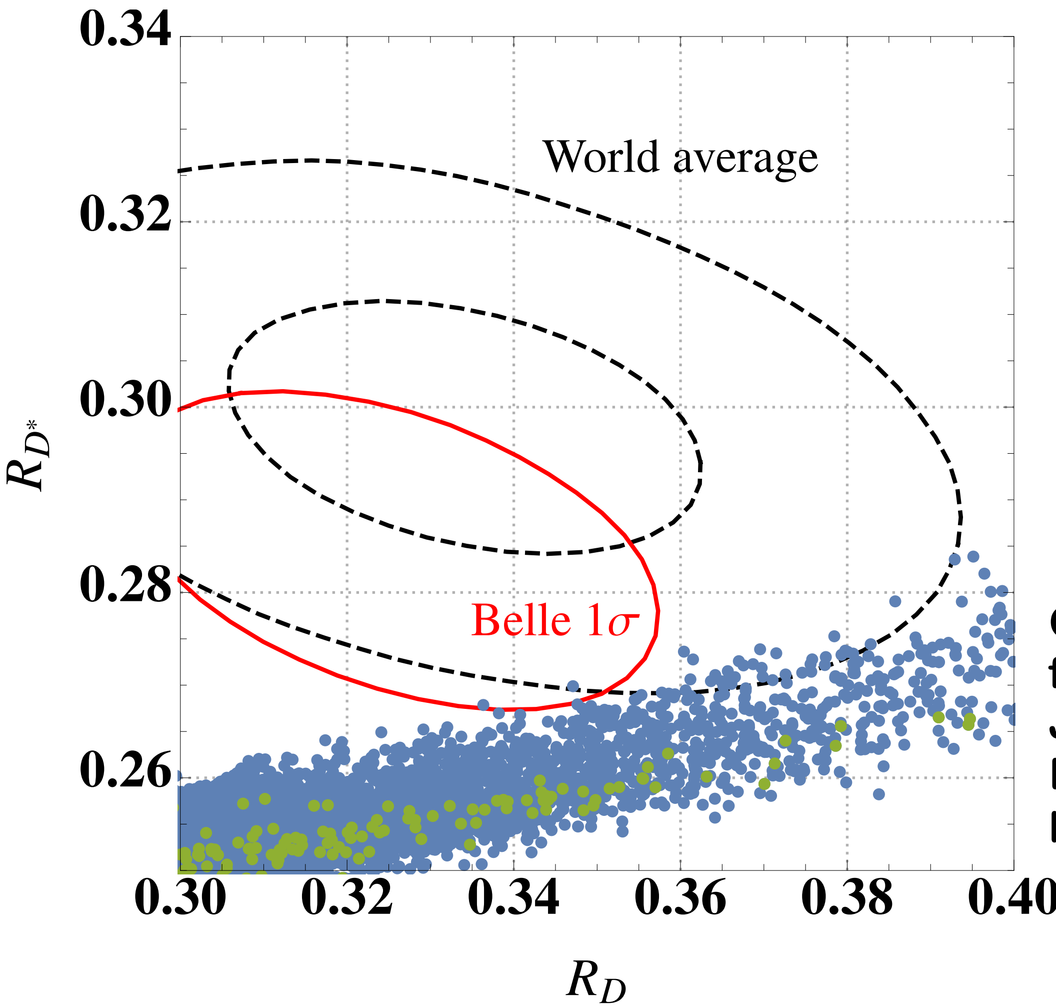






We performed a detailed phenomenological study of the BN model *per se*, and then examined one rad. nu mass model containing (3,1,-1/3) LQ to see if it could simultaneously explain anomalies & generate neutrino masses correctly.

The (3,1,-1/3) LQ for $R_{D(*)}$ without nu mass connection:



Simultaneous fit.

Green points lie in 2σ region for $R_{K(*)}$.

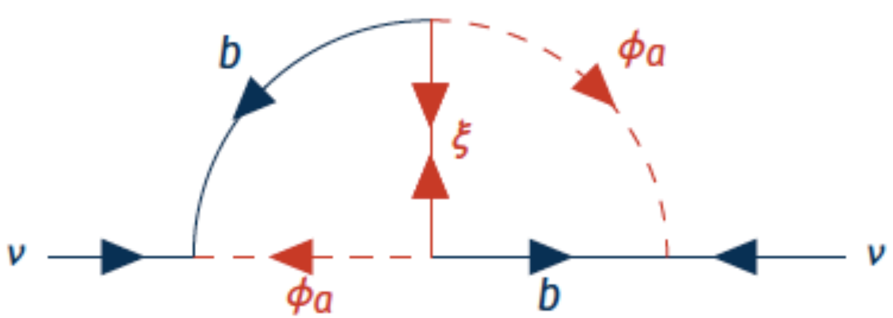
Calculation performed following Bardhan et al JHEP 01 (2017) 125. I'll comment more on this later.

Overall: This very simple model improves agreement with the data, but isn't perfect.

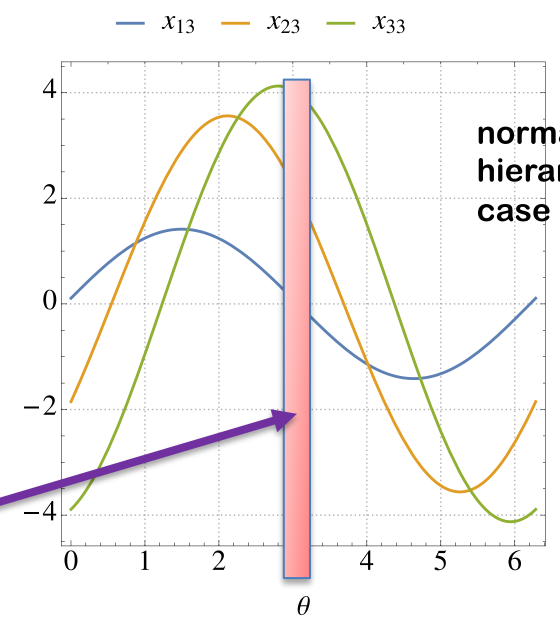
Case study for adding a neutrino mass connection:

Chose existing model that has 2 copies of Φ~(3,1,-1/3) and a Majorana fermion ξ~(8,1,0). Generates dim-9 op. $\mathcal{O}_{11} = LLQd^cQd^c$

Angel+ (2013)



2-loop nu mass diagram



Casas-Ibarra-type parameter

θ ~ 3±nπ req. due to μN→eN constraint.

Resulting x-ratios OK for $R_{D(*)}$ but <u>not</u> for $R_{K(*)}$

4. A next-to-minimal model

Bigaran, Gargalionis, RV: 1906.01870

Adding the φ~(3,3,-1/3) scalar leptoquark

Recall from d=7 operator models:

Scalar	Scalar	Operator
(3,2,1/6)	(3,1,-1/3)	$O_{3,8}$
(3,2,1/6)	(3,3,-1/3)	O_3
Fermion	**Scalar**	**Operator**
(3,2,-5/6)	(3,1,-1/3)	$O_{3,8}$
(3,2,-5/6)	(3,3,-1/3)	O_3

The simultaneous presence of the two LQs is not required at d=7 for nu mass, but you can put them together.

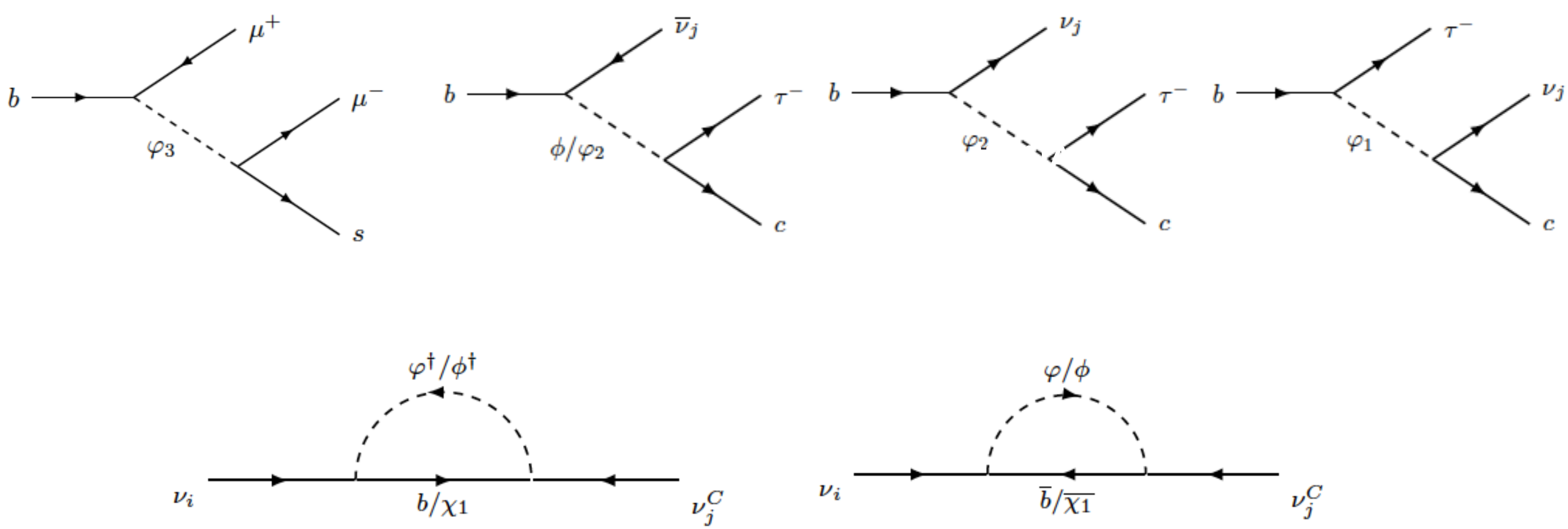
Are there models where they are required? Yes!! (See in a moment.)

New particles: $\chi_L + \chi_R \sim (3,2,-5/6)$

vector-like fermion doublet ~ $\begin{pmatrix} \chi_1 : Q = -1/3 \\ \chi_2 : Q = -4/3 \end{pmatrix}$

$\Phi \sim (3,1,-1/3)$ **and** $\varphi \sim (3,3,-1/3)$

leptoquark scalars



b→s,c both at tree-level. **1-loop neutrino mass.**

Yukawa couplings:

Gauge basis:

$$\mathcal{L}_{\rm int} = m_Q\overline{\chi}\chi + Y_b\overline{d_R}H\chi_L + \left(\lambda_{\phi L}\phi^\dagger - \lambda_{\varphi L}\varphi^\dagger\right)\overline{L_L^c}Q_L + \lambda_R\overline{e_R^c}u_R\phi^\dagger + \left(\lambda_{\chi\phi}\phi - \lambda_{\chi\varphi}\varphi\right)\overline{\chi_R}L_L + \text{h.c}$$

Charged-fermion mass basis; neutrino flavour basis:

$$\mathcal{L}_{int} = \left(y^{\chi\varphi}\overline{\chi_{2,R}}\breve{\nu}_L + x^{L\varphi}\overline{e_L^C}\hat{d}_L\right)\varphi_3^\dagger + \left(y^{\chi\varphi}\overline{\hat{\chi}_{1,R}}e_L - y^{L\varphi}\overline{\breve{\nu}_L^C}u_L\right)\varphi_1^\dagger + \frac{1}{\sqrt{2}}y^{\chi\varphi}\left(\overline{\hat{\chi}_{1,R}}\breve{\nu}_L - \overline{\chi_{2,R}}e_L\right)\varphi_2 + \frac{1}{\sqrt{2}}\left(y^{L\varphi}\overline{e_L^C}u_L + x^{L\varphi}\overline{\breve{\nu}_L^C}\hat{d}_L\right)\varphi_2^\dagger + \left(x^{L\phi}\overline{\breve{\nu}_L^C}\hat{d}_L - y^{L\phi}\overline{e_L^C}u_L + y^{R\phi}\overline{e_R^C}u_R\right)\phi^\dagger + y^{\chi\phi}\left(\overline{\hat{\chi}_{1,R}}\breve{\nu}_L + \overline{\chi_{2,R}}e_L\right)\phi + \text{h.c.}$$

$$x^{L\varphi} \equiv y^{L\varphi}\mathbf{V}, \quad \text{and} \quad x^{L\phi} \equiv y^{L\phi}\mathbf{V}.$$

V = CKM

Parameter dependence of neutrino mass matrix:

$$(m_\nu)_{ij} = \frac{3m_{\chi_1} m_b}{16\pi^2} m_{b\chi} \times \left[2(x_{i3}^{L\phi} y_j^{\chi\phi} + y_i^{\chi\phi *} x_{j3}^{L\phi *}) \frac{\ln(\frac{m_{\chi_1}}{m_\phi})}{m_{\chi_1}^2 - m_\phi^2} + (x_{i3}^{L\varphi} y_j^{\chi\varphi} + y_i^{\chi\varphi *} x_{j3}^{L\varphi *}) \frac{\ln(\frac{m_{\chi_1}}{m_\varphi})}{m_{\chi_1}^2 - m_\varphi^2} \right]$$

It turns out that having the isotriplet LQ dominate neutrino mass generation provides significantly larger parameter space that passes all constraints and permits anomalies to be addressed.

Rank-2 mass matrix at this level of approximation.

Casas-Ibarra-like parameterisation:

$$\mathbf{m}_\nu = \mathbf{U}^\dagger \mathrm{diag}(m_1, m_2, m_3) \mathbf{U}^* = m_2 \mathbf{u}_2^\dagger \mathbf{u}_2^* + m_k \mathbf{u}_k^\dagger \mathbf{u}_k^*.$$

u_i are columns of PMNS matrix, including phases. Choose k=3 for normal hierarchy.

$$\mathbf{m}_\nu = \frac{m_0}{2} \left[\left(\frac{\mathbf{x}_{\mathbf{L}\eta}}{\zeta} + \zeta \mathbf{y}_{\chi\eta}^\dagger \right) \left(\frac{\mathbf{x}_{\mathbf{L}\eta}}{\zeta} + \zeta \mathbf{y}_{\chi\eta}^\dagger \right)^T - \left(\frac{\mathbf{x}_{\mathbf{L}\eta}}{\zeta} - \zeta \mathbf{y}_{\chi\eta}^\dagger \right) \left(\frac{\mathbf{x}_{\mathbf{L}\eta}}{\zeta} - \zeta \mathbf{y}_{\chi\eta}^\dagger \right)^T \right]_{\eta = \varphi}$$

$$\mathbf{x}_{\mathbf{L}\varphi} = x_{j3}^{L\varphi}, \quad \mathbf{y}_{\chi\varphi} = y_i^{\chi\varphi}$$

$$m_0 = \frac{3 m_{b_\chi} m_b m_{\chi_1}}{16\pi^2 (m_{\chi_1}^2 - m_\varphi^2)} \ln\left(\frac{m_{\chi_1}}{m_\varphi} \right)$$

ζ = Casas-Ibarra free complex parameter
Important for pheno.

$$\mathbf{x}_{\mathbf{L}\eta} = \frac{\zeta}{\sqrt{2m_0}} (\sqrt{m_2} \mathbf{u_2}^* + i\sqrt{m_3} \mathbf{u_3}^*),$$

$$\mathbf{y}_{\chi\eta}^\dagger = \frac{1}{\zeta\sqrt{2m_0}} (\sqrt{m_2} \mathbf{u_2}^* - i\sqrt{m_3} \mathbf{u_3}^*).$$

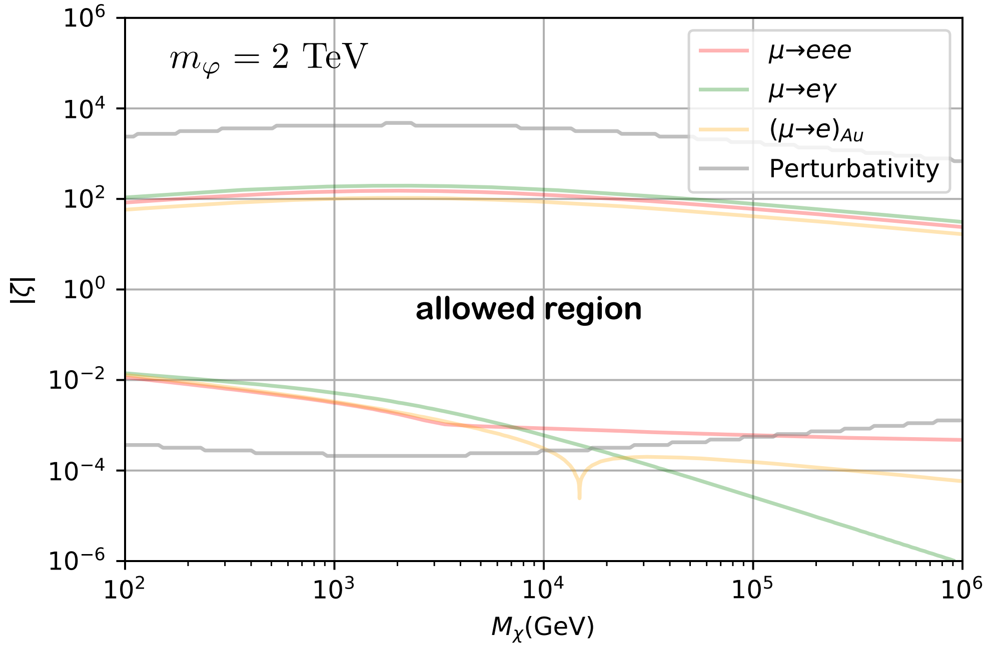
Relevant parameters for fitting the anomalies:

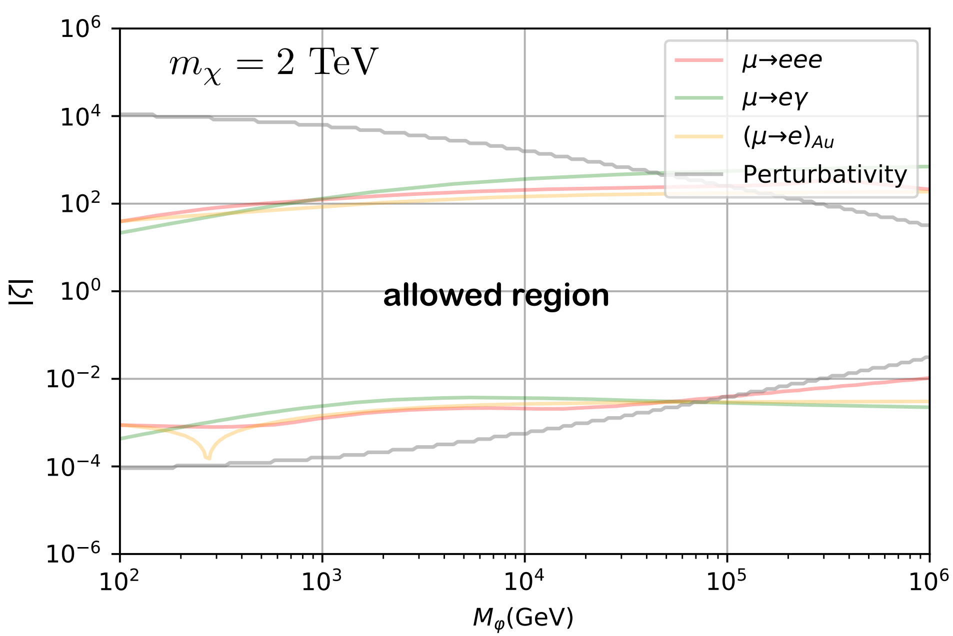
$$\mathbf{x}^{L\phi} = \begin{pmatrix} 0 & 0 & x_{13}^{L\phi} \\ 0 & 0 & x_{23}^{L\phi} \\ 0 & 0 & x_{33}^{L\phi} \end{pmatrix}, \quad \mathbf{y}^{R\phi} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_{23}^{R\phi} \\ 0 & y_{32}^{R\phi} & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{x}^{L\varphi} = \begin{pmatrix} 0 & 0 & x_{13}^{L\varphi} \\ 0 & x_{22}^{L\varphi} & x_{23}^{L\varphi} \\ 0 & 0 & x_{33}^{L\varphi} \end{pmatrix}$$

With φ solely driving neutrino mass generation: $x_{13}^{L\phi}, \quad x_{23}^{L\phi} = 0$

Flavour symmetry connection? Explain these textures?

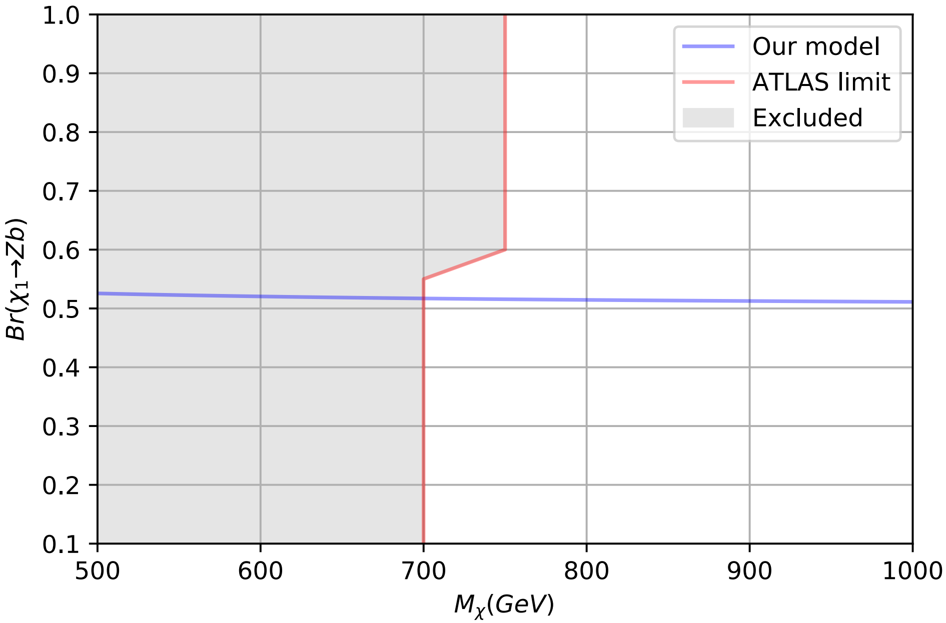
Charged lepton flavour constraints fitting just to neutrino mass and mixing:





Gives indicative range of $10^{-2} < |\zeta| < 10^{2}$

Collider bounds:



$$m_{\chi_1} > 700 \text{ GeV}$$

$$m_{\chi_2} > 1350 \text{ GeV}$$

In our model LQs must couple 3rd gen. quarks to muons:

$tt\mu\mu$ **searches → m > 1.3 TeV**

φ **must couple s to μ:**

dimuon+dijet searches → m > 1.5 TeV

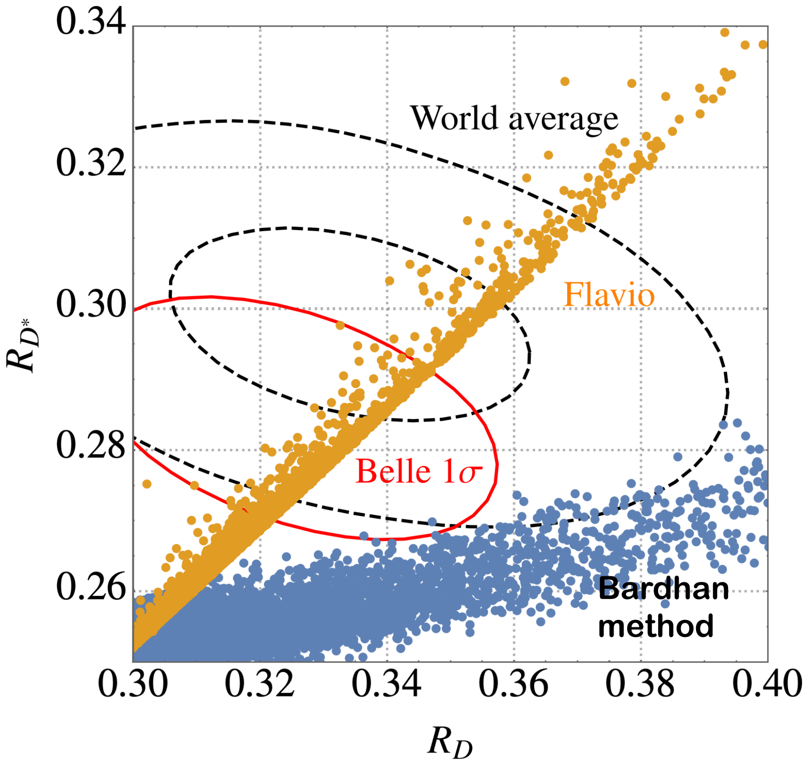
List of all constraints we imposed:

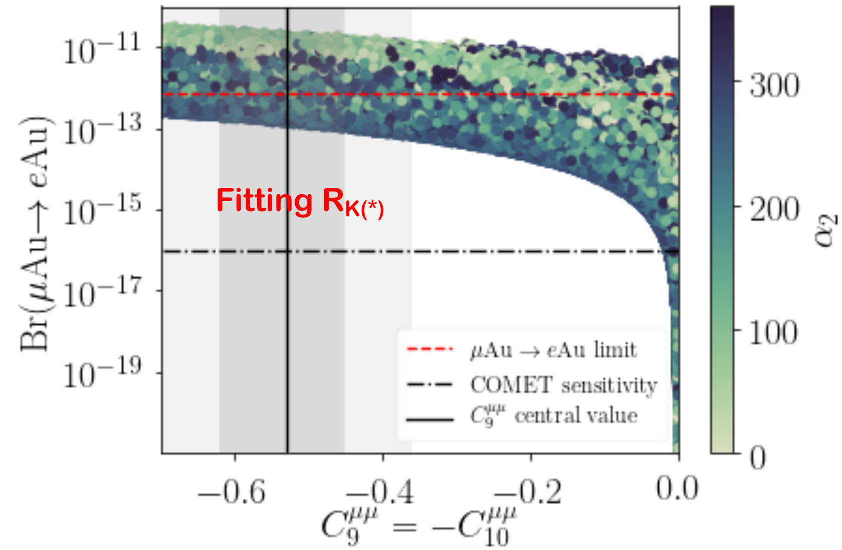
PROCESS	LIMITS
$\mathrm{Br}(\mu \to e\gamma)$	$< 4.2 \times 10^{-13}$
$\mathrm{Br}(\mu \to 3e)$	$< 1.0 \times 10^{-12}$
$\frac{\sigma(\mu\mathrm{Au}\to e\mathrm{Au})}{\sigma(\mu\mathrm{Au}\to\mathrm{capture})}$	$< 7.0 \times 10^{-13}$
$\mathrm{Br}(\tau \to e\gamma)$	$< 3.3 \times 10^{-8}$
$\mathrm{Br}(\tau \to \mu\gamma)$	$< 4.4 \times 10^{-8}$
$\mathrm{Br}(\tau \to 3\mu)$	$< 2.1 \times 10^{-8}$
$\mathrm{Br}(\tau \to 3e)$	$< 2.7 \times 10^{-8}$

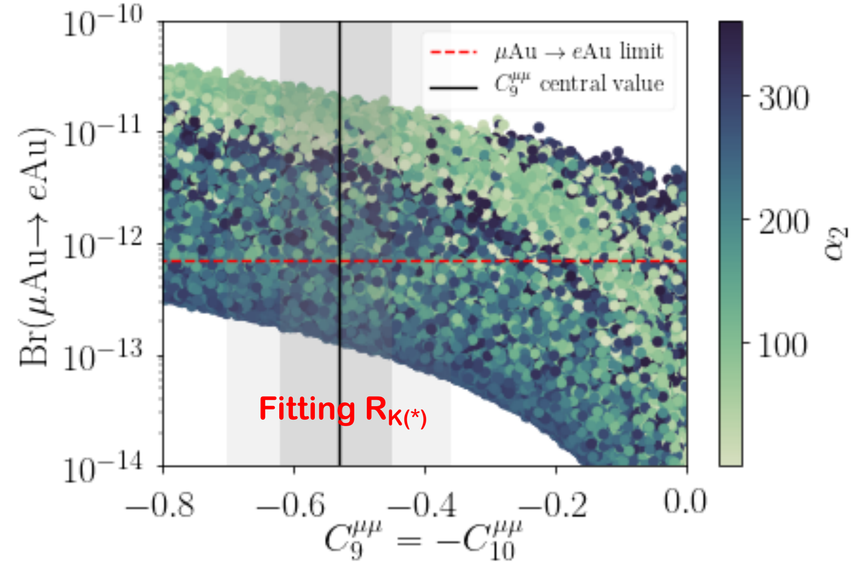
Process	Quantity	Requirement
$ss \to \mu\mu$	$\lvert x_{22}^{L\varphi}\rvert$	$< 0.41 m_\varphi/$ TeV [69]
$bb \to \mu\mu$	$\lvert x_{23}^{L\varphi}\rvert$	$< 0.58 m_\varphi/$ TeV [69]
$ss \to \tau\tau$	$\lvert x_{32}^{L\varphi}\rvert$	$< 0.54 m_\varphi/$ TeV [69]
$bb \to \tau\tau$	$\lvert x_{33}^{L\varphi}\rvert$	$< 0.80 m_\varphi/$ TeV [69]
$bb \to ee$	$\lvert x_{13}^{L\varphi}\rvert$	$< 0.44 m_\varphi/$ TeV [104]
$Z \to bb$	C_{Hd}^{33}	$\in [-0.38, 0.03]$ [107]
$\tau \to \eta e$	Br	$< 9.2 \cdot 10^{-8}$
$\tau \to \pi e$	Br	$< 8.0 \cdot 10^{-8}$
$\tau \to \phi\mu$	Br	$< 8.4 \cdot 10^{-8}$
$Z \to e^\pm\mu^\mp$	Br	$< 7.5 \cdot 10^{-7}$
$Z \to e^\pm\tau^\mp$	Br	$< 9.8 \cdot 10^{-6}$
$Z \to \mu^\pm\tau^\mp$	Br	$< 1.2 \cdot 10^{-5}$
$Z \to \ell_i\ell_i$	g_L	$\in [-8.5, 12] \cdot 10^{-4}$
$Z \to \ell_i\ell_i$	g_R	$\in [-5.4, 6.7] \cdot 10^{-4}$
$Z \to \nu_i\nu_i$	N_ν	within 2.9840 ± 0.0164
$D^0 \to \mu\mu$	Br	$< 7.6 \cdot 10^{-9}$ [115]
$B^+ \to K^+ e^\pm\mu^\mp$	Br	$< 9.1 \cdot 10^{-8}$
$B^0 \to K^{0*} e^\pm\mu^\mp$	Br	$< 1.8 \cdot 10^{-7}$
$B_s \to \mu^\pm e^\mp$	Br	$< 5.4 \cdot 10^{-9}$
$B \to D\ell\nu$	$R_D^{\mu/e} = \frac{\mathrm{Br}(B\to D\mu\nu)}{\mathrm{Br}(B\to De\nu)}$	within 0.995 ± 0.090 [119]
$B \to D^*\ell\nu$	$R_{D^*}^{e/\mu} = \frac{\mathrm{Br}(B\to D^* e\nu)}{\mathrm{Br}(B\to D^*\mu\nu)}$	within 1.04 ± 0.10 [120]
B_s–$\bar{B}_s$ mixing	C_{B_s}	$\in [0.942, 1.288]$ [118]
$B \to K\nu\nu$	$r_K^{\nu\nu} = \frac{\mathrm{Br}}{\mathrm{Br}_{\mathrm{SM}}}$	< 3.9 [116]
$B \to K^*\nu\nu$	$r_{K^*}^{\nu\nu} = \frac{\mathrm{Br}}{\mathrm{Br}_{\mathrm{SM}}}$	< 2.7 [116]
$b \to s\gamma$	Br	$\in [-0.17, 0.24]$ [121]
$B_c \to \tau\nu$	Br	$< 30\%$ [122]
$K \to \ell\nu$	$r_K^{\mu/e} = \frac{\mathrm{Br}(K\to e\nu)}{\mathrm{Br}(K\to\mu\nu)}$	within $(2.488 \pm 0.018) \cdot 10^{-5}$

Simultaneous fit to anomalies and nu mass/mixing while obeying all the preceding constraints:

$R_{D^{(*)}}$ fit same as for minimal model.
Tree level isosinglet LQ.







μ→e conversion most important constraint. COMET and Mu2e experiments will probe much of the allowed parameter space!

Populating theory space by *exploding!* operators

Gargalionis, RV: manuscript in preparation

My student John Gargalionis has automated the operator-opening procedure: exotics are scalars, vector-like fermions, Majorana fermions.

The program generates all minimal tree-level openings at mass dim d, then filters the models to eliminate those that also generate a an operator of dimension <d that would thus dominate.

This has been completed for d=9,11.

Total number of models is in the thousands …

Amongst the d=11 models, but not for d=9, there are ~60 candidates that *require* $(3,1,-1/3)_S$ and $(3,3,-1/3)_S$ for neutrino mass generation!

$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$, $(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$ + exotics			leading operator(s) $\mathcal{O}_i$
$(\mathbf{3}, \mathbf{3}, 2/3)_S$			$25, 73$
$(\mathbf{3}, \mathbf{4}, 1/6)_S$			$25, 73$
$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)_S$	$(\mathbf{1}, \mathbf{1}, 1)_S$		$21, 25, 30, 31, 72, 73$
$(\mathbf{6}, \mathbf{3}, 1/3)_S$	$(\bar{\mathbf{6}}, \mathbf{3}, 2/3)_S$		47
$(\bar{\mathbf{3}}, \mathbf{3}, 4/3)_S$	$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)_S$	$(\mathbf{1}, \mathbf{1}, 1)_S$	$21, 31, 72$
$(\bar{\mathbf{6}}, \mathbf{3}, 2/3)_S$	$(\bar{\mathbf{6}}, \mathbf{2}, 1/6)_S$	$(\mathbf{6}, \mathbf{1}, 1/3)_S$	$25, 73$
	$\vdots$		$\vdots$
$(\bar{\mathbf{6}}, \mathbf{3}, 2/3)_S$	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)_F$		47
$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)_F$	$(\mathbf{1}, \mathbf{4}, 1/2)_F$	$(\bar{\mathbf{3}}, \mathbf{4}, 5/6)_F$	47
$(\mathbf{3}, \mathbf{3}, 2/3)_S$	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)_F$	$(\bar{\mathbf{3}}, \mathbf{2}, 1/6)_F$	25
$(\mathbf{3}, \mathbf{3}, 2/3)_S$	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)_F$	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)_F$	47
$(\bar{\mathbf{6}}, \mathbf{3}, 2/3)_S$	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)_F$	$(\mathbf{1}, \mathbf{3}, 1)_F$	$28, 69$
	$\vdots$		$\vdots$

A selection of scalar-only (upper) and scalar-plus-fermion (lower) models derived from dimension-11 operators including both scalar leptoquarks $(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$ and $(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$. There are ~ 60 such models in total.

Selection of generated operators. See Babu+Leung (2001) & de Gouvêa+Jenkins (2008) for others.

$$\mathcal{O}_{21} = LLLe^cQu^cHH \qquad \mathcal{O}_{25} = LLQd^cQu^cHH \qquad \mathcal{O}_{47} = LLQQ\bar{Q}\bar{Q}HH$$

$$\mathcal{O}_{30} = LL\bar{L}\overline{e^c}\,\bar{Q}\overline{u^c}HH \qquad \mathcal{O}_{31} = LL\bar{Q}\overline{d^c}\,\bar{Q}\overline{u^c}\,HH \qquad \mathcal{O}_{72} = LLLe^cHQu^cH$$

5. Final remarks

Radiative neutrino mass models are being studied systematically by *exploding!* $\Delta L=2$ effective operators.

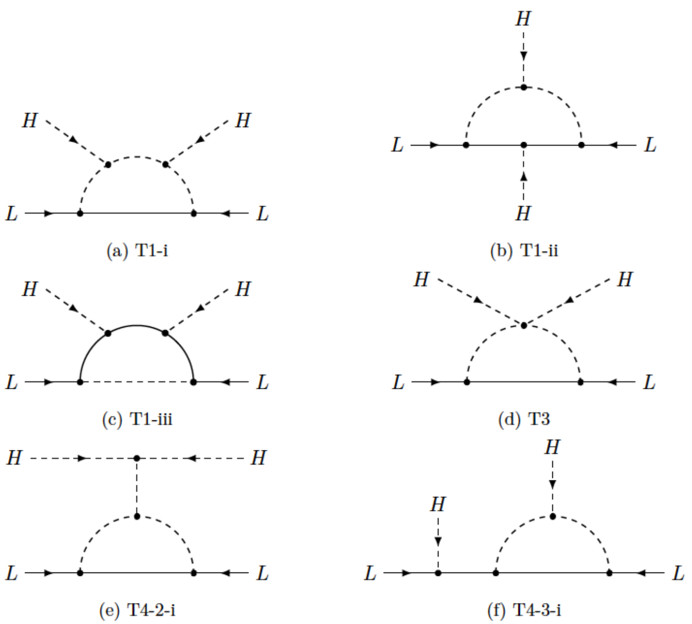
The techniques being invented will be applicable in other contexts such as SMEFT.

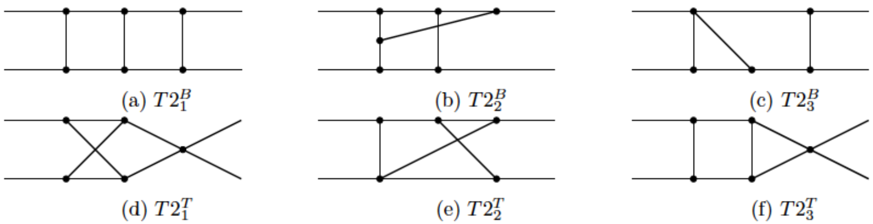
These models are readily connected with the flavour anomlies. Let's hope!

Spectrum of relevant experiments:
charged-lepton no. violation (COMET/Mu2e especially),
quark-flavour, LHC, g-2, neutrino oscillation,
dark matter, …

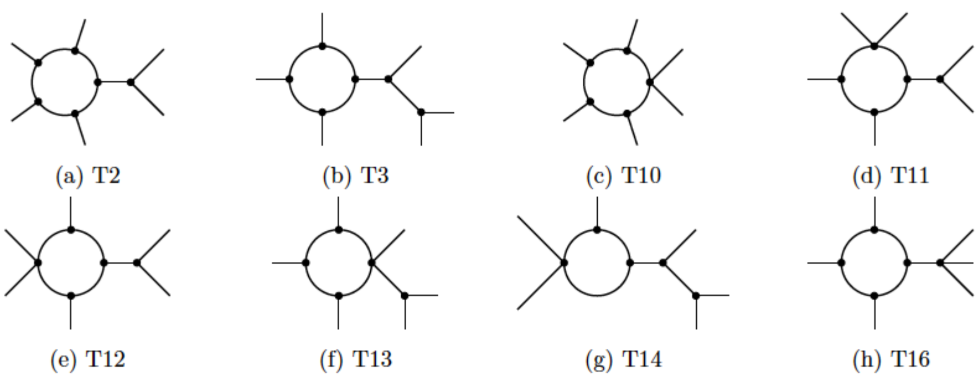
Back-up slides

Classification of Weinberg operator openings





1- and 2-loop topologies for O_1



1-loop topologies for O_1'

Effective operators and constraints for Bauer-Neubert model

$$\mathcal{L} \supset x_{ij}\nu_L^i d_L^j \phi^\dagger - z_{ij} e_L^i u_L^j \phi^\dagger + y_{ij}\bar{e}_R^i \bar{u}_R^j \phi + H.c.$$

$$z = xV_{\text{CKM}}^\dagger$$

Mass-basis fields except nu which are flavour-basis.

The b→sμμ box diagrams give the effective operators:

$$\mathscr{O}_{LL}^\mu \equiv \frac{1}{2}(\mathscr{O}_9^\mu - \mathscr{O}_{10}^\mu) = (\bar{s}\gamma^\mu P_L b)(\bar{\mu}\gamma_\mu P_L \mu),$$

$$\mathscr{O}_{LR}^\mu \equiv \frac{1}{2}(\mathscr{O}_9^\mu + \mathscr{O}_{10}^\mu) = (\bar{s}\gamma^\mu P_L b)(\bar{\mu}\gamma_\mu P_R \mu)$$

$$C_{LL}^{\phi,\mu} = \frac{m_t^2}{8\pi\alpha m_\phi^2}|z_{23}|^2 - \frac{\sqrt{2}}{64\pi\alpha G_F m_\phi^2 V_{tb}V_{ts}^*}\sum_i x_{i3}x_{i2}^* \sum_j |z_{2j}|^2,$$

$$C_{LR}^{\phi,\mu} = \frac{m_t^2}{16\pi\alpha m_\phi^2}|y_{23}|^2 \left[\ln\frac{m_\phi^2}{m_t^2} - f\left(\frac{m_t^2}{m_W^2}\right)\right] - \frac{\sqrt{2}}{64\pi\alpha G_F m_\phi^2 V_{tb}V_{ts}^*}\sum_i x_{i3}x_{i2}^* \sum_j |y_{2j}|^2,$$

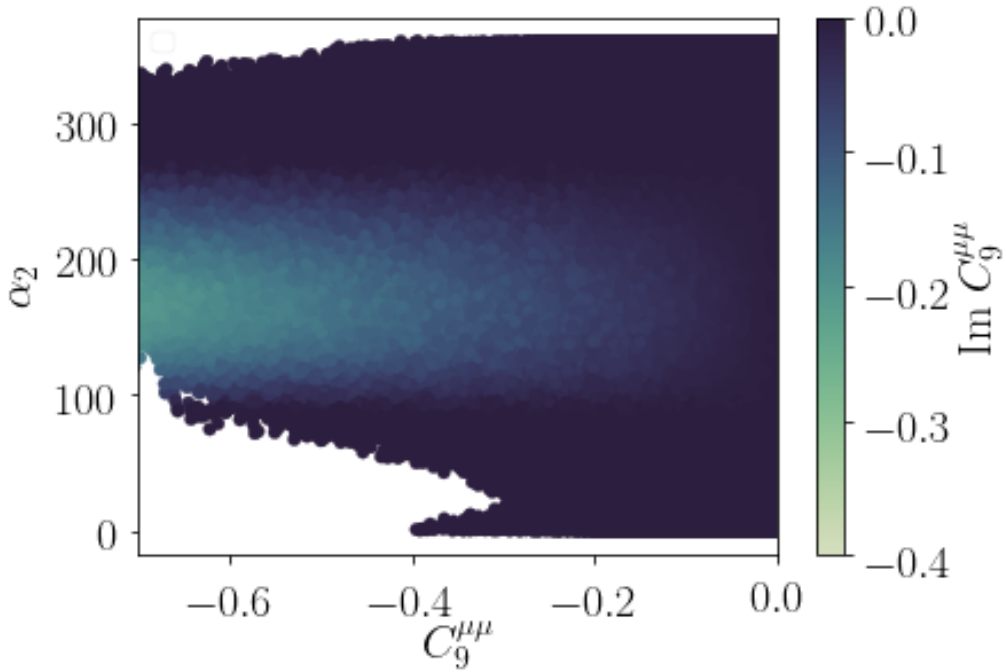
$$f(x) = 1 - \frac{3}{x-1} + \frac{3}{(x-1)^2}\ln x.$$

Wilson coefficients

Constraints:

- $K^+ \to \pi^+\nu\nu$
- $\mu N \to eN$
- $B \to K\nu\nu$
- B_s–$\bar{B}_s$ mixing
- Precision EW measurements
- $D^0 \to \mu\mu$, $D^+ \to \pi^+\mu\mu$
- $P \to (P')\ell\nu$, $\tau \to P\nu$ + LFU ratios
- $\tau \to \mu\mu\mu$, $\tau \to \mu\gamma$

Role of α_2:



$100^o < \alpha_2 < 300^o$ preferred

rate for $\mu \to e$ conversion $\propto |y_{11}^{L\varphi *}\, y_{21}^{L\varphi}|^2 \propto |-1.08906 + \sin(\alpha_2 + \delta_{\rm CP})|^2$

Some proposed scalar leptoquark models for resolving the flavour anomalies.

Scalar LQ models	$b \to c$	$b \to s$	Example References	Notes
$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)_S$	Tree level	Tree level	1408.1627, 1412.1791	
$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)_S$	Tree level	Loop level	1511.01900, 1704.05849	
$(\mathbf{3}, \mathbf{2}, 7/6)_S$	Tree level	Loop level	1309.0301, 1704.05835	$b \to s$ incompatible with $b \to c$
$(\mathbf{3}, \mathbf{2}, 1/6)_S + (1, 1, 0)_F$	Tree level	Tree level	1608.08501	$b \to c$ requires complex couplings. Predicts $R_{K^*} > 1$
$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)_S + (\bar{\mathbf{3}}, \mathbf{1}, 1/3)_S$	Tree level	Loop level	1803.10972, 1706.07808	
$(\bar{\mathbf{3}}, \mathbf{2}, 7/6)_S + (\mathbf{3}, \mathbf{1}, 5/3)_V$	Tree level	Loop level	1805.04917	
$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)_S + (\bar{\mathbf{3}}, \mathbf{1}, 1/3)_S + (\mathbf{3}, \mathbf{2}, 7/6)_S$	Tree level	Tree level	1703.03251	

A selection of models from the literature containing scalar leptoquarks explaining the flavour anomalies. The references are an incomplete list of works that utilise the given leptoquark(s).